

Performance Analysis of Finite Population Cellular System Using Channel Sub-rating Policy

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Abstract This paper illustrates a channel sub-rating allocation policy for finite population cellular system. In this policy, certain channels are permitted to transiently divide into two channels at half the original rate to adapt handoff calls. As a result, one half can be employed to serve the existing call and the other half to serve the handoff request so that we can get rid of forced termination of calls. We propose two system models based on fractional guard channel and sub-rating channel allocation policies with and without queueing of handoff calls which give a significant improvement in the QoS of the cellular network. New call blocking probability, handoff call forced termination probability and the degraded call quality are also investigated to examine the effects of the sub-rating on the QoS. Our policies have the least forced termination probability and the call incompleteness probability when compared with the other policies.

Keywords Finite Population, Sub-rating, Dropping Probability, Forced Termination, Channel Allocation, QoS

1 Introduction

In wireless network, call admission control plays a significant role in providing quality of service. In order to reduce the network congestion and call dropping we can limit the number of call connection in to the network by using channel allocation policies. A good channel allocation policy has to balance the handoff dropping and new call blocking in order to provide the desired Quality of Service (QoS) requirement [1-5].

Due to user's mobility, channel allocation policy becomes much more elaborated in wireless network. Base station is responsible for communication service to mobile users in its area called cell. When a new call originate in the current cell, if an idle channel is available, it will be allotted for communication between mobile user and the base station and a new call is admitted for service. If a channel can not be allotted to the new call, the new call may be blocked and cleared from the sys-

tem. When a new call requires a channel, it will hold the channel either until it is completed in the cell or until the mobile station moves out of the cell. If the mobile user does not finish the service in the originating cell, it will move out of the cell and attempt to acquire a channel in another cell. If a cell does not acquire either buffer or a channel, it will be forced to terminate. If the handoff is successful, then the call will continue in the new cell and the same procedure repeats.

In the guard channel policy, some channels are reserved for handoff calls. When there are no free channels available in the cell new call are blocked. When the handoff calls finds that all channels are filled, they are queued according to various queueing discipline and wait for an occupied channel to be free [3]. The existing policies can effectively reduce the handoff dropping probability but at the cost of increase of new call blocking probability. Channel assignment policy called the sub-rating is brought in to address handoff call problem in the personnel cellular system [6]. In this policy, certain channel are allowed to be temporarily divide in to two channels at half of the original rate to adapt handoff calls. When all channels are filled at the moment of handoff call arrival, this sub-rating policy will be activated. When a sub-rated channel is freed, it forms into an original full rated channel by combining with other sub-rated channel [7].

Using the channel sub-rating, it is shown that the forced termination probability is greatly reduced with a little increase of new call blocking probability [8]. Channel sub-rating is also applied for handoff calls in CDMA cellular system [9]. In all of the aforementioned studies, the channel sub-rating is only used without quality preference. The proposed policy which combines fractional guard channel and sub-rating with and without queueing of handoff calls gives a significant improvement in the QoS of the cellular network. New call blocking probability and handoff call forced termination probability are computed to evaluate the proposed call admission control policy. The degraded calls due to half rate channel is discussed to study the effect of the sub-rating on QoS.

The rest of this paper is organized as follows. Section 2 presents system model. Analysis of Finite Population Sub-Rating (FPSR) policy and performance indices are

given in Section 3. Section 4 gives analysis of Finite Population Sub-Rating (FPSR) Queueing policy. Numerical results are discussed in Section 5. Section 6 concludes the paper.

2 System Model

The proposed models consider channel allocation policy for a homogeneous cellular system with finite number of population. We focus our study to a single cell with predefined $C_h + C$ number of channels with population size K . When K is significantly larger than $C_h + C$, it can be assumed to be constant with little loss of accuracy. When all the channels busy, every cell has a facility for sub-rating channels to facilitate more number of handoff calls to continue. To make a proper tradeoff, new calls are given an acceptance priority after certain threshold channel C_h . In Finite Population Sub-Rating (FPSR) policy without queueing of handoff calls, new calls are accepted with probability β after threshold C_h . When all $C_h + C$ channels are filled then sub-rating of channel starts. Sub-rating policy blocks the new call attempts, but accommodate handoff calls if $C_h + C \leq i < C_h + 2C$. That is, when a handoff call arrives, sub-rating policy sub-rates a channel into two channels to accommodate handoff calls. If $i = C_h + 2C$, all C channels are sub-rated and new handoff calls are force terminated. When all the sub-rated channels are occupied, handoff calls queued in a finite size buffer in Finite Population Sub-Rating policy with queueing of handoff calls. The system model for FPSR channel allocation polices are depicted in Figure 1.

