

Analysis and Controller Design of a Torsional D.C. Drive under Resonance Conditions

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Abstract The performance of an electric drive not only depends upon electrical parameters but is also significantly affected by mechanical features of drive like, elasticity of shaft, misalignment of shaft, eccentricity of driven load. For precise determination of performance, specially under dynamic conditions, the effect of these mechanical factors should be included in the analysis. The work reported in this paper deals with the analysis of a D.C. motor drive with elastic coupling and periodically varying load torque. Periodic variation of load torque may due to mechanical features or may be the characteristics of the driven load itself. A mathematical model of the drive system is developed and the system equations are expressed in state model form. Simulink models of the drive are developed and performance of drive system is studied for constant as well as pulsating load torque conditions. Under some specific conditions, system goes under resonance. Conditions leading to resonance are identified and drive performance under normal as well as resonance conditions is analyzed. The analyses presented reveal that mechanical failure of the drive system may occur due to excessive shear stress developed in the shaft under resonance conditions. Design of a controller is suggested to protect the drive from such failures under resonance condition.

Keywords D.C. Drive, Torsional Oscillations, Pulsating Load Torques, Resonance, Controller

effect of elasticity of shaft is mainly to produce torsional oscillations in the system. The effect of other factors is to impose cyclic rotational disturbances in the form of either impact loads or periodic change in load torques. Bishop and Mayer [2] have emphasized the need of accurate modeling of total system including the system non-linearity and component interactions. Drive system disturbance sources such as pulsating torques, imbalances and switching transients of the drive motor, impact and cyclic effects of load and mechanical inaccuracies are discussed. The effect of torsional elements on transient analysis of large induction motor drives is discussed by Buckley [3]. Mayer [4] has also discussed the various sources of excitation of torsional oscillations for cement industry drives. The authors in earlier papers [5, 6], have analyzed the performance of torsional D.C. drives and obtained closed form solution for motor speed and current under transient and steady state conditions. The same author has also presented [7] stability analysis of phase controlled closed-loop D.C. drive with elastic coupling using D-Partition Technique.

To the best of author's knowledge, no work has so far been reported on study of performance of D.C. drives with elastic coupling and supplying pulsating load torques in resonance condition. Work presented in this paper is an effort in this direction. In this paper, performance of this type of drive is analyzed in transient as well as steady state conditions, conditions leading to resonance are identified, possible causes of mechanical failure of system are discussed and a controller is designed to protect the drive from such mechanical failures

1. Introduction

The study of these mechanical factors has attracted the attention of researchers in the past. A comprehensive description of various mechanical factors affecting the performance of electric drives is given by Carter [1]. The factors discussed include elasticity of shaft, back lash, misalignment, bending of shaft and unbalance of rolls. The

2. Mathematical Model of Drive System

The system analysed consists of a D.C. separately excited motor supplied with a constant D.C. voltage and connected to the load through an elastic shaft as shown in Fig.1. The system is represented by a two rotor system. The moments of inertia J_1 , J_2 and damping B_1 , B_2 of motor and load

respectively, are considered separately. The shaft is considered to be elastic in nature having elasticity of shaft C . The motor torque T_e is considered to be positive whereas the load torque T_L is considered to be negative. The angular positions at motor and load ends are θ_1 and θ_2 respectively. For a perfectly rigid shaft, values of θ_1 and θ_2 are same, but for an elastic shaft θ_1 and θ_2 are different and this gives rise to twist ($\theta_1 - \theta_2$) in the shaft in running condition.

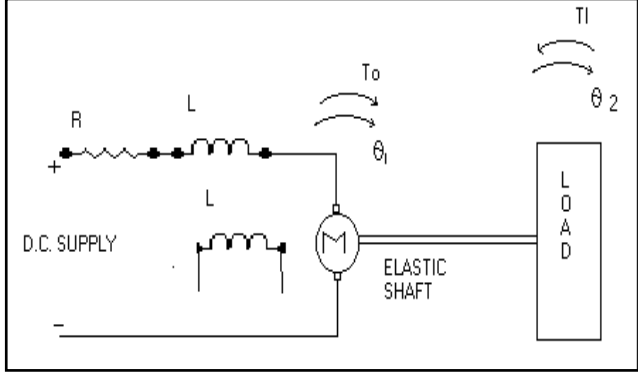


Figure 1. Torsional DC drive electro-mechanical system.

