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Common Fixed Point Results for Four Mappings Satisfying Almost generalized $(S,T)-$contractive Condition in Partially Ordered Fuzzy Metric Spaces

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Abstract In this paper, we define the concept of almost generalized $(S,T)-$ contractive condition, and prove some common fixed point results for four mappings satisfying almost generalized $(S,T)-$ contractive condition in partially ordered fuzzy metric spaces.

Keywords Common fixed point, Ordered metric space, Almost contraction, Weakly annihilator maps, Dominating maps

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1 Introduction

The concept of fuzzy sets was introduced initially by Zadeh [18] in 1965. Since then, to use this concept in topology and analysis, many authors have expansively developed the theory of fuzzy sets and application. George and Veeramani [5] and Kramosil and Michalek [9] have introduced the concept of fuzzy topological spaces induced by fuzzy metric. Many authors [3, 4, 7, 11, 13, 15, 16] have studied different properties, for e.g, topological, fixed point properties and applications of fuzzy (probabilistic) metric spaces and also its generalized and different versions. Recently, Kumar [10] proved a common fixed point theorem for a pair of weakly compatible maps under E.A. property and Wadhwa et al. [17] defined a E. A. like property and proved common fixed point theorems in fuzzy metric spaces.

Definition 1.1. [14] A binary operation $*:[0,1]\times[0,1]\longrightarrow[0,1]$ is a continuous t-norm if it satisfies the following conditions:

1. $*$ is associative and commutative,

2. $*$ is continuous,

3. $a*1=a$ for all $a\in[0,1]$,

4. $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0, 1]$.

Two typical examples of a continuous t-norm are $a * b = ab$ and $a * b = \min\{a, b\}$.

Definition 1.2. [5] A 3-tuple $(X, M, *)$ is called a *fuzzy metric space* if X is an arbitrary (non-empty) set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$, satisfying the following conditions for each $x, y, z \in X$ and $t, s > 0$,

1. $M(x, y, t) > 0$,
2. $M(x, y, t) = 1$ if and only if $x = y$,
3. $M(x, y, t) = M(y, x, t)$,
4. $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
5. $M(x, y, .) : (0, \infty) \longrightarrow [0, 1]$ is continuous.

Let $(X, M, *)$ be a fuzzy metric space. For $t > 0$, the *open ball* $B(x, r, t)$ with center $x \in X$ and radius $0 < r < 1$ is defined by

$$B(x, r, t) = \{y \in X : M(x, y, t) > 1 - r\}.$$

If $(X, M, *)$ is a fuzzy metric space, let τ be the set of all $A \subset X$ with $x \in A$ if and only if there exist $t > 0$ and $0 < r < 1$ such that $B(x, r, t) \subset A$. Then τ is a topology on X (induced by the fuzzy metric M). This topology is Hausdorff and first countable. A sequence $\{x_n\}$ in X converges to x if and only if $M(x_n, x, t) \to 1$ as $n \to \infty$, for each $t > 0$. It is called a Cauchy sequence if for each $0 < \varepsilon < 1$ and $t > 0$, there exists $n_0 \in \mathbb{N}$ such that $M(x_n, x_m, t) > 1 - \varepsilon$ for each $n, m \geq n_0$. The fuzzy metric space $(X, M, *)$ is said to be *complete* if every Cauchy sequence is convergent. A subset A of X is said to be F-bounded if there exist $t > 0$ and $0 < r < 1$ such that $M(x, y, t) > 1 - r$ for all $x, y \in A$.

Example 1.3. Let $X = \mathbb{R}$. Put $a * b = ab$ for all $a, b \in [0, 1]$. For each $t \in (0, \infty)$, define

$$M(x, y, t) = \frac{t}{t + |x - y|}$$

for all $x, y \in X$.

Lemma 1.4.[6] Let $(X, M, *)$ be a fuzzy metric space. Then $M(x, y, t)$ is non-decreasing with respect to t, for all x, y in X.

Definition 1.5. Let $(X, M, *)$ be a fuzzy metric space. Then M is said to be *continuous* on $X^2 \times (0, \infty)$ if

$$\lim_{n\to\infty} M(x_n, y_n, t_n) = M(x, y, t),$$

whenever a sequence $\{(x_n, y_n, t_n)\}$ in $X^2 \times (0, \infty)$ converges to a point $(x, y, t) \in X^2 \times (0, \infty)$. i.e.

$$\lim_{n\to\infty} M(x_n, x, t) = \lim_{n\to\infty} M(y_n, y, t) = 1 \text{ and } \lim_{n\to\infty} M(x, y, t_n) = M(x, y, t).$$

Lemma 1.6. Let $(X, M, *)$ be a fuzzy metric space. Then M is a continuous function on $X^2 \times (0, \infty)$.

