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Solution of the Relativistic Bound State Problem for Hadrons

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Abstract A second order extension of a generalised QED Lagrangian (including boson-boson coupling) has been used to describe $q\bar{q}$ hadrons. Assuming massless elementary fermions (quantons) this results in a finite theory without open parameters, which may be regarded as a fundamental description of the strong interaction. Two potentials are deduced, a boson-exchange potential and one, which can be identified with the known confinement potential in hadrons. This formalism has been applied the mesonic systems $\omega(782)$, $\Phi(1020)$, $J/\psi(3097)$ and $\Upsilon(9460)$ with a good description of their masses. The most important results are: 1. The confinement of hadrons is not due to colour, but is a general property of relativistic bound states. 2. Massive quarks in the Standard Model (QCD) are understood as effective fermions with a mass given by the binding energy in the boson-exchange potential.

Keywords Bound state description of had-rons based on a second order Lagrangian with massless fermions (quantons) and boson-boson coupling, Confinement and boson-exchange potential, Quarks understood as effective fermions with masses given by bound state energies, Higgs-bosons and supersymmetry not needed

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1. Introduction

To study fundamental forces, nature provides us with hadrons and leptons, which form the constituents of matter, but also with composite systems, nuclei, atoms, and gravitational states in form of solar and galactic systems. For the description of these stable and massive systems as bound states a relativistic theory is needed, since the elementary constituents of these states are relativistic. However, relativistic bound state problems are generally difficult to solve, see e.g. Salpeter and Bethe [1], and could not be tackled so far for particle bound states (see the discussion in ref. [2]).

Instead, powerful effective theories have been developed, which are contained (except gravitation) in the Standard Model of particle physics [3] (SM). In these divergent first order field theories particle bound states are included effectively by a number of parameters including nine masses of 'elementary' particles. Only in QED bound states have been calculated from the Coulomb potential. But also in this theory the magnitude of the coupling constant

α is not understood from first principles.

To understand the underlying mechanisms as well as the parameters needed in first order theories, a more fundamental theory should exist, in which these features are explained. If we demand further a real physical understanding of the development of particle systems in the universe, this theory should most likely be finite, since nature develops in a smooth way without singularities. Such a theory is expected to be based on a Lagrangian including higher order fields. This can be seen for example from the mass of light hadrons, which is much larger than the underlying quark masses.

However, in spite of an evident need for higher order theories, there is a strong belief that Lagrangians describing fundamental forces can be only first order. This is correct for divergent field theories, since the inclusion of higher order terms destroys the renormalisability of the theory. But this argument is not valid, if a finite theory is constructed. Another argument against the use of higher order theories is that in such theories Lagrangians with higher field derivatives [4] are required, which can lead to unphysical solutions (ghosts). But it should be realised that the problem is not the use of higher order theories in principle, but to find a form of the Lagrangian, in which all important criteria of relativistic theories are respected, like gauge invariance and energy-momentum conservation.

Recently, a second order theory has been developed by extending the QED Lagrangian by boson-boson coupling [5]. This formalism fulfills the above criteria of a relativistic theory and can be regarded as a fundamental description of the electric force in light atoms. However, for hadronic bound states the requirements are still higher and ask also for massless elementary fermions (quantons). Then the mass of $q\bar{q}$ bound states has to be entirely due to binding energy.

An interesting question is, whether in a fundamental theory of hadrons the colour degree of freedom is really needed (since hadrons are colour neutral). The answer depends on confinement. If it would be due to colour (as often assumed [6]), a non-Abelian theory with colour would be needed. However, already in the description of atomic systems a confinement potential has been found [5], suggesting that confinement is a basic property of bound states of relativistic particles. Preliminary results from an application to other fundamental forces can be found in ref. [7].

2. Theoretical framework

The Lagrangian may be written in the form

$$\mathcal{L} = \frac{1}{\tilde{m}^2}\bar{\Psi}\ i\gamma_\mu D^\mu D_\nu D^\nu \Psi\ -\ \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\ , \tag{1}$$

where $\tilde{m}$ is a mass parameter and Ψ in general a two-component fermion field $\Psi = (\Psi^+\ \Psi^o)$ and $\bar{\Psi} = (\Psi^-\ \bar{\Psi}^o)$ with charged and neutral part (here only charged fermions are considered). Vector boson fields A_μ with generalised charge coupling g (of self-consistent amplitude) are contained in the covariant derivatives $D_\mu = \partial_\mu - igA_\mu$ and the Abelian field strength tensor $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$.

