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Shear Strength of Normal and Light Weight Reinforced Concrete Slender Beams without Web Reinforcement

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Abstract There is no general consensus or accepted theory for evaluating the ultimate shear capacity of reinforced concrete beams without web reinforcement as a result the requirements in most of Codes of practice are provided in the form of empirical equations for predicting the shear capacity of reinforced concrete beams. In this paper, a study is conducted to evaluate the predictive accuracy of 6 empirical equations used in different Code of practice to predict the shear capacity of reinforced concrete slender beams. Empirical equations used in some Codes are identified to be superior to other equations. In addition, a study was also conducted to assess predictive accuracy of 17 empirical equations proposed in the literature by several researchers to predict the shear capacity of reinforced concrete slender beams. Among these 17 empirical equations some equations are identified to be superior to the other proposed equations. On the basis of experimental results of reinforced concrete beams having shear span to depth ratio a/d ≥2.5, empirical equations are proposed which include basic parameters i.e. concrete compressive strength, shear span to depth ratio and ratio of longitudinal reinforcement. The coefficient of correlation (COR) for proposed empirical equation for predicting the shear capacity of reinforced concrete beams having depth d < 300mm and d ≥ 300mm without web reinforcement comes out to be 0.869 and 0.953 respectively.

Keywords Empirical Equations, Shear Strength, Slender Beams, Concrete Compressive Strength

1. Introduction

Extensive research over the years on the combined effects of flexure and shear on the resistance capacity of the structure has not yielded a generalized theory of combined flexure shear for computing the resistance capacity of reinforced concrete members [1], [2]. As a result, the design for shear is uncoupled with respect to the flexural design.

Most of the code of practices uses empirical equations to estimate the shear capacity of reinforced concrete beams. In addition to the equations in the Codes, there are number of empirical equations proposed in the literature by different researchers. Empirical equations developed from experimental results for calculating Vc involves different influencing parameters based on the variable considered in the experimental program by the researcher. Each researcher has selected different influencing parameters as there is no general consensus or accepted theory for evaluating the ultimate shear capacity of reinforced concrete beams without web reinforcement.

In this paper, Design equations used in six (6) Design Codes of practice were evaluated using the experimental data contained in ACCESS shear database [3].Predictive accuracy of 17empirical equations proposed in the literature for predicting the shear capacity of reinforced concrete slender beams a/d> 2.5, were studied using the experimental data contained in ACCESS shear database [3]. On the basis of results, for slender reinforced concrete beams, empirical equations used in some Codes are identified to be superior to others. Among the proposed empirical equations in the literature, equations that use the use $(f'c)^{1/3}$ function and include depth factor are found to be superior to others.

On the basis of experimental results of reinforced concrete beams [3] having shear span to depth ratio a/d≥ 2.5, empirical equations are proposed which include basic parameters i.e. concrete compressive strength $f'c$, shear span to depth ratio a/d and ratio of longitudinal reinforcement ρ. The coefficient of correlation (COR) for proposed empirical equation for predicting the shear capacity of reinforced concrete beams having depth d < 300mm and d ≥ 300mm without web reinforcement comes out to be 0.869 and 0.953 respectively.

2. Evaluation of Design Equations

Equations 1 to 6 shows empirical equations in different Codes of practice along with their limits of applicability used to predict the shear capacity of reinforced concrete slender beams. For the study of predictive accuracy of the Code equations, experimental data for slender beams from the shear database [3] was used.

