Advances in Signal Processing 1(3): 44-47, 2013 http://www.hrpub.org
DOI: 10.13189/asp.2013.010302

Against Phase Velocities of Elastic Waves in Thin Orthotropic Cylindrical Shell

A.A. Kleshchev

St. Petersburg State Marine Technical University, st. Lotsmanskaya 3, St. Petersburg, 190008, Russia
*Corresponding Author: alexalex-2@yandex.ru

Abstract In paper is received the characteristic equation for the determine of wave numbers of phase velocities of elastic waves in the thin cylindrical shell with the help of the dynamic theory of the elasticity for the orthotropic medium and of the hypothesis of thin shells.

Keywords Theory of Elasticity, Phase Velocity, Wave Number, Characteristic Equation, Boundary Conditions

1. Introduction

Based at the use of the dynamic theory of the elasticity for the anisotropic medium and with the help of the hypothesis of thin shells is determined the characteristic equation for wave numbers of elastic waves in the thin orthotropic cylindrical shell.

2. The Dynamic Theory of the Elasticity for the Orthotropic Medium

Let's consider the infinite thin orthotropic cylindrical shell. The harmonic elastic wave is spread along axis Z, that is the axis of the symmetry of the second order. The orthotropic elastic medium is characterized by the nine elastic moduluses [1 – 4]:

$$A_{11}, A_{12}, A_{13}, A_{22}, A_{23}, A_{33}, A_{44}, A_{55}, A_{66}.$$

Components of the displacement vector $\vec{U}(U_r, U_\varphi, U_z)$ can be presented in the form of the serieses [2 – 4]:

$$U_r = \exp(i \bullet k \bullet z) \bullet \sum_{m=0}^{\infty} \cos(m \bullet \varphi) \bullet U_m(r);$$

$$U_\varphi = \exp(i \bullet k \bullet z) \bullet \sum_{m=1}^{\infty} \sin(m \bullet \varphi) \bullet V_m(r); \qquad (1)$$

$$U_z = \exp(i \bullet k \bullet z) \bullet \sum_{m=0}^{\infty} \cos(m \bullet \varphi) \bullet W_m(r),$$

where k is the wave number of the elastic wave [2 – 4]. Equations of the dynamic balance in displacements are [2 – 4]:

$$\frac{\partial^2 U_r}{\partial r^2} + \frac{a_1}{r} \bullet \frac{\partial^2 U_\varphi}{\partial \varphi \partial r} + a_2 \bullet \frac{\partial^2 U_z}{\partial z \partial r} +$$
$$\frac{a_3}{r} \bullet \frac{\partial^2 U_\varphi}{\partial \varphi \partial r} - \frac{a_3}{r} \bullet \frac{\partial U_\varphi}{\partial \varphi} + \frac{a_3}{r^2} \bullet \frac{\partial^2 U_r}{\partial^2 \varphi} +$$
$$a_4 \bullet \frac{\partial^2 U_r}{\partial z^2} + a_4 \bullet \frac{\partial^2 U_z}{\partial r \partial z} + \frac{1}{r} \bullet \frac{\partial U_r}{\partial r} +$$
$$+ \frac{a_2}{r} \bullet \frac{\partial U_z}{\partial r} - \frac{a_5}{r^2} \bullet U_r - \frac{a_6}{r} \bullet \frac{\partial U_z}{\partial z} + a_7 \bullet U_r = 0, \qquad (2)$$

where

$$a_1 = \frac{A_{12}}{A_{11}}; a_2 = \frac{A_{13}}{A_{11}}; a_3 = \frac{A_{66}}{A_{11}}; a_4 = \frac{A_{55}}{A_{11}}; a_5 = \frac{A_{22}}{A_{11}};$$

$a_6 = \frac{A_{23}}{A_{11}}; a_7 = \frac{\rho\omega^2}{A_{11}}; \rho -$ the density of the ma-terial of the shell; $\omega -$ the angular frequency.

$$\frac{\partial^2 U_\varphi}{\partial r^2} + \frac{1}{r} \bullet \frac{\partial^2 U_r}{\partial \varphi \partial r} + \frac{a_8}{r} \bullet \frac{\partial^2 U_r}{\partial r \partial \varphi} + \frac{a_9}{r} \bullet \frac{\partial^2 U_\varphi}{\partial \varphi^2} +$$
$$\frac{a_9}{r} \bullet \frac{\partial U_r}{\partial \varphi} + \frac{a_{10}}{r} \bullet \frac{\partial^2 U_z}{\partial z \partial \varphi} + \frac{a_{11}}{r} \bullet \frac{\partial^2 U_z}{\partial \varphi \partial z} +$$
$$a_{11} \bullet \frac{\partial^2 U_\varphi}{\partial z^2} + \frac{1}{r} \bullet \frac{\partial U_\varphi}{\partial r} -$$
$$- \frac{1}{r^2} \bullet U_\varphi + \frac{1}{r^2} \bullet \frac{\partial U_r}{\partial \varphi} + a_{12} \bullet U_\varphi = 0, \qquad (3)$$

