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Decomposition of Complex Signal on Lorenz Components

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Abstract Algorithm of complex signal decomposition on elementary components having Lorenz form is proposed.The non-linear minimization problem to the problem of linear equation solving. The number of components is the necessary aprioir information. The algorithm can be combined with the method of statistical regularization. The results of numerical experiments are represented.

Keywords Complex Signal Decomposition, Lorenz Form, Statistical Regularization

1. Formulation of the Problem.

The complex signal decomposition on elementary component is the problem of the interest of many fields: spectroscopy, chemometric, radiophysics etc. [1-6]. Рассмотрим Algorithm of complex signal decomposition on elementary components having Lorenz form with some transformation of non-linear minimization problem to the problem of linear equation solving. The number and the form of components are the necessary apriory information.

$$\Phi(\omega)=\sum_{i=1}^{m} a_i L(\omega,b_i,c_i)=\sum_{i=1}^{m} a_i \frac{1}{(\omega-c_i)^2+b_i} \tag{1}$$

Here a_i, b_i, c_i are the elementary component parameters to be determined (a is the amplitude, c is the center position and b is the halfwidth) , m is the number of components.

2. Description of the Method

Multiplying the both parts on the multiplier

$$\prod_{i=1}^{m}(\omega^2-2\omega c_i+c_i^2+b_i^2), \tag{2}$$

we have

$$\Phi(\omega)\prod_{j=1}^{m}(\omega^2-2\omega c_j+c_j^2+b_j^2)=\sum_{i=1}^{m} a_i L(\omega,b_i,c_i)\prod_{j=1}^{m}{}'(\omega^2-2\omega c_j+c_j^2+b_j^2) \tag{3}$$

Here and later ’ near the product symbol indicates that multiplier with i=j is eliminate of the product. (3) can be represented in the form:

$$\Phi(\omega)\prod_{j=1}^{m}(\omega-W_j)(\omega-W_j^*)=\sum_{i=1}^{m} a_i \prod_{j=1}^{m}{}'(\omega-W_j)(\omega-W_j^*). \tag{4}$$

Roots W_j , W_j^* can be found from:

$$\omega^2 - 2\omega c_j + c_j^2 + b_j^2 = (\omega - W_j)(\omega - W_j^*). \qquad (5)$$

Let us to introduce the new variables x_i, z_k *(i=1,2,....,2m)*, *(k=1,2,...,2m-1)*, defined by:

$$x_1 = \sum_{j=1}^{m} (W_j + W_j^*)$$

$$x_2 = \sum_{i=1}^{m} \sum_{j=1}^{m} {}' (\left|W_j\right|^2 + W_i W_j + \ldots + W_i^* W_j^*) \qquad (6)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots$$

$$x_{2m} = \prod_{j=1}^{m} \left|W_j\right|^2$$

$$z_1 = \sum_{i=1}^{m} a_i$$

$$z_2 = \sum_{i=1}^{m} a_i \sum_{j=1}^{m} {}' (W_j + W_j^*) \qquad (7)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots$$

$$z_{2m-1} = \sum_{i=1}^{m} a_i \prod_{j=1}^{m} W_j W_j^*$$

Taking into account (5.6) and (5.7) the equation (5.1) can be represented in the form:

$$\begin{aligned} &\Phi(\omega)(\omega^{2m} + x_1 \omega^{2m-1} + \ldots + x_{2m}) = \\ &= z_1 \omega^{2m-2} + z_2 \omega^{2m-3} + \ldots + z_{2m-1}. \end{aligned} \qquad (8)$$

(8) is the system of linear equations with *4m-1* unknown quantities: *2m* unknown quantities x_j and *2m-1* unknown quantities z_k. The system can be solved using *4m-1* known values of $\Phi(\omega_i)$.

$$W^{2m} + x_1 W^{2m-1} + x_2 W^{2m-2} + \ldots + x_{2m} = 0 \qquad (9)$$

The unknown parameters are

$$c_j = \mathrm{Re} W_j, b_j = \mathrm{Im} W_j. \qquad (10)$$

After that the parameters a_i can be found from linear equations system.

$$\Phi(\omega_i) = \sum_{j=1}^{m} L_{ij} a_j, \qquad (11)$$

Where the matrix L_{ij} is determined as

$$L_{ij} = \frac{1}{(\omega_j - W_i)(\omega_j - W_i^*)}. \qquad (12)$$

The complex polynomial (5.9) roots can be found by Berstow method [7] .
For noise removal can be used the regularized multistep support vector method [8].

