

# Rating of Planar Kinematic Chains Using Design Parameters

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**Abstract** A comparative analysis of kinematic chains and their mechanisms at the conceptual stage of design is essential for the designers. There are large number of distinct chains available in each category, i.e., with specified number of links and degree of freedom. Chains with distinct structures can be expected to possess different characteristics which are not quite obvious. The designer must have some idea about the expected behaviour of the chain at least in comparative sense, so that he can pick up the best chain in order to obtain best performance. To accomplish this, the designer should be able to read the characteristics of the chain based on their topology. Using this belief, in the present work an attempt is made and a method is proposed to compare the chains from the effective utilization of link design parameters, quality of motion and function generation point of view. It provides a simple quantitative estimate to compare all the eligible chains. The concepts developed are applied to planar in-parallel robots, which are gaining in importance.

**Keywords** Kinematic Chain (KC), Chain Link Value, Chain Joint Value, Chain Flow Value

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## 1. Introduction

Kinematic chain is a combination of links connected by different kinematic pairs and can be divided into two categories i.e. serial and parallel. Although serial chains have limited choice but in parallel, there are distinct chains available, such as 16 in eight links single degree of freedom, 40 in nine links 2 degree of freedom, 98 in 10 links 3 degree of freedom and more than 6000 single DOF with twelve links [1, 2]. Therefore complexity of kinematic chains increases with the number of the links of the chains with the same number and type of links. There are no guidelines to select the best possible chain for the specified task such as function generation, path generation or robot manipulator application, despite the availability of many chains with distinct structure. Until now, most of the work is directed to study isomorphism

among chains and to know the type of freedom [3-7], work relating to this aspect is hardly seen in the literature. Some work reported earlier [8-10] deals with the dimensional aspects and not with the structural influence. Rao and Rao [11, 12] proposed link-loop Hamming values for rating of the kinematic chains. Rao [13] presented a pseudo genetic algorithm for evaluation of kinematic chains. Rao [14] using the concept of fuzzy logic proposed a numerical measure to compare the chain characteristics like symmetry, parallelism and mobility. Srinath and Rao [15] proposed the concept of correlation to evaluate the performance of the chains.

Exact static and dynamic behavior of linkages in an absolute sense cannot be predicted unless the link shapes, dimensions, masses and their distribution etc., are known [16]. However, it should be possible to compare different chains with same number of links and joints for their expected behavior on the basis of their structure, without having to perform the actual static and dynamic analysis. Structure of a chain depends upon the link assortment, i.e., type of link, their number, and their adjacency. Each link is associated with certain number of design parameters. Design parameters are the link lengths, angles etc. which are necessary to form a link. For example a binary link has one design parameter a ternary link has three parameters, a quaternary link has five parameters and a quinary link has seven design parameters. Performance of a chain depends upon effective use of its design parameters. Selection of the input link or link to be connected to the actuator is important from the point of view of transmission and resolution. In the chain that consist of the same type and number of links the type of joints control the quality of motion generation. So keeping all these aspects of kinematic chains a selection (index) criterion is proposed for kinematic chains from the structural error point of view which provide a simple quantitative estimate to compare all the eligible chains.

## 2. Definitions of Terminology

The following definitions are to be understood clearly

before applying this method. Various definitions with their abbreviations are given below.

- i. Degree of link (D): A numerical value for the link, based on its connectivity to other links. Therefore quaternary link has degree equal to four and ternary link has degree equal to three.
- ii. Link value ( $L_v$ ): For a particular link it is defined as the ratio of number of design parameters associated with the link to the degree of that link. The link values of various types of links are determined by Eq.1 and listed in Table-1.

$$\text{Link value} = \frac{\text{Number of link design parameters}}{\text{Degree of the link}} \quad (1)$$

- iii. Joint value ( $J_v$ ): For a particular joint it is defined as the ratio of product of degree of the links to the number of links connected at that joint. It is denoted by  $J_v$ . For a planar kinematic chain having only simple joints, the joint values of various joints are determined by Eq.2 and listed in Table-2.

$$\text{Joint value} = [D_i \times D_j] \div 2 \quad (2)$$

- iv. Chain link value (CLV): For a planar kinematic chain it is defined as the sum of all the link values of the chain. It represents the effective utilization of link design parameters in the chain and determined by Eq.3

$$\text{CLV} = a_2 + b_3 + c_4 + d_5 \quad (3)$$

Here; a, b, c, d are the link values and  $n_2, n_3, n_4, n_5$  are the number of binary, ternary, quaternary and quinternary links respectively. For example the CLV of chain 1 (a) is  $(0.5 \times 4 + 1 \times 4) = 6$  and CLV of chain 1 (b) is  $(0.5 \times 5 + 1 \times 2 + 1.25 \times 1) = 5.75$ .