We insert $D^\mu = \partial^\mu - igA^\mu$ and $D_\nu D^\nu = \partial_\nu\partial^\nu - ig(A_\nu\partial^\nu + \partial_\nu A^\nu) - g^2 A_\nu A^\nu$ in eq. (1) and obtain for the first term of $\mathcal{L}$

$$\begin{aligned}\mathcal{L}_1 = \frac{1}{\tilde{m}^2}\bar{\Psi}\ i\gamma_\mu D^\mu D_\nu D^\nu\Psi = \frac{i}{\tilde{m}^2}\bar{\Psi}\ \gamma_\mu\partial^\mu\partial_\nu\partial^\nu\Psi + \frac{g}{\tilde{m}^2}\bar{\Psi}\ \gamma_\mu A^\mu\partial_\nu\partial^\nu\Psi \\ +\frac{g}{\tilde{m}^2}\bar{\Psi}\ \gamma_\mu\partial^\mu A_\nu\partial^\nu\Psi + \frac{g}{\tilde{m}^2}\bar{\Psi}\ \gamma_\mu\partial^\mu\partial_\nu A^\nu\Psi - \frac{ig^2}{\tilde{m}^2}\bar{\Psi}\ \gamma_\mu A^\mu A_\nu\partial^\nu\Psi \\ -\frac{ig^2}{\tilde{m}^2}\bar{\Psi}\ \gamma_\mu A^\mu\partial_\nu A^\nu\Psi - \frac{ig^2}{\tilde{m}^2}\bar{\Psi}\ \gamma_\mu\partial^\mu A_\nu A^\nu\Psi - \frac{g^3}{\tilde{m}^2}\bar{\Psi}\ \gamma_\mu A^\mu A_\nu A^\nu\Psi\ .\end{aligned} \tag{2}$$

The gauge condition $\partial_\mu A^\mu = 0$ used for simpler Lagrangians (as in QED) is replaced in our case by $\partial(\partial_\nu A^\nu) = 0$.

In eq. (2) the number of field derivatives and boson couplings varies between the first and last term. This shows that the various terms are related to different kinetic situations, pointing to a rather complex dynamics of the system.

Contributions to stationary solutions have been studied by using the standard method of evaluating fermion matrix elements (or ground state expectation values) of field operators [8] derived from generalised Feynman diagrams. These can be written in the form $\mathcal{M}(p'-p) = < g.s.|\ K(q)\ |g.s. >\sim \bar{\psi}(p')\ K(q)\ \psi(p)$, where $\psi(p)$ is a fermionic wave function $\psi(p) = \frac{1}{\tilde{m}^{3/2}}\Psi(p_1)\Psi(p_2)$ and $K(q=p'-p)$ a kernel, which is expressed by $K(q) = \frac{1}{\tilde{m}^{2(n+1)}}\ [O^n(q)\ O^n(q)]$, where n is the number of boson fields and derivatives in eq. (2) (in the present case n=3).

For the construction of stationary states we expect contributions mainly from terms of the Lagrangian (2), which contain static fields (without derivatives). This is the case only for the last term $\mathcal{L}_{1,8} = -\frac{1}{\tilde{m}^2}\bar{\Psi}\ g^3\gamma_\mu A^\mu A_\nu A^\nu\Psi$ and leads to a matrix element $\mathcal{M}_{3g}$, which contains three boson fields on the right and left

$$\mathcal{M}_{3g} = \frac{-\alpha^3}{\tilde{m}^8}\ \bar{\psi}(p')\gamma_\mu A^\mu(q)A_\nu(q)A^\nu(q)\ A_\sigma(q)A^\sigma(q)\gamma_\rho A^\rho(q)\psi(p)\ , \tag{3}$$

where $\alpha = g^2/4\pi$. A comparable matrix element in a first order theory may be written in the form $\mathcal{M}_{f.o.} = \frac{-\alpha}{\tilde{m}^4}\ \bar{\psi}(p')\gamma_\mu A^\mu(q)\ \gamma_\rho A^\rho(q)\psi(p)$, giving rise to a (boson-exchange) interaction of vector structure $v_v(q)\sim\alpha A_\mu(q)A^\rho(q)$ (but only for equal times of the two boson fields, which means in the non-relativistic limit). Differently, in eq. (3) three interactions of scalar and vector structure $V_\mu^\nu(q)\sim\alpha A_\mu(q)A^\nu(q)$ are involved. In a dual picture the two boson fields, which appear twice (on the left and right side of $\mathcal{M}_{3g}$) can be regarded (analoguous to the fermion wave function $\psi(p)$) as bosonic (quasi) wave functions[1] $W_\mu^\nu(q) = \frac{1}{\tilde{m}}A_\mu(q)A^\nu(q)$. The fact that boson fields can be combined to wave functions leads quite naturally to a finite theory, in which the wave functions are normalised.

The physical picture of $\mathcal{M}_{3g}$ is that for the lowest energy state of a relativistic bound state system the fermions interact only inside the boson density (which leads to a boundary condition discussed below) and feel therefore three interactions. A single boson-exchange interaction is possible only in dynamical situations, see the terms 2-4 in eq. (2), which do not lead to a bound state potential.

