Universal Journal of Physics and Application 1(2): 71-82, 2013 http://www.hrpub.org
DOI: 10.13189/ujpa.2013.010204

Non dynamical Quantitative Approach to Nuclear Stability Based upon Nucleon and Quark Content of Nuclei

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Abstract In this paper a new quantitative explanation is presented for nuclear stability based upon the quark content of nuclei. At Hagedorn temperature, considering the freely interacting up and down quarks inside nuclei during nucleon formation, by counting the number of possible nucleon formations(NWN), it is concluded that the most probable abundance (stability) occurs at N=Z without Coulomb interaction. Adding Coulomb effect before and after nucleon formation, results in an amount of deviation from N=Z, which is in excellent agreement with experimental stability chart of nuclei.

Keywords Nuclei, Stability, Quark

PACS 21. 65. Qr, 12. 38. Mh

1. Introduction

Nuclear stability is described in the chart of stable nuclide and is shown in Fig. (1). It can be seen in this chart that the stable nuclei and their isotopes locate around N=Z line and more deviation is observed as the nuclei become heavier. Such a deviation is due to the fact that more numbers of neutrons are required to compensate the Coulomb repulsion between protons [1, 2]. In nuclear shell model, which is the most successful existing model in predicting the spin, parity and other characteristics of nuclei, based upon Pauli Exclusion Principle, each energy state is filled with 2 protons and 2 neutrons due to nucleon spin directions. Therefore it is expected that the nuclei have the same number of protons and neutrons in the ground state [3, 4]. In this model the deviation from N=Z line is attributed qualitatively to the Coulomb force and no quantitative relation is presented between number of protons and neutrons in stable nuclide. For example why there are 118 neutrons and 79 protons in the gold nuclei and not other possible ratios.

In the liquid drop model, which gives the best result for the binding energy and nuclear stability, there is a quantitative explanation for such natural distribution of nucleons inside nuclide. In this model in addition to the volume, surface and pairing effects, however the Coulomb and asymmetry terms are introduced as two terms in the binding energy expression in a phenomenological manner, in the context of liquid drop model assumptions. The constants corresponding to each term are determined experimentally. Therefore more accurate measurements may result in a new set of constants [5, 6].

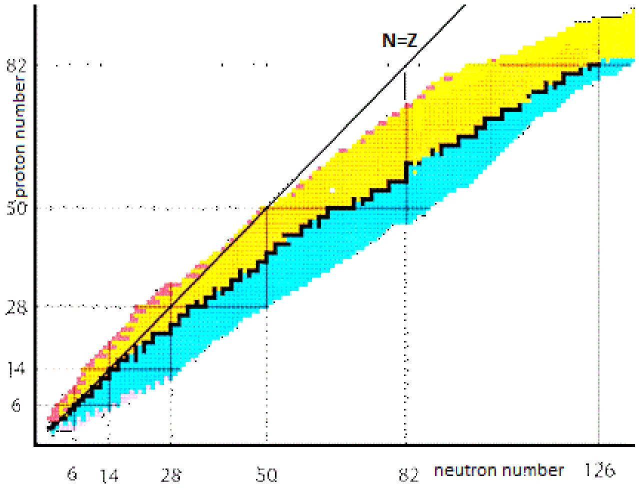

Figure 1. chart of nuclides

In our approach the constituent quarks of nucleons are not primarily bounded at above Hagedorn temperature but act as freely interacting particles, participating in the nucleon formation. If the quark-gluon plasma (QGP) is considered as a thermodynamical media then it should proceed toward maximum disorder. It should be investigated how such a system approaches equilibrium. The thermodynamical state is a stable system with a maximum probability state, i.e., the most probable state with a maximum number of complexions. In this research paper, the nuclear stability concept is extended from the nucleons to the quarks properties. Alone this is a point of duality since the nuclei show properties of the nucleons as will also be explained here by reviewing a model based on quarks statistics, while the new results to be reported here show properties which are determined by quarks in nuclei as if the nucleons are fully split into their quark constituents. It should be noted that in this model, the most stable nuclei occur around N=Z. By considering the Coulomb effect before and after nucleon formation (at about 160 Mev), the amount of deviation from N=Z line is determined to be in an excellent agreement with the experimental data. This model has been able to determine all magic numbers in a natural manner and predict a new magic number namely 184 [7]. In addition to magic numbers, the nuclear binding energy is also given in a simple expression in terms of A and Z and up quark mass [8]. In our previous article, using quark model the existing amount of deviation between theoretical and experimental values of deuteron magnetic dipole moment is significantly improved [9].