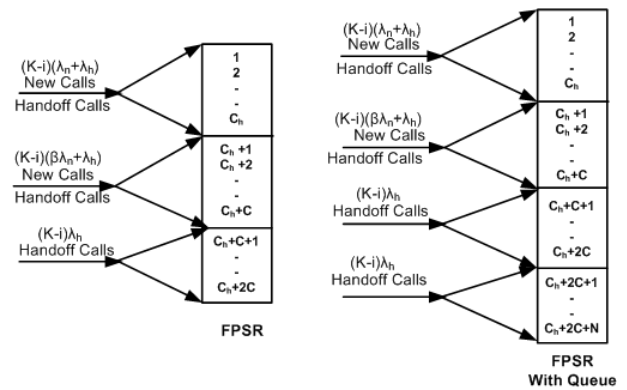


Figure 1. System model for channel allocation in FPSR policies.

son process with mean rate λ_n and λ_h , respectively. Duration of call is assumed to be exponentially distributed with mean $1/\mu$. Here both the type of calls are allowed up to C_h channels. Then new calls are accepted with probability β up to $C_h + C$ channels along with handoff calls. Further incoming handoff calls are allowed by sub-rating C number of channels but new calls are blocked. When a sub-rated channel released it combine with another sub-rated channel to form a complete channel. Let us assume the state of the target cell is j , where j represents the number of active users in the target cell. The effective incoming call rate is λ_j , where $0 \leq j < C_h + 2C$ is given by

$$\lambda_j = \begin{cases} (K-j)(\lambda_n + \lambda_h) & : 0 \leq j < C_h, \\ (K-j)(\beta\lambda_n + \lambda_h) & : C_h \leq j < C_h + C, \\ (K-j)\lambda_h & : C_h + C \leq j < C_h + 2C. \end{cases}$$

Algorithm 1 Algorithm for Channel Allocation Using FPSR Policy.

```

BC(t) is Number of Busy Channel at Time t.
if Handoff Call AND BC(t) < Ch + C then
    Allocate a Channel;
    BC(t)=BC(t)+1;
    K = K - 1;
else
    if ( BC(t)= Ch + C)
    AND (Sub-rate Channel < Ch + 2C) then
        Allocate a Channel;
        BC(t)=BC(t)+1;
        K = K - 1;
    else
        Drop the Call;
    end if
end if
if New Call AND BC(t) < Ch then
    Allocate a Channel;
    BC(t)=BC(t)+1;
    K = K - 1;
    if (BC(t) > Ch) AND (BC(t) < Ch + C) then
        Allocate a Channel with Probability β;
        K = K - 1;
    else
        Block the Call;
    end if
end if
    
```

3 Analysis of FPSR Policy

In FPSR policy we assumes that both new and handoff call are generated from K population according to Pois-

The channel allocation policy for FPSR policy is shown in Algorithm 1. Let P_j be the steady state probability that there are j channels busy and the service rate is $j\mu$. The state transition diagram that describes the system is shown in Figure 2. From the state transition diagram in Figure 2, the steady state probability P_j is determined as follows:

$$P_j = \begin{cases} \binom{K}{j} \left(\frac{\lambda_n + \lambda_h}{\mu} \right)^j P_0 & : 0 < j \leq C_h, \\ \binom{K}{j} \left(\frac{\lambda_n + \lambda_h}{\mu} \right)^{C_h} \left(\frac{\beta\lambda_n + \lambda_h}{\mu} \right)^{j-C_h} P_0 & : C_h < j \leq C_h + C, \\ \binom{K}{j} \left(\frac{\lambda_n + \lambda_h}{\mu} \right)^{C_h} \left(\frac{\beta\lambda_n + \lambda_h}{\mu} \right)^C \left(\frac{\lambda_h}{\mu} \right)^{j-C_h-C} P_0 & : C_h + C < j \leq C_h + 2C. \end{cases}$$

and using normalization condition $\sum_{i=0}^{C_h+2C} P_j = 1$, we get P_0 as

$$P_0 = \left[1 + \sum_{j=1}^{C_h} \binom{K}{j} \left(\frac{\lambda_n + \lambda_h}{\mu} \right)^j + \left(\frac{\lambda_n + \lambda_h}{\mu} \right)^{C_h} \times \left\{ \sum_{j=C_h+1}^{C_h+C} \binom{K}{j} \left(\frac{\beta\lambda_n + \lambda_h}{\mu} \right)^{j-C_h} + \sum_{i=C_h+C+1}^{C_h+2C} \binom{K}{j} \left(\frac{\beta\lambda_n + \lambda_h}{\mu} \right)^C \left(\frac{\lambda_h}{\mu} \right)^{j-C_h-C} \right\} \right]^{-1}. \quad (1)$$