- v. Chain joint value (CJV): For a planar kinematic chain it is define as the sum of all the joint values of the chain. It represents the quality of motion and function generation of the chain and determine Eq.4

$$\text{CJV} = e_{22} + f_{23} + g_{24} + h_{33} + i_{34} + j_{n44} \quad (4)$$

Here; e, f, g, h, i, j are the joint values and  $n_{22}, n_{23}, n_{24}, n_{33}, n_{34}$  and  $n_{44}$  are number of binary-binary, binary-ternary, binary-quaternary, ternary-ternary, ternary-quaternary, quaternary-quaternary joints respectively. For example the CJV of chain 1 (a) is  $(2 \times 2 + 3 \times 4 + 4 \times 5) = 34$  and CJV of chain 2 (b) is CJV is  $(2 \times 1 + 3 \times 6 + 4 \times 5) = 33.5$ .

- vi. Link flow matrix (FM): For an n-link planar kinematic chain it is an n x n square matrix defined by Eq.5

$$[\text{FM}] = \{F_{ij}\} \text{ n x n} \quad (5)$$

Here;  $F_{ij}$  = {Minimum number of joints between link i and j = 0 if i is equal to j}

Thus the form [FM] matrix will be

$$[\text{FM}] = \begin{bmatrix} 0 & F_{12} & F_{13} & - & F_{1n} \\ F_{21} & 0 & F_{23} & - & F_{2n} \\ - & - & - & - & - \\ F_{n1} & F_{n2} & F_{n3} & - & 0 \end{bmatrix}$$

- vii. Chain flow value (CFV): For a planar kinematic chain it is defined as sum of all the elements of the

link flow matrix. It represents the quality of distribution of motion from one link to the other links.

**Table 1.** Link values of various types of links

Type of link	Link value $L_v$
Binary	$1 \div 2 = 0.5$
Ternary	$3 \div 3 = 1.0$
Quaternary	$5 \div 4 = 1.25$
Quinternary	$7 \div 5 = 1.4$

**Table 2.** Joint values of various types of joints

Type of joint	Joint value $J_v$
Binary-binary	$[2 \times 2] \div 2 = 2.0$
Binary-ternary	$[2 \times 3] \div 2 = 3.0$
Binary-quaternary	$[2 \times 4] \div 2 = 4.0$
Ternary-ternary	$[3 \times 3] \div 2 = 4.5$
Ternary-quaternary	$[3 \times 4] \div 2 = 6.0$
Quaternary-quaternary	$[4 \times 4] \div 2 = 8.0$

## 3. Rating of Planar Kinematic Chains

### 3.1. Method

A simple method is proposed for rating of planar kinematic chains. The method consists of the following hierarchy of the three criterions:

- (i) Compare the CLV of the chains under consideration; chains with lower CLV is better or rated higher.
- (ii) It is clear that chains with the same number of links, DOF and with same link assortment have identical CLV in that case chain with lower CJV is better or rated higher.
- (iii) For the chains with equal number and type of links and joints, both CLV and CJV are identical in that case chain flow path value (CFV) becomes the criterion for rating. Chains with lower CFV are better or rated higher.

### 3.2. Proposed Method Basis

- i. The first criterion is based on the observation that design parameters in high connectivity link are not effectively employed [15]. For example in quaternary link five design parameters have no relative motion among themselves where as the single design parameter of a binary link is fully effective. Therefore in the proposed method a binary link assign a link value of 0.5 as compared to 1.4 for quinternary link. Lower CLV for a chain interprets that it has more binary links or less high connectivity (degree) links and has more effective utilization of link design parameters.

- ii. The second criterion is based on the observation that the type of joint also controls the quality of path and function generation, i.e. direct joining of links with higher connectivity is disadvantageous [11, 12]. Therefore in the proposed method a binary- binary joint assign a joint value of 2 as compared to 8 for quaternary-quaternary joint. Lower CJV for a chain interprets that it has less direct joining of links with higher connectivity links and has better quality generation.
- iii. The third criterion is based on the observation that layout of the links in the chain also control the motion from one link to the other links, it reaches easily through shorter routes [11, 12]. Lower CFV for a chain interprets that it has more uniform distribution of motion among its links.