It can be seen that to reflect the effect of concrete compressive strength $f'c$ on the shear capacity of reinforced concrete beams, ACI Code [4] Eq. 1, Canadian Code [5] Eq.2 and New Zealand Code [6] Eq. 3 use function (f'c)½, whereas the Euro code EC2 [7] Eq.4 , Espanish Code EHE-99 [8] Eq. 5 and CEB-FIP Model Code [9] Eq.6 use function (f'c)⅓. The influence of size effect on the shear capacity is not included in the equations of ACI Code [4] Eq. 1, and New Zealand Code [6] Eq. 3, whereas the equations of the other Codes have terms that accommodate the influence of size effect.

$$\vartheta_{cr} = 0.16\sqrt{f'_c} + 17.2\rho\frac{Vd}{M} \qquad \text{for a/d} \geq 2.5 \tag{1}$$

$$\vartheta_{cr} = 0.2\sqrt{f'_c} \qquad \text{for d} \leq 300\text{mm} \tag{2}$$

$$\vartheta_{cr} = \left(\frac{260}{1000+d}\right)\sqrt{f'_c} \geq 0.1\sqrt{f'_c} \qquad \text{for d>300mm}$$

$$\vartheta_{cr} = (0.07 + 10\rho)\sqrt{f'_c} \qquad \text{for a/d >2.0} \tag{3}$$

$$\vartheta_c = \frac{0.18}{\gamma_c}K(100\rho_l f_{ck})^{1/3} + 0.15\sigma_{cp} \tag{4}$$

$$\vartheta_c \min = 0.035k^{3/2} f_{ck}{}^{1/2}$$

where

$f_{ck} \leq 100\ MPa$

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2,$$

where d is in mm

$$\rho_l = \frac{A_s}{b_w d} \leq 0.02$$

$$\vartheta_c = 0.12\xi(\rho_s f_{ck})^{1/3} - 0.15\sigma_{cd} \tag{5}$$

where

$$\xi = 1 + \sqrt{\frac{200}{d}}$$

$$\vartheta_c = 0.12\left(1 + \sqrt{\frac{200}{d}}\right)\left(\frac{3d}{a_s}\right)^{1/3}(\rho_s f_{ck})^{1/3} - 0.15\sigma_{cd} \tag{6}$$

where

N_d = Factored Axial Force

Ac = Area of concrete

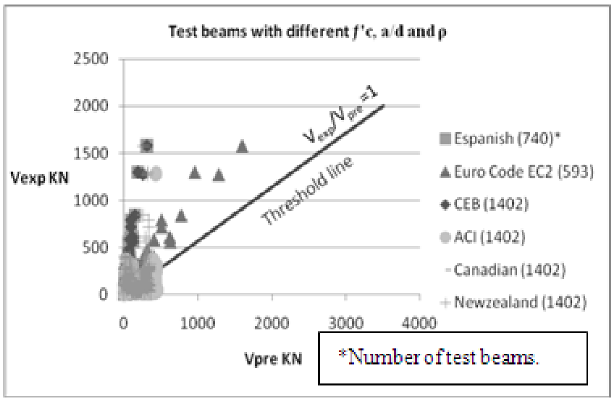

Figure1. Comparison of prediction of Code equations with experimental results.

Fig. 1 shows the plot of the experimental (measured) ultimate shear force (V_{exp}) and predicted ultimate shear force (V_{pre}), for all the 6 code equations, along with the threshold line ($V_{exp}/V_{pre} = 1$). It can be seen from Fig. 1, the (V_{exp}/V_{pre}) of CEB-FIP Model Code [9] Eq. 6 and Espanish EHE-99 Code [8] Eq. 5 are much greater than 1, which shows that these two codes significantly underestimate the shear capacity of reinforced concrete slender beams, as compared to ACI Code [4] Eq. 1, Euro Code EC2 [7] Eq. 4, New Zealand Code [6] Eq. 3 and Canadian Code [5] Eq. 2.

Table 1. Summary of results for the Average Margin of Safety $(V_{exp}/V_{pre})_{avg}$ of empirical equations used in different Codes for predicting the shear capacity of normal strength and high strength reinforced concrete slender beams.

