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Fixed and Periodic Point Theorems on Symmetric Spaces

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Abstract This paper proves the existence of periodic and fixed points for self maps satisfying some contractive conditions in symmetric space and also we prove coincidence and fixed points without continuity requirement satisfying a slightly more general Seghal's contractive conditions with suitable example.

Keywords Symmetric Space, $(E.A)$ property, Weak commutativity, Compatible Mappings, Coincidence Point, Periodic Point, Fixed Point

Mathematics Subject Classification: 47H10,54H25

1. Introduction with Preliminaries

In [1], the authors gave a notion $(E.A)$ which generalizes the concept of noncompatible mappings in metric spaces, and they proved some common fixed-point theorems for noncompatible mappings under strict contractive conditions. In [3], The Author promoted commutative maps as a tool for generalizing. Subsequently, a variety of variations and generalizations of Theorem 1 in [3] which utilized the commuting map concept appeared. In [4,6] the authors proved some common fixed-point theorems for strict contractive noncompatible mappings in metric spaces. Clearly, commuting mappings are weakly commuting and weakly commuting pairs are compatible; examples in [5] and [11] show that neither converse is true. Some common fixed point theorems due to Aamri and El Moutawakil [1],Pant and Pant [4] proved for strict contractive mappings in metric spaces are extended to symmetric (semi-metric) spaces under tight conditions. we present a few results that establish the existence of common periodic points for a pair of maps on a symmetric space when the maps have a unique common fixed point. These results are supported by suitable examples. In [9] B. E. Rhodes collected good number of contractive inequalities considered by various authors and established implications and non-implications among them. In this paper we obtain Some results on coincidence and fixed points without continuity requirements satisfying a slightly more general Seghal's contractive conditions.

Definition 1.1: A symmetric on a set X is a function $d: X\times X \to [0,\infty)$ such that for all $x, y \in X$

(i) $d(x,y)=0 \Leftrightarrow x=y$

(ii) $d(x,y)=d(y,x)$

Examples:

$$(1)\ Let\ X=[-1,1]$$

$$d(x,y)=|x-y| \quad if\ x\neq y\neq 0$$

$$=0 \qquad if\ x=y$$

$$d(x,0)=d(0,x)=\frac{|x|}{2}$$

(2) *Let* $X = [0,1]$

$$d(x,y) = \frac{1}{x} + \frac{1}{y} \quad if \;\; 0 \neq x \neq y \neq 0$$
$$= 0 \qquad if \; x = y$$
$$d(x,0) = d(0,x) = x$$

If d is symmetric on a set X, then for $x \in X$ and $\varepsilon > 0$, we write $B(x,\varepsilon) = \{y \in X : d(x,y) < \varepsilon\}$. A topology $\tau(d)$ on X is given by $U \in \tau(d)$ if and only if for each $x \in X$, $B(x,\varepsilon) \subset U$ for some $\varepsilon > 0$. A set $S \subset X$ is a neighborhood of $b \in X$ iff there exists $U \in \tau(d)$ such that $b \in U \subset S$. A symmetric d is a semi-metric if for each $x \in X$ and for each $\varepsilon > 0, B(x,\varepsilon)$ is a neighborhood of x in the topology $\tau(d)$.

Definition 1.2: A semi-metric space X is a topological space whose topology $\tau(d)$ on X is induced by semi-metric d . In what follows symmetric space as well as semi-metric space will be denoted by (X,d).

The distinction between a symmetric and a semi-metric is evident as one can easily
Construct a symmetric d such that $B(x,\varepsilon)$ need not be a neighborhood of x in $\tau(d)$.

For a symmetric d on X the following two axioms were given by Wilson [8]:

$$(W3): \textit{For a sequence} \{x_n\} \textit{in } X \textit{ and } x,y \in X,$$
$$\lim_{n\to\infty} d(x_n,x) = 0 \;\; and \;\; \lim_{n\to\infty} d(x_n,y) = 0 \; imply \; x = y$$

$$(W4): \textit{For a sequence} \{x_n\},\{y_n\} \textit{in } X \textit{ and } x \in X,$$
$$\lim_{n\to\infty} d(x_n,x) = 0 \;\; and \;\; \lim_{n\to\infty} d(y_n,x_n) = 0 \;\; imply \;\; \lim_{n\to\infty} d(y_n,x) = 0$$

Example:

1. Let $X = \Re$;

$$d(x,y) = \begin{cases} |x-y| & ; \; If \;\; x \neq y \\ 1 & ; \; If \;\; x = y \end{cases}$$

$$d(x,0) = d(0,x) = \frac{|x-1|}{3} \;\; if \; x \neq 1$$

Note that the function d is not a metric as triangular inequality fails.

