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Coding theory on $(h(x), g(y))$-extension of Fibonacci p-numbers polynomials

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Abstract In this paper, we define $(h(x), g(y))$-extension of the Fibonacci p-numbers. We also define golden $(p, h(x), g(y))$-proportions where p $(p = 0, 1, 2, 3, \cdots)$ and $h(x)(> 0)$, $g(y)(> 0)$ are polynomials with real coefficients. The relations among the code elements of a new Fibonacci matrix, $G_{p,h,g}$, $(p = 0, 1, 2, 3, \cdots)$, $h(x)$ (> 0), $g(y)$ (> 0) coincide with the relations among the code matrix for all values of p and $h(x) = m(> 0)$ and $g(y) = t(> 0)$ [8]. Also, the relations among the code matrix elements for $h(x) = 1$ and $g(y) = 1$, coincide with the generalized relations among the code matrix elements for Fibonacci coding theory [6]. By suitable selection for the initial terms in $(h(x), g(y))$-extension of the Fibonacci p-numbers, a new Fibonacci matrix, $G_{p,h,g}$ is applicable for Fibonacci coding/decoding. The correct ability of this method, increases as p increases but it is independent of $h(x)$ and $g(y)$. But $h(x)$ and $g(y)$ being polynomials, improves the cryptography protection. And complexity of this method increases as the degree of the polynomials $h(x)$ and $g(y)$ increases. We have also find a relation among golden $(p, h(x), g(y))$-proportion, golden $(p, h(x))$-proportion and golden p-proportion.

Keywords Coding theory, (h(x); g(y))-Extension of Fibonacci

1 Introduction

In 13th century, Italian mathematician Leonardo discovered the Fibonacci numbers. First of all, the Fibonacci numbers anticipated the method of recursive relations, one of the most powerful methods of combinatory analysis. Later the Fibonacci numbers were found in many natural objects and phenomena. Now a days Fibonacci numbers [2,10,11] are used in sciences, arts and more recently in combinatorial design theory, high energy physics, information and coding theory [5,7].
The Fibonacci numbers $F_n(n = 0, \pm 1, \pm 2, \pm 3, \ldots)$ satisfy the recurrence relation

$$F_{n+1} = F_n + F_{n-1} \tag{1}$$

with initial terms $F_1 = F_2 = 1$.

Table 1. Fibonacci numbers, F_n

$n \rightarrow$	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12
F_n	-3	2	-1	1	0	1	1	2	3	5	8	13	21	34	55	89	144

We take the ratio of two adjacent numbers and direct this ratio towards infinity. We derive the following unexpected result:
$\lim_{n\longrightarrow\infty} \frac{F_n}{F_{n-1}} = \mu = \frac{1+\sqrt{5}}{2}$
where μ is the golden mean.
Stakhov [1] introduced Fibonacci p-numbers given by the following recurrence relation:

$$F_p(n) = F_p(n-1) + F_p(n-p-1) \text{ for } n > p+1 \tag{2}$$

with initial terms

$$F_p(1) = F_p(2) = F_p(3) = \cdots = F_p(p+1) = 1 \tag{3}$$

where $p = 0, 1, 2, 3, \cdots$
The Fibonacci p-numbers can be represented by binomial coefficients as follows:

$$F_p(n+1) = ^n C_0 + ^{n-p} C_1 + ^{n-2p} C_2 + ^{n-3p} C_3 + ^{n-4p} C_4 + \cdots + ^{n-kp} C_k + \cdots \tag{4}$$

where the binomial coefficients $^{n-kp}C_k = 0$ for the case $k > n - kp$.
For $p = 0$ the equation (4) reduces to the well-known formula of combinatorial analysis:

$$2^n = ^n C_0 + ^n C_1 + ^n C_2 + ^n C_3 + ^n C_4 + \cdots + ^n C_n$$

In fact, when $p = 1$ we obtain the Fibonacci numbers

$$F_1(n) = F(n) = F_n \tag{5}$$

For calculations of Fibonacci p-numbers for all values of n, we consider the recurrence relation

$$F_p(n) = F_p(n-1) + F_p(n-p-1) \tag{6}$$

with initial terms

$$F_p(1) = F_p(2) = F_p(3) = \cdots = F_p(p+1) = 1. \tag{7}$$

Considering (7) as initial term then from (6) we have

$$F_p(p+1) = F_p(p) + F_p(0) \tag{8}$$

Since $F_p(p+1) = F_p(p) = 1$. Therefore, $F_p(0) = 0$.
Continuing this process by writing $n = p, p-1, \cdots, 2$ in (6) we get
$F_p(0) = F_p(-1) = F_p(-2) = \cdots = F_p(-p+1) = 0$.
When $n = 1$, (6) gives