Code	No. of Beams used for Evaluation	*Average Margin of Safety (Vexp/Vpre)avg
CEB-FIP Model	1402	7.214
Espanish EHE-99	740	6.539
ACI	1402	1.314
Eurocode EC2	593	1.273
NewZealand Code	1402	1.207
Canadian Code	1402	1.209

Table 1 shows the summary of results in terms of the comparison of the average Margin of Safety $(V_{exp}/V_{pre})_{avg}$ of design equations used in different Codes of practice for predicting the shear capacity of reinforced concrete slender beams. The number of beams, whose data was used, varies for each case, because of the relative constraints or the limits in the respective empirical equations of the Codes. From Table 1 it can be seen that the average Margin of Safety $(V_{exp}/V_{pre})_{avg}$ for CEB-FIP Model Code [9]Eq. 6 and Espanish EHE-99 Code [8] Eq. 5 is 7.214 and 6.539 respectively which is much higher than $(V_{exp}/V_{pre})_{avg}$ values, when using the ACI Code [4] Eq. 1, Euro Code EC2 [7] Eq. 4, New Zealand Code [6] Eq. 3 and Canadian Code [5] Eq. 2.which are 1.325, 1.273, 1.207 and 1.209 respectively. Thus the CEB-FIP Model Code [9] Eq. 6 and Espanish EHE-99 Code [8] Eq. 5 estimations are significantly more conservative (order of 6 or higher) as compared to other codes and are not considered in the further evaluation.

2.1 Effect of Influencing Factors

Two major factors are studied, the concrete compressive strength and the size effect. Table 2 shows the summary of results showing the Average Margin of Safety $(V_{exp}/V_{pre})_{avg}$ with coefficient of correlation (COR) for Normal Strength Concrete (NSC) having f'c < 40MPa and High Strength Concrete (HSC) having f'c ≥ 40MPa reinforced slender beams. It can be seen from Table 2 that in case of NSC beams, COR for Euro code EC2 Code [7] Eq. 4 is 0.974, which is higher as compared to New Zealand Code [6] Eq. 3 , Canadian Code [5] Eq. 2 and ACI Code [4] Eq. 1, which are

0.855, 0.932 and 0.899 respectively.

In case of HSC beams, COR for Euro code EC2 [7] Eq. 4 is 0.974 which is higher as compared to New Zealand Code [6] Eq. 3 , Canadian Code [5] Eq. 2 and ACI Code [4] Eq. 1 which are 0.882, 0.932 and 0.90 respectively. It should be noted that Euro code EC2 [7] Eq. 4 equation uses cubic root function (*f*'c) ⅓ rather than the square root function (*f*'c) ½ used by ACI Code [4] Eq. 1, Canadian Code [5] Eq. 2 and Newzealand Code [6] Eq. 3 to reflect the effect of the concrete compressive strength *f*'c on the shear capacity of reinforced concrete beams. This implies that (*f*'c)½ function used in the ACI Code [4] Eq. 1, Canadian Code [5] Eq. 2 and Newzealand Code [6] may not be adequate to reflect the effect of the *f*'c on the shear capacity of high strength reinforced concrete beams.

Table 2. Summary of results showing the Average Margin of Safety with coefficient of correlation for NSC and HSC reinforced slender beams.

Code	Strength of Concrete			
	No. of Beams used for evaluation	NSC HSC	Average Margin of Safety	COR
Eurocode EC2	593	370	1.60	0.974
		223	1.14	0.974
New Zealand Code	1402	951	1.25	0.855
		451	1.12	0.882
Canadian Code	1402	951	1.28	0.932
		451	1.04	0.932
ACI Code	1402	951	1.38	0.899
		451	1.16	0.907