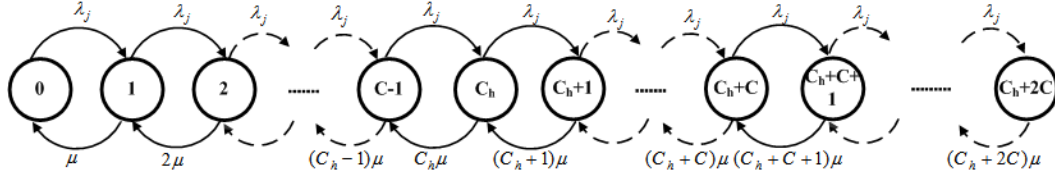


Figure 2. State transition model for FPSR policy.

3.1 Performance Indices

The blocking probability of a new call is the sum of the probabilities that the state number of the base station is larger than or equal to C . Hence

$$B_n = (1 - \beta) \sum_{j=C_h}^{C_h+C-1} P_j + \sum_{j=C_h+C}^{C_h+2C} P_j. \quad (2)$$

The dropping probability of handoff calls is given by

$$B_h = P_{C_h+2C}. \quad (3)$$

The expected number of busy channels is obtained as

$$E[N] = \sum_{i=1}^{C_h+2C} iP_i. \quad (4)$$

The degradation ratio of the call quality is the portion of the call holding time that the mobile user experiences the degraded call quality. The expected number of the sub-rated channels is given as the degradation ratio of the call quality and can be expressed as

$$E[D] = \sum_{i=C_h+C+1}^{C_h+2C} \frac{2(i - C_h - C)}{i} P_i. \quad (5)$$

4 Analysis of FPSR Queueing Policy

FPSR Queueing policy is an extension of FPSR policy. When all the sub-rated channels are occupied the incoming handoff calls are queued in a finite size queue N . The duration of call is assumed to be exponentially distributed with mean $1/\mu$. The residency time of a call in a queue is also exponentially distributed with mean $1/\mu_q$. The arrival request λ_j , of a call to the system at different states is given by

$$\lambda_j = \begin{cases} (K - j)(\lambda_n + \lambda_h) : 0 \leq j < C_h, \\ (K - j)(\beta\lambda_n + \lambda_h) : C_h \leq j < C_h + 2C, \\ (K - j)\lambda_h : C_h + 2C \leq j < C_h + 2C + N. \end{cases}$$

Let j as the state of the base station when j is the total number of channels used in the cell and the number of handoff calls present in the queue. The state transition diagram is shown in Figure 3. The FPSR Queueing channel allocation policy is shown in Algorithm 2.

Let P_j represent the steady state probability that the base station is in state j . Using the birth-death pro-

cesses, the steady state probability P_j is found to be

$$P_j = \begin{cases} \binom{K}{j} \left(\frac{\lambda_n + \lambda_h}{\mu} \right)^j P_0 & : 0 \leq j \leq C_h, \\ \binom{K}{j} \left(\frac{\lambda_n + \lambda_h}{\mu} \right)^{C_h} \left(\frac{\beta\lambda_n + \lambda_h}{\mu} \right)^{j-C_h} P_0 & : C_h < j \leq C_h + C, \\ \binom{K}{j} \left(\frac{\lambda_n + \lambda_h}{\mu} \right)^{C_h} \left(\frac{\beta\lambda_n + \lambda_h}{\mu} \right)^C \left(\frac{\lambda_h}{\mu} \right)^{j-C_h-C} P_0 & : C_h + C + 1 \leq j \leq C_h + 2C, \\ \frac{K!}{(K-j)!(C_h+2C)!} \left(\frac{\lambda_n + \lambda_h}{\mu} \right)^{C_h} \left(\frac{\beta\lambda_n + \lambda_h}{\mu} \right)^C \left(\frac{\lambda_h}{\mu} \right)^C & \\ \quad \times \prod_{m=1}^{j-C_h-2C} \left(\frac{\lambda_h}{(C_h+2C)\mu + m\mu_q} \right) & \\ : C_h + 2C + 1 \leq j \leq C_h + 2C + N. \end{cases}$$

