

Fuzzy Models of "Costs" and "Profit"

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Abstract This article describes construction of fuzzy models "Costs" and "Profit" for operational decisions in the economic system management by introducing linguistic variables and application of expert rules.

Keywords Model, Linguistic Variable, Economic System, Costs, Profit, Expert Analysis

1. Introduction

Profit reflects the effectiveness of the company and it shows how the sales proceeds of product (goods and services) exceed the production costs. In a free market economy, the company's management takes his own decisions and plans volume of production, so modelling of a "Costs-Profit" system is an actual problem, because it allows to develop an effective production plan and pricing of products.

As part of the research work "Application of information technology for the economic system characteristics calculation using fuzzy logic" we have developed the "Costs" and "Profit" models which were applied on a real company which sells hardware and software products and performs services in IT. The models are constructed using linguistic variables that approximate human reasoning.

2. Methodology

2.1 Costs and Profit as Characteristics of an Economic System

The free market economy orients enterprises to serve market demand and specific requests of customers. Focusing the manufacturing processes to products types that are in demand is necessary to have profits for the company. The market is characterized by a constant desire to improve production efficiency. It implies freedom of decision-making by the people who are responsible for the results of the company and its subsidiaries; it requires constant adjustment of goals and planning programs depending on market conditions. This requires a special control system, typical for

market conditions and priorities and mentality of Russian business, because many Russian companies underestimate costs in accounting and it implies significant economic losses for their activity.

The scientific method of costs classification is of great importance for the correct organization of accounting. Places of origin, centres of responsibility, types of expenses and sources group production costs. Fixed and variable costs are used for break-even analysis and its related indicators, and for production optimization. Variable costs vary in proportion to production or sales volumes, whereas costs for a single unit of output are constant.

Fixed costs in total do not vary with the level of business activity, but they decrease with volume of production or sales increasing, if they are calculated to the unit of production.

As for profits, there are usually gross (balance sheet, total) and net profit calculated, remaining after gross profits taxes and other deductions.

The company where this research has been applied mostly works with corporate customers. Prices of hardware, software and customer services are determined depending on various factors. For example, the price of an accounting system will be formed as follows: the costs plus 20-25% (depending on total costs and the client financial capabilities).

In a free market system there is a dynamic interaction between profits and costs. Their values, as well as value of others attributes, can be considered as quantitative and qualitative variables using the tools of fuzzy logic.

2.2 Formalization of the Profit Management Procedure

For calculating profit growth rates we suggest to use the linguistic variable

$$c = c(x, T(x), G, M),$$

where x is the variable name; $T(x)$ is the set of names (terms) of the linguistic values x , each of which is a fuzzy set on the set X ; G is a syntactic rule for new terms formation with the ligaments of "and", "or", and localizing modifiers clarifying values of individual characteristics, like "very," "almost," etc.; M is the semantic rule for

associating each value with its concept [1].

We propose to introduce two linguistic variables: COSTS and PROFIT. The linguistic variable COSTS has the terms "small", "medium", "large"; the values of the linguistic variable PROFIT are chosen between the terms of "low", "medium", and "high".

There are several methods for constructing fuzzy set membership functions by expert estimates. Using direct methods the expert directly defines the rules for determining values of the membership function $\mu_A(x)$, which characterize the element x . Usually, the direct methods are used to describe the concepts that have measurable properties, such as height, weight, volume. In this case the direct definition of the membership function values is more appropriate. With indirect methods the values of membership functions are chosen by preselected conditions. In that case the expert input is treated as raw data suitable for further processing. Additional conditions may be imposed to the form of received information, as well as to the processing procedure [2].