Table 3 shows the summary of results showing the Average Margin of Safety $(V_{exp}/V_{pre})_{avg}$ with COR for NSC and HSC reinforced slender beams including the size effect. It can be also seen from Table 3 that in case of beams with effective depth $d < 300$mm the COR for Euro code EC2 (2002) Code [7] Eq. 4 is 0.985, which is higher as compared to Canadian Code [5] Eq. 2 (1994), ACI Code [4] Eq. 1 (2008) and New Zealand Code [6] Eq. 3 (1995) which are 0.932, 0.90 and 0.674 respectively. In case of beams with effective depth $d \geq 300$mm the COR or Euro code EC2 [7] Eq. 4 is 0.975, which is higher as compared to Canadian Code [5] Eq. 2 , ACI Code [4] Eq. 1 and New Zealand Code [6] Eq. 3 which are 0.938, 0.90 and0.899 respectively. It should be noted that the Euro code EC2 [7] Eq. 4 and Canadian code [5] Eq. 2 equations that are identified to have higher values of coefficient of correlation COR use depth factor in their respective expressions, whereas ACI Code [4] Eq. 1 and New Zealand Code [6] Eq. 3 equations do not use depth factor in their relative expressions. Although the COR of Euro code EC2 Code [7] Eq. 4 is higher, however the applicability over the number of beams is limited, due to constraints or the limits in the empirical equation as compared New Zealand Code [6] Eq. 3, Canadian Code [5] Eq. 2 and ACI Code [4] Eq. 1 for which a larger number of test beams were used to assess the predictive accuracy.

3. Evaluation of Empirical Equations Proposed in the Literature

Number of empirical equations has been proposed in the literature for predicting the shear capacity of NSC and HSC beams without web reinforcement. Equations 7 to 23 shows empirical equations proposed by Karim et al [11] Eq. 7, Daejoong Kim et al [13] Eq. 8, K.N.Smith [19] Eq. 9 and Eq.10 , ASCE-ACI – Committee 426 [1] Eq.11, Crist [21] Eq. 12, Daiz De Cossio et al [22] Eq. 13, Jin-Keun Kim et al [14] Eq. 14, Eq. 15 and Eq. 16, Shuaib et al [17] Eq. 18, Zsutty's [20] Eq. 18 ,Mphonde et al [18] Eq. 19 ,Gastebled et al [10] Eq. 20, Kaiss et al [15] Eq.21 , S. Sarkar et al [12] Eq. 22 and Okamura [16] Eq. 23 , along with their limits of applicability.

Table 3. Summary of results showing the Average Margin of Safety with coefficient of correlation for NSC and HSC reinforced slender beams including the size effect.

Code	Size Effect			
	No. of Beams used for evaluation	d<300mm d≥300mm	Average Margin of Safety	COR
Eurocode EC2	593	292	1.35	0.985
		301	1.20	0.975
New Zealand Code	1402	1046	1.19	0.674
		356	1.23	0.899
Canadian Code	1402	1046	1.31	0.932
		356	0.90	0.938
ACI Code	1402	1046	1.44	0.884
		356	0.92	0.916

It can be seen that effect of $f'c$ on the shear capacity of reinforced concrete beams is accommodated through use of $(f'c)^{1/2}$ function in the equations proposed by Karim et al [11] Eq. 7, Daejoong Kim et al [13] Eq. 8, K.N.Smith [19] Eq. 9 and Eq.10 , ASCE-ACI – Committee 426 [1] Eq.11, Crist [21] Eq. 12 and Daiz De Cossio et al [22] Eq. 13, through use of $(f'c)^{1/3}$ function in the equations proposed by Jin-Keun Kim et al [14] Eq. 14, Eq. 15 and Eq. 16, Shuaib et al [17] Eq. 18, Zsutty's [20] Eq. 18 and Mphonde et al [18] Eq. 19 and through the use of $f'c^{0.35}$, $f'c^{0.38}$, $f'c^{0.55}$ and $f'c$ functions in the equations proposed by Gastebled et al [10] Eq. 20, Kaiss et al [15] Eq.21 , S. Sarkar et al [12] Eq. 22 and Okamura [16] Eq. 23 respectively.

