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Calculation of Deuteron Magnetic Dipole Moment Using Constituent Quarks for All Possible, Δ^{++}, Δ^{+}, Δ^{0}, Δ^{-}, *p* and *n* Baryon Formations

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Abstract In the quark like model, deuteron is considered as a system of six quarks clustering into two baryons out of several possible baryon states. The deuteron wave function is written in terms of all possible baryon combinations and the quark constituent of each baryon. In addition to (p, n) pair the (Δ^{++},Δ^{-}), (Δ^{+},Δ^{0}), (Δ^{+},n), (p,Δ^{0}) pairs are also considered. The expectation value of the deuteron magnetic dipole moment is calculated and the value $\mu_D = 0.8554514\mu_N$ is obtained. This finding is in a better agreement with the experimental value as compared to the value given in the shell model.

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1 Introduction

According to the present quark model, the hadrons are made of quarks with three generations and six flavors. Each quark has its own spin, charge, bare mass and different effective masses in baryons and mesons [1]. In the shell model, the magnetic dipole moment of the deuteron μ_D is calculated in the ground state $(l=0)$, by using the proton and neutron spin g-factors namely, $g_{sp} = 5.5856912$ and $g_{sn} = -3.8260837$ [2]. In the shell model, the obtained value of μ_D is given as $\mu_D = 0.879804\mu_N$ where μ_N stands for the nuclear magenton [3]. The experimental measured value of μ_D is given as: $\mu_D^{\exp} = 0.8574376\mu_N$ [3-5]. In order to reduce the difference between the measured and calculated values, it was assumed that the deuteron ground state is a combination of *l=0* and *l=2* states [3].To improve the calculated value of magnetic dipole moment of deuteron, the spin-orbit coupling is also considered in the nuclear Shell model [6-8]. By using the same spin g-factor for the proton and neutron, the magnetic dipole moments of other nuclei were also determined and resulted in a larger difference between the measured and calculated values. In order to achieve a better match, it was assumed that the spin g-factors of proton and neutron are different from the bound proton and neutron and their approximation relation is, $g_s^{bound} \cong 0.6 g_s^{free}$ [9].

In this work a different approach is chosen and the deuteron constituent quarks are considered. In this quark model, the constituent quarks are not completely bounded and continuously participate in the process of baryon creation and annihilation. The two baryons inside deuteron do not have to be only proton and neutron, they can be one of these pairs, and (p,n). Although the masses of the Delta baryons are greater than the nucleon mass, but by taking into account the Heisenberg uncertainty principle between energy and time, it is possible for deuteron to be made of two Delta baryons for very short time. It is noticeable that the Delta baryons decay in strong mode and their life - times are about $0.6\times10^{-23}s$ [1], therefore, the mass change of the deuteron can be few hundreds of Mev. For only (p,n) pair formation, the calculations are recently published [10]. Based upon this quark model of nuclei, all magic numbers are determined [11]. In this model, the nuclear binding energy is also given in a simpler expression [12].

2 Determination of Deuteron Wave Function

Based upon the assumption that deuteron is made of 3 up and 3 down quarks, clustering continuously into a pair of baryons, the deuteron wave function is obtained. The following pairs are considered, (Δ^{++},Δ^{-}), (Δ^{+},Δ^{0}), (Δ^{+},n), (p,Δ^{0}) and (p,n). The total deuteron wave function is written as:

$$\psi(D) = c(\Delta^{++},\Delta^{-})\psi(\Delta^{++},\Delta^{-}) + c(\Delta^{+},\Delta^{0})\psi(\Delta^{+},\Delta^{0}) + c(\Delta^{+},n)\psi(\Delta^{+},n) \\ +c(p,\Delta^{0})\psi(p,\Delta^{0}) + c(p,n)\psi(p,n) \tag{1}$$

In Eq. (1), the square of each coefficient represents the probability of each baryon pair formation. These coefficients are determined later. Each term of Eq. (1) consists of four independent parts namely, spin, flavor, color and space, therefore [1, 10]:

$$\psi = \psi(space)\psi(spin)\psi(flavor)\psi(color) \tag{2}$$

The total wave function is anti-symmetric (under exchange of two baryons in nuclei or two quarks in baryon). Although the functional dependency of the ground state of the space part is not known but it is symmetric due to the fact that $l=0$ for all quarks. The three colors are generators of SU(3) color symmetry and three colors together makes one decuplet and two octet and one color singlet namely,

$$3\otimes 3\otimes 3 = 10\oplus 8\oplus 8\oplus 1 \tag{3}$$

Naturally made particles are colorless and color singlet and in SU(3) color singlet is anti-symmetric [13]. Therefore each term of the deuteron wave function is written in terms of baryon wave functions as follow:

$$\psi = \psi(space)\left(|b_1\rangle|b_2\rangle - |b_2\rangle|b_1\rangle\right)/\sqrt{2} \tag{4}$$

Where, $|b_1\rangle$ and $|b_2\rangle$ are wave functions of the two baryon states which form the deuteron.