Where

$$a_8 = \frac{A_{12}}{A_{66}}; a_9 = \frac{A_{22}}{A_{66}}; a_{10} = \frac{A_{23}}{A_{66}}; a_{11} = \frac{A_{44}}{A_{66}}; a_{12} = \frac{\rho\omega^2}{A_{66}}.$$

$$\frac{\partial^2 U_r}{\partial z \partial r} + \frac{\partial^2 U_z}{\partial r^2} + \frac{a_{13}}{r^2} \bullet \frac{\partial^2 U_z}{\partial \varphi^2} + \frac{a_{13}}{r} \bullet \frac{\partial^2 U_\varphi}{\partial z \partial \varphi} +$$
$$a_{14} \bullet \frac{\partial^2 U_r}{\partial r \partial z} + \frac{a_{15}}{r} \bullet \frac{\partial^2 U_\varphi}{\partial \varphi \partial z} + \frac{(a_{15}+1)}{r} \bullet \frac{\partial U_r}{\partial z} + \quad (4)$$
$$\frac{1}{r} \bullet \frac{\partial U_z}{\partial r} + a_{16} \bullet \frac{\partial^2 U_z}{\partial z^2} + a_{17} \bullet U_z = 0,$$

where

$$a_{13} = \frac{A_{44}}{A_{55}}; a_{14} = \frac{A_{13}}{A_{55}}; a_{15} = \frac{A_{23}}{A_{55}}; a_{16} = \frac{A_{33}}{A_{55}}; a_{17} = \frac{\rho\omega^2}{A_{55}}.$$

Now if components of the displacement vector $\overrightarrow{U}$ taken from (1) substitute in (2) – (4), then we receive fol-lowing equations for radial functions $U_m(r), V_m(r), W_m(r)$ [2]:

$$\frac{\partial^2 U_m}{\partial r^2} + \frac{m \bullet (a_1 + a_3)}{r} \bullet \frac{\partial V_m}{\partial r} + (a_2 + a_4) \bullet i \bullet k \bullet \frac{\partial W_m}{\partial r} -$$
$$\frac{m \bullet (a_3 + a_5)}{r^2} \bullet V_m - \frac{a_3}{r^2} \bullet m^2 \bullet V_m - a_4 \bullet k^2 \bullet U_m + \frac{1}{r} \bullet \frac{\partial U_m}{\partial r} +$$
$$+ \frac{(a_2 - a_6)}{r} \bullet i \bullet k \bullet W_m + (a_7 - \frac{a_5}{r^2}) U_m = 0; \quad (5)$$

$$\frac{\partial^2 V_m}{\partial r^2} - \frac{m \bullet (1 + a_8)}{r} \bullet \frac{\partial U_m}{\partial r} - \frac{m^2 \bullet a_9}{r} \bullet V_m -$$
$$\frac{m \bullet a_9}{r} \bullet U_m + \frac{1}{r} \bullet \frac{\partial V_m}{\partial r} - \frac{m \bullet (a_{10} + a_{11})}{r} \bullet i \bullet k \bullet W_m - \quad (6)$$
$$a_{11} \bullet k^2 \bullet V_m + (a_{12} - \frac{1}{r^2}) \bullet V_m - \frac{m}{r^2} \bullet U_m = 0;$$

$$(1 + a_{14}) \bullet i \bullet k \bullet \frac{\partial U_m}{\partial r} + \frac{\partial^2 W_m}{\partial r^2} - \frac{m^2 \bullet a_{13}}{r^2} \bullet W_m +$$
$$m \bullet i \bullet k \bullet \frac{1}{r} \bullet (a_{13} + a_{15}) \bullet V_m + i \bullet k \bullet \frac{(1 + a_{15})}{r} \bullet U_m +$$
$$\frac{1}{r} \bullet \frac{\partial W_m}{\partial r} - k^2 \bullet$$
$$\bullet a_{16} \bullet W_m + a_{17} \bullet W_m = 0. \quad (7)$$

Boundary conditions [normal (σ_r) and tangent $(\tau_{r\varphi}, \tau_{rz})$ stresses are equal zero at external $(r = a)$ and in-ternal $(r = b)$ surfaces of the elastic shell] are added to equations (5) – (7) [2]:

$$\frac{\partial U_m}{\partial r} + \frac{a_1}{r} \bullet m \bullet V_m + \frac{a_1}{r} \bullet U_m + a_2 \bullet i \bullet k \bullet W_m = 0;$$
$$[r=a;r=b] \quad (8)$$

$$\frac{\partial V_m}{\partial r} - \frac{1}{r} \bullet V_m - \frac{m}{r} \bullet U_m = 0; \quad [r=a;r=b] \quad (9)$$

$$i \bullet k \bullet U_m + \frac{\partial W_m}{\partial r} = 0; \quad [r=a;r=b] \quad (10)$$