$d(-1,2) \nleq d(-1,0) + d(0,2)$

Consider the sequences $x_n = 1 - \frac{1}{n}$, $n = 1,2,3,..$

Clearly, $\lim\limits_{n\to\infty} d\left(1-\frac{1}{n},1\right) = \lim\limits_{n\to\infty} d\left(1-\frac{1}{n},0\right)$

But $1 \neq 0$, thus $(W3)$ does not satisfy.

2. $X = [0,1]$

$$d(x,y)=\begin{cases} x+y & ;\ If\ \ x\neq y \\ 0 & ;\ If\ \ x=y \end{cases}$$

$$d(x,0)=d(0,x)=\frac{x}{3}$$

Then (X,d) is a symmetric space which is not a metric as triangular inequality fails.

$$d(\frac{1}{2},1)\not\leq d(\frac{1}{2},0)+d(0,1)$$

Consider the sequences $x_n=\frac{1}{n}$, $y_n=\frac{1}{n+1}$ then

$$\lim_{n\to\infty} d\left(\frac{1}{n},\frac{1}{n+1}\right)=\lim_{n\to\infty} d\left(\frac{1}{n},0\right)=0$$

$$\Rightarrow \lim_{n\to\infty} d\left(\frac{1}{n+1},0\right)=0$$

$$\therefore\ (W4)\ \text{holds.}$$

Definition 1.3: [2] A pair of self-mappings (f,g) on a symmetric (semi-metric) space (X,d) is said to be R-weakly commuting if there exists some real number $R>0$ such that $d(f\,g\,x,g\,f\,x)\leq R\,d(f\,x,g\,x)$ for all $x\in X$, where as the pair (f,g) is said to be point wise R-weakly commuting if given $x\in X$ there exists $R>0$ such that $d(f\,g\,x,g\,f\,x)\leq R\,d(f\,x,g\,x)$.

Here it may be noted that on the points of coincidence *R*-weak commutativity is equivalent to commutativity and remains a necessary minimal condition for the existence of common fixed points of contractive type mappings.

Definition 1.4: [5] A pair of self-mappings (f,g) on a symmetric (semi-metric) space (X,d) is said to be compatible if $\lim_{n\to\infty} d(f\,g\,x_n,g\,f\,x_n)=0$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} f(x_n)=\lim_{n\to\infty} g(x_n)=t\in X.$

Here it may be noted that *R*-weakly commuting mappings need not be compatible.

Definition 1.5: [10] A pair of self-mappings (f,g) on a symmetric (semi-metric) space (X,d) is said to be weakly compatible (or coincidentally commuting) if $f\,x=g\,x$ implies $f\,g\,x=g\,f\,x$.

Definition 1.6: [1] A pair of self-mappings (f,g) on a symmetric (semi-metric) space (X,d) is said to enjoy property $(E.A)$ if there exists a sequence $\{x_n\}$ such that $\lim_{n\to\infty} f(x_n)=\lim_{n\to\infty} g(x_n)=t,\ for\ some\ t\in X$.

Clearly non compatible pairs satisfy property $(E.A)$.

Definition 1.7: Let T be a self-mapping of X . A point $x\in X$ is called a periodic

Point of T, if there exists a positive integer m such that $T^m x=x$.

$(C.1)$: $\lim d(x_n,y_n)=0=\lim d(x_n,x)\Rightarrow \lim d(y_n,x)=0$

$(C.2)$: $\lim d(x_n,x)=0,\lim d(y_n,x)=0\Rightarrow \lim d(x_n,y_n)=0$

2. Point of Coincidence, Fixed points

Theorem 2.1: Let (X,d) be a symmetric (semi-metric) space with the $(W3)$ or a Hausdorff semi-metric space. Let

(f,g) be a pair of self maps of X that has the $(E.A)$ property. Then f *and* g have a point of coincidence if

$$d(gx,gy) < \max\{d(fx,gx), d(fy,gy), d(fx,fy)\} \qquad \textbf{(2.1.1)}$$

$f(X)$ is a closed subset of X,

Then f *and* g have a point of coincidence.