The total wave function of the fermions is anti-symmetric. The spin part is obtained from Clebsch – Gordan's method as follow [14]:

$$\left.\begin{array}{c} |\tfrac{3}{2},\tfrac{3}{2}\rangle = (\uparrow\uparrow\uparrow) \\ |\tfrac{3}{2},\tfrac{1}{2}\rangle = (\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow)/\sqrt{3} \\ |\tfrac{3}{2},\tfrac{-1}{2}\rangle = (\uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow + \downarrow\downarrow\uparrow)/\sqrt{3} \\ |\tfrac{3}{2},\tfrac{-1}{2}\rangle = (\downarrow\downarrow\downarrow) \end{array}\right\} \text{for spin } \tfrac{3}{2},\ (\psi_s) \tag{5}$$

$$\left.\begin{array}{c} |\tfrac{1}{2},\tfrac{1}{2}\rangle_{12} = (\uparrow\downarrow - \downarrow\uparrow)\uparrow/\sqrt{2} \\ |\tfrac{1}{2},\tfrac{-1}{2}\rangle_{12} = (\uparrow\downarrow - \downarrow\uparrow)\downarrow/\sqrt{2} \end{array}\right\} \text{for spin } \tfrac{1}{2},\ (\psi_{12}) \tag{6}$$

$$\left.\begin{array}{c} |\tfrac{1}{2},\tfrac{1}{2}\rangle_{23} = \uparrow(\uparrow\downarrow - \downarrow\uparrow)/\sqrt{2} \\ |\tfrac{1}{2},\tfrac{-1}{2}\rangle_{23} = \uparrow(\uparrow\downarrow - \downarrow\uparrow)/\sqrt{2} \end{array}\right\} \text{for spin } \tfrac{1}{2},\ (\psi_{23}) \tag{7}$$

All members of the spin 3/2 state are symmetric and for 1/2 state are partially anti-symmetric and change sign under exchange of two particles. There are also the following combinations, made from particles 1 and 3:

$$\left.\begin{array}{c} |\tfrac{1}{2},\tfrac{1}{2}\rangle_{13} = (\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow)/\sqrt{2} \\ |\tfrac{1}{2},\tfrac{-1}{2}\rangle_{13} = (\uparrow\downarrow\downarrow - \downarrow\downarrow\uparrow)/\sqrt{2} \end{array}\right\} \text{for spin } \tfrac{1}{2},\ (\psi_{13}) \tag{8}$$

which are not independent and can be constructed from the two previous combinations namely,

$$|\ \rangle_{13} = |\ \rangle_{12} + |\ \rangle_{23} \tag{9}$$

The flavor part of the baryons, $\psi(flavor)$ is similar to the spin part due to the fact that these baryons are made of 2 kinds of quarks namely, u and d and belong to the SU(2) group. In fact by substituting ↑ spin for *u* and ↓ spin for *d*, the flavor part is studied. As stated before, the color function is anti-symmetric for baryons and the space function is symmetric, therefore $\psi(flavor)\psi(spin)$ must be symmetric. Considering the fact that deuteron's spin is equal to 1 and the Z-component of spin is -1, 0, 1, we can count the total number of all possible baryon pairs given in Eq. (1), forming the deuteron. For example the possibilities in the case of (Δ^{++},Δ^{-}) pair in formation of the deuteron, are obtained in the appendix. The total number of possible states in this case equals to 50.

The same is for the other pairs involved. The total numbers of possible states are counted and then those states with spin other than one are eliminated due to the fact that, deuteron has spin 1. The obtained results are summarized in table 1.