2. Nuclear Stability around N=Z line

It is believed that, the hot quark-gluon plasma (QGP) existed right after big bang and by relativistic expansion cool down and cluster to form protons and neutrons. In the continued process of expansion, nucleation takes place and different nuclei are formed. Let us assume quarks inside nuclei are at first, free interacting particles participating in the nucleon formation. First let us consider a nucleus with only two nucleons inside. This nucleus is made of 6 quarks. If both nucleons are protons, then there are 4 up and 2 down quarks involved. If we denote them by d_1, d_2, u_1, u_2, u_3 and u_4, then the number of ways two protons are formed is given as follow:

$$\{(u_1u_2d_1)(u_3u_4d_2)\},\{(u_1u_2d_2)(u_3u_4d_1)\},$$
$$\{(u_1u_3d_1)(u_2u_4d_2)\},\ \{(u_1u_3d_2)(u_2u_4d_1)\},$$
$$\{(u_1u_4d_1)(u_2u_3d_2)\},\{(u_1u_4d_2)(u_2u_3d_1)\} \tag{1}$$

Each ( ) denotes a nucleon and { } denotes a nucleus. So there are only 6 ways (color will be considered later). Now let us consider nuclei with one proton and one neutron. This case involves 3 up and 3 down quarks, namely, d_1, d_2, d_3, u_1, u_2 and u_3. In this case we obtain 9 different ways of nucleon formation as follow:

$$\{(u_1u_2d_1)(u_3d_2d_3)\},\{(u_1u_2d_2)(u_3d_1d_3)\},\{(u_1u_2d_3)(u_3d_1d_2)\},$$
$$\{(u_1u_3d_1)(u_2d_2d_3)\},\ \{(u_1u_3d_2)(u_2d_1d_3)\},\ \{(u_1u_3d_3)(u_2d_1d_2)\},$$
$$\{(u_2u_3d_1)(u_1d_2d_3)\},\{(u_2u_3d_2)(u_1d_1d_3)\},\{(u_2u_3d_3)(u_1d_1d_2)\} \tag{2}$$

Now if the color of quarks is considered, each nucleon is formed in 3 different ways. Therefor there are 6×3^2 number of ways to form 2 neutrons or 2 protons and 9×3^2 number of ways to form one neutron and one proton. As can be seen, the process of nuclei formation for one proton and one neutron is more probable than for 2 protons or 2 neutrons. Since the color factor only multiplies the "number of ways" by a constant, it is neglected here.

Now let us consider a nucleus with 3 nucleons and calculate the number of ways of nuclei formation (NWN);

a. Nuclei with 3 protons (6 up and 3 down quarks) then,

$$NWN = \left[\binom{6}{2}\times 3\right]\left[\binom{4}{2}\times 2\right]\left[\binom{2}{2}\times 1\right]\frac{1}{3!} = 90 \tag{3}$$

In Eq. (3) the combination relation $\binom{m}{n} = \frac{m!}{n!(m-n)!}$ is used and the factor $\left[\binom{6}{2}\times 3\right]$ is for the number of ways of the first nucleon and $\left[\binom{4}{2}\times 2\right]$ is the number of ways for the second nucleon formation and so on. The factor $\frac{1}{3!}$ stands for the indistinguishability of protons.

b. Nuclei with 2 protons and one neutron (5 up and 4 down quarks)

$$NWN = \left[\binom{5}{2}\times 4\right]\left[\binom{3}{2}\times 3\right]\left[1\times\binom{2}{2}\right]\frac{1}{2!\times1!} = 180 \tag{4}$$

In Eq. (4) the factor $\frac{1}{2!\times1!}$ stands for the indistinguishability of protons and neutrons.