The nature of load torque variation is periodic in nature as shown in Fig. (2). This variation in load torque may be due to various mechanical factors discussed earlier or may be due to the characteristics of the driven mechanism itself. The load torque consists of two components, uniform component T_{L0} superimposed by a pulsating component T_{L1} as shown in the Fig.2. The frequency of load torque pulsation is ω_L .

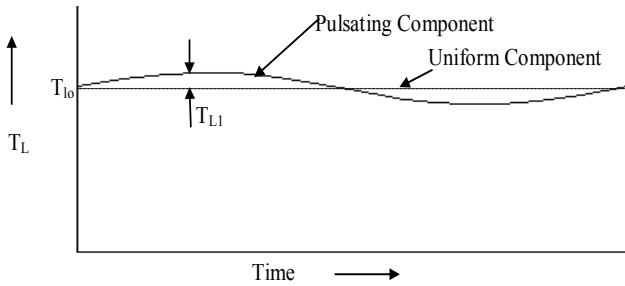


Figure 2. Periodic variation of load torque.

The equations governing the performance of the system consist of a voltage-current equation of the motor, and the dynamic equations of motion, as given below:

$$V = L \frac{di}{dt} + Ri + K_m \dot{\theta}_1 \quad (1)$$

$$T_e = J_1 \ddot{\theta}_1 + B_1 \dot{\theta}_1 + C(\theta_1 - \theta_2) \quad (2)$$

$$-T_L(t) = J_2 \ddot{\theta}_2 + B_2 \dot{\theta}_2 + C(\theta_2 - \theta_1) \quad (3)$$

$$\text{where } T_e = K_e i \quad (4)$$

$$\text{and } T_L(t) = T_{L0} + T_{L1} \sin(\omega_L t) \quad (5)$$

The symbols used in Eqn. (1) to Eqn. (5) are given in Appendix-A. The above system equations can be also written in the form of state model form as:

$$\dot{x} = Ax + Bu \quad (6)$$

The values of matrices A , B , u and x are given in Appendix-B.

The characteristic equation is given as $|sI - A| = 0$, which is of the form:

$$A_1 s^5 + A_2 s^4 + A_3 s^3 + A_4 s^2 + A_5 s + A_6 = 0 \quad (7)$$

The values of (A_1, \dots, A_6) depend on the elements of $[A]$ matrix.

For the system under analysis, the values of motor parameters as well as mechanical parameters are given in Appendix-A. From the characteristics equation, the eigen values of system can be calculated as:

$$s_{1,2} = -12.5699893 \pm 7.888014j.$$

$$s_3 = 0.0$$

$$s_{4,5} = -0.08001069 \pm 519.8234292j$$

The system undamped natural frequency of oscillations is related to the real and imaginary part of the Eigen values:

$$\omega_{n1} = \sqrt{(\alpha_1^2 + \beta_1^2)} \quad \text{and} \quad \omega_{n2} = \sqrt{(\alpha_2^2 + \beta_2^2)}$$

where

$$\alpha_1 = -12.5699893 \quad \& \quad \beta_1 = \pm 7.888014.$$

$$\alpha_2 = -0.08001069 \quad \& \quad \beta_2 = \pm 519.8234292$$

The value β_1 refers to the frequency of damped natural frequency of oscillation of the system and depends mainly on its electrical parameters such as L and R . Here damping is more frequency of oscillation is less. The value β_2 refers to the frequency of damped natural frequency of torsional oscillation of the system and depends mainly on its mechanical parameters such as J_1 , J_2 and B_1 , B_2 and C . In this case damping is less and frequency of oscillation is more. The undamped natural frequency of torsional oscillations of the system ω_n , which is very close to β_2 , is given as:

$$\omega_n = \sqrt{C \left(\frac{1}{J_1} + \frac{1}{J_2} \right)} \quad (8)$$

The D.C. drive system with an elastic shaft and pulsating load torque, under specific operating conditions, exhibits peculiar performance as large peak in armature current and speed are observed. Such a situation arises when the frequency of the load torque pulsation approaches the frequency β_1 or β_2 . This phenomenon is referred to as "RESONANCE".