Table 1. The number of possible ways that deutron can be formed by each baryon pair

Baryon pair	Δ^{++},Δ^{-}	Δ^{+},Δ^{0}	Δ^{+},n	p,Δ^{0}	p,n
Number of states	15	45	270	270	432

As a result of the precise counting of the number of states, the coefficients introduced in Eq. (1) are determined as follow:

$$c(\Delta^{++},\Delta^{-}) = (5/344)^{\frac{1}{2}},\ c(\Delta^{+},\Delta^{0}) = (15/344)^{\frac{1}{2}},\ c(\Delta^{+},n) = (90/344)^{\frac{1}{2}}$$
$$c(p,\Delta^{0}) = (90/344)^{\frac{1}{2}},\ c(p,n) = (144/344)^{\frac{1}{2}} \tag{10}$$

Since Δ^{++}, Δ^{+}, Δ^{0} and Δ^{-} all have spin equal to $3/2$, and also their spin wave function are symmetric under quark exchange, therefore their flavor part is also symmetric. But the spin wave function of proton and neutron possess mixed symmetry, because of their spin $1/2$ properties. As a result, their flavor part must also have mixed symmetry. So, the multiplication of their spin and flavor wave function become symmetric. Therefore, the nucleon wave function is as follow:

$$\psi(nucleon) = \frac{\sqrt{3}}{3}[\psi_{12}(flavor)\psi_{12}(spin) + \psi_{23}(flavor)\psi_{23}(spin) + \psi_{13}(flavor)\psi_{13}(spin)] \tag{11}$$

For example, the multiplication of the flavor and spin parts of the proton is written as in Eq. (12).

$$|\,p:\tfrac{1}{2},\tfrac{1}{2}>=\{\tfrac{1}{2}(\uparrow\downarrow\uparrow-\downarrow\uparrow\uparrow)(udu-duu)+\tfrac{1}{2}(\uparrow\uparrow\downarrow-\uparrow\downarrow\uparrow)(uud-udu)+ \tfrac{1}{2}(\uparrow\uparrow\downarrow-\downarrow\uparrow\uparrow)(uud-duu)\}\frac{\sqrt{3}}{2}=\frac{2}{3\sqrt{2}}(u(\uparrow)u(\uparrow)d(\downarrow)-\frac{1}{3\sqrt{2}}(u(\uparrow)u(\downarrow)d(\uparrow)- \frac{1}{3\sqrt{2}}(u(\downarrow)u(\uparrow)d(\uparrow)+\ldots \tag{12}$$

Similarly, neutron wave function is obtained by exchange of $u \leftrightarrow d$ in the proton wave function. Now by substituting Eq. (10) into Eq. (1), the total deuteron wave function is written as:

$$\psi(D)=\frac{1}{\sqrt{2}}\{\sqrt{\tfrac{5}{344}}(|\,\Delta^{++}>|\,\Delta^{-}>-|\,\Delta^{-}>|\,\Delta^{++}>)+\sqrt{\tfrac{15}{344}}(|\,\Delta^{+}>|\,\Delta^{0}>-|\,\Delta^{0}>|\,\Delta^{+}>) +\sqrt{\tfrac{90}{344}}(|\,\Delta^{+}>|\,n>-|\,n>|\,\Delta^{+}>)+\sqrt{\tfrac{90}{344}}(|\,p>|\,\Delta^{0}>-|\,\Delta^{0}>|\,p>)+\sqrt{\tfrac{144}{344}}(|\,p>|\,n>-|\,n>|\,p>)\} \tag{13}$$

For example, based upon the constituent quark's spin, the first term in Eq. (13) is expanded as follow:

$$|\,\psi(\Delta^{++},\Delta^{-}):1,1>=\sqrt{\tfrac{3}{10}}\left\{\tfrac{1}{\sqrt{2}}(|\,\Delta^{++}:\tfrac{3}{2}\tfrac{3}{2}>|\,\Delta^{-}:\tfrac{3}{2}\tfrac{-1}{2}>-|\,\Delta^{-}:\tfrac{3}{2}\tfrac{-1}{2}>|\,\Delta^{++}:\tfrac{3}{2}\tfrac{3}{2}>)\right\} +\sqrt{\tfrac{3}{10}}\left\{\tfrac{1}{\sqrt{2}}(|\,\Delta^{++}:\tfrac{3}{2}\tfrac{-1}{2}>|\,\Delta^{-}:\tfrac{3}{2}\tfrac{3}{2}>-|\,\Delta^{-}:\tfrac{3}{2}\tfrac{3}{2}>|\,\Delta^{++}:\tfrac{3}{2}\tfrac{-1}{2}>)\right\} -\sqrt{\tfrac{4}{10}}\left\{\tfrac{1}{\sqrt{2}}(|\,\Delta^{++}:\tfrac{3}{2}\tfrac{1}{2}>|\,\Delta^{-}:\tfrac{3}{2}\tfrac{1}{2}>-|\,\Delta^{-}:\tfrac{3}{2}\tfrac{1}{2}>|\,\Delta^{++}:\tfrac{3}{2}\tfrac{1}{2}>)\right\} \tag{14}$$