### 4. Application to Planar Kinematic Chains

The proposed method is explained with the help of four eight link, one DOF, kinematic chains possessing different and same link assortment as shown in Fig. 1 and Fig. 2 respectively.

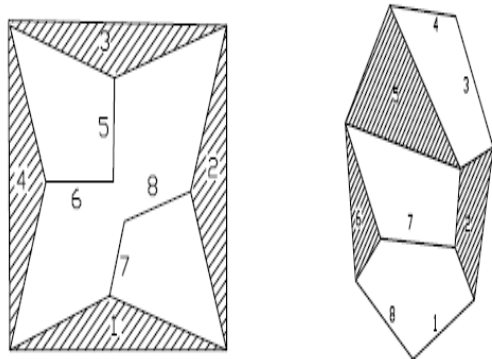


Figure 1. Eight links, one DOF kinematic chains with different link assortment

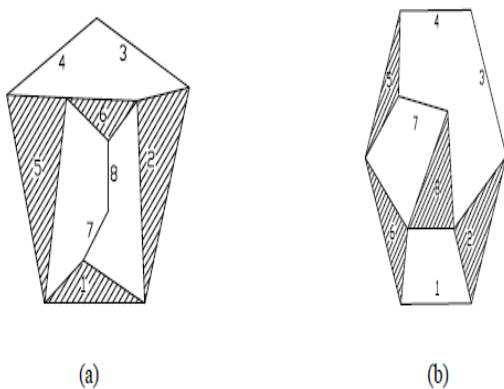


Figure 2. Eight links, one DOF kinematic chains with same link assortment

**Step-1: Computation of CLV**

For chain 1 (a);  $n_2 = 4, n_3 = 4$ , and CLV computed by Eq. 3 is  $(0.5 \times 4 + 1 \times 4) = 6$

For chain 1 (b);  $n_2 = 5, n_3 = 2, n_4 = 1$ , and CLV is  $(0.5 \times 5 + 1 \times 2 + 1.25 \times 1) = 5.75$

For chain 2 (a);  $n_2 = 4, n_3 = 4$ , and CLV computed by Eq. 3 is  $(0.5 \times 4 + 1 \times 4) = 6$

For chain 2 (b);  $n_2 = 4, n_3 = 4$ , and CLV is  $(0.5 \times 4 + 1 \times 4) = 6$

**Step-2: Determination of CJV**  
On comparing the CLV it is observed that chain's shown in Fig. 1(a), Fig. 2 (a) and Fig. 2 (b) has the identical values. So CJV of these chains are only required to compute.

For chain 1 (a)  $n_{22} = 2, n_{23} = 4$  and  $n_{33} = 4$ , and CJV computed by Eq. 4 is  $(2 \times 2 + 3 \times 4 + 4.5 \times 4) = 34$

For chain in Fig. 2 (a)  $n_{22} = 2, n_{23} = 4, n_{33} = 4$  and CJV is  $(2 \times 2 + 3 \times 4 + 4.5 \times 4) = 34$

For chain in Fig. 2 (b)  $n_{22} = 1, n_{23} = 6, n_{33} = 3$  and CJV is  $(2 \times 1 + 3 \times 6 + 4.5 \times 3) = 33.5$

**Step-3: Determination of CFV**  
On comparing the CJV it is observed that chain's in Fig. 1(a) and Fig. 2 (a) has the equal values. So CFV of these chains are only required to compute.

Link flow matrix for chain in Fig. 1 (a)

$$\begin{bmatrix} 0 & 1 & 2 & 1 & 3 & 2 & 1 & 2 \\ 1 & 0 & 1 & 2 & 2 & 3 & 2 & 1 \\ 2 & 1 & 0 & 1 & 1 & 2 & 3 & 2 \\ 1 & 2 & 1 & 0 & 2 & 1 & 2 & 3 \\ 3 & 2 & 1 & 2 & 0 & 1 & 4 & 3 \\ 2 & 3 & 2 & 1 & 1 & 0 & 3 & 4 \\ 1 & 2 & 3 & 2 & 4 & 3 & 0 & 1 \\ 2 & 1 & 2 & 3 & 3 & 4 & 1 & 0 \end{bmatrix}$$

Link flow matrix for chain in Fig. 2 (a)

$$\begin{bmatrix} 0 & 1 & 2 & 2 & 1 & 2 & 1 & 2 \\ 1 & 0 & 1 & 2 & 2 & 1 & 2 & 2 \\ 2 & 1 & 0 & 1 & 2 & 2 & 3 & 3 \\ 2 & 2 & 1 & 0 & 1 & 2 & 3 & 3 \\ 1 & 2 & 2 & 1 & 0 & 1 & 2 & 2 \\ 2 & 1 & 2 & 2 & 1 & 0 & 2 & 1 \\ 1 & 2 & 3 & 3 & 2 & 2 & 0 & 1 \\ 2 & 2 & 3 & 3 & 2 & 1 & 1 & 0 \end{bmatrix}$$

The link flow value for chain 1 (a) is 112 and for chain 2 (a) is 100.