Proof. See Proposition 1 of [12]

Definition 1.7. [8] Let A and S be mappings from a fuzzy metric space $(X, M, *)$ into itself. Then the mappings are said to be *weak compatible* if they commute at a coincidence point, that is, $Ax = Sx$ implies that $ASx = SAx$.

Definition 1.8 ([2]). Let X be a nonempty set. Then $(X, M, *, \preceq)$ is called an ordered fuzzy metric space iff:
(i) $(X, M, *)$ is a fuzzy metric space,
(ii) $(X, \preceq)$ is partial ordered set.

Definition 1.9 ([1]). Let $(X, \preceq)$ be a partial ordered set. $x, y \in X$ are called comparable if $x \preceq y$ or $y \preceq x$ holds.

Also very recently M. Abbas, T. Nazir and S. Radenović [2] introduced the new concepts in a partial ordered set as follows:

Definition 1.10 ([1]). Let $(X, \preceq)$ be a partially ordered set. A mapping f is called weak annihilator of g if $fgx \preceq x$ for all $x \in X$.

Definition 1.11 ([1]). Let $(X, \preceq)$ be a partially ordered set. A mapping f is called dominating if $x \preceq fx$ for all $x \in X$.

2 Results

The aim of this paper is to present a common fixed point result for four mappings which satisfy almost generalized $(S,T)-$ contractive condition in partially ordered fuzzy metric spaces.

For this purpose we need the following definitions:

Definition 2.1. Let f, g, S and T be self maps on a fuzzy metric space $(X,M,*)$, then f and g are said to satisfy almost generalized $(S,T)-$ contractive condition if there exists $\delta \in [0,1)$ such that

$$M(fx,gy,t) \geq L^{\delta}(x,y,t), \qquad x,y \in X \ and \ t>0 \tag{1}$$

where

$$L(x,y,t) = \min\Big\{M(Sx,Ty,t), M(fx,Sx,t), M(gy,Ty,t), \sqrt{M(Sx,gy,2t)*M(fx,Ty,2t)}\Big\}.$$

Theorem 2.2. Let $(X,M,*,\preceq)$ be an ordered complete fuzzy metric space such that $a*b \geq a.b$ for every $a,b \in [0,1]$. Let f, g, S and T be self maps on X, with $f(X) \subseteq T(X)$ and $g(X) \subseteq S(X)$ and dominating maps f and g are weak annihilators of T and S, respectively.

Suppose that f and g satisfy almost generalized $(S,T)-$contractive condition (1) for every two comparable elements $x,y \in X$.

If for a nondecreasing sequence $\{x_n\}$ with $x_n \preceq y_n$ for all n and $y_n \to u$ implies that $x_n \preceq u$ and furthermore

(a) $\{f,S\}$ and $\{g,T\}$ are weakly compatible,

(b) one of $f(X)$, $g(X)$, $S(X)$ and $T(X)$ is a closed subspace of X,

then f, g, S and T have a common fixed point.

Further, If we assume that the set of common fixed points of f,g,S and T is well ordered then f, g, S and T have a unique common fixed point.

Proof. Let x_0 be in X. Since $f(X) \subseteq T(X)$ and $g(X) \subseteq S(X)$, we can find the sequences $\{x_n\}$ and $\{y_n\}$ in X such that $y_{2n} = fx_{2n} = Tx_{2n+1}$, and $y_{2n+1} = gx_{2n+1} = Sx_{2n+2}$, $n \geq 0$.

By given assumptions, $x_{2n} \preceq fx_{2n} = Tx_{2n+1} \preceq fTx_{2n+1} \preceq x_{2n+1}$, and $x_{2n+1} \preceq gx_{2n+1} = Sx_{2n+2} \preceq gSx_{2n+2} \preceq x_{2n+2}$. Thus, for all $n \geq 0$ we have $x_n \preceq x_{n+1}$.