In Eq. (14) the coefficients before each terms are the Clebsch-Gordon coefficients. By considering the constituent quark's flavor in addition to the spin, the first term of the total deuteron wave function, which is given in Eq.(14), is expanded and given in the appendix. For other baryon pairs, the same procedure is considered for the wave function of deuteron and the total wave function is obtained.

3 Determination of the Deuteron Magnetic Dipole Moment

In the absence of the orbital motion, the magnetic dipole moment of deuteron is simply the vector sum of the six constituent quarks namely,

$$\mu = \mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5 + \mu_6 \tag{15}$$

The terms of this equation depend upon the flavor of quarks (since up and down quarks have different magnetic dipole moments) and also depend upon the spin direction of six quarks.

The magnetic dipole moment of spin 1/2 particle with mass m and charge q is given [1, 3] as

$$\mu = \frac{q}{mc} S \tag{16}$$

Substituting for spin 1/2, (all of the quarks have spin1/2) therefore:

$$\mu = \frac{q\hbar}{2mc} S \tag{17}$$

Now for up and down quarks μ_u and μ_d are obtained as:

$$\mu_u = \frac{2}{3}\frac{e\hbar}{2m_u c}, \qquad \mu_d = \frac{-1}{3}\frac{e\hbar}{2m_d c} \tag{18}$$

Therefore the deuteron magnetic dipole moment is given as:

$$\begin{aligned} \mu_D &= < D:1,1 \,|\, \mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5 + \mu_6 \,|\, D:1,1 > \\ &- \tfrac{2}{\hbar}\sum_{i=1}^{6} < D:1,1 \,|\, \mu_i s_{iz} \,|\, D:1,1 > \end{aligned} \tag{19}$$

The deuteron wave function is given in section 2. The first term of this wave function is $\sqrt{\frac{5}{344}}\sqrt{\frac{1}{20}}[uuuddd(\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow)]$ therefore,

$$\begin{aligned} &\tfrac{2}{\hbar}(\textstyle\sum_{i=1}^{6} \mu_i s_{iz})[uuudddu(\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow)] \\ &= (\mu_u + \mu_u + \mu_u + \mu_d - \mu_d - \mu_d)[uudddu(\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow)] \\ &= (3\mu_u^{\Delta} - \mu_d^{\Delta})[uuuddd(\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow)] \end{aligned} \tag{20}$$

And the expectation value for the magnetic dipole moment of this first term is,

$$(\sqrt{\tfrac{1}{1376}})^2 \tfrac{2}{\hbar}(\textstyle\sum_{i=1}^{6} < uuuddd(\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow) \,|\, \mu_i s_{iz} \,|\, uuuddd(\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow) >) = \tfrac{1}{1376}(9\mu_u^{\Delta} - \mu_d^{\Delta}) \tag{21}$$

Similar calculations are carried out for each remaining terms of the deuteron wave function and finally the magnetic dipole moment of deuteron is obtained as follow:

$$\mu_D = < D:1,1 \,|\, \mu \,|\, D:1,1 > = \tfrac{174}{344}(\mu_u^p + \mu_d^p) + \tfrac{140}{344}(\mu_u^{\Delta} + \mu_d^{\Delta}) = 0.8554514\mu_N \tag{22}$$

where

$$\mu_u^p = \frac{2}{3}\frac{e\hbar}{2m_u^p c} = \frac{2}{3}\frac{m_p}{m_u^p}\frac{e\hbar}{2m_p c} = \frac{2m_p}{3m_u^p}\mu_N = 1.7231956\mu_N \tag{23}$$

$$\mu_u^{\Delta} = \frac{2}{3}\frac{e\hbar}{2m_u^{\Delta} c} = \frac{2}{3}\frac{m_p}{m_u^{\Delta}}\frac{e\hbar}{2m_p c} = \frac{2m_p}{3m_u^{\Delta}}\mu_N = 1.5219465\mu_N \tag{24}$$

$$\mu_d^p = \frac{-1}{3}\frac{e\hbar}{2m_d^p c} = \frac{-m_p}{3m_d^p}\mu_N = -0.8615978\mu_N \tag{25}$$

$$\mu_d^{\Delta} = \frac{-1}{3}\frac{e\hbar}{2m_d^{\Delta}c} = \frac{-m_p}{3m_d^{\Delta}}\mu_N = -.7609732\mu_N \tag{26}$$