The γ-matrices can be removed (contracted) by adding a matrix element with interchanged μ and ρ (according to $\frac{1}{2}(\gamma_\mu\gamma_\rho+\gamma_\rho\gamma_\mu) = g_{\mu\rho}$). Further, an equal time requirement of the two boson fields (to reach overlap) allows to replace all fermion four-vectors[2] by three-vectors in momentum or r-space. Correspondingly, the boson wave functions $W_\mu^\nu(q)$ and the remaining (boson-exchange) interaction $V_\mu^\nu(q)$ are reduced to $w_{s,v}(q)$ and $v_v(q)\sim w_v(q)$, which are two-dimensional. This yields

$$\mathcal{M}_{3g} = \frac{-\alpha^3}{\tilde{m}^5}\ \bar{\psi}(p')w_{s,v}(q)\ v_v(q)\ w_{s,v}(q)\psi(p)\ . \tag{4}$$

Writing the matrix element by $\mathcal{M}_{3g} = \bar{\psi}(p')\ V_{3g}(q)\ \psi(p)$ we obtain a three-boson potential

$$V_{3g}^{s,v}(q) = \frac{-\alpha^3}{\tilde{m}^2}\ w_{s,v}^2(q)v_v(q)\ . \tag{5}$$

Fourier transformation to r-space leads to a folding potential

$$V_{3g}^{s,v}(r) = -\frac{\alpha^3\hbar}{\tilde{m}}\int dr' w_{s,v}^2(r')\ v_v(r-r')\ . \tag{6}$$

Such a form has been used to describe elastic and inelastic hadron processes [9].

The bosonic part of eq. (4) can also be written in the form of a matrix element, in which the wave functions $w_{s,v}(q)$ are connected by $v_v(q)$

$$\mathcal{M}^g = \frac{-\alpha^3}{\tilde{m}^2}\ w_{s,v}(q)v_v(q)w_{s,v}(q). \tag{7}$$

This matrix element shows binding of two bosons in the potential $v_v(q)$, consequently $\partial^2 w_s(q)$ is related to their kinetic energy. The contribution from the vector part $\partial^2 w_v(q)$ cancels out as a consequence of the gauge condition. Below it will be shown that this implies also the existence of a static two-boson potential $V_{2g}(q)$.

From the general structure of the fermion matrix element in eq. (4) one can see that there are two fundamental s-states (with quantum numbers ($J^\pi = 1^-$)) with scalar and vector boson wave functions $w_{s,v}(r)$ and corresponding fermion wave functions[3] $\psi_{s,v}(r)\sim w_{s,v}(r)$ normalised by $4\pi\int r^2dr\ \psi_{s,v}(r) = 1$. Further, there are 0^+ states with p-wave functions $\psi_{L=1}(r)$, which are not considered in the present paper.

[1] leading to boson (quasi) densities $W^2(q)$ with dimension $[GeV]^2$.
[2] in a $(t,\vec{r})$ representation
[3] for the radial wave functions $\bar{\psi}(r) = \psi(r)$.

To evaluate the potentials $V_{3g}^{s,v}(r)$, the boson wave functions $w_{s,v}(r)$ have to be determined. To achieve this, a geometric boundary condition can be formulated by requiring that the interaction takes place inside the volume of the strongest bound state. As a consequence, the corresponding boson-exchange potential (6) should be proportional to the density $w_s^2(r)$, leading to

$$c\ w_s^2(r) \sim |V_{3g}^v(r)|\ . \tag{8}$$

Another condition is orthogonality of the fermion wave functions. Both conditions can be satisfied by boson wave functions $w_s(r)$ and $w_v(r)$, which are approximated by

$$w_s(r) = w_{s_o}\ exp\{-(r/b)^\kappa\} \tag{9}$$

and

$$w_v(r) = w_{v_o}\ [w_s(r) + \beta R\ \frac{dw_s(r)}{dr}]\ , \tag{10}$$

where $w_{(s,v)_o} = [2\pi \int rdr\ w_{s,v}^2(r)]^{-1}$ and $\beta R = -\int r^2 dr\ w_s(r)/\int r^2 dr\ [dw_s(r)/dr]$ (elimination of spurious bosonic motion, $< r_{w_s} >= 0$). The boson-exchange interaction $v_v(r)$ is given by $v_v(r) = -\hbar\ w_v(r)$.

To generate a stable bound state, the potential $V_{3g}^{s,v}(r)$ is not sufficient to keep the bosons confined. The other terms of the Lagrangian (2) show kinematic situations, in which bosons and/or fermions are in motion. Nevertheless, term 6 may be written in the form $\mathcal{L}_{1,6} = -\frac{ig^2}{\tilde{m}^2}\bar{\Psi}\ \gamma_\mu A^\mu(\partial_\nu A^\nu)\Psi - \frac{ig^2}{\tilde{m}^2}\bar{\Psi}\ \gamma_\mu A^\mu A_\nu\ \partial^\nu\Psi$ and gives rise to another bound state potential.

The first term of $\mathcal{L}_{1,6}$ leads to

$$\mathcal{M}_{2g} = \frac{\alpha^2}{\tilde{m}^8}\bar{\psi}(p')\gamma_\mu A^\mu(q)(\partial_\nu A^\nu(q))\ \gamma_\rho A^\rho(q)(\partial_\sigma A^\sigma(q))\psi(p)\ . \tag{11}$$

Using the gauge condition we can write $(\partial_\nu A^\nu(q))(\partial_\sigma A^\sigma(q)) = \frac{1}{2}\partial_\nu[\partial_\sigma(A_\mu A^\mu)^\sigma]^\nu$. After contracting the γ-matrices and reducing the fermion and boson vectors by one dimension as discussed for $\mathcal{M}_{3g}$, this yields

$$\mathcal{M}_{2g} = \frac{\alpha^2}{2\tilde{m}^6}\bar{\psi}(p')\ w_s(q)\partial^2 w_s(q)\ \psi(p)\ . \tag{12}$$

Since the two bosons are bound, see eq. (7), $\partial^2 w_s(q)/2\tilde{m}$ is related to their kinetic energy distribution. According to the virial theorem this implies also the existence of a static two-boson potential $V_{2g}(q)$.