Proof: In a semi metric space (X,d) the sequence $\{x_n\}$ converges to x in $\tau(d)$ iff $d(x_n, x) \to 0$. To substantiate this, suppose $x_n \to x$

And let $\varepsilon > 0$. suppose $S(x,\varepsilon)$ is a neighborhood of x there exists $U \in \tau(d)$ such that $x \in U \subset S(x,\varepsilon)$. Since $x_n \to x$ there is a $m \in N$ (the natural number)

Such that $x_n \in U \subset S(x,\varepsilon)$ for $n \geq m$ so $d(x_n, x) < \varepsilon$ for $n \geq m$.

i.e $d(x_n, x) \to 0$. The converse part is obvious in view of the definition of $\tau(d)$.

By $(E.A)$ property , there must exists a sequence $\{x_n\}$ in X with $t \in X$ such that $\lim_{n\to\infty} f(x_n) = \lim_{n\to\infty} g(x_n) = t \in X$. As $f(X)$ is d-closed, every convergent sequence of points of $f(X)$ has its limit in $f(X)$, therefore $\lim_{n\to\infty} f(x_n) = b = f(a) = \lim_{n\to\infty} g(x_n)$ *for some* $a \in X$ which in turn yields that $t = f(a) \in f(X)$. Now we assert that $f(a) = g(a)$. If it is not so then in view of (2.1.1),

one gets $d(gx_n, ga) < \max\{d(fx_n, gx_n), d(fa, ga), d(fx_n, fa)\}$

which on letting $n \to \infty$, reduces to

$d(fa, ga) < d(fa, ga)$ which is a contradiction.

Therefore $f(a) = g(a)$, which shows that $'a'$ is a point of coincidence for f *and* g.

The same proof works for the alternate statement. This completes the proof.

The following variant of Theorem 2.1 also remains true.

Theorem 2.2: *Theorem* 2.1 *remains true if* d *-closedness (*$\tau(d)$*-closedness) of* $f(X)$ *is replaced by* d *-closedness (*$\tau(d)$*-closedness) of* $g(X)$ along with $g(X) \subset f(X)$ *retaining the rest of the hypotheses.*

Proof. Since f *and* g enjoy property $(E.A)$, we know that there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} f(x_n) = \lim_{n\to\infty} g(x_n) = g(a) = t \in X$ for some $a \in X$ as $g(X)$ is a d-closed subset of X. Now due to $g(X) \subset f(X)$ one can find some $b \in X$ such that $g(a) = f(b)$. We claim that $g(a) = g(b)$. Suppose on contrary that $g(a) \neq g(b)$ then using (2.1.1) one obtains

$$d(gx_n, gb) < \max\{d(fx_n, gx_n), d(fb, gb), d(fx_n, fb)\}$$

which on letting $n \to \infty$, reduces to

$d(ga, gb) < d(ga, gb)$ which is a contradiction.

Therefore $ga = gb = fb$

Theorems 2.1 and 2.2 ensure common fixed point instead of point of coincidence if contractive condition (2.1.1) is replaced by a slightly weaker condition. In this regard we have

Theorem 2.3: *In the setting of Theorems* 2.1 *and* 2.2, f *and* g *have a unique common* fixed point provided f *and* g are weakly compatible and satisfy the contraction condition (2.1.1) for all

$$x \neq y \in X,\ d(gx,gy) < \max\{d(fx,gx), d(fy,gy), d(fx,fy)\} \qquad \textbf{(2.3.1)}$$

Proof. In view of Theorems 2.1 and 2.2, f *and* g have a point of coincidence 'a'

i.e., $f(a) = g(a)$. Now due to weak compatibility one can write $fg(a) = ff(a) = gg(a) = gf(a)$. If

$g\,g(a) = g(a)$ then (2.3.1) implies

$$d(g\,a, g\,g\,a) < \max\{d(f\,a, g\,a), d(f\,g\,a, g\,g\,a), d(f\,a, f\,g\,a)\}$$
$$= d(g\,a, g\,g\,a)$$

which is a contradiction. Hence $g\,a = g\,g\,a = g\,f\,a = f\,g\,a = f\,f\,a$, which shows that $g\,a$ is a common fixed point of f *and* g. Uniqueness of the common fixed point follows easily.