Suppose first that $y_m = y_{m+1}$ for some m. Then, the sequence $\{y_n\}$ is constant for $n \geq m$. Indeed, let $m = 2k$. Then $y_{2k} = y_{2k+1}$ and we obtain from (1) that

$$\begin{aligned} M(y_{2k+1},y_{2k+2},t) &= M(gx_{2k+1},fx_{2k+2},t) \\ &= M(fx_{2k+2},gx_{2k+1},t) \\ &\geq L^{\delta}(x_{2k+2},x_{2k+1},t), \end{aligned}$$

where

$$\begin{aligned} L(x_{2k+2},x_{2k+1},t) = & \min\Big\{M(Sx_{2k+2},Tx_{2k+1},t), M(fx_{2k+2},Sx_{2k+2},t), M(Tx_{2k+1},gx_{2k+1},t), \\ & \sqrt{M(Sx_{2k+2},gx_{2k+1},2t)*M(fx_{2k+2},Tx_{2k+1},2t)}\Big\} \\ = & \min\Big\{M(y_{2k+1},y_{2k},t), M(y_{2k+2},y_{2k+1},t), M(y_{2k},y_{2k+1},t), \\ & \sqrt{M(y_{2k+1},y_{2k+1},2t)*M(y_{2k+2},y_{2k},2t)}\Big\} \\ = & \min\Big\{1, M(y_{2k+2},y_{2k+1},t), 1, \sqrt{1*M(y_{2k+2},y_{2k},2t)}\Big\}. \end{aligned}$$

Since

$$\begin{aligned} M(y_{2k},y_{2k+2},2t) &\geq M(y_{2k},y_{2k+1},t)*M(y_{2k+1},y_{2k+2},t) \\ &\geq M(y_{2k},y_{2k+1},t).M(y_{2k+1},y_{2k+2},t) = 1.M(y_{2k+1},y_{2k+2},t) \\ &\geq M^2(y_{2k+1},y_{2k+2},t), \end{aligned}$$

then it follows from the last inequality and (1) that

$$M(y_{2k+1}, y_{2k+2}, t) \geq M^{\delta}(y_{2k+1}, y_{2k+2}, t),$$

Since $\delta \in [0, 1)$, we have $M(y_{2k+1}, y_{2k+2}, t) = 1$.Hence $y_{2k+1} = y_{2k+2}$.

Similarly, if m=2k+1 one can easily obtains that $y_{2k+2} = y_{2k+3}$. Thus, in this case $\{y_n\}$ turns out to be eventually a constant sequence and y_{2n} is the common fixed point of f, g, S and T.

Suppose now that $M(y_n, y_{n+1}, t) < 1$ for each n. We shall prove that for each $n \geq 1$,

$$M(y_n, y_{n+1}, t) \geq M^{\delta}(y_{n-1}, y_n, t). \tag{2}$$

Since x_{2n} and x_{2n+1} are comparable, from (1) , we obtain

$$\begin{aligned} M(y_{2n}, y_{2n+1}, t) &= M(fx_{2n}, gx_{2n+1}, t) \\ &\geq L^{\delta}(x_{2n}, x_{2n+1}, t), \end{aligned} \tag{3}$$

where

$$\begin{aligned} L(x_{2n}, x_{2n+1}, t) = & \min\Big\{M(Sx_{2n}, Tx_{2n+1}, t), M(fx_{2n}, Sx_{2n}, t), M(Tx_{2n+1}, gx_{2n+1}, t), \\ & \sqrt{M(Sx_{2n}, gx_{2n+1}, 2t) * M(fx_{2n}, Tx_{2n+1}, 2t)}\Big\} \\ = & \min\Big\{M(y_{2n-1}, y_{2n}, t), M(y_{2n}, y_{2n-1}, t), M(y_{2n}, y_{2n+1}, t), \sqrt{M(y_{2n-1}, y_{2n+1}, 2t) * M(y_{2n}, y_{2n}, 2t)}\Big\} \\ = & \min\Big\{M(y_{2n-1}, y_{2n}, t), M(y_{2n}, y_{2n+1}, t), \sqrt{M(y_{2n-1}, y_{2n+1}, 2t) * 1}\Big\}. \end{aligned}$$