In a transformation to r-space the bosonic part of eq. (12) gives rise to a Hamiltonian of the form

$$-\frac{\alpha^2\tilde{m} < r_{w_s}^2 > F_{2g}}{4}\ \Big(\frac{d^2 w_s(r)}{dr^2} + \frac{2}{r}\frac{dw_s(r)}{dr}\Big) + V_{2g}(r)\ w_s(r) = E_i\ w_s(r)\ , \tag{13}$$

where the factor F_{2g} is due to the Fourier transformation of the boson kinetic energy, $< r_{w_s}^2 >$ the radius square of the boson density and $w_s(r)$ the Fourier transform of $w_s(q)$. The potential $V_{2g}(r)$ is given by

$$V_{2g}(r) = \frac{\alpha^2\tilde{m} < r_{w_s}^2 > F_{2g}}{4}\ \Big(\frac{d^2 w_s(r)}{dr^2} + \frac{2}{r}\frac{dw_s(r)}{dr}\Big)\frac{1}{w_s(r)} + E_o\ , \tag{14}$$

where $E_o = 0$ is used to make a connection to the vacuum (state without binding between the quantons and therefore $E_{vac} = 0$). A similar potential involving $w_v(q)$ deduced from $\mathcal{L}_{1,7}$ yields negligible contribution to the binding energy. All other terms of the Lagrangian (2) do not contribute to bound state potentials.

The implications of using massless fermions are very strong and can be summarized as follows: First, the vacuum of the theory is the absolute vacuum with average energy $E_{vac} = 0$. This is consistent with the low energy density of the universe deduced from astrophysical observations. Second, the lowest energy solution in $V_{2g}(r)$ is the vacuum and therefore $E_o = E_{vac} = 0$. By this condition the absolute height of $V_{2g}(r)$ is fixed. Third, by rewriting eq. (12) in the form $\mathcal{M}_{2g} = \frac{\alpha^2}{2\tilde{m}^6}w_s(q)\{\bar{\psi}(p')\psi(p)\}\partial^2 w_s(q)$, one can see that fermion-antifermion pairs can be created during the dynamical overlap of two fluctuating boson fields. By this mechanism stable particles can be created out of the absolute vacuum. These facts are consistent with the requirement for a fundamental theory.

An important fact is that $V_{2g}(r)$ can be identified with the confinement potential in hadron potential models [10]. This will be shown in a comparison with the confinement potential from lattice QCD simulations [11] and the discussion of quark masses.

$V_{2g}(r)$ can also be written in a different form

$$V_{2g}(r) = \frac{\alpha^2 \hbar^2 F_{2g}}{4\tilde{m}} \Big(\frac{d^2 w_s(r)}{dr^2} + \frac{2}{r}\frac{dw_s(r)}{dr}\Big) \frac{1}{w_s(r)} + E_o \ . \tag{15}$$

This leads to the condition

$$Rat = \frac{\hbar^2}{\tilde{m}^2 < r^2_{w_s} >} = 1 \ . \tag{16}$$

A last constraint is related to energy-momentum conservation in relativistic systems, indicating that for binding in $V^s_{3g}(r)$ the total energy of the system is not increased, the negative fermion and boson binding energies E^s_f and E_g have to be compensated by the root mean square momenta of the corresponding potentials

$$< q^2_{V_{3g}} >^{1/2} + < q^2_{v_v} >^{1/2} = -(E^s_f + E_g) \ . \tag{17}$$

However, for the confinement potential $V_{2g}(r)$ this condition is not valid. Therefore, the constraint (17) can be applied only for the binding potentials $V(q) = V_{3g}(q)$ and $v_v(q)$ with $< q^2_V >= f_{red} \int dq\ q^3 V(q)/ \int dq\ qV(q)$ and $f_{red} = E^s_f/(E^s_f + E_g)$.

The fermion mass of the system is defined by the energy to balance binding

$$M^{s,v}_n = -E^{3g}_{f_{s,v}} + E^{2g}_{f_n} \ , \tag{18}$$

where $E^{3g}_{f_{s,v}}$ is the negative binding energy in $V^{s,v}_{3g}(r)$ (for these potentials only the lowest state is discussed here) and $E^{2g}_{f_n}$ are positive binding energies for different (excited) states in $V_{2g}(r)$. This shows two types of mass generation, binding in the Coulomb like potential $V_{3g}(r)$ and dynamical mass generation in $V_{2g}(r)$.

In the whole formalism there are finally four constraints, orthogonality of the total wave functions and the boundary conditions (8), (16) and (17), by which **all** open parameters - shape parameter κ, slope (or size) parameter b and the coupling constant α - are determined within rather small ambiguities. In addition, the different flavour states in the quark model[4] can be related by a vacuum sum rule similar to that applied in ref. [5], which indicates that in principle a complete solution of the relativistic bound state problem for all states is achieved. Below it will be shown that the need for massless elementary fermions in the present formalism is entirely consistent with the requirement of finite quark masses in the SM.