2.3 An Indirect Method for Constructing Membership Functions Based on Rank Estimates

Indirect methods for determining the values of the membership functions are used in cases when there are no basic measurable properties to define a fuzzy set. This method was developed by A. P. Rothstein, based on the idea that the distribution of membership values of universal set elements according to their ranks. The rank of an element $x_i \in X$ is the value $r_s(x_i)$, which is characterization of the importance of this element in the formation of the properties in fuzzy term S [3]. Let's assume the following rule: the higher the rank of an element, the greater the degree of membership. Also let's introduce the notation

$$r_s(x_i) = r_i, \mu_s(x_i) = \mu_i, i = (1, n).$$

We are going to use scale of relative importance (according to Saaty) for expert reviews:

- 1 – equal importance (two activities contribute equally to the objective);
- 3 – weak importance of one over another (experience and judgement slightly favour one activity over another);
- 5 – essential or strong importance (experience and judgement slightly favour one activity over another);
- 7 – demonstrated importance (an activity is strongly favoured and its dominance demonstrated in practice);
- 9 – absolute importance (the evidence favouring one activity over another is of the highest possible order of affirmation);

- 2, 4, 6, 8 – intermediate values between the two adjacent judgments (when compromise is needed) [4].

Let's take four values for the linguistic variable COSTS:

$$x_1 = 1000, x_2 = 18000, x_3 = 45000, x_4 = 50000.$$

The ratios of ranks to these elements for the fuzzy terms of "small", "medium", "large" are showed in the Table 1

Table 1. Relations between elements and ranks for the linguistic variable COSTS

	small	medium	large
r1	9	3	1
r2	3	9	2
r3	1	2	5
r4	1	1	9

The next step of determination of the membership degree $\mu_s(x_i)$ is the calculation of relative score of ranks, which form the matrix A :

$$A = \begin{bmatrix} 1 & \frac{r_2}{r_1} & \frac{r_3}{r_1} & \dots & \frac{r_n}{r_1} \\ \frac{r_1}{r_2} & 1 & \frac{r_3}{r_2} & \dots & \frac{r_n}{r_2} \\ \frac{r_1}{r_3} & \frac{r_2}{r_3} & \frac{r_3}{r_3} & \dots & \frac{r_n}{r_3} \\ \dots & \dots & \dots & \dots & \dots \\ \frac{r_1}{r_n} & \frac{r_2}{r_n} & \frac{r_3}{r_n} & \dots & 1 \end{bmatrix}$$

The calculations are performed for all the terms of the linguistic variable COSTS. The matrices of these ranks are displayed in the Table 2.1 - 2.3.

Table 2.1- 2-3. Rank matrices for the linguistic variable COSTS.

small					medium					Large				
	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4
x_1	1	0.33	0.11	0.11	x_1	1	3	0.66	0.33	x_1	1	2	5	9
x_2	3	1	0.33	0.33	x_2	0.33	1	0.22	0.11	x_2	0.5	1	2.5	4.5
x_3	9	3	1	1	x_3	1.5	4.5	1	1	x_3	0.2	0.4	1	1
x_4	9	3	1	1	x_4	3	9	2	1	x_4	0.11	0.22	0.55	1

Using the data obtained according to the conditions:

$$\mu_1 + \mu_2 + \dots + \mu_n = 1,$$

$$\mu_{n-1} = \frac{r_{n-1}}{r_n} \mu_n,$$

we have found the elements of the membership function for each term (values on the assessments scale):

$$\left. \begin{aligned} \mu_1 &= \left(1 + \frac{r_2}{r_1} + \frac{r_3}{r_1} + \dots + \frac{r_n}{r_1} \right)^{-1} \\ \mu_2 &= \left(\frac{r_1}{r_2} + 1 + \frac{r_3}{r_2} + \dots + \frac{r_n}{r_2} \right)^{-1} \\ &\dots\dots\dots \\ \mu_n &= \left(\frac{r_1}{r_n} + \frac{r_2}{r_n} + \frac{r_3}{r_n} + \dots + 1 \right)^{-1} \end{aligned} \right\}$$

The calculated values of the membership function are displayed in the Table 3.

Table 3. Values of the membership function for the linguistic variable COSTS

	small	medium	large
1000	0.64	0.20	0.06
18000	0.21	0.60	0.12
45000	0.07	0.13	0.38
50000	0.07	0.07	0.53

The obtained values of membership functions (the vertical axis) in normalized form is presented at the Chart 1.