c. For nuclei with one proton and 2 neutrons,
NWN=180

d. Nuclei with 3 neutrons,
NWN=90

In general the following formula provides the number of ways of nuclei formation for Z protons and A-Z neutrons,

$$NWN = \left[\binom{A+Z}{2}(2A-Z)\right]\left[\binom{A+Z-2}{2}(2A-Z-1)\right]\cdots\left[\binom{A-Z+2}{2}(2A-2Z+1)\right]\times\left[(A-Z)\binom{2A-2Z}{2}\right]\Big[(A-$$
$$Z{-}12A{-}2Z{-}22\cdots122\times1Z!A{-}Z! \tag{5}$$

In a more compact form, Eq. (5) is given as:

$$NWN = \frac{(A+Z)!(2A-Z)!}{(A-Z)!2^A(Z)!} \tag{6}$$

Since the relative magnitude of this quantity is important, we can divide it by NWN (Z=0) and calculate the following quantity:

$$\frac{NWN}{NWN\ (Z=0)} = \frac{(A+Z)!(2A-Z)!}{(A-Z)!(Z)!(2A)!} \tag{7}$$

In nuclei with the same A, the number of possible ways, are an indication of their possibility to exist namely, their abundance. The calculated numerical values of Eq. (7) for several nuclei are given in appendix.

3. Calculation of Coulomb Effect on Nuclear Stability

The effect of electromagnetic interaction among the quarks is considered in two different time intervals. a) During the nuclei formation, b) After nuclei is formed

a. Before nucleon formation quarks interact with each other to form a nucleon. Neutron is a neutral particle and Coulomb force between two oppositely charged particles always agree to form a neutral one, therefore Coulomb force is in the same direction as the strong force. But for proton since it is positively charged, the Coulomb force opposes the strong force. Therefore it should be subtracted from the strong force. Also Coulomb force is $\frac{1}{137}$ weaker than the strong force. Therefore for neutron formation a factor of $\left(1+\frac{1}{137}\right)$ and for proton formation a factor of $\left(1-\frac{1}{137}\right)$ should be multiplied.

Now let us consider the color factor. Since each nucleon can be formed in three different ways (for example in case of proton there are $u_r u_g d_b$, $u_g u_b d_r$, $u_b u_r d_g$ where r, b and g stands for red, blue and green color), the proton formation has a factor of $\left(1-\frac{1}{137}\right)^3$ and for neutron formation a factor of $\left(1+\frac{1}{137}\right)^3$ is multiplied and Eq. (7) is then generalized to

$$NWNC1 = \frac{(A+Z)!(2A-Z)!}{(A-Z)!Z!(2A)!}(1-\alpha)^Z(1+\alpha)^{A-Z} \tag{8}$$

Where $\alpha = \frac{1}{137}$

b). Now let us consider the nucleons are formed in a nuclei. The Coulomb force still exists between protons and strong force also exists between nucleons and has impact upon the stability of nuclei. In order to calculate the electromagnetic and strong nuclear force the following assumptions are made:

1. The electromagnetic force is long-ranged and the strong force between nucleons is short-ranged.
2. The strength of the electromagnetic force is α times strong force (where, $\alpha = \frac{1}{137}$).
3. The nucleons inside nuclei have an ordered lattice and each nucleon interacts in strong force only with neighboring nucleons.

Let us consider a lattice for nucleon similar to carbon atoms in a diamond lattice as shown in figure 2.

The number of interactions of each nucleon with other nucleons in electromagnetic and strong cases are counted and calculated. In order to achieve that, let us consider a nuclide with Z protons. Each proton interacts with Z-1 other protons via electromagnetic force, therefore the mutual number of interactions are $\frac{Z-1}{2}$. Moreover each proton interacts with four neighboring nucleons strongly, therefore the number of strong interactions are $\frac{4}{2} = 2$. So each proton causes the relation (8) to be multiplied by a factor $\left(2-\alpha\frac{Z-1}{2}\right)$ and for Z protons a by factor of $2^Z\left(1-\frac{\alpha(Z-1)}{4}\right)^Z$. For each neutron since there is no electromagnetic interaction, only the 2^N factor is multiplied. Therefore for all nucleons a factor of $2^A\left(1-\frac{\alpha(Z-1)}{4}\right)^Z$ is multiplied by equation (8). Since the factor 2^A is a constant for all nuclei with the same A, it can be neglected and the following relation is obtained

$$NWNC = \frac{(A+Z)!(2A-Z)!}{(A-Z)!Z!(2A)!}(1-\alpha)^Z(1+\alpha)^{A-Z}\left(1-\frac{\alpha(Z-1)}{4}\right)^Z \tag{9}$$