It can be seen that that the issue of size effect is addressed only in equations proposed by Okamura et al [16] Eq. 23, Shuaib et al [17] Eq. 18, Jin-Kuen-Kim et al [14] Eq. 14, Eq. 15 and Eq. 16 and Gastebled et al [10] Eq. 20.

$$\vartheta_c = 0.4 + \sqrt{f'_c \rho \frac{d}{a}(10 - 3\,A_d)} \qquad \text{For } \frac{a}{d} \geq 2.5,\ A_d = 2.5 \tag{7}$$

$$\vartheta_u = 0.2\left(1 - \sqrt{\rho}\right)\left(\frac{d}{a}\right)^r \left[\sqrt{f'_c} + 1020\rho^{0.9}\left(\frac{d}{a}\right)^{0.6}\right] \qquad \text{Where } r = \left(\frac{d}{a}\right)^{0.6} \rho^{-.01} \tag{8}$$

r= internal moment arm length index

$$\vartheta_{cr} = 2.6\sqrt{f'_c} + 3409\frac{Vd\rho}{M} \tag{9}$$

$$\vartheta_{cr} = 1.74\sqrt{f'_c} + 4550\frac{Vd\rho}{M} \tag{10}$$

$$\vartheta_c = (0.8 + 100\rho)\frac{\sqrt{f'_c}}{12} \qquad \leq 0.179\sqrt{f'_c}\,\text{MPa} \tag{11}$$

$$\vartheta_{cr} = 2.27\sqrt{f'_c} + 2905\frac{Vd\rho}{M} \tag{12}$$

$$\vartheta_{cr} = 2.14\sqrt{f'_c} + 4600\frac{Vd\rho}{M} \tag{13}$$

$$\vartheta_u = 15.5 f'_c{}^{\propto/3} \rho^{3/8}\left(0.4 + \frac{d}{a}\right)\left(\frac{1}{\sqrt{d}} + 0.07\right) \qquad \text{For d } \geq 250\,mm\ (9.84\,in) \tag{14}$$

For $\frac{a}{d} \geq 3, \propto = 1$

$$\vartheta_u = 3.5\, f'_c{}^{1/3} \rho^{3/8}\left(0.4 + \frac{d}{a}\right)\lambda(d) \qquad \frac{a}{d} \geq 3 \tag{15}$$

Where, $\lambda(d) = \frac{1}{\sqrt{1+0.008d}} + 0.18$

$$\vartheta_u = 19.4\, f'_c{}^{\propto/3} \rho^{3/8}\left(0.4 + \frac{d}{a}\right)\left(\frac{1}{\sqrt{d}} + 0.07\right) \qquad \text{For d } \geq 250\,mm\ (9.84\,in) \tag{16}$$

For $\frac{a}{d} \geq 3, \propto = 1$

$$\vartheta_u = \eta\left[1.8\left(f'_c \rho \frac{d}{a}\right)^{0.333}\right] \qquad \text{For } 3 \leq \frac{a}{d} \leq 6, \tag{17}$$

$$\eta = 1 - 0.00265\left[\frac{(d - 135.9)^{0.85}}{\left(\frac{a}{d}\right)^{0.63}}\right]$$

For $\frac{a}{d} \leq 3$

$$\eta = 1 - 0.03985\left[\frac{(d-135.9)^{0.8}}{\left(\frac{a}{d}\right)^{2.84}}\right]$$

$$\vartheta_u = 2.3(f'_c\rho\frac{d}{a})^{0.333} \qquad \frac{\mathrm{a}}{\mathrm{d}} \geq 2.5 \tag{18}$$

$$\vartheta_u = 0.366\sqrt[3]{f'_c} + 0.49 \qquad \frac{a}{d} = 3.5 \tag{19}$$

$$\vartheta_c = 0.15\left(\frac{37.41}{\sqrt{d}}\right)\left(\frac{3d}{a_s}\right)^{1/3}(100\rho_s)^{1/6}\left(1 - \sqrt{\rho_s}\right)^{2/3} f'^{\,0.35}_c \tag{20}$$