Corollary 2.4: Let (X, d) be a symmetric (semi-metric) space that enjoys $(W3)$ (the Hausdorff separation axiom).Let g be a self map of X such that for all $x \neq y \in X$

$d(g\,x, g\,y) < \max\{d(x, g\,x), d(y, g\,y), d(x, y)\}$ then g has a unique fixed point.

Proof: If we take $f = I$ the identity mapping in above theorem 2.3, and follow a similar proof as that in theorem 2.3 , we can establish this corollary 2.4.

In [12], P Sumati kumari presented a few results that establish the existence of common periodic points for a pair of maps on a symmetric (metric) space when the maps have a unique common fixed point. These results are supported by suitable examples.

Theorem 2.5[12]: Let f be a self map of a symmetric space (X, d) satisfying

$$d(fx, fy) < \max\{d(x, y), d(x, f(x)), d(y, f(y)), d(x, f(y)), d(y, f(x))\}$$

For each $x, y \in X\ (x \neq y)$ for which the right hand side of above inequality is not zero. Then $u \in X$ is a periodic point of f if and only if u is the unique fixed point of f.

By using Corollary 2.4 and Theorem 2.5 we have the following theorem

Theorem 2.6: Let g be a self map on a symmetric space (X, d) satisfying $d(gx, gy) < \max\{d(x, y), d(x, g(x)), d(y, g(y))\}$ For each $x, y \in X\ (x \neq y)$ for which the right hand side of above inequality is not zero. Then $u \in X$ is a periodic point of g if and only if u is the unique fixed point of g.

To illustrate the above theorem we have the following Example

Example: Let $X = [0, 1)$ and $d(x, y) = |x - y|^2$.

The inequality can be easily checked.

Then next theorem involves a function $\phi : \Re^+ \to \Re^+$ which satisfies the following conditions:

(i) ϕ is non decreasing on $\Re^+$,

(ii) $0 < \phi(t) < t$ *for each* $t \in (0, \infty)$.

Theorem 2.7: Let A, B, S *and* T *be self-mappings of a symmetric (semi-metric) space* (X, d) *that enjoy* $(W3)$ (*the Hausdorff 's* T_2 separation axiom*)* and $\phi(0) = 0$. *Suppose that*

(i) $A(X) \subset T(X), B(X) \subset S(X)$

The pair (B,T) enjoys the property $(E.A)$(or alternatively the pair (A,S) enjoys the property $(E.A)$)

The following inequality holds

$$d(Ax, By) \leq \phi(m(x, y)), \tag{2.7.1}$$

$m(x, y) = \max\{d(Sx, Ax), d(Ty, By), d(Sx, Ty)\}$

$S(X)$ is d -closed ($\tau(d)$ -closed) subset of X (or alternatively $T(X)$ is a d -closed ($\tau(d)$ -closed) subset of X.

Then pairs (A, S) has a point of coincidence u and the pair (B,T) has a point of coincidence w.

Proof: Since the pair (B,T) enjoys the property $(E.A)$, there exists a sequence $\{x_n\} \subset X$ such that $\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = t \in X$. Since $B(X) \subset S(X)$

For each x_n there exists y_n such that $Bx_n = Sy_n$. Thus in all $Bx_n \to t$, $Sy_n \to t$ *and* $Tx_n \to t$. Now we assert that $Ay_n \to t$. Suppose , to obtain a contradiction in each case.

Case (i) $Ay_m = Bx_m$ for only finitely many m.