Here on the basis of hierarchy of the three criteria discussed above among these four chains, chain in Fig. 1 (b) have the highest rating 1 and chain in Fig. 1 (a) have the least rating 4. Chain in Fig. 2 (b) and in Fig. 2 (a) are rated second and third respectively.

### 5. Application to Platform- Type Robots

In the recent past open- chain linkages have received greater attention for robot arms. Each joint in these robot arms is actuated independently. While possessing many

advantages, such as large work space and maneuverability, they do suffer from disadvantages like less rigidity, accumulation of mechanical errors from shoulder to the end- effectors, control problems, etc. The alternative to the open chain robot arms is the in- parallel actuator arrangement often referred to as the platform –type, whose application is reported by many investigators [18]. In what follows only planar robots are dealt with. The method explained earlier helps the designer in selecting the best chain for use as a platform-type robot. To illustrate let us consider two 8 link, 2 DOF chains shown in Fig.3. For the chain shown in Fig. 3(a) and in Fig. 3 (b) the CLV is 5 for both the chains and the CJV are 24 and 24.5 respectively, so chain in Fig. 3 (a) is better as compared to chain in Fig. 3 (b) from the viewpoint of quality of path and function generation. For further illustration, let us consider three 9 link, 2 DOF chains shown in Fig. 4. Here the CLV for chain in Fig. 4 (a) is 6.5, for chain in Fig. 4(b) is 6.25 and for chain in Fig. 4 (c) is 6, therefore, easy to understand that chain (b) is better but chain (c) is the best, from the viewpoint of effective use of link design parameters. Comparison of the present results with the results reported by Rao and Jagdeesh [17] for the same chains reveals that the chains that have higher CLV and/or CJV are dynamically more sensitive and also present difficulties from the viewpoint of balancing.

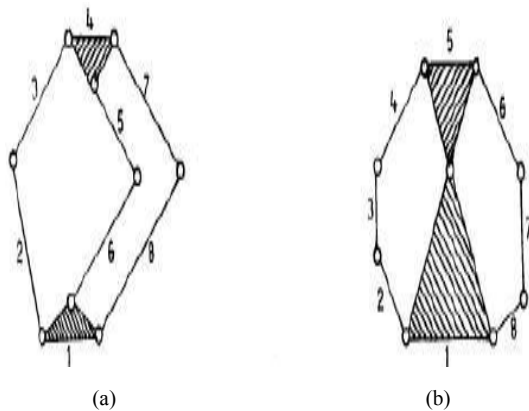


Figure 3. Eight links Three DOF kinematic chains

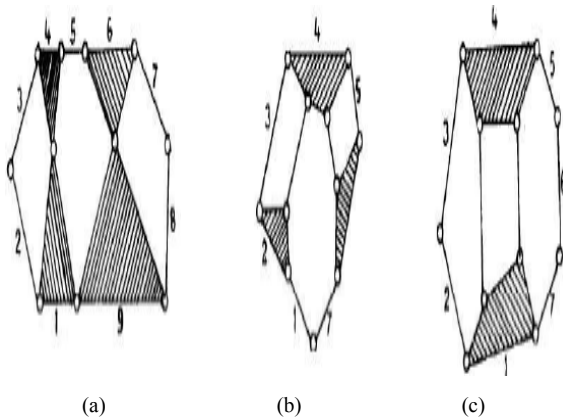


Figure 4. Nine links Two DOF kinematic chains

## 6. Conclusions

In this paper a method is proposed to compare the chains from the point of view of effective utilization of link design parameters, quality of motion and function generation, and the quality of distribution of motion from one link to the other links. The method provides a simple quantitative estimate to compare all the eligible chains. This study provides some idea about the expected behaviour of the chain at least in comparative sense, to the designer so that he can pick up the best chain in order to obtain best performances at the conceptual stage of design.

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