$$F_p(1) = F_p(0) + F_p(-p) \tag{9}$$

Since $F_p(1) = 1$, $F_p(0) = 0$. Therefore, $F_p(-p) = 1$.
Representing Fibonacci p-numbers $F_p(0)$, $F_p(-1)$, $\cdots$, $F_p(-p+1)$ in form of (6) we get
$F_p(-p-1) = F_p(-p-2) = \cdots = F_p(-2p+1) = 0$.
Also, by substituting $n = -p+1, -p, -p-1$ in (6), we have $F_p(-2p) = -1$, $F_p(-2p-1) = 1$, $F_p(-2p-2) = 0$.
So, we summarize above the following table:

Table 2. Fibonacci p-numbers, $F_p(n)$

$n \rightarrow$	0	-1	.	.	-p+1	-p	-p-1	.	.	-2p+1	-2p	-2p-1	-2p-2
$F_p(n)$	0	0	.	.	0	1	0	.	.	0	-1	1	0

Thus, we get Fibonacci p-numbers, $F_p(n) = F_p(n-1) + F_p(n-p-1)$ for $p = 0, 1, 2, 3, \cdots$ and $n = 0, \pm1, \pm2, \pm3, \ldots$
where $F_p(1) = F_p(2) = F_p(3) = \cdots = F_p(p+1) = 1$.
Also, from (6) and (7) we have

$$F_p(1) + F_p(2) + F_p(3) + F_p(4) + F_p(5) + \cdots + F_p(n) = F_p(n+p+1) - 1 \tag{10}$$

For case $p = 0$, the formula (10) reduces to the well known formula for the binary numbers

$$2^0 + 2^1 + 2^2 + 2^3 + \cdots + 2^{n-1} = 2^n - 1 \tag{11}$$

For case $p = 1$, the classical Fibonacci numbers F_n satisfy the following formulae:
(a) $F_1 + F_2 + F_3 + \cdots + F_n = F_{n+2} - 1$
(b) $F_1 + F_3 + F_5 + \cdots + F_{2n+1} = F_{2n}$
(c) $F_2 + F_4 + F_6 + \cdots + F_{2n} = F_{2n+1} - 1$
(d) $F_1^2 + F_2^2 + F_3^3 + \cdots + F_n^2 = F_n F_{n+1}$
(e) $F_n^2 + F_{n+1}^2 = F_{2n+1}$
Divide (6) by $F_p(n-p-1)$ and consider $\lim_{n\longrightarrow\infty} \frac{F_p(n)}{F_p(n-1)} = u$.
We have characteristic equation:

$$u^{p+1} - u^p - 1 = 0 \tag{12}$$

The only one positive root, μ_p of (12) is called golden p-proportion. The golden p-proportion possess the following remarkable properties:
(a) $1 \leq \mu_p \leq 2$

Table 3. Golden p-proportion(mean), μ_p

$p \rightarrow$	0	1	2	3	4	5	6	7	8	9	10
μ_p	2.000	**1.618**	1.465	1.380	1.324	1.285	1.255	1.232	1.213	1.197	1.184

(b) $\mu_p^n = \mu_p^{n-1} + \mu_p^{n-p-1} = \mu_p \mu_p^{n-1} = \mu_p^r \mu_p^{n-r}, \quad r = 1, 2, 3, \cdots, n$

Stakhov [1] proves that the golden p-proportion represents a new class of irrational numbers which express some unknown mathematical properties of the Pascal triangle. Clearly, such mathematical results are of fundamental importance for the development of modern sciences.
The generalized Fibonacci numbers [12,13,14] based on the relation

$$F_m(n) = mF_m(n-1) + F_m(n-2) \tag{13}$$

with initial terms

$$F_m(0) = 0, F_m(1) = 1 \tag{14}$$

where m (> 0) and $n = 0, \pm 1, \pm 2, \pm 3, \ldots$.
The m-extension of Fibonacci p-numbers [15] defined by the recurrence relation

$$F_{p,m}(n) = mF_{p,m}(n-1) + F_{p,m}(n-p-1) \tag{15}$$

with initial terms

$$F_{p,m}(1) = a_1, F_{p,m}(2) = a_2, F_{p,m}(3) = a_3, \cdots, F_{p,m}(p+1) = a_{p+1} \tag{16}$$