3. Hypothesis of Thin Shells

The fellow parameter $\xi = \frac{z}{R_0}$ can be used for thin shells, where $R_0 = \frac{a+b}{2}$ is the middle radius and $z = r - R_0$ is the coordinate taking from the middle surface [2 – 6]:

$$U_m(r) = \sum_{n=0}^{N_1} x_n \bullet \xi^n;$$
$$V_m(r) = \sum_{n=0}^{N_1} y_n \bullet \xi^n; \quad (11)$$
$$W_m(r) = \sum_{n=0}^{N_1} z_n \bullet \xi^n.$$

We substitute decompositions in boundary conditions (8) – (10) and receive 6 equations relative to $3 \bullet (N_1 + 1)$ unknown coefficients x_n, y_n, z_n [2]:

$$R_0^{-1}\bullet\sum_{n=0}^{N_1}x_n\bullet n\bullet(\xi_1)^{n-1}+a_1\bullet m\bullet(R_0+0,5\bullet h)^{-1}\bullet\sum_{n=0}^{N_1}y_n\bullet(\xi_1)^n+a_1\bullet(R_0+0,5\bullet h)^{-1}\sum_{n=0}^{N_1}x_n\bullet(\xi_1)^n+a_2\bullet i\bullet k\bullet$$

$$\bullet\sum_{n=0}^{N_1}z_n\bullet(\xi_1)^n=0, \qquad (12)$$

Where

$$\xi_1=\frac{a-R_0}{R_0};$$

$$h=a-b.$$

$$R_0^{-1}\bullet\sum_{n=9}^{N_1}x_n\bullet n\bullet(-\xi_1)^{n-1}+a_1\bullet m\bullet(R_0-0,5\bullet h)^{-1}\bullet\sum_{n=0}^{N_1}y_n\bullet(-\xi_1)^n+a_1\bullet(R_0-0,5\bullet h)^{-1}\sum_{n=0}^{N_1}x_n\bullet(-\xi_1)^n+a_2\bullet i\bullet k\bullet$$

$$\sum_{n=0}^{N_1}z_n\bullet(-\xi_1)^n=0; \qquad (13)$$

$$R_0^{-1}\bullet\sum_{n=0}^{N_1}y_n\bullet n\bullet(\xi_1)^{n-1}-(R_0+0,5\bullet h)^{-1}\bullet\sum_{n=0}^{N_1}y_n\bullet(\xi_1)^n-m\bullet(R_0+0,5\bullet h)^{-1}\bullet\sum_{n=0}^{N_1}x_n\bullet(\xi_1)^n=0; \qquad (14)$$

$$R_0^{-1}\bullet\sum_{n=0}^{N_1}y_n\bullet n\bullet(-\xi_1)^{n-1}-(R_0-0,5\bullet h)^{-1}\bullet\sum_{n=0}^{N_1}y_n\bullet(-\xi_1)^n-m\bullet(R_0-0,5\bullet h)^{-1}\bullet\sum_{n=0}^{N_1}x_n\bullet(-\xi_1)^n=0; \qquad (15)$$

$$i\bullet k\bullet\sum_{n=0}^{N_1}x_n\bullet(\xi_1)^n+R_0^{-1}\bullet\sum_{n=0}^{N_1}z_n\bullet n\bullet(\xi_1)^{n-1}=0; \qquad (16)$$

$$i\bullet k\bullet\sum_{n=0}^{N_1}x_n\bullet(-\xi_1)^n+R_0^{-1}\bullet\sum_{n=0}^{N}z_n\bullet n\bullet(-\xi_1)^{n-1}=0. \qquad (17)$$