and using normalization condition $\sum_{i=0}^{C_h+2C+N} P_j = 1$, we get P_0 as

$$P_0 = \left[1 + \sum_{j=1}^{C_h} \binom{K}{j} \left(\frac{\lambda_n + \lambda_h}{\mu} \right)^j + \left(\frac{\lambda_n + \lambda_h}{\mu} \right)^{C_h} \times \left\{ \sum_{j=C_h+1}^{C_h+C} \binom{K}{j} \left(\frac{\beta\lambda_n + \lambda_h}{\mu} \right)^{j-C_h} + \left(\frac{\beta\lambda_n + \lambda_h}{\mu} \right)^C \sum_{j=C_h+C+1}^{C_h+2C} \binom{K}{j} \left(\frac{\lambda_h}{\mu} \right)^{j-C_h-C} + \left(\frac{\beta\lambda_n + \lambda_h}{\mu} \right)^C \sum_{i=C_h+2C+1}^{C_h+2C+N} \frac{K!}{(K-j)!(C_h+2C)!} \times \left(\frac{\lambda_h}{\mu} \right)^{j-C_h-2C} \prod_{m=1}^{j-C_h-2C} \left(\frac{\lambda_h}{(C_h+2C)\mu + m\mu_q} \right) \right]^{-1} \quad (6)$$

4.1 Performance Indices

The probability that a new call fails on obtaining a channel or finds all channels are busy is B_n and computed as

$$B_n = (1 - \beta) \sum_{i=C_h}^{C_h+C-1} P_i + \sum_{i=C_h+C}^{C_h+2C+N} P_i. \quad (7)$$

The probability that a handoff call finds the queue buffer is full on its arrival is B_h and computed as

$$B_h = P_{C_h+2C+N}. \quad (8)$$

The handoff attempt will fail either if there is no free position in the queue or any reasons it abandons the queue. This can be achieved as

$$B_{fh} = P_{C_h+2C+N} + \sum_{i=0}^{N-1} \frac{(P_{C_h+2C+N})(i+1)\mu_q}{(C_h+2C)\mu + (i+1)\mu_q}. \quad (9)$$

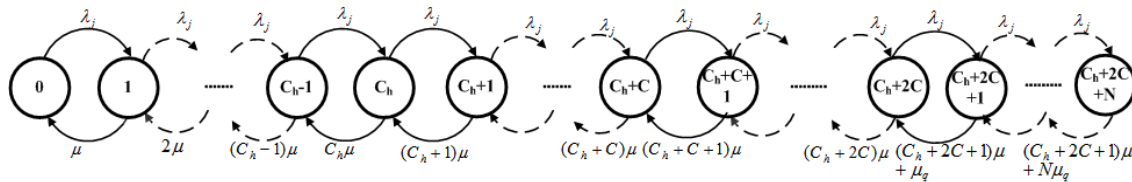


Figure 3. State transition model for FPSR Queueing policy.

Expected queue length of handoff call is

$$L_q = \sum_{i=C_h+2C+1}^{C_h+2C+N} [i - (C_h + 2C)] P_i. \quad (10)$$

We define the queued handoff call waiting time as the time interval between time waiting in the queue and time of successfully access of an arbitrarily free channel. Given that the system was at state j when call arrived, we denote the average waiting time of a queued handoff call by W_{av} . This is given by

$$W_{av} = \frac{\sum_{j=M}^{M+N-1} \left(-\frac{1}{\mu_q} \ln \frac{(M)\mu}{(M)\mu + (j - C_h - 2C + 1)\mu_q} \right) P_j}{\sum_{j=M}^{M+N-1} P_j}, \quad (11)$$

where $M = C_h + 2C$.

The expected number of busy channels is obtained as

$$E[N] = \sum_{i=1}^{C_h+2C} iP_i. \quad (12)$$

The expected number of the sub-rated channels is given as the degradation ratio of the call quality and can be expressed as

$$E[D] = \sum_{i=C_h+C+1}^{C_h+2C} \frac{2(i - C_h - C)}{i} P_i. \quad (13)$$

The relative penalty from blocking and dropping of calls, we define the overall blocking probability of cost function (CF) as follows,

$$CF = \frac{\lambda_n}{\lambda_n + \lambda_h} B_n + \frac{\lambda_h}{\lambda_n + \lambda_h} B_{fh}. \quad (14)$$

5 Numerical Results

In this section, we present numerical result of the FPSR policies and compare the result with FPGC policy.