3. Application to $q\bar{q}$ mesons

Within the above formalism calculations have been performed for $q\bar{q}$ mesons of different flavour structure, $\omega(782)$, $\Phi(1020)$, charmonium $J/\Psi(3097)$ and bottonium $\Upsilon(9460)$ including excited states. The potentials $V_{3g}(r)$ and $V_{2g}(r)$ have been determined by adjusting all open parameters, κ, slope parameter b and the coupling constant α, to fulfill the constraints discussed above. Remaining uncertainties have been reduced by fine-adjustment of the factor F_{2g} in the confinement potential $V_{2g}(r)$ to fit the spectrum of radial excitations.

Results on the radial dependence of densities and potentials are given in fig. 1 for the $\omega(782)$ system. In the upper part the interaction $v_v(r)$ is given by the solid line. Compared to a Coulomb like potential $v_{coul}(r) \sim \hbar/r$ (dot-dashed line) there are no divergencies for $r \to 0$ and ∞, in agreement with the demand of a finite theory.

In the middle part a comparison of the density $w^2_s(r)$ (dot-dashed line) with the potentials $V^s_{3g}(r)$ (dashed line) and $V^v_{3g}(r)$ (solid line) is made. We see that condition (8) for the vector potential is reasonably well fulfilled at larger radii. This indicates that the bosonic wave functions $w_s(r)$ and $w_v(r)$ are reasonably well described by the forms in eq. (9) and (10).

[4]the notion of flavour from the quark model is kept in the present approach to characterise systems of different slope parameter b.

Table 1. Results for mesonic systems, $\omega(782)$, $\Phi(1020)$, $J/\psi(3097)$, and $\Upsilon(9460)$ including excited states, in comparison with the data [3]. Masses and binding energies are given in GeV, b in fm, and the mean radius squares in fm^2. α_{eq} is the equivalent coupling constant in a Coulomb like potential.

System	M_1^s	M_2^s	M_3^s	M_4^s	M_1^v	M_1^{exp}	M_2^{exp}	M_3^{exp}
ω	0.78	1.42	1.93		1.36	0.782	1.42±0.03	
Φ	1.02	1.68	2.20		2.04	1.02	1.68±0.02	
J/ψ	3.10	3.69	4.16	4.58	9.63	3.097	3.686	4.16±0.02
Υ	9.46	10.02	10.46	10.8	27.2	9.46	10.023	10.355

System	κ	b	α	α_{eq}^*	E_g	E_{3g}^s	m_{quark}	$< r_{w_s}^2 >$
ω	1.4	0.589	0.65	0.05	-0.026	-0.013	0.0065	0.256
Φ	1.4	0.450	1.50	0.59	-0.43	-0.216	0.108	0.150
J/ψ	1.4	0.148	2.40	2.42	-4.82	-2.42	1.21	0.016
Υ	1.4	0.049	2.59	2.89	-18.4	-9.0	4.5	0.0017

* $\alpha_{eq} = \sum_{s,v} \int dr\ V_{3g}^{s,v}(r) / \int dr\ V_{coul}(r)$

In the lower part of fig. 1 the deduced confinement potential $V_{2g}(r)$ is shown. It is characterized by a close to linear form at larger radii, as expected from ref. [10, 11]. Resulting masses and parameters for different systems are given in table 1. Although the binding energies are quite different, in all cases a satisfactory agreement of the various quantities is obtained, which fulfill all boundary conditions. A similar plot as in fig. 1 is shown for the bottonium system in fig. 2. Apart from a very different radial extent of the two systems the only important difference is the relative size of the confinement potential, which is drastically reduced for the heavy system due to a very different dynamics.

It should be realised that the different systems in table 1 correspond to different flavours (in the quark model). Here they are characterised only by a different slope parameter b. This shows that the flavour degree of freedom of hadrons is naturally included in the present formalism. As detailed in ref. [5] for states in light atoms, the different flavour states can be related by a vacuum potential sum rule. A preliminary analysis has also been performed for leptons [12], in which the flavour degree of freedom could be described similarly. The direct consequence of these results is that a description of the flavour degree of freedom does not require another explanation, as the existence of supersymmetric particles proposed in supersymmetric extensions of the SM, see ref. [3].

A comparison of the deduced confinement potential $V_{2g}(r)$ with the lattice QCD simulations of Bali et al. [11] (solid points with error bars) is shown in fig. 3. This potential has the same form as the confinement potential $V_{conf}(r) \sim -\alpha/r + l \cdot r$ deduced from potential models [10]. The fact that very similar results are deduced from theories with and without colour indicates clearly that the confinement of hadrons is not due to colour (as assumed in ref. [6] without clear understanding of the mechanisms involved), but represents a general property of relativistic bound states.

The question of a vector or scalar structure of the confinement potential can be studied by looking at the splitting of p-wave states in charmonium and bottonium, see ref. [13]. From the existing data neither a vector nor a scalar structure is found, supporting strongly a derivative structure of the potential $V_{2g}(r)$, as found in the present approach.