The rest cof equations can be received, by substitution of decompositions (11) in equations (5) – (7) and by equated of coefficients at the fellow parameter ξ [2 – 4, 6]:

$$x_{n+2}\bullet(n+2)\bullet(n+1)+x_{n+1}\bullet(n+1)\bullet(2n+1)+x_n\bullet[n^2-m^2\bullet a_3-a_5+R_0^2\bullet(a_7-k^2\bullet a_4)]+x_{n-1}\bullet2\bullet R_0^2\bullet$$
$$\bullet(a_7-k^2\bullet a_4)+x_{n-2}\bullet R_0^2\bullet(a_7-k^2\bullet a_4)+y_{n+1}\bullet(n+1)\bullet m\bullet(a_1+a_3)+y_n\bullet m\bullet[n\bullet(a_1+a_3)-a_3-a_5]+z_{n+1}\bullet$$
$$\bullet i\bullet k\bullet(a_2+a_4)\bullet R_0\bullet(n+1)+z_n\bullet R_0\bullet i\bullet k\bullet[2\bullet n\bullet(a_2+a_4)+a_2-a_6]+z_{n+1}\bullet R_0\bullet i\bullet k\bullet[(n-1)\bullet(a_2+a_4)+a_2-$$
$$-a_6]=0; \qquad (18)$$

$$-x_{n+1}\bullet m\bullet(1+a_8)\bullet(n+1)-x_n\bullet m\bullet[n\bullet(1+a_8)+R_0\bullet a_9+1]-x_{n-1}\bullet R_0\bullet m\bullet a_9+y_{n+2}\bullet(n+2)\bullet(n+1)+y_n\bullet$$
$$\bullet[n^2-R_0\bullet m^2\bullet a_9-1+R_0^2\bullet(a_{12}-k^2\bullet a_{11})]+y_{n-1}\bullet R_0\bullet(2\bullet R_0\bullet a_{12}-m^2\bullet a_9-2\bullet R_0\bullet a_{11}\bullet k^2)-z_n\bullet R_0\bullet i\bullet k\bullet$$
$$\bullet m\bullet(a_{10}+a_{11})-z_{n-1}\bullet R_0\bullet i\bullet k\bullet m\bullet(a_{10}+a_{11})=0; \qquad (19)$$

$$x_{n+1}\bullet i\bullet k\bullet(1+a_{14})\bullet R_0\bullet(n+1)+x_n\bullet i\bullet k\bullet R_0\bullet[2\bullet n\bullet(1+a_{14})+1+a_{15}]+x_{n-1}\bullet i\bullet k\bullet R_0\bullet[(n-1)\bullet(1+a_{14})+$$
$$+1+a_{15}]+y_n\bullet i\bullet k\bullet m\bullet R_0\bullet(a_{15}+a_{13})+y_{n-1}\bullet i\bullet k\bullet m\bullet R_0\bullet(a_{15}+a_{13})+z_{n+2}\bullet(n+1)\bullet(n+2)+z_{n+1}\bullet(n+1)\bullet$$
$$\bullet(2\bullet n+1)+z_n\bullet[n^2-m^2\bullet a_{13}+R_0^2\bullet(a_{17}-k^2\bullet a_{16})]+z_{n-1}\bullet2\bullet R_0^2\bullet(a_{17}-k^2\bullet a_{16})+z_{n-2}\bullet R_0^2\bullet(a_{17}-k^2\bullet$$
$$\bullet a_{16})=0, \qquad (20)$$

where

It is necessary to use $3\bullet(N_1+!)-6$ of equations (18) – (20), but for $n=0$ and $n=1$ coefficients with negative indexes are equal to zero. Then in common with equations (12) – (17) the homogeneous system of $3\bullet(N_1+1)$ equations relative to coefficients x_n, y_n, z_n is formed. Then we expand the determinant of this system and let this determinant is equal zero we receive the characteristic equation for wave numbers k of elastic waves with the mode m in the orthotropic cylindrical shell.

4. Conclusions

In the paper were found the characteristic equation for wave numbers of elastic waves in thin orthotropic cylindrical shell with the help of the dynamic theory of the elasticity for the orthotrpic medium and of the hypothesis of thin shells.

Acknowledgments

The work was supported as part of research under State Contract no P242 of April 21, 2010, within the Federal Target Program "Scientific and scientific pedagogical personnel of innovative Russia for the 2009 – 2013".

REFERENCES

[1] S. G. Lekhnitsky. The theory of elasticity of anisotropic body. M.: Science. 1977. P. 416.

[2] A. A. Kleshchev. Against phase velocities of elastic waves in thin orthotropic cylindrical shell. // Coll. Proc. Russ. Acoust. Soc. XI session. M.: GEOS. 2001. V. 1. P. 241 – 244.

[3] A. A. Kleshchev. Diffraction and propagation of waves in elastic mediums and bodies. S.-Pb.: Vlas. 2002. P. 156.

[4] A. A. Kleshchev. Diffraction, radiation and propagation of elastic waves. S.-Pb.: Profprint. 2006. P. 160.

[5] E. L. Shenderov. Radiation and scattering of sound. L.: Shipbuilding. 1989. P. 302.

[6] A. A. Kleshchev. Against phase velocities of elastic waves in thin transversely isotropic cylindrical shell. // Coll. Proc. Russ. Acoust. Soc. X session. M.: GEOS. 2000. V. 1. P. 206 – 210