$$\vartheta_{cr} = 1.8\left(f'_c\rho\frac{V_u d}{M_u}\right)^{0.38} \qquad \text{For } \tfrac{a}{d} \geq 2 \tag{21}$$

$$\vartheta_n = 3.05\left(f'_c\rho\frac{d}{a}\right)^{0.55} \qquad For\,\tfrac{a}{d} > 2,\; 40 < f'_c < 110\text{ MPa} \tag{22}$$

$$\vartheta_c = 0.20\frac{\rho^{1/3}}{d^{1/4}}f'_c\left[0.75 + \frac{1.40}{a/d}\right] \tag{23}$$

Table 4. Summary of results for the Average Margin of Safety of empirical equations proposed in literature for predicting the shear capacity of concrete in normal and high beams.

Authors of the proposed empirical equations, published in the literature	No. of Beams used for Evaluation	** Average Margin of Safety **$(V_{exp}/V_{pre})_{avg}$**
Okamura et al	1402	2.677
Shuaib H et al	1401	1.470
Kaiss F. Sarsam et al	1402	1.460
K.N.Smith et al (B)	1402	1.317
Jin-Keun Kim et al (A)	897	1.306
Gastebled et al	1402	1.194
S. Sarkar et al	1402	1.180
Crist	1402	1.117
Daizet al	1402	1.107
Jin-Keun Kim et al(C)	897	1.106
Zsutty	1402	1.050
Jin-Keun Kim et al(B)	897	1.044
ASCE-ACI Committee 426	1402	1.004
K.N.Smith et al (A)	1402	0.973
Karim et al	1402	0.921
Daejoong Kim et al	1402	0.746
Mphonde et al	19	0.696

** listed in the order of descending order of Average Margin of Safety

Table 4 shows the summary of results in terms of the comparison of the average Margin of Safety's$(V_{exp}/V_{pre})_{avg}$of different proposed equations for predicting the shear capacity of reinforced concrete slender beams (a/d> 2.5). From Table 4, it can be seen that the average Margin of Safety's $(V_{exp}/V_{pre})_{avg}$ for the empirical equation proposed by Mphonde et al [18] Eq. 19 is 0.67 which is the least among all the empirical equations, with its applicability on only 19 test beams due to its limits. It can be also seen from Table 4 that although the empirical equations proposed by Jin-Keun Kim et al [14] Eq. 14, Eq. 15 and Eq. 16, have relatively low values of the average Margin of Safety's $(V_{exp}/V_{pre})_{avg}$ as compared to other proposed empirical equations but its applicability is limited to only 897 test beams as compared to 1402 test beams used for other proposed empirical equations. Therefore in the further evaluation, the empirical equations proposed by Mphonde et al [18] and Jin-Keun Kim et al [14] are not considered.

Table 5 shows the summary of results for the Average Margin of Safety $(V_{exp}/V_{pre})_{avg}$ and the size effect with respective Coefficient of correlation (COR), using the empirical equations of empirical equations proposed in literature for both normal strength concrete (NSC) and high strength reinforced concrete (NSC) slender beams.In case of NSC reinforced slender beams,, the equations proposed by Gastebled et al [10] Eq. 20, Shuaib et al [17] Eq. 18 and Kaiss et al [15] Eq.21, have higher values of coefficient of correlation (COR) which are 0.959,and 0.938 respectively as compared to other proposed equations (Table 5).

In case of HSC reinforced concrete slender beams, the empirical equations proposed by Shuaib et al [17] Eq. 18, Gastebled et al [10] Eq. 20 and Daejoong Kim et al [13] Eq. 8 have higher values of coefficient of correlation (COR) which are 0.967, 0.950 and 0.948 respectively as compared to other proposed equations (Table 5).

Table 5. Summary of results for the Average Margin of Safety and the size effect with respective Coefficient of correlation (COR), using the empirical equations of empirical equations proposed in literature for both normal and high strength reinforced concrete slender beams.