In equations (22) to (26) μ_N stands for nuclear magenton, $m_p = 938.280\frac{mev}{c^2}$, $m_u^{\Delta} = m_d^{\Delta} = 411\frac{mev}{c^2}$ and $m_u^p = m_d^p = m_u^n = m_d^n = 330\frac{mev}{c^2}$ [1,12].

4. Conclusion

The existing deviation of the previously found theoretical and experimental values of the magnetic dipole moment of deuteron has been improved. The experimental value for μ_D is $0.8574376\mu_N$. In the shell model, for $l = 0$ the value of magnetic dipole moment of deuteron is $\mu_D = 0.8748046\mu_N$ whereas our finding for μ_D is $0.8554514\mu_N$ which is in a better agreement with the experimental measurement.

Appendix

A - The possibilities in the case of $(\Delta^{++}, \Delta^{-})$ pair in formation of the deuteron

$|\Delta^{++}:\frac{3}{2},\frac{3}{2}>|\Delta^{-}:\frac{3}{2},\frac{-1}{2}>=(uuu)(\uparrow\uparrow\uparrow)\times\frac{1}{\sqrt{3}}(ddd)(\uparrow\downarrow\downarrow+\downarrow\uparrow\downarrow+\downarrow\downarrow\uparrow)$ 3 possible states

$|\Delta^{++}:\frac{3}{2},\frac{3}{2}>|\Delta^{-}:\frac{3}{2},\frac{-3}{2}>=(uuu)(\uparrow\uparrow\uparrow)\times(ddd)(\downarrow\downarrow\downarrow)$ only one possible state

$|\Delta^{++}:\frac{3}{2},\frac{1}{2}>|\Delta^{-}:\frac{3}{2},\frac{1}{2}>=\frac{1}{\sqrt{3}}(uuu)(\uparrow\uparrow\downarrow+\uparrow\downarrow\uparrow+\downarrow\uparrow\uparrow)\times\frac{1}{\sqrt{3}}(ddd)(\uparrow\uparrow\downarrow+\uparrow\downarrow\uparrow+\downarrow\uparrow\uparrow)$ 9 possible states

$|\Delta^{++}:\frac{3}{2},\frac{1}{2}>|\Delta^{-}:\frac{3}{2},\frac{-1}{2}>=\frac{1}{\sqrt{3}}(uuu)(\uparrow\uparrow\downarrow+\uparrow\downarrow\uparrow+\downarrow\uparrow\uparrow)\times\frac{1}{\sqrt{3}}(ddd)(\uparrow\downarrow\downarrow+\downarrow\uparrow\downarrow+\downarrow\downarrow\uparrow)$ 9 possible states

$|\Delta^{++}:\frac{3}{2},\frac{1}{2}>|\Delta^{-}:\frac{3}{2},\frac{-3}{2}>=\frac{1}{\sqrt{3}}(uuu)(\uparrow\uparrow\downarrow+\uparrow\downarrow\uparrow+\downarrow\uparrow\uparrow)\times(ddd)(\downarrow\downarrow\downarrow)$ 3 possible states

$|\Delta^{++}:\frac{3}{2},\frac{-1}{2}>|\Delta^{-}:\frac{3}{2},\frac{3}{2}>=\frac{1}{\sqrt{3}}(uuu)(\uparrow\downarrow\downarrow+\downarrow\uparrow\downarrow+\downarrow\downarrow\uparrow)\times(ddd)(\uparrow\uparrow\uparrow)$ 3 possible states

$|\Delta^{++}:\frac{3}{2},\frac{-1}{2}>|\Delta^{-}:\frac{3}{2},\frac{1}{2}>=\frac{1}{\sqrt{3}}(uuu)(\uparrow\downarrow\downarrow+\downarrow\uparrow\downarrow+\downarrow\downarrow\uparrow)\times\frac{1}{\sqrt{3}}(ddd)(\uparrow\uparrow\downarrow+\uparrow\downarrow\uparrow+\downarrow\uparrow\uparrow)$ 9 possible states