If $M(y_{2n}, y_{2n+1}, t) \leq M(y_{2n-1}, y_{2n}, t) < 1$, then

$$\begin{aligned} M(y_{2n-1}, y_{2n+1}, 2t) &\geq M(y_{2n-1}, y_{2n}, t) * M(y_{2n}, y_{2n+1}, t) \\ &\geq M(y_{2n-1}, y_{2n}, t).M(y_{2n}, y_{2n+1}, t) \\ &\geq M^2(y_{2n}, y_{2n+1}, t), \end{aligned}$$

then it follows from the last inequality and (3) that

$$M(y_{2n}, y_{2n+1}, t) \geq M^{\delta}(y_{2n}, y_{2n+1}, t).$$

Since $M(y_{2n}, y_{2n+1}, t) < 1$ and $0 < \delta < 1$, it is a contradiction.
Hence, $M(y_{2n}, y_{2n+1}, t) > M(y_{2n-1}, y_{2n}, t)$ and from (3) we have

$$M(y_{2n}, y_{2n+1}, t) \geq M^{\delta}(y_{2n-1}, y_{2n}, t).$$

By similar arguments we can show that,

$$M(y_{2n}, y_{2n-1}, t) \geq M^{\delta}(y_{2n-1}, y_{2n-2}, t).$$

So, (2) holds for each n. That is, for $n \geq 1$

$$M(y_n, y_{n+1}, t) \geq M^{\delta}(y_{n-1}, y_n, t).$$

Hence, for $n \geq 1$ it follows that

$$M(y_n, y_{n+1}, t) \geq M^{\delta}(y_{n-1}, y_n, t) \geq \cdots \geq M^{\delta^n}(y_0, y_1, t).$$

For $m > n$ and $t > 0$ there exists $t_1 > 0$ such that $t_1 \leq \frac{t}{m-n}$, hence by the triangle inequality, we have

$$\begin{aligned} M(y_m, y_n, t) &\geq M(y_m, y_n, (m-n)t_1) \\ &\geq M(y_n, y_{n+1}, t_1) * M(y_{n+1}, y_{n+2}, t_1) * \cdots * M(y_{m-1}, y_m, t_1) \\ &\geq M^{\delta^n}(y_0, y_1, t_1).M^{\delta^{n+1}}(y_0, y_1, t_1). \cdots .M^{\delta^{m-1}}(y_0, y_1, t_1) \\ &= M^{\delta^n+\delta^{n+1}+\cdots+\delta^{m-1}}(y_0, y_1, t_1) \\ &\geq M^{\frac{\delta^n}{1-\delta}}(y_0, y_1, t_1) \longrightarrow 1, \end{aligned}$$

which implies that $\{y_n\}$ is a Cauchy sequence and since X is complete, there exists a point y in X such that $\lim_{n\to\infty} y_n = y$.

Therefore,

$$\lim_{n\to\infty} fx_{2n} = \lim_{n\to\infty} Tx_{2n+1} = \lim_{n\to\infty} gx_{2n+1} = \lim_{n\to\infty} Sx_{2n+2} = y.$$

Assume that $T(X)$ is closed, there exists $u \in X$ such that $y = Tu$. Suppose $M(gu, y, t) < 1$ for some $t > 0$.

Also, $x_{2n} \preceq fx_{2n}$, and $fx_{2n} \to y$ as $n \to \infty$, $x_{2n} \preceq y$, and since dominating map f is weak annihilator of T we obtain

$$x_{2n} \preceq y = Tu \preceq fTu \preceq u,$$

From (1) we have

$$M(fx_{2n}, gu, t) \geq L^{\delta}(x_{2n}, u, t), \tag{4}$$

where

$$\begin{aligned} L(x_{2n}, u, t) = & \min\Big\{M(Sx_{2n}, Tu, t), M(fx_{2n}, Sx_{2n}, t), M(gu, Tu, t), \\ & \sqrt{M(Sx_{2n}, gu, 2t) * M(fx_{2n}, Tu, 2t)}\Big\} \\ = & \min\Big\{M(Sx_{2n}, y, t), M(fx_{2n}, Sx_{2n}, t), M(gu, y, t), \\ & \sqrt{M(Sx_{2n}, gu, 2t) * M(fx_{2n}, y, 2t)}\Big\} \\ \to & \min\Big\{M(y, y, t), M(y, y, t), M(gu, y, t), \sqrt{M(y, gu, 2t) * M(y, y, 2t)}\Big\} \\ = & M(gu, y, t), \end{aligned}$$

since $M(y, gu, 2t) \geq M(y, gu, t) \geq M^2(y, gu, t)$.

Now, letting $n \to \infty$ in (4), we get

$$M(y, gu, t) \geq M^{\delta}(gu, y, t).$$

Since $M(gu, y, t) < 1$ and $0 < \delta < 1$, it is a contradiction. Hence $M(gu, y, t) = 1$ for all $t > 0$. $gu = y$. Thus $gu = Tu = y$. Since the maps g and T are weakly compatible, we have $gy = gTu = Tgu = Ty$.