In a rather fundamental description of the strong interaction one should be able to derive also information on the dynamics of the systems in question, related to mass distributions and decay probabilities. Experimentally, the widths of the considered mesonic states are quite different, 8.4 MeV for $\omega(782)$, 4.3 MeV for $\Phi(1020)$, ~87 keV for $J/\psi(3097)$, and ~53 keV for $\Upsilon(9460)$, see ref. [3]. The small widths of these states is explained in the quark model by a different quark structure involved in heavy quarks, which cannot decay into light quarks. In the present model the width is related to the dynamical structure of the system, given by kinetic energy distributions $T_{2g}(q)$ and $T_{3g}(q)$, which are the Fourier transforms of the kinetic energy distributions $T_{2g}(r)$ and $T_{3g}(r)$, which are given by $T_{2g}(r) = V_{2g}(r)$ and $T_{3g}(r) = \frac{1}{2} < r^2 > (d^2V_{3g}(r)/dr^2 + \frac{2}{r}\ dV_{3g}(r)/dr)$.

The mass distributions $T_{3g}(q)$ calculated from the boson-exchange potential $V_{3g}(r)$ are rather bround, with a width of about 1 GeV for $\omega(782)$ and 3 GeV for $J/\psi(3097)$. Very different, the mass distributions $T_{2g}(q)$ obtained from the confinement potential $V_{2g}(r)$ are extremely narrow and may be far below the experimental widths. Unfortunately, in numerical Fourier transformations of $V_{2g}(r)$ oscillations appear, which can be damped only in a radial interpolation of logarithmic form, but in addition an extremely fine grid and expansion up to large radii is needed to obtain a peak comparable to the experimental width. Therefore, this problem should be studied in detail, possibly using better Fourier transformation techniques.

4. Discussion of quark masses

An important point is the need for finite quark masses in the SM (QCD), which should be understood in the present more fundamental approach. These masses have been estimated in different models, as e.g. in QCD inspired potential models [10] (more details can be found in ref. [3]). The empirical form $V_{conf}(r) \sim -\alpha/r + l \cdot r$ assumed in these models is consistent with $V_{2g}(r)$; therefore, the quark masses can be related directly to the binding energy in $V^s_{3g}(r)$. This leads to the relation[5]

$$m_{quark} = -\frac{1}{2}\ E^s_{3g}\ . \tag{19}$$

The resulting quark masses are given in table 1 and are compared with the extracted masses [3] in fig. 3. An excellent agreement is obtained. This is clear indication that the need for massive fermions (quarks) in the first order theory (QCD) is perfectly consistent with the assumption of massless elementary fermions in the present approach. Thus, the quarks can be understood as **effective fermions** with masses related to the binding energy in the bound state potential $V^s_{3g}(r)$.

Together with the results above this shows that both the flavour degree of freedom of hadrons as well as the quark masses are well understood in the present description. Therefore, there is no need to engage other mechanisms for their interpretation. In particular, the Higgs-mechanism (which demands an extra background field [14] adopted from the theory of supraconducters and ferromagnets in solids) is not needed. Similarly, also supersymmetric extensions of the SM, which predict a new regime of super-symmetric particles at high energies, are needless. This confirms the general view that a fundamental theory must have a very simple symmetry structure.

5. Conclusion

Although the SM yields an excellent description of many particle properties, it is an effective theory with parameters (as the quark masses), which are not understood. To get a correct insight into the nature of these parameters, a fundamental theory of hadrons is required, in which all parameters and assumptions of the effective theory can be understood. In the present approach the long standing relativistic bound state problem as well as the confinement problem is solved. The obtained results give clear insight into the mechanisms, in which bound states of relativistic particles are formed, with the surprising results that in a rather fundamental descriptions of hadrons the colour degree of freedom as well as Higgs and supersymmetric fields are not needed. This opens up the way to a new understanding of the structure of fundamental forces.

[5]this expression is independent of using an Abelian or non-Abelian structure of the Lagrangian. In an Abelian theory massive "quarks" have the same charge as the quantons in eq. (1).

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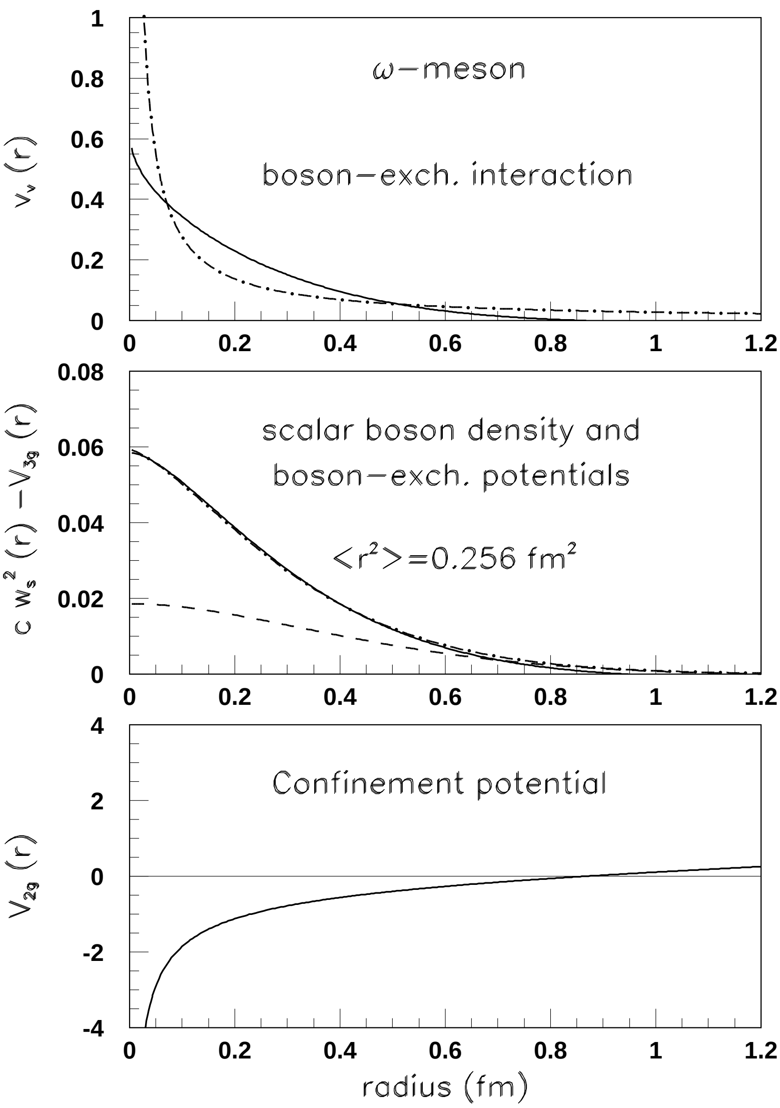