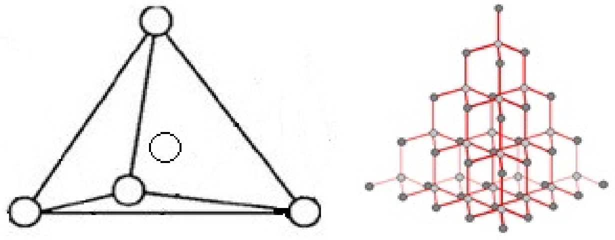

Figure 2. Sample of ordered lattices formed by nucleons inside nuclei

Now for a constant A, the maximum value of NWNC give us the most stable nuclei. The results of our calculations for several nuclei are given in appendix. For mass numbers up to 150, our findings are in good agreement with existing stable nuclei in nature. For heavy nuclei (A > 150), since the radius of nuclei increases, the electromagnetic effect is expected to decrease. Therefore a radius dependent coefficient of form C(r) is introduced in Eq. (9) and the multiplying factor becomes $\left(1-C(r)\frac{\alpha(Z-1)}{4}\right)^Z$. The coefficient c(r) is an indication of the dependency of the electromagnetic interaction upon nuclear radius. This coefficient is less than one for heavy nuclei. Our investigation indicates that c(r) is function of r^{-1}. Let us write, $C(r) = \left(\frac{120}{A}\right)^{\frac{1}{3}}$, where 120 stands for a medium nuclei mass number. It is noted that our results are not so sensitive to the choice of 120, it can be selected between 110 to 140 but 120 gives the best fit. In liquid drop model of the nucleus, the Coulomb term in binding energy is also given in terms of the inverse of nuclear radius due to the dependency of Coulomb potential upon the inverse radius of the nuclei. Therefore, our results are sensitive to the nuclear radius. Then Eq. (9) becomes,

$$NWNC = \frac{(A+Z)!(2A-Z)!}{(A-Z)!Z!(2A)!}(1-\alpha)^Z(1+\alpha)^{A-Z}\Bigg(1- \quad 120A13\alpha Z{-}14Z \tag{10}$$

Our results for different stable nuclei are plotted in Fig. (3) where comparison are made with existing experimental data.