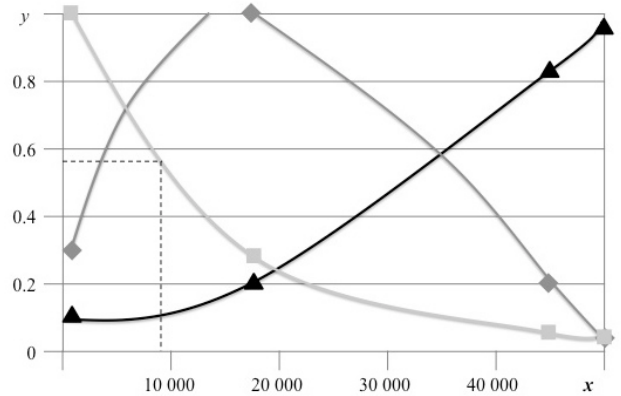


Chart 1. The model “Costs” (◆ - "small", ■ - "medium", ▲ - "large").

On the horizontal axis there are prices in roubles.

For calculating the values of the membership function for the linguistic variable PROFIT we used a similar procedure. For the next values of variable x :

$$x_1 = 1000, x_2 = 6000, x_3 = 12000, x_4 = 50000,$$

We obtained the relations between elements and ranks in fuzzy terms which are showed in the Table 4.

Table 4. Relations between elements and ranks for the linguistic variable PROFIT

	small	medium	large
r1	9	1	1
r2	1	9	1
r3	1	2	3
r4	1	1	9

The values are obtained for the rank matrices are shown in the tables 5.1 - 5.3.

Table 5.1 – 5.3. Rank matrices for the linguistic variable PROFIT.

low					medium					high				
	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4
x_1	1	0.11	0.11	0.11	x_1	1	9	2	1	x_1	1	1	3	9
x_2	9	1	1	1	x_2	0.11	1	0.22	0.11	x_2	1	1	3	9
x_3	9	3	1	1	x_3	0.5	4.5	1	0.5	x_3	0.33	0.33	1	3
x_4	9	3	1	1	x_4	1	9	2	1	x_4	0.11	0.11	0.33	1

Also the elements of membership functions for all terms have been found (Table 6).

Table 6. Values of the membership function for the linguistic variable PROFIT.

	low	medium	high
1000	0.75	0.08	0.07
6000	0.08	0.69	0.07
12000	0.08	0.15	0.21
50000	0.08	0.08	0.64

The membership function (the vertical axis) in the normalized form is shown in the next chart (Chart 2). The horizontal axis indicates the price in roubles.

An example. Let the cost of software installation be 9000 roubles. Then, according to the "Costs" chart (Chart 1), the membership degree of the term "small" on a scale of assessments will be 0.55. We apply the rule "low costs correspond to low profit". Accordingly, on the graph "Profit" (Chart 2) the intersection of the term "low" with the estimate of 0.55 corresponds to a value of about 3000 roubles.

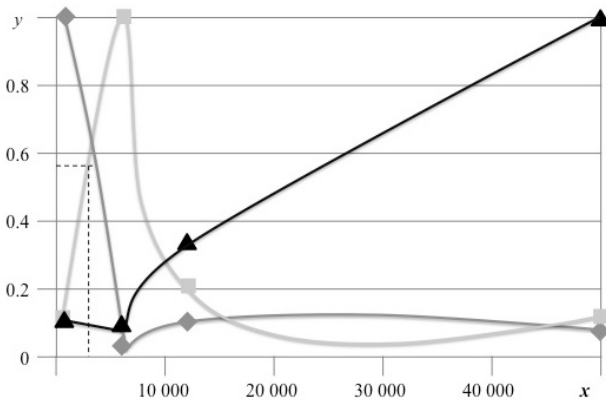


Chart 2. The model "Profit" (◆- "small", ■- "medium", ▲ - "large")

3. Conclusion

In many practical situations, the operating decisions that are based on approximate reasoning can be quite successful. That results enable acquiring the necessary information about economic system characteristics and making informed decisions.

Our results were highly appreciated by the administrative personnel of the company and they were found quite sufficient for operational management of the current production process. Currently this method of costs and profits calculation is applied within Prime LLC, which sells hardware, software and IT services to its customers.

The goal of this paper is to apply existing analytical methods to practical research. The full description of using approach could be found in the special literature, links of which was done in the bibliography.

The standard approach using three term-sets will be modified in following research (including increasing of the number of term-sets).

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