$|\Delta^{++}:\frac{3}{2},\frac{-1}{2}>|\Delta^{-}:\frac{3}{2},\frac{-1}{2}>=\frac{1}{\sqrt{3}}(uuu)(\uparrow\downarrow\downarrow+\downarrow\uparrow\downarrow+\downarrow\downarrow\uparrow)\times\frac{1}{\sqrt{3}}(ddd)(\uparrow\downarrow\downarrow+\downarrow\uparrow\downarrow+\downarrow\downarrow\uparrow)$ 9 possible states

$|\Delta^{++}:\frac{3}{2},\frac{-3}{2}>|\Delta^{-}:\frac{3}{2},\frac{3}{2}>=(uuu)(\downarrow\downarrow\downarrow)\times(ddd)(\uparrow\uparrow\uparrow)$ only one possible state

$|\Delta^{++}:\frac{3}{2},\frac{-3}{2}>|\Delta^{-}:\frac{3}{2},\frac{1}{2}>=(uuu)(\downarrow\downarrow\downarrow)\times\frac{1}{\sqrt{3}}(ddd)(\uparrow\uparrow\downarrow+\uparrow\downarrow\uparrow+\downarrow\uparrow\uparrow)$ 3 possible states

B - The flavor and spin part of the wave function of the first term of the deuteron based upon its quarks content

$$|\psi(\Delta^{++},\Delta^{-}):1,1>=\frac{1}{\sqrt{20}}\{uuuddd(\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow)+uuuddd(\uparrow\uparrow\uparrow\downarrow\uparrow\downarrow)+uuuddd(\uparrow\uparrow\uparrow\downarrow\downarrow\uparrow)$$
$$-ddduuu(\uparrow\downarrow\downarrow\uparrow\uparrow\uparrow)-ddduuu(\downarrow\uparrow\downarrow\uparrow\uparrow\uparrow)-ddduuu(\downarrow\downarrow\uparrow\uparrow\uparrow\uparrow)\}$$
$$+\frac{1}{\sqrt{20}}\{uuuddd(\uparrow\downarrow\downarrow\uparrow\uparrow\uparrow)+uuuddd(\downarrow\uparrow\downarrow\uparrow\uparrow\uparrow)+uuuddd(\downarrow\downarrow\uparrow\uparrow\uparrow\uparrow)-ddduuu(\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow)$$
$$-ddduuu(\uparrow\uparrow\uparrow\downarrow\uparrow\downarrow)-ddduuu(\uparrow\uparrow\uparrow\downarrow\downarrow\uparrow)\}$$
$$-\sqrt{\tfrac{2}{90}}\{uuuddd(\uparrow\uparrow\downarrow\uparrow\uparrow\downarrow)+uuuddd(\uparrow\uparrow\downarrow\uparrow\downarrow\uparrow)+uuuddd(\uparrow\uparrow\downarrow\downarrow\uparrow\uparrow)+uuuddd(\uparrow\downarrow\uparrow\uparrow\uparrow\downarrow)$$
$$+uuuddd(\uparrow\downarrow\uparrow\uparrow\downarrow\uparrow)+uuuddd(\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow)+uuuddd(\downarrow\uparrow\uparrow\uparrow\uparrow\downarrow)+uuuddd(\downarrow\uparrow\uparrow\uparrow\downarrow\uparrow)$$
$$+uuuddd(\downarrow\uparrow\uparrow\downarrow\uparrow\uparrow)-ddduuu(\uparrow\uparrow\downarrow\uparrow\uparrow\downarrow)-ddduuu(\uparrow\uparrow\downarrow\uparrow\downarrow\uparrow)-ddduuu(\uparrow\uparrow\downarrow\downarrow\uparrow\uparrow)$$
$$-ddduuu(\uparrow\downarrow\uparrow\uparrow\uparrow\downarrow)-ddduuu(\uparrow\downarrow\uparrow\uparrow\downarrow\uparrow)-ddduuu(\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow)-ddduuu(\downarrow\uparrow\uparrow\uparrow\uparrow\downarrow)$$
$$-ddduuu(\downarrow\uparrow\uparrow\uparrow\downarrow\uparrow)-ddduuu(\downarrow\uparrow\uparrow\downarrow\uparrow\uparrow)\}$$

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