Suppose $M(gy, y, t) < 1$ for some $t > 0$.

From (1) we have

$$M(fx_{2n}, gy, t) \geq L^{\delta}(x_{2n}, y, t), \tag{5}$$

where

$$\begin{aligned} L(x_{2n}, y, t) = & \min\Big\{M(Sx_{2n}, Ty, t), M(fx_{2n}, Sx_{2n}, t), M(gy, Ty, t), \sqrt{M(Sx_{2n}, gy, 2t) * M(fx_{2n}, Ty, 2t)}\Big\} \\ = & \min\Big\{M(Sx_{2n}, gy, t), M(fx_{2n}, Sx_{2n}, t), 1, \sqrt{M(Sx_{2n}, gy, 2t) * M(fx_{2n}, y, 2t)}\Big\} \\ \to & \min\Big\{M(y, gy, t), M(y, y, t), 1, \sqrt{M(y, gy, 2t) * 1}\Big\} \\ = & M(y, gy, t), \end{aligned}$$

since $M(y,gy,2t) \geq M(y,gy,t) \geq M^2(y,gy,t)$. Now, letting $n \to \infty$ in (5), we get

$$M(y,gy,t) \geq M^\delta(y,gy,t).$$

Since $M(y,gy,t) < 1$ and $0 < \delta < 1$, it is a contradiction. Hence $M(y,gy,t) = 1$ for all $t > 0$. Hence $gy = y$. Thus $gy = Ty = y$.

Since $g(X) \subseteq S(X)$ there exists a point $v \in X$ such that $gy = Sv$. Suppose that $M(fv,Sv,t) < 1$ for some $t > 0$.. Since $y \preceq gy = Sv \preceq gSv \preceq v$ implies $y \preceq v$. From (1) we have

$$M(fv,Sv,t) = M(fv,gy,t) \geq L^\delta(v,y,t), \tag{6}$$

where

$$\begin{aligned} L(v,y,t) =& \min\Big\{M(Sv,Ty,t), M(fv,Sv,t), M(gy,Ty,t), \sqrt{M(Sv,gy,2t) * M(fv,Ty,2t)}\Big\} \\ =& \min\Big\{1, M(fv,Sv,t), 1, \sqrt{1 * M(fv,Sv,2t)}\Big\} \\ =& M(fv,Sv,t), \end{aligned}$$

since $M(fv,Sv,2t) \geq M(fv,Sv,t) \geq M^2(fv,Sv,t)$.

Now from (6), we get

$$M(fv,Sv,t) = M(fv,gy,t) \geq M^\delta(fv,Sv,t).$$

Since $M(fv,Sv,t) < 1$ and $0 < \delta < 1$, it is a contradiction. Hence$M(fv,Sv,t) = 1$ for all $t > 0$. Hence $fv = Sv$. Since f and S are weakly compatible, $fy = fSv = Sfv = Sy$. Thus y is a coincidence point of f and S.

Suppose $M(fy,y,t) < 1$ for somr $t > 0$. From (1) we get

$$\begin{aligned} M(fy,y,t) &= M(fy,gy,t) \\ &\geq L^\delta(y,y,t) = M^\delta(fy,y,t) \end{aligned}$$

It is a contradiction. Hence $M(fy,y,t) = 1$. for all $t > 0$.Hence $fy = y$. Thus $fy = Sy = y$. Thus y is a common fixed point of f, g, S and T. The proofs for the cases in which $S(X)$ or $f(X)$ or $g(X)$ is closed are similar.

If the set of common fixed points of f, g, S and T is well ordered then clearly (1) implies the uniqueness of the common fixed point and the proof is complete. □

Now we give an example to support our result.