Figures 4 to 10 represent different performance measures of FPSR policy in terms of graphs. The comparative results with other policies are also presented. The basic system parameters taken as $C_h=6$, $C=2$, population size $K=40$ to 120 and $\beta=0.6$. Figures 4 and 5 show the behavior of new call blocking probability B_n and call dropping probability B_h , respectively as a function of offered load for various acceptance probabilities. For both the case, with increase of offered load the blocking probability or dropping probabilities increases. It can be seen from Figure 4 that for a higher value of β blocking probability minimizes due to admission of more new

calls where as the reverse can be seen in Figure 5. But the gain in blocking probability is remarkable as corresponding to loss in dropping probability. Hence, β need a proper set up to achieve a better result for the system.

Figures 6 and 7 demonstrate the effect of population

Algorithm 2 Algorithm for Channel Allocation Using FPSR Queueing Policy.

```

BC(t) is Number of Busy Channel at Time t.
if Handoff Call AND BC(t) < Ch + C then
    Allocate a Channel;
    BC(t)=BC(t)+1;
    K = K - 1;
else
    if ( BC(t)= Ch + C)
        AND (Sub-rate Channel < Ch + 2C) then
            Allocate a Channel;
            BC(t)=BC(t)+1;
            K = K - 1;
        else
            if BC(t)= Ch + 2C AND Queue length < N then
                Allocate a Channel and Place in Queue;
                K = K - 1;
            else
                Drop the Call;
            end if
        end if
    end if
if New Call AND BC(t) < Ch then
    Allocate a Channel;
    BC(t)=BC(t)+1;
    K = K - 1;
    if (BC(t) > Ch) AND (BC(t) < Ch + C) then
        Allocate a Channel with Probability  $\beta$ ;
        K = K - 1;
    else
        Block the Call;
    end if
end if
    
```

size on blocking and dropping probability for different values of channel threshold C_h . As both the calls admitted up to C_h from a finite population K , changing both the parameters gives significant effect. It can be observed that both blocking and dropping probability increases with increase of population size. The figures indicate that the large value of C_h effectively reduces the blocking and dropping probabilities. Hence, better trade off of C_h can serve better.

Figure 8 plots degradation $E[D]$ verses population for different loads. When population increases the quality degradation increases as more channels are sub-rated. It can be seen that an user experiences degraded voice quality on heavy offered load. The lower degradation value is due to the fact that sub-rated channels only allow handoff calls access and up graded back immediately after conversation is complete.

Figures 9 and 10 represent a comparative analysis between a FPGC and FPSR policies in terms of blocking and dropping probabilities, respectively. Under similar

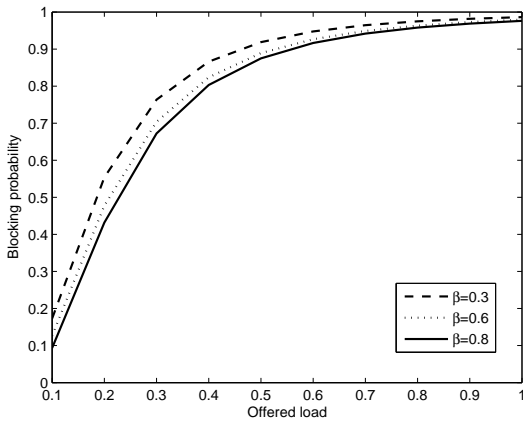


Figure 4. Effect of offered load on blocking probability.

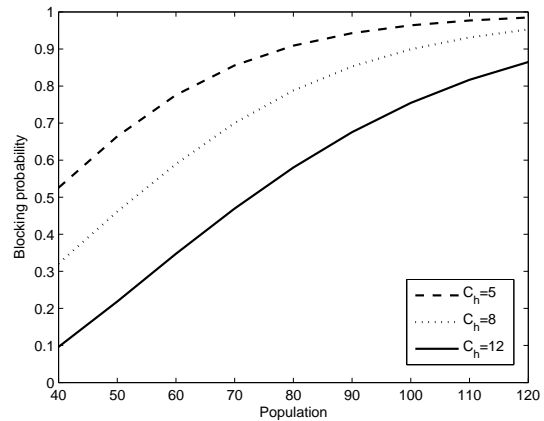


Figure 6. Effect of population size on blocking probability.