Figure 1. Self-consistent solution for the $\omega(782)$ meson system. Upper part: Interaction $w_v(r)$ in comparison with the Coulomb potential, given by solid and dot-dashed lines, respectively. Middle part: Bosonic density $w_s^2(r)$ and potential $|V_{3g}^v(r)|$ given by the overlapping dot-dashed and solid lines, respectively, matched by the condition (8); $|V_{3g}^s(r)|$ is shown by dashed line. Lower part: Deduced confinement potentials $V_{2g}(r)$.

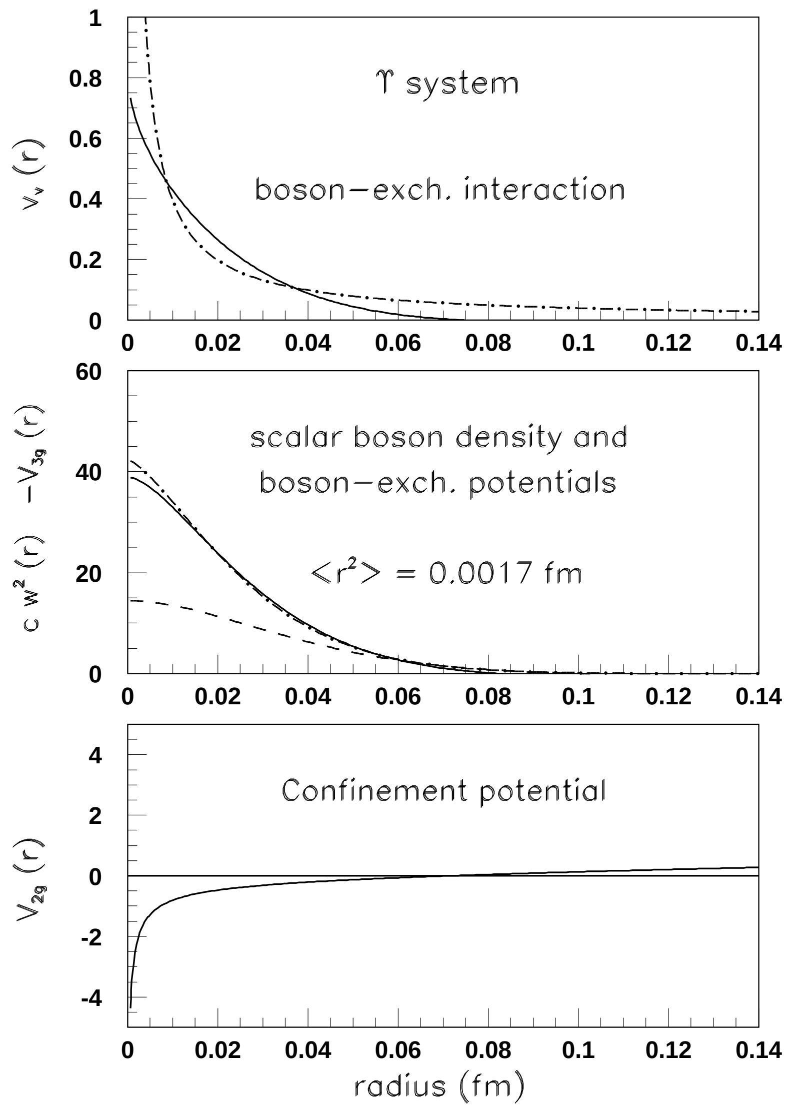

Figure 2. Same as fig. 1 for the bottonium system ϒ(9460).