Author	Strength of concrete			
	No. of Beams used for Evaluation	(NSC) (HSC)	Average Margin of Safety	COR
Gastebled , May	1402	951	1.24	0.959
		451	1.08	0.950
Shuaib et al	1401	950	1.51	0.938
		451	1.37	0.967
Kaiss et al	1402	951	1.52	0.938
		451	1.33	0.942
Daejoong Kim et al	1402	951	0.74	0.936
		451	0.76	0.948
Sarkar et al	1402	951	1.27	0.947
		451	0.97	0.938
Zsutty's	1402	951	1.08	0.933
		451	0.98	0.941
ASCE-ACI -426	1402	951	1.05	0.930
		451	0.9	0.929
Karim et al	1402	951	0.96	0.930
		451	0.85	0.943
K.N.Smith et al(B)	1402	951	1.38	0.906
		451	1.18	0.913
K.N.Smith et al(A)	1402	951	1.025	0.899
		451	0.86	0.907
Crist	1402	951	1.17	0.899
		451	0.98	0.907
Daiz De Cossio, Seiss	1402	951	1.16	0.904
		451	0.99	0.911
Okamura et al	1402	951	3.15	0.966
		451	1.68	0.919

In Table 5, generally the equations which uses cubic power or power lesser than square root on *f*'c to reflect the effect of the concrete compressive strength *f*'c on the shear capacity of reinforced concrete slender beams have higher values of COR as compared to the equations which use power equal to or higher than square on *f*'c to reflect the effect of the concrete compressive strength *f*'c on the shear capacity of reinforced concrete beams. Exception in the proposed equation of Daejoong Kim et al [13] Eq. 8 which uses square power on *f*'c and has a COR value of 0.948.

Table 6 also shows the summary of results Average Margin of Safety $(V_{exp}/V_{pre})_{avg}$ with respective COR's for reinforced concrete slender beams with effective depth $d < 300$mm and with effective depth $d \geq 300$mm. For reinforced concrete slender beams with effective depth $d < 300$mm, empirical equations proposed by Zsutty's [20] Eq. 18, Kaiss et al [15] Eq.21, Karim et al [11] Eq. 7 and Shuaib et al [17] Eq. 18 have higher values of COR's, which are 0.901, 0.896 and 0.896 respectively as compared to other proposed equations (Table 6). For 355 reinforced concrete slender beams with an effective depth $d \geq 300$mm, empirical equation proposed by Shuaib et al (1986) and Gastebled et al have the higher values of COR's, which are 0.967 and 0.965 respectively as compared to other proposed equations (Table 6). It is important to note that empirical equations proposed by Shuaib et al [17] Eq. 18 and Gastebled et al [10] Eq. 20 use the size effect and depth factor variable in their expressions to reflect the shear capacity of reinforced concrete slender beams.

Table 6. Summary of results for the Average Margin of Safety and the size effect with respective Coefficient of correlation (COR), using the empirical equations of empirical equations proposed in literature for both normal and high strength reinforced concrete slender beams.

Author	Size Effect			
	No. of Beams used for Evaluation	d<300mm d≥300mm	Average Margin of Safety $\left(\frac{v_{exp}}{v_{pre}}\right)$	COR
Gastebled , May	1402	1046	1.21	0.869
		356	1.15	0.965
Shuaib et al	1401	1046	1.51	0.896
		355	1.35	0.967
Kaiss et al	1402	1046	1.55	0.896
		356	1.17	0.948
Daejoong Kim et al	1402	1046	0.77	0.868
		356	0.65	0.949
Sarkar et al	1402	1046	1.24	0.857
		356	0.98	0.948
Zsutty's	1402	1046	1.12	0.901
		356	0.84	0.863
ASCE-ACI -426	1402	1046	1.03	0.760
		356	0.91	0.951
Karim et al	1402	1046	0.98	0.896
		356	0.74	0.947
K.N.Smith et al(B)	1402	1046	1.43	0.892
		356	0.95	0.922
K.N.Smith et al(A)	1402	1046	1.07	0.884
		356	0.68	0.916
Crist	1402	1046	1.23	0.884
		356	0.78	0.916
Daiz De Cossio, Seiss	1402	1046	1.21	0.890
		356	0.79	0.890
Okamura et al	1402	1046	2.83	0.790
		356	2.22	0.879