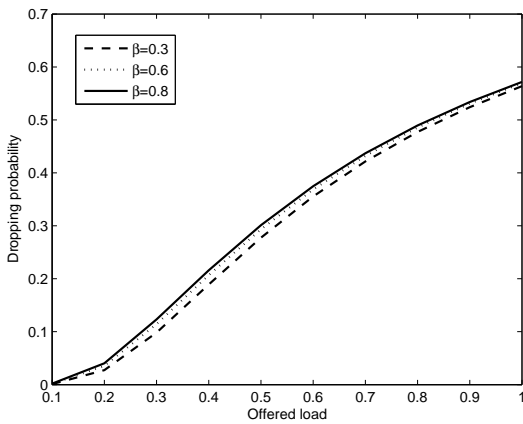


Figure 5. Effect of offered load on dropping probability.

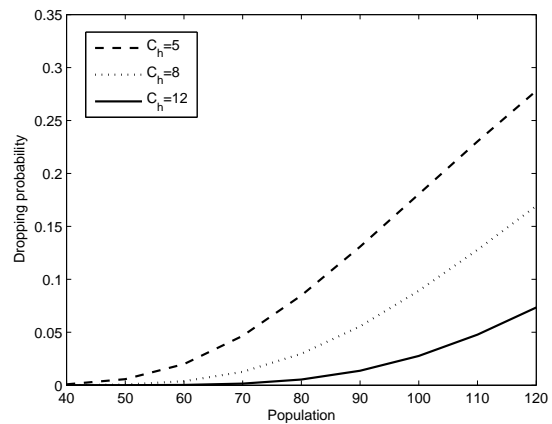


Figure 7. Effect of population size on dropping probability.

resource condition for both the policies, with increase of population size blocking as well as dropping probability increases. It can be observed from Figure 9 that FPSR policy performs better than FPGC policy by admitting new calls in guard channel region with probability β . This significant gain in blocking probability is compensated by increase in dropping probability. Figure 10 reflects that FPSR has higher dropping probability than FPGC policy but this loss of dropping probability is almost negligible compared to gain in blocking probability which sustains the stability of the system.

Figures 11 to 15 show different performance indices of FPSR Queueing policy. Figures 11 and 12 present the effect of sub-rating on blocking and dropping probabilities, respectively, for different channel thresholds. The basic system parameters for these plots are taken as $C_h = 10$, $C = 6$, $\beta = 0.6$ and $N = 5$. We can observe that as sub-rating channel C increases, both blocking and dropping probabilities decrease as more channels are available with a degraded quality. But this can serve more number of handoff calls as well as new calls with probability β . Both plots group the performance using bars for different channel thresholds in particular sub-rated level. When C_h increases, both blocking and dropping performance decrease. Hence, C and C_h need a proper tradeoff in achieving a better result for new and handoff calls.

Figure 13 illustrates the dependence of the number of waiting handoff calls in queue on the population size.

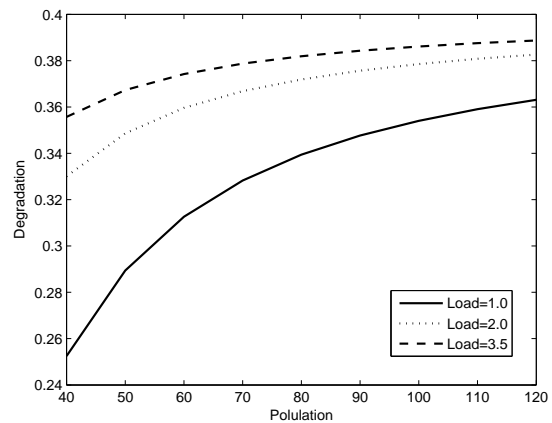


Figure 8. Effect of population size on degradation of voice quality.

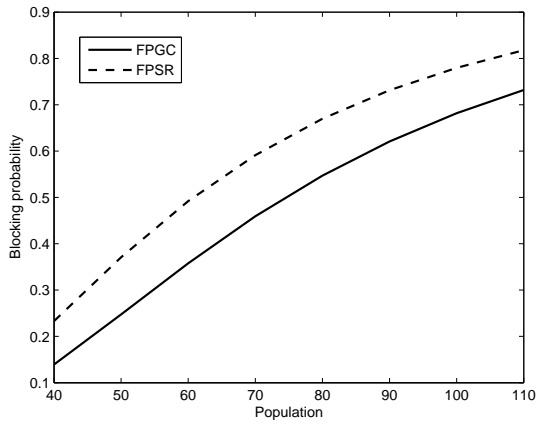


Figure 9. Population size versus blocking probability.