Example 2.3. Consider the nonnegative real numbers $X = [0, \infty)$ equipped with the standard fuzzy metric $M(x,y,t) = e^{-\frac{|x-y|}{t}}$, $x, y \in X$, $t > 0$ with $a * b = a.b$ for every $a, b \in [0,1]$. Suppose that "$\leq$" be the usual ordering on $\mathbb{R}$. We define a new ordering "$\preceq$" on X as follows:

$$x \preceq y \quad \Leftrightarrow \quad y \leq x, \quad \forall x, y \in X.$$

It is easy to see that $(X, M, *, \preceq)$ is an ordered complete fuzzy metric space. Let f, g, S and $T : X \to X$ be define by

$$\begin{aligned} f(x) &= \ln(1+x), \qquad g(x) = \ln(1+\frac{x}{2}), \\ T(x) &= e^x - 1, \qquad S(x) = e^{2x} - 1. \end{aligned}$$

For each $x \in X$, we have $1 + x \leq e^x$ and $1 + \frac{x}{2} \leq e^x$ so $f(x) = \ln(1+x) \leq x$ and $g(x) = \ln(1+\frac{x}{2}) \leq x$, which follows that, $x \preceq f(x)$ and $x \preceq g(x)$. Thus f and g are dominating maps. Also note that for each $x \in X$ we have $fT(x) = f(e^x - 1) = \ln e^x = x \geq x$ and $gS(x) = g(e^{2x} - 1) = \ln(\frac{e^{2x}+1}{2}) \geq x$, which implies that $fT(x) \preceq x$ and $gS(x) \preceq x$. Thus f and g are weak annihilators of T and S, respectively.

In order to show that f, g, S and T do satisfy the contractive condition (1) in Theorem 2.2., using mean value theorem we have

$$\begin{aligned} M(fx,gy,t) = e^{-\frac{|f(x)-g(y)|}{t}} &= e^{-\frac{\left|\ln(1+x)-\ln(1+\frac{y}{2})\right|}{t}} \geq e^{-\frac{1}{2}\frac{|2x-y|}{t}} \\ &= (e^{-\frac{|2x-y|}{t}})^{\frac{1}{2}} \geq M^{\frac{1}{2}}(Sx,Ty,t) \\ &\geq L^{\frac{1}{2}}(x,y,t), \qquad x, y \in X \end{aligned}$$

Thus the condition (1) of Theorem 2.2 is satisfied. One can easily verify all the remaining conditions of Theorem 2.2. Moreover, 0 is a unique common fixed point of f, g, S and T.

Corollary 2.4. Let $(X, M, *, \preceq)$ be an ordered complete fuzzy metric space such that $a * b \geq a.b$ for every $a, b \in [0,1]$. Let f and T be self maps on X, with $f(X) \subseteq T(X)$ and dominating map f is weak annihilators of T.

Suppose that there exists $\delta \in [0,1)$ such that

$$M(fx, fy, t) \geq L^{\delta}(x, y, t)$$

where

$$L(x,y,t) = \min\Big\{M(Tx,Ty,t), M(fx,Tx,t), M(fy,Ty,t), \sqrt{M(Tx,fy,2t) * M(fx,Ty,2t)}\Big\}.$$

for every two comparable elements $x, y \in X$ and $t > 0$.

If for a nondecreasing sequence $\{x_n\}$ with $x_n \preceq y_n$ for all n and $y_n \to u$ implies that $x_n \preceq u$ and furthermore

(a) $\{f, T\}$ is weakly compatible,

(b) one of $f(X)$ and $T(X)$ is a closed subspace of X,

then f and T have a common fixed point.

Remark 2.5. It is easy to see that for every map $T : X \to X$, $\{T, I_X\}$ is weakly compatible, and I_X is dominating map, where I_X is identity map on X, so by taking $f = g = I_X$ in Theorem (2.2) we have the following result.

Corollary 2.6. Let $(X, M, *, \preceq)$ be an ordered complete fuzzy metric space such that $a * b \geq a.b$ for every $a, b \in [0,1]$. Let S and T be surjective self maps on X, such that $S(x) \preceq x$ and $T(x) \preceq x$ for all $x \in X$, and suppose that there exists $\delta \in [0,1)$ such that

$$M(x, y, t) \geq L^{\delta}(x, y, t)$$

where

$$L(x,y,t) = \min\Big\{M(Sx,Ty,t), M(x,Sx,t), M(y,Ty,t), \sqrt{M(Sx,y,2t) * M(x,Ty,2t)}\Big\}.$$

for every two comparable elements $x, y \in X$ and $t > 0$.

If for a nondecreasing sequence $\{x_n\}$ with $x_n \preceq y_n$ for all n and $y_n \to u$ implies that $x_n \preceq u$

then S and T have a common fixed point.

3 Discussions and Conclusions

In this paper, we obtained a common fixed point theorem for four mappings satisfying almost contraction condition in fuzzy metric spaces. We have also given an example to illustrate our main theorem.

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