3. Numerical Experiments

For investigation the quality of algorithm the mathematical experiments were carried out. The base of the modal signal is the complex contour consisting of two or three Lorenz components. The “experimental” curves were simulated by the random noise generator. Mean square deviation

σ_2 of the modal signal from the calculated with reconstructed parameters was used as disparity measure.

$$\sigma_2 = \frac{1}{n}\sum_{i=1}^{n}\left[\Phi(\omega_i) - \Phi_B(\omega_i)\right]^2,$$

where $\Phi(\omega)$ is the real complex signal , $\Phi_B(\omega)$ is the couture calculated with reconstructed parameters.

If the noise level η is 8 percent from the maximum of the real signal the solution became senseless: a1=0,58 (the real is 1), a2=2,98(2), b1=0(1), b2=2,46(2), c1=1,5(3), c2=9,99(9). But after using regularization the solution became more better .a1=0,86, a2=2,47, b1=0,77, b2=2,2, c1=3,1, c2=9,27

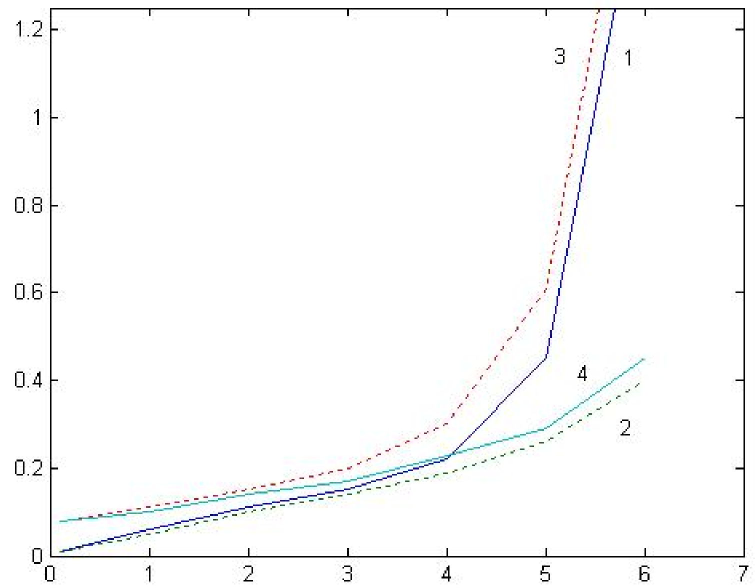

Figure 1. The dependence of mean square deviation σ_2 from the noise level for two lorenzians separation (curve 1 – without regularization and 2 – with regularization, the real parameters are a1=1, b1=1, c1=3, a2=2, b2=2, c2=9) and three lorenzians (curve 3 – without regularization and 4 – with regularization, a1=1, b1=1, c1=3, a2=2, b2=1, c2=8, a3=1, b3=1, c3=12).

With the components number m increasing the quality of reconstruction falls (fig/1 curves 3 and 4 for m=3). But the using of regularization improves the quality of the solution.

That method is very sensible to the number of components a'priori setting and if m is incorrect the solution can be unmeaning. Tab.1 represents the values of reconstructed parameters in dependence of a'priori setting number of components (the real number is three) .

The quality of reconstruction in dependence of the distance between their maximums were investigated. Some results are represented at the tab.2.

Table 1. The values of reconstructed parameters in dependence of a'priori etting number of components. The level of noise is 0,05 percent from the maximum of the curve.

m	a1,b1,c1	a2,b2,c2	a3,b3,c3	b1	b2	b3	c1	c2	c3
3	1.0060 1.0039 3.0006	2.0372	0.8562	1.0039	0.9980	0.9980	3.0006	7.9977	12.003
4	1.0039	1.7583	-7.496	1.0045	1.9767	0.9991	2.9992	5.0310	8.0021
5	0.9922	0.3279	17.571	0.9981	0	1.0041	2.9994	7.0101	8.0012

Table.2 The parameters reconstruction with different distance between the centers (D) of the elementary components. The noise level is 0.5 percent of the maximum.

D	a1	a2	b1	b2	c1	c2
6	1,0009	2,0007	1,0000	2,0001	3,0003	9,0000
4	1,0005	2,0018	1,0001	2,0003	4,9924	9,0007
2	1,0021	2,0051	1,0611	1,9924	6,9101	9,0232
1.75	0,9722	2,3716	1,0119	2,0339	7,2525	8,9228
1.25	0,9338	2,3754	1,0125	1,9818	7,6311	9,1237
1	0,9325	2,5716	1,1610	1,9827	7,8123	9,1042
0.75	1,2349	1,2427	0,7324	2,0107	8,4241	8,8111

4. Conclusions

The represented method permits to get sufficient accurate values of components parameters even for near moved elementary curves. Using the regularization makes it possible to work in the presence of experimental noise. But the number of components must be set exactly.

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