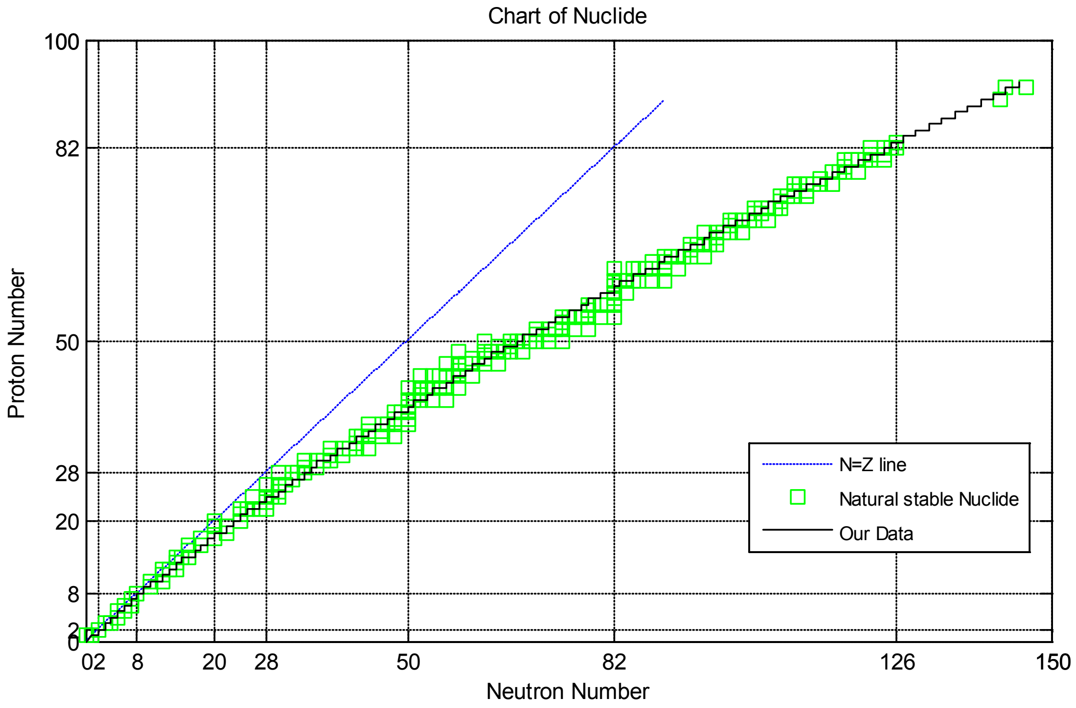

Figure 3. predictions of our data (based upon the relation (10)) for stable nuclides compared to natural stable nuclides

4. Conclusion

Our findings in figure 3 are in excellent agreement with the stable chart of nuclides found experimentally. It is shown in this article, that the concept of nuclear stability at room temperature is attributed to the quark properties at the Hagedorn temperature (below which quarks are bounded inside nucleons) in addition to the nucleon properties. Based upon quarks statistics, the maximum number of nucleon formation from constituent quark, is attributed to the maximum nuclear stability. This analysis along with our previous analysis of the origin of magic numbers and also the binding energy in terms of up quark mass and determination of nuclear magnetic dipole moment indicate that nuclear quark model in which nuclei is considered in terms of constituent quarks instead of constituent nucleons, might be able to provide a more natural and much simpler explanation about different aspects of nuclei such as stability, decay and magnetic dipole moment.