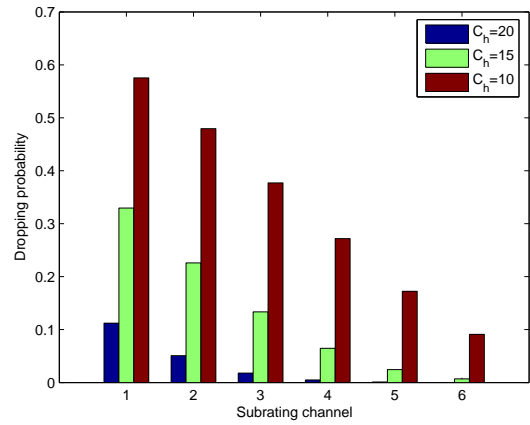


Figure 12. Effect of sub-rating on dropping probability.

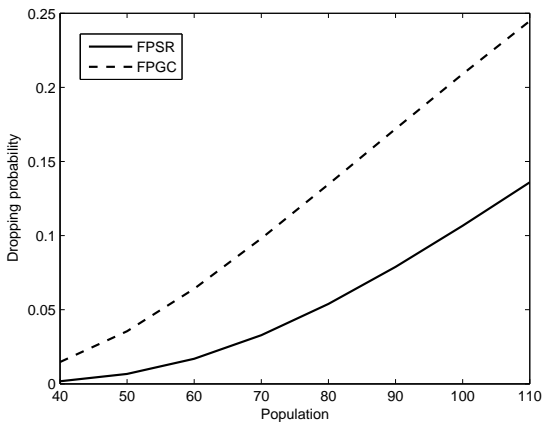


Figure 10. Population size versus dropping probability.

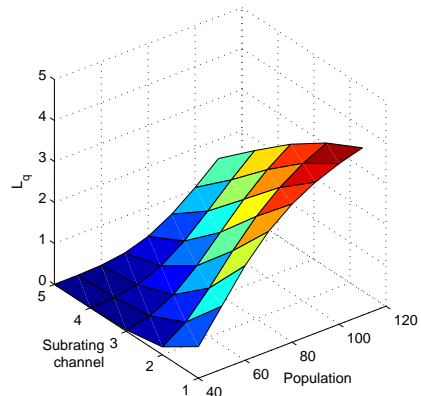


Figure 13. Queue length for different values of sub-rating and population.

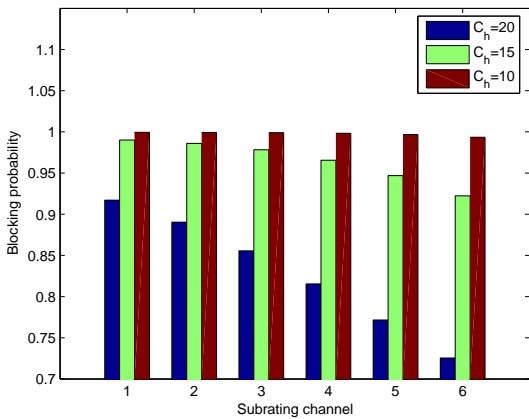


Figure 11. Effect of sub-rating on blocking probability.

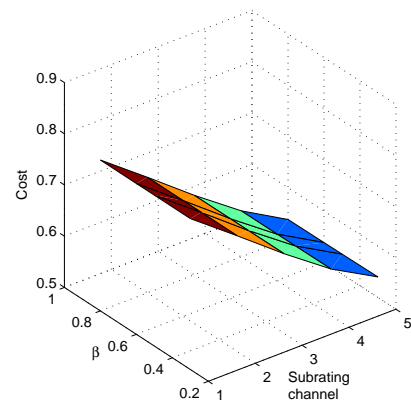


Figure 14. Cost factor for different values of sub-rating and β .