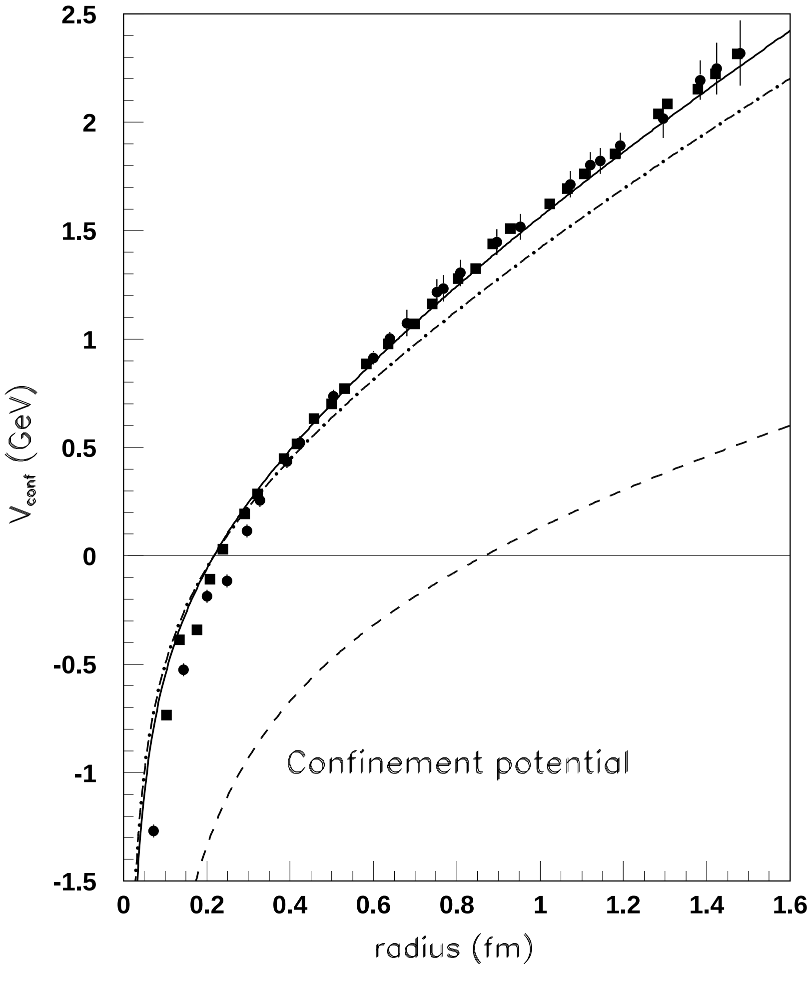

Figure 3. Comparison of the confinement potential with lattice QCD calculations. $V_{2g}(r)$ calculated for the two mesonic systems $\omega(782)$ and charmonium J/ψ, given by dashed and dot-dashed line, respectively. The latter, multiplied with a factor 1.1 (solid line) shows an excellent agreement with lattice gauge simulations [11] (solid points).

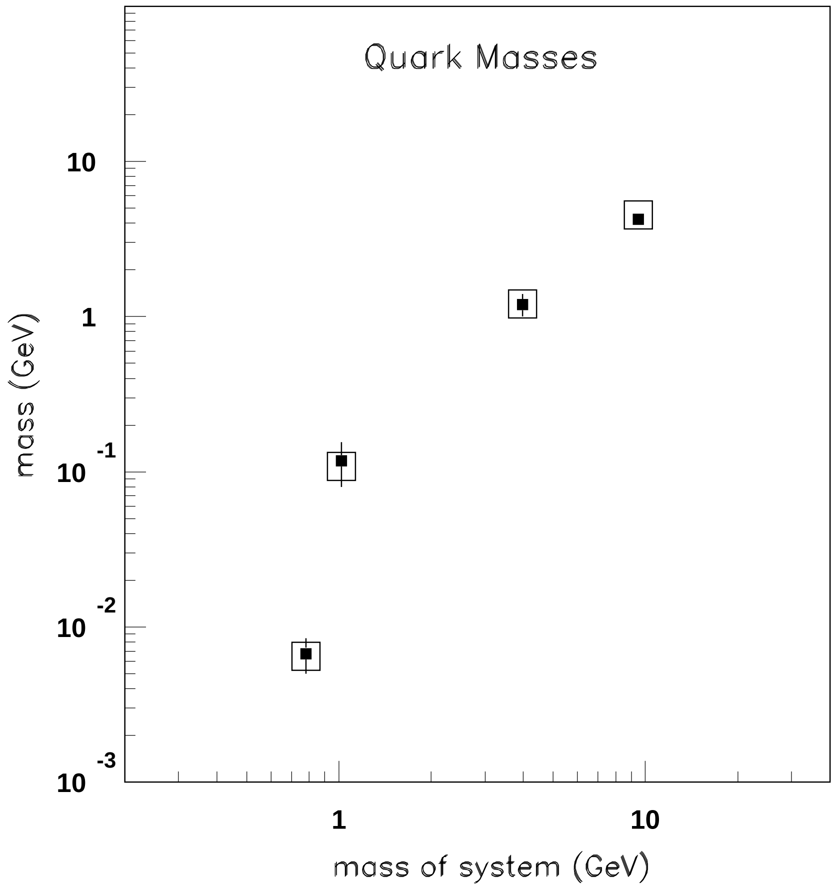

Figure 4. Quark masses as a function of the g.s. masses. The open squares show the present results using eq. (19), the solid squares with error bars give the extracted values from other sources, see ref. [3].