Appendix

Table of the number of ways of nucleon formation based upon the relations (7) and (9) for several nuclei

```
A           Z        N      NWN/NWN(Z=0)          NWNC
     [2]     [0]     [2]     [              1]     [              1]
     [2]     [1]     [1]     [ 1.5000e+000]     [1.4357e+000]
     [2]     [2]     [0]     [              1]       [9.5366e-001]

     [3]     [0]     [3]     [              1]     [              1]
     [3]     [1]     [2]     [              2]     [1.9143e+000]
     [3]     [2]     [1]     [              2]     [1.9073e+000]
     [3]     [3]     [0]     [              1]       [9.4671e-001]

     [4]     [0]     [4]     [              1]     [              1]
     [4]     [1]     [3]     [ 2.5000e+000]     [2.3929e+000]
     [4]     [2]     [2]     [ 3.2143e+000]     [3.0653e+000]
     [4]     [3]     [1]     [ 2.5000e+000]     [2.3668e+000]
     [4]     [4]     [0]     [              1]       [9.3636e-001]
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[5]     [0]     [5]     [              1]     [            1]
[5]     [1]     [4]     [              3]     [2.8714e+000]
[5]     [2]     [3]     [ 4.6667e+000]     [4.4504e+000]
[5]     [3]     [2]     [ 4.6667e+000]     [4.4180e+000]
[5]     [4]     [1]     [              3]     [2.8091e+000]
[5]     [5]     [0]     [              1]     [9.2272e-001]

[6]     [0]     [6]     [              1]     [            1]
[6]     [1]     [5]     [ 3.5000e+000]     [3.3500e+000]
[6]     [2]     [4]     [ 6.3636e+000]     [6.0687e+000]
[6]     [3]     [3]     [ 7.6364e+000]     [7.2294e+000]
[6]     [4]     [2]     [ 6.3636e+000]     [5.9587e+000]
[6]     [5]     [1]     [ 3.5000e+000]     [3.2295e+000]
[6]     [6]     [0]     [              1]     [9.0593e-001]

[7]     [0]     [7]     [              1]     [            1]
[7]     [1]     [6]     [              4]     [3.8286e+000]
[7]     [2]     [5]     [ 8.3077e+000]     [7.9227e+000]
[7]     [3]     [4]     [ 1.1538e+001]     [1.0924e+001]
[7]     [4]     [3]     [ 1.1538e+001]     [1.0804e+001]
[7]     [5]     [2]     [ 8.3077e+000]     [7.6657e+000]
[7]     [6]     [1]     [              4]     [3.6237e+000]
[7]     [7]     [0]     [              1]     [8.8616e-001]

[8]     [0]     [8]     [              1]     [            1]
[8]     [1]     [7]     [ 4.5000e+000]     [4.3072e+000]
[8]     [2]     [6]     [ 1.0500e+001]     [1.0013e+001]
  A   Z N      NWN/NWN(Z=0)            NWNC
[8]     [3]     [5]     [ 1.6500e+001]     [1.5621e+001]
[8]     [4]     [4]     [ 1.9038e+001]     [1.7827e+001]
[8]     [5]     [3]     [ 1.6500e+001]     [1.5225e+001]
[8]     [6]     [2]     [ 1.0500e+001]     [9.5123e+000]
[8]     [7]     [1]     [ 4.5000e+000]     [3.9877e+000]
[8]     [8]     [0]     [              1]     [8.6360e-001]

[9]     [0]     [9]     [              1]     [            1]
[9]     [1]     [8]     [              5]     [4.7857e+000]
[9]     [2]     [7]     [ 1.2941e+001]     [1.2341e+001]
[9]     [3]     [6]     [ 2.2647e+001]     [2.1440e+001]
[9]     [4]     [5]     [ 2.9441e+001]     [2.7568e+001]
[9]     [5]     [4]     [ 2.9441e+001]     [2.7166e+001]
[9]     [6]     [3]     [ 2.2647e+001]     [2.0517e+001]
[9]     [7]     [2]     [ 1.2941e+001]     [1.1468e+001]
[9]     [8]     [1]     [ 5.0000e+000]     [4.3180e+000]
[9]     [9]     [0]     [              1]     [8.3849e-001]

[10]     [ 0]     [10]     [              1]   [            1]
[10]     [ 1]     [ 9]     [ 5.5000e+000]     [5.2643e+000]
[10]     [ 2]     [ 8]     [ 1.5632e+001]     [1.4907e+001]
[10]     [ 3]     [ 7]     [ 3.0105e+001]     [2.8501e+001]
[10]     [ 4]     [ 6]     [ 4.3387e+001]     [4.0626e+001]
[10]     [ 5]     [ 5]     [ 4.8810e+001]     [4.5038e+001]
[10]     [ 6]     [ 4]     [ 4.3387e+001]     [3.9306e+001]
[10]     [ 7]     [ 3]     [ 3.0105e+001]     [2.6678e+001]
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[10]    [ 8]    [ 2]    [ 1.5632e+001]    [1.3499e+001]
[10]    [ 9]    [ 1]    [ 5.5000e+000]    [4.6117e+000]
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[11]    [ 1]    [10]    [           6]    [5.7429e+000]
[11]    [ 2]    [ 9]    [ 1.8571e+001]    [1.7711e+001]
[11]    [ 3]    [ 8]    [ 3.9000e+001]    [3.6922e+001]
[11]    [ 4]    [ 7]    [ 6.1579e+001]    [5.7660e+001]
[11]    [ 5]    [ 6]    [ 7.6632e+001]    [7.0710e+001]
[11]    [ 6]    [ 5]    [ 7.6632e+001]    [6.9423e+001]
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[11]    [ 8]    [ 3]    [          39]    [3.3680e+001]
[11]    [ 9]    [ 2]    [ 1.8571e+001]    [1.5572e+001]
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REFERENCES

[1] K.Krane, “Introductory Nuclear Physics”, John Wiley and Sons, New York (1988).

[2] R. Javaheri, the General Science journal, (2008).

[3] B. Alex Brown, “the Nuclear Shell Model Towards the Drip Lines”, Progress in Particle and Nuclear Physics 47, 517, (2001).

[4] C.B. Dover, A. Gal, J.M. Richard, Physics Letters B 344, 433-435, (1995).

[5] K. Pomorski and J. Dudek, Phys. Rev. C67, 044316 (2003).

[6] G. Royer and C. Gautier, Phys. Rev. C73, 067302 (2006).

[7] N. Ghahramany, H. Hora, G.H. Miley, M. Ghanaatian, M. Hooshmand, K. Philbert and F. Osman, Physics Essay21, 3, (2008).

[8] N. Ghahramany, Sh. Gharaati and M. Ghanaatian, Physics of Elementary Particles and Atomic Nuclei Theory8, 97-106, (2011).

[9] N. Ghahramany,E. Yazdankish, Commun. Theor. Phys. 59 (2013) 579–582.