where p (≥ 0) is integer, m (> 0), $n > p+1$ and a_1, a_2, a_3, $\cdots$, a_{p+1} are arbitrary real or complex numbers.
The Fibonacci polynomials [4] are defined by the recurrence relation

$$F_n(x) = xF_{n-1}(x) + F_{n-2}(x), \quad n \geq 3 \tag{17}$$

with initial terms

$$F_1(x) = 1, F_2(x) = x. \tag{18}$$

The $h(x)$-Fibonacci polynomials [3] (where $h(x)$ is a polynomial with real coefficients) are defined by the recurrence relation

$$F_{h,n+1}(x) = h(x)F_{h,n}(x) + F_{h,n-1}(x), \quad n \geq 1 \tag{19}$$

with initial terms

$$F_{h,0}(x) = 0, F_{h,1}(x) = 1. \tag{20}$$

Basu, Prasad et al. [9] introduce $h(x)(> 0)$ Fibonacci p-numbers polynomials, $F_{p,h}(n, x)$ by the recurrence relation

$$F_{p,h}(n, x) = h(x)F_{p,h}(n-1, x) + F_{p,h}(n-p-1, x) \tag{21}$$

with initial terms

$$F_{p,h}(1, x) = b_1, F_{p,h}(2, x) = b_2, F_{p,h}(3, x) = b_3, \cdots, F_{p,h}(p+1, x) = b_{p+1} \tag{22}$$

where p (≥ 0) is integer, $h(x)$ (> 0) is a polynomial with real coefficients, $n > p+1$ and b_1, b_2, b_3, $\cdots$, b_{p+1} are arbitrary real or complex numbers.
In this paper, we introduce $h(x)(> 0)$, $g(y)(> 0)$ Fibonacci p-numbers polynomials, $F_{p,h,g}(n, x, y)$ by the recurrence relation

$$F_{p,h,g}(n, x, y) = h(x)F_{p,h,g}(n-1, x, y) + g(y)F_{p,h,g}(n-p-1, x, y) \tag{23}$$

with initial terms

$$F_{p,h,g}(1, x, y) = c_1, F_{p,h,g}(2, x, y) = c_2, F_{p,h,g}(3, x, y) = c_3, \cdots, F_{p,h,g}(p+1, x, y) = c_{p+1} \tag{24}$$

where p (≥ 0) is integer, $h(x)$ (> 0), $g(y)$ (> 0) are polynomials with real coefficients, $n > p+1$ and c_1, c_2, c_3, $\cdots$, c_{p+1} are arbitrary real or complex numbers.

2 Connection among Golden $(p, h(x), g(y))$-proportion, Golden $(p, h(x))$-proportion and Golden p-proportion

The characteristic equation of the $(h(x), g(y))$-extension of the Fibonacci p-numbers is

$$u^{p+1} - h(x)u^p - g(y) = 0 \tag{25}$$

Equation (25) has only one positive root $u = \mu_{p,h(x),g(y)}$ is called golden $(p, h(x), g(y))$-proportion.
The characteristic equation of the $h(x)$-extension of the Fibonacci p-numbers is

$$u^{p+1} - h(x)u^p - 1 = 0 \tag{26}$$

The equation (26) has only one positive root $u_1 = \mu_{p,h(x)}$, called golden $(p, h(x))$-proportion.
The characteristic equation of the Fibonacci p-numbers is

$$u^{p+1} - u^p - 1 = 0 \tag{27}$$

The equation (27) has only one positive root $u_2 = \mu_p$ called golden p-proportion.
When $g(y) = 1$, $\mu_{p,h(x),1}$ coincides with, $\mu_{p,h(x)}$, golden $(p, h(x))$-proportion. Also when $h(x) = 1$, $g(y) = 1$, $\mu_{p,h(x),g(y)}$ coincides with, μ_p, golden p-proportion.
Then u, u_1, u_2 satisfy the equation

$$u_1 - u = u_1^{\frac{log(u_2-1)}{log u_2}} - g(y)u^{\frac{log(u_2-1)}{log u_2}} \tag{28}$$

$\mu_{p,h(x),g(y)}$, golden $(p, h(x), g(y))$-proportion extends infinitely a number of new mathematical constants or in other words we say that golden $(p, h(x), g(y))$-proportion is a wide generalization of golden $(p, h(x))$-proportion.