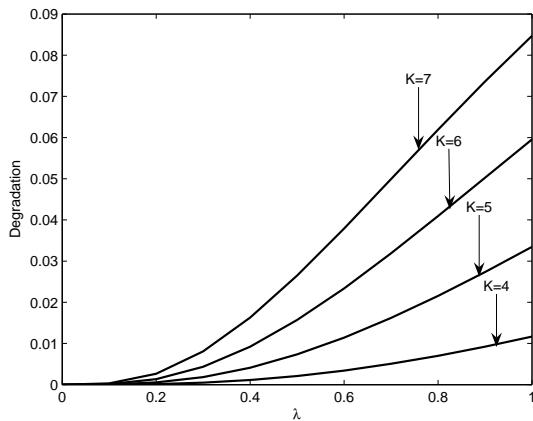


Figure 15. Effect arrival rate on voice degradation.

varying K from 40 to 120 and the number of sub-rating channels varying from 1 to 5. We observe that for fixed population size the queue length decreases as the sub-rating increases. Further, with fixed sub-rating it increases when the population size increases. Hence, we can setup an admissible sub-rating channel and population size in the cell in order to have lower number of calls in queue.

The variation in the cost factor for different values of the sub-rating channel size C and the acceptance probability is shown in Figure 14. We varied the sub-rating channel size C from 1 to 5, while the acceptance probability β is varied from 0.2 to 1.0. It is observed that for fixed acceptance probability the cost decreases when the sub-rating channel size increases. Further with fixed sub-rating channel size the cost increases when the acceptance probability increases. To accomplish this, we can carefully setup the number of sub-rating channels and the acceptance probability in the system in order to ensure the minimum cost.

Figure 15 shows the effect of arrival rate λ on voice degradation quality for different population size. We observe that as the arrival rate of new calls and handoff calls increases, voice quality degradation also increases. For fixed arrival rate, as population size increases voice quality degradation increases.

6 Conclusion

In this paper, we have presented a finite population channel sub-rating policy. When all channels are busy, an incoming handoff request is served by dividing a full rate channel into two half rate channels temporarily. One of the half rate channel serve the existing call and the other serve the handoff request so that we can get rid of forced termination of calls. This sub-rate channel policy may be reserved for handoff calls without having to disturb another user's call. For short durations during a call, the power consumption is not effected significantly or the voice degradation quality might be tolerable. We proposed two system models based on fractional guard

channel and sub-rating channel allocation policies with and without queueing of handoff calls which give a significant improvement in the QoS of the cellular network. New call blocking probability and handoff call forced termination probability are computed to evaluate the proposed call admission control policy. The derived formula may be helpful to reduce the overall call blocking probability by providing the tolerable number of threshold channels and selecting the right releasing function. The degraded call quality due to the half rate channel is also investigated to examine the effects of the sub-rating on the QoS. Our policies have the least forced termination probability and the call incompleteness probability when compared with the other policies.

REFERENCES

- [1] C. Chang, C. j. Chang. Analysis of a hierarchical cellular system with reneging and dropping for waiting new calls and handoff calls, *IEEE Transactions on Vehicular Technology*, Vol. 48, No. 4, 1091–1099, 1999.
- [2] Y. Fang, Y. Zhang. Call admission control schemes and performance analysis in wireless mobile networks, *IEEE Transactions on Vehicular Technology*, Vol. 51, No. 2, 371–382, 2002.
- [3] W. Feng, M. Kowada. Performance analysis of wireless mobile networks with queueing priority and guard channels, *International Transactions in Operational Research*, Vol. 15, No. 4, 481–508, 2008.
- [4] M. Kulavaratharajah, A. Aghvami. Teletraffic performance evaluation of microcellular personal communication networks (PCN) with prioritized handoff procedures, *International Transactions in Operational Research*, Vol. 48, No. 1, 137–152, 1999.
- [5] V. Lau, S. Maric. Mobility of queued call requests of a new call queueing technique for cellular systems, *IEEE Transactions on Vehicular Technology*, Vol. 47, No. 2, 480–488, 1998.
- [6] W. Li, A. Alfa. A PCS network with correlated arrival process and splitted-rate channels, *IEEE Journal on Selected Areas in Communications*, Vol. 17, 1318–1325, 1999.
- [7] Y. Lin, A. Noerpel. The sub-rating channel assignment strategy for PCS hand-offs, *IEEE Transactions on Vehicular Technology*, Vol. 45, No. 1, 122–130, 1996.
- [8] Y. Lin, S. Mohan, A. Noerpel. Queueing priority channel assignment strategies for handoff and initial access for a PCS network, *IEEE Transactions on Vehicular Technology*, Vol. 43, No. 3, 704–712, 1994.
- [9] S. Yamanaka, K. Shimohara. Performance analysis of CDMA cellular networks with channel sub-rating, *International Journal of Communication Systems*, Vol. 19, No. 3, 317–333, 2006.