4. Proposed Empirical Equation

On the basis of shear data base of the experimental test results [3], an empirical equation is developed for predicting the shear capacity of reinforced concrete beams having shear span to depth ratio a/d≥ 2.5.

For d < 300 mm and $\frac{a}{d} \geq 2.5$

$$\vartheta = 0.35 \left(\frac{a}{d} f'c\right)^{0.33} \rho^{0.1} \tag{24}$$

For d ≥ 300 mm and a/d ≥2.5

$$\vartheta = \xi\ 0.35 \left(\frac{a}{d} f'c\right)^{0.33} \rho^{0.1} \tag{25}$$

where

$$\xi = \frac{17.32}{\sqrt{d}}$$

The proposed empirical equations (Eq. 24 and Eq. 25), contains basic parameters i.e. concrete compressive strength f'c , shear span to depth ratio a/d and ratio of longitudinal reinforcement ρ. In addition to these basic parameters, proposed equation also uses depth factor ξ to account the effect of size effect on the shear capacity of reinforced concrete beams without web reinforcement. In order to assess the predictive accuracy of proposed empirical equations (Eq. 24 and Eq. 25), test results of 1085 reinforced concrete beams without web reinforcement from ACCESS shear database (Rafeeqi et al 2011) were used. The COR for Eq. 24 comes out to be 0.869. For the predictive accuracy of Eq. 25 test results of 393 reinforced concrete beams without web reinforcement from ACCESS shear database [3] were used. The COR for Eq. 25 comes out to be 0.953. Although the COR of the proposed empirical equation is less as compared to equations of Shuaib et al [17] Eq. 18 and Gastebled et al [10] Eq. 20, the applicability of the proposed equation is over a larger number of beams (393) as opposed to 355 beams for equations proposed by Shuaib et al [17] Eq. 18 and Gastebled et al [10] Eq. 20.

5. Summary and Conclusions

From the evaluation study of design equations used in different codes of practice to predict the shear capacity of reinforced concrete slender beams, the following conclusions can be drawn;

1) The predictions of CEB-FIP Model Code and Espanish EHE-99 are much more conservative as compared to ACI Code, Euro code EC2, New Zealand Code and Canadian Code.
2) For NSC and HSC beams, the predictive accuracy of Euro code EC2, ACI Code, Canadian Code and New Zealand Code is comparable.
3) The (f'c)½ function used in the ACI Code, Canadian Code and New Zealand Code seems to be inadequate to reflect the effect of the f'c on the shear capacity for higher strength concrete.
4) The (f'c)⅓ function used in Euro code EC2 seems to be better in reflecting the effect of the f'c on the shear capacity.
5) The issue of size effect is addressed in equations of Euro code EC2 and Canadian Code. The equations used by, Euro code EC2 and Canadian Code have higher values of coefficient of correlation (COR) for beams with effective depth d ≥ 300mm as compared to ACI Code and New Zealand Code equations which do not have a depth factor in their respective expressions.

From the evaluation study of proposed empirical equations published in the literature following conclusions can be drawn;

1) the shear capacity of reinforced concrete slender beams, considering all beams (NSC as well as HSC beams) with d < 300 mm as well as with d < 300 mm, overall the empirical equation proposed by Shuaib et al provides the highest COR and thus is considered to be the superior to the other equations.
2) An empirical equation is developed that is applicable to test data of 393 beams. For beams with effective depth d < 300 mm, the COR for the proposed equation is 0.869 and for beams having effective depth d ≥ 300 mm, the COR for the proposed equation is 0.953.

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