3 Fibonacci $G_{p,h,g}$ matrix

In this paper, we define a new Fibonacci $G_{p,h,g}$ matrix of order $(p + 1)$ on the $(h(x), g(y)$-extension of the Fibonacci p-numbers where p (≥ 0) is integer and $h(x)$ (> 0), $g(y)$ (> 0)

$$G_{p,h,g} = \begin{pmatrix} F_{p,h,g}(2) & F_{p,h,g}(1) & \cdot\;\cdot\;\cdot\;\cdot\;\cdot & F_{p,h,g}(3-p) & F_{p,h,g}(2-p) \\ F_{p,h,g}(2-p) & F_{p,h,g}(1-p) & \cdot\;\cdot\;\cdot\;\cdot\;\cdot & F_{p,h,g}(3-2p) & F_{p,h,g}(2-2p) \\ \cdot & \cdot & \cdot\;\cdot\;\cdot\;\cdot\;\cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot\;\cdot\;\cdot\;\cdot\;\cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot\;\cdot\;\cdot\;\cdot\;\cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot\;\cdot\;\cdot\;\cdot\;\cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot\;\cdot\;\cdot\;\cdot\;\cdot & \cdot & \cdot \\ F_{p,h,g}(0) & F_{p,h,g}(-1) & \cdot\;\cdot\;\cdot\;\cdot\;\cdot & F_{p,h,g}(1-p) & F_{p,h,g}(-p) \\ F_{p,h,g}(1) & F_{p,h,g}(0) & \cdot\;\cdot\;\cdot\;\cdot\;\cdot & F_{p,h,g}(2-p) & F_{p,h,g}(1-p) \end{pmatrix} \tag{29}$$

The initial terms c_1, c_2, c_3, $\cdots$, c_{p+1} are in such a manner that $Det\ G_{p,h,g} = (-1)^p$ which is independent of $h(x)$ and $g(y)$ and nth power of $G_{p,h,g}$,

$$G^n_{p,h,g} = \begin{pmatrix} F_{p,h,g}(n+1) & F_{p,h,g}(n) & \cdot\;\cdot\;\cdot & F_{p,h,g}(n-p+2) & F_{p,h,g}(n-p+1) \\ F_{p,h,g}(n-p+1) & F_{p,h,g}(n-p) & \cdot\;\cdot\;\cdot & F_{p,h,g}(n-2p+2) & F_{p,h,g}(n-2p+1) \\ \cdot & \cdot & \cdot\;\cdot\;\cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot\;\cdot\;\cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot\;\cdot\;\cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot\;\cdot\;\cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot\;\cdot\;\cdot & \cdot & \cdot \\ F_{p,h,g}(n-1) & F_{p,h,g}(n-2) & \cdot\;\cdot\;\cdot & F_{p,h,g}(n-p) & F_{p,h,g}(n-p-1) \\ F_{p,h,g}(n) & F_{p,h,g}(n-1) & \cdot\;\cdot\;\cdot & F_{p,h,g}(n-p+1) & F_{p,h,g}(n-p) \end{pmatrix} \tag{30}$$

and $Det\ G^n_{p,h,g} = (-1)^{np}$ which is independent of $h(x)$ and $g(y)$. We choose c_1, c_2, c_3, $\cdots$, c_{p+1} in such a manner that the matrix $G_{p,h,g}$ and nth power of $G_{p,h,g}$ satisfied (29) and (30) respectively. Then matrix, $G_{p,h,g}$, is applicable for Fibonacci coding/decoding. When $g(y) = 1$ and $c_1 = 1$, $c_2 = h(x)$, $c_3 = h^2(x)$, $\cdots$, $c_{p+1} = h^p(x)$ then (29) and (30) satisfy cheerfully [9].

4 Conclusion

In this paper, we define $(h(x), g(y))$-extension of Fibonacci p-numbers and golden $(p, h(x), g(y))$-proportion. We also established a relation among Golden $(p, h(x), g(y))$-proportion, Golden $(p, h(x))$-proportion and Golden p-proportion. The research work can be develop for finding the suitable initial terms c_1, c_2, c_3, $\cdots$, c_{p+1} in such a manner that $G_{p,h,g}$ matrix applied for Fibonacci coding/decoding method. The correct ability of this method increases as p increases but it is independent of $h(x)$, $g(y)$ and for large value of p, it is approximately to 100%. For $g(y) = 1$, properties of $G_{p,h,g}$, $G^n_{p,h,g}$ matrix coincide with the properties of $G_{p,h}$, $G^n_{p,h}$ matrix respectively [9]. The relations among the code matrix elements for $h(x) = 1$ and $g(y) = 1$, coincide with the generalized relations among the code matrix elements for Fibonacci coding theory [6].

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