Let $Ay_m \neq Bx_m$ for all $m \geq n_0$ where n_0 is a positive integer. Then for $m \geq n_0$,

$$d(Ay_m, Bx_m) \leq \phi(\max\{d(Sy_m, Ay_m), d(Tx_m, Bx_m), d(Sy_m, Tx_m)\})$$
$$= \phi(\max\{d(Bx_m, Ay_m), d(Tx_m, Bx_m), d(Bx_m, Tx_m)\})$$
$$= \phi(\max\{d(Bx_m, Ay_m), d(Tx_m, Bx_m)\})$$

But $\phi(d(Ay_m, Bx_m)) < d(Ay_m, Bx_m)$

Hence $\max\{d(Bx_m, Ay_m), d(Bx_m, Tx_m) \neq d(Bx_m, Ay_m)$

Hence $\max\{d(Bx_m, Ay_m), d(Bx_m, Tx_m) \neq d(Bx_m, Tx_m)$

Hence $d(Ay_m, Bx_m) \leq \phi(d(Bx_m, Tx_m))$

But $\phi(t) \leq t \ \forall t \in [0, \infty)$

Hence $d(Ay_m, Bx_m) \leq d(Bx_m, Tx_m)$

Letting $m \to \infty$ and using $(C.2)$ we get,

$$\limsup_{n\to\infty} d(Ay_m, Bx_m) \leq \lim_{m\to\infty} d(Bx_m, Tx_m)$$
$$= 0 \text{ by } (C.2)$$

And the fact that $\lim Bx_m = \lim Tx_m = t$

Hence $\lim Sup\, d(Ay_m, Bx_m) = 0$ which implies $\lim d(Ay_m, Bx_m) = 0$

$$\Rightarrow \lim d(Ay_m) = \lim Bx_m \text{ by } (C.1)$$
$$\therefore \lim Ay_m = t$$

Suppose $Ay_m = Bx_m$ for infinitely many values of m

Let the sequence $K = \{k_1 < k_2 < k_3 < ....\}$ have the property that $Ay_{k_i} = Bx_{k_i}$ for $i = 1,2,....$ and $J = \{j_1 < j_2 < j_3 < ....\}$ have the property that $Ay_{j_i} \neq Bx_{j_i}$ for $i = 1,2,....$ and let $K \cup J = N$.

Then the sequence $Ay_{k_i} \to t$ since the sequence $Bx_{k_i} \to t$. If J is a finite set then we may suppose $J = \phi$ and conclude $Ay_m \to t$. Otherwise we can conclude that $Ay_{j_i} \to t$ as in case (i).

Since $K \cup J = N$ and since $Ay_{k_i} \to t$ and also $Ay_{j_i} \to t$,

It is clear that $Ay_m \to t$.

Suppose that $S(X)$ is a d *-closed* ($\tau(d)$ *-closed*) subset of X then $Sy_n \to t$ and one can find a point $u \in X$ such that $Su = t$ now we suppose that $Au \neq Su$

$$d(Au, Bx_n) \leq \phi\left(\max\{d(Su, Au), d(Tx_n, Bx_n), d(Su, Tx_n)\}\right)$$

Which on letting $n \to \infty$, yields

$$d(Au, Su) \leq \phi\left(d(Su, Au)\right) < d(Su, Au)$$

Which is a contradiction

Hence $Au = Su$

Also $A(X) \subset T(X)$, there exists $w \in X$ such that $Au = Tw.$ we assert that $Tw = Bw.$

If not, then using inequality (2.7.1), one gets

$$\begin{aligned} d(Au, Bw) &\leq \phi\left(\max\{d(Su, Au), d(Tw, Bw), d(Su, Tw)\}\right) \\ &< d(Tw, Bw) \\ &= d(Au, Bw) \end{aligned}$$

Which is a contradiction.

Hence $Au = Su = Bw = Tw.$

Which shows that the pairs (A, S) *and* (B, T) *have a point of coincidence* u & w *respectively.*

The proof is similar if we consider the case when pair (A, S) enjoys property $(E.A)$,

And $T(X)$ is d *-closed* ($\tau(d)$ *-closed*) subset of X.

Hence it is omitted. This completes the proof.

Theorem 2.8: *In the setting of Theorem* 2.7 *,* A, B, S *and* T *have a unique common fixed point provided one adds the weak compatibility of the pair* (A, S) (*or weak compatibility of the pair* (B, T) *and satisfying the contractive* *condition* (2.7.1) *for* $x \neq y \in X$,

$$d(Ax, By) \le \phi(m(x, y)) \tag{2.8.1}$$

$$m(x, y) = \max\{d(Sx, Ax), d(Ty, By), d(Sx, Ty)\}$$

Proof: In view of Theorem 2.7, one concludes that $Au = Su = Bw = Tw.$

Now the weak compatibility of (A, S) implies that $ASu = SAu$ *and* $AAu = ASu = SAu = SSu$. Suppose that $Au \neq AAu$ then using (2.7.1), one gets

$$d(Au, AAu) = d(AAu, Bw)$$
$$\le \phi(\max\{d(SAu, AAu), d(Tw, Bw), d(SAu, Tw)\})$$
$$< d(Au, AAu)$$

Which is a contradiction. Thus $Au = AAu = SAu$, then Au is the common fixed point of A and S. Also Au is a common fixed point of the pair (B, T). Uniqueness of the common fixed point follows easily. The proof is similar in the other case. This completes the proof.

We now give an example to illustrate the above theorem.

Example: Consider $X = [0, 1]$ equipped with the symmetric $d(x, y) = (x - y)^2$. Define

$$Ax = Bx = \frac{x}{1+x},\ \textit{if}\ 0 \le x \le 1, \quad Sx = Tx = x\ ,\ \textit{if}\ 0 \le x \le 1,\ \textit{and}\ \ \phi : \Re^+ \to \Re^+\ \textit{as}$$

$$\phi(t) = \begin{cases} \dfrac{t}{1+t}, & \textit{if}\quad 0 \le t \le 1, \\[2mm] \dfrac{t}{2}, & \textit{if}\quad t > 1. \end{cases}$$

then

$$A(X) = B(X) = \left[0, \frac{1}{2}\right] \subset [0, 1] = S(X) = T(X),$$

ϕ is non decreasing and $0 < \phi(t) < t$ for all $t \in (0, \infty)$. Since d induces the usual topology therefore $(W3)$ is satisfied. The pair (A, S) satisfies the $(E.A)$ property as there is a sequence $\left\{\frac{1}{n}\right\} \subset [0, 1]$ such that

$$\lim_{n\to\infty} A\left(\frac{1}{n}\right) = \lim_{n\to\infty} \frac{1}{n+1} = \lim_{n\to\infty} S\left(\frac{1}{n}\right) = \lim_{n\to\infty} \frac{1}{n} = 0 \in X$$

Also (A, S) is weakly compatible as $S0 = A0 \Rightarrow AS0 = SA0 . S(X) = [0, 1]$ is a d - closed subset of X.

In order to verify contractive condition (2.6.1), *If* $x = 0$ *and* $0 < y \le 1$, then

$$d(Ax, By) = \left(\frac{y}{1+y}\right)^2 < \frac{y^2}{1+y^2} = \phi(y^2) = \phi(d(Sx, Ty)) \le \phi(m(x, y))$$

In case $x \ne y$ *and* $0 < x < y \le 1$, *then*

$$d(Ax, By) = \left(\frac{x}{1+x} - \frac{y}{1+y}\right)^2 = \frac{(x-y)^2}{((1+x)(1+y))^2} \le \left(\frac{x-y}{1+(x-y)}\right)^2$$

$$< \frac{(x-y)^2}{1+(x-y)^2} = \phi\left((x-y)^2\right) = \phi(d(Sx, Ty)) \le \phi(m(x, y)).$$

Thus all the conditions of Theorems 2.7 and 2.8 are satisfied and 0 is the coincidence as well as common fixed point of the pair *(A, S)*.

Corollary 2.9: Let f *be self map of a symmetric* (*semi-metric*) *space* (X, d) *that enjoys* $(W3)$ (*the Hausdorffness of* $\tau(d)$) *and satisfying*

$$d(f\,x, f\,y) \le \phi(m(x, y)),$$

Where $m(x, y) = \max\{d(x, f\,x), d(y, f\,y), d(x, y)\}$. Then f has a unique fixed point.

Proof: If we take $A = B = f$ and $S = T = I$ an identity mapping in above theorem 2.8, and follow the similar proof as that in theorem 2.8, we can establish this corollary 2.9.

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