3. Work presented

(A) Analysis under normal conditions

The system performance under resonance condition has been studied for normal and the resonance conditions. For

these operating conditions, performance is obtained as discussed below:

The analysis of a D.C. motor drive with an elastic coupling is presented for a motor fed by a constant D.C. voltage. For normal operation, ω_L is taken as zero for constant load torque and $\omega_L=10$ rad., for pulsating load torque. The performance is obtained using Simulink model shown in Appendix-C. For a typical set of drive system parameters given in Appendix-A, the performance in terms of armature current, motor speed and twist in the shaft for the drive is determined in transient as well as steady state conditions.

(i) Performance of D.C. drive with constant voltage supply and constant load-torque

Voltage applied to armature: 200 V

Constant load torque, $T_{L0}=75\%$ of full load torque

In this case a constant voltage is applied to the armature and the load torque has only a uniform component T_{L0} .

Fig. 3(a) and 3(b) show the variation of armature current in transient and steady state, while Fig. 4(a) and (b) show the variation of motor speed in transient and steady state respectively. Similar variations for shaft twist are shown in Fig. 5 (a) and Fig. 5 (b).

It is observed that the armature current, motor speed and twist in steady state consist of an oscillating component superposed on a constant component. The oscillating component is varying at the frequency of torsional oscillations β_2 of the system, which is due to the elasticity of the shaft. The oscillations of frequency β_1 are not noticeable.

The twist produces shear stress Q in the shaft given by:

$$Q=G*(\text{twist})*d/2l$$

The average value of the twist, for the case under study, is equal to $1.5*10^{-3}$ radians and the resulting shear stress is equal to 19.125 kg/cm². which is small and well under permissible limit.

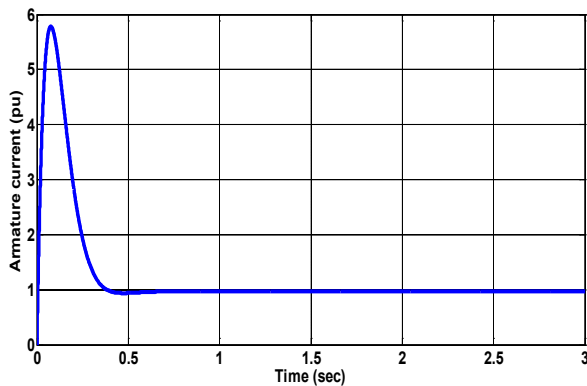


Figure 3(a). Transient armature current response for constant load torque.

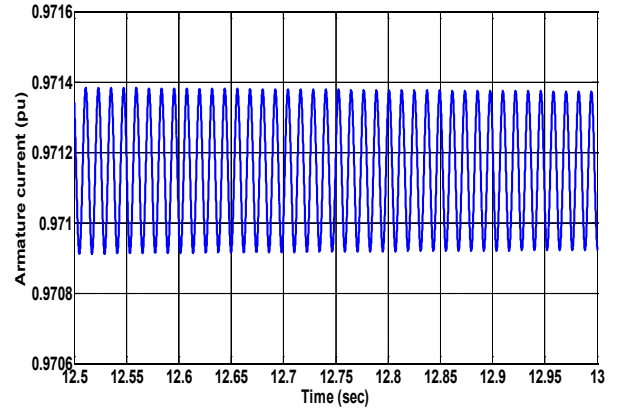


Figure 3(b). Steady state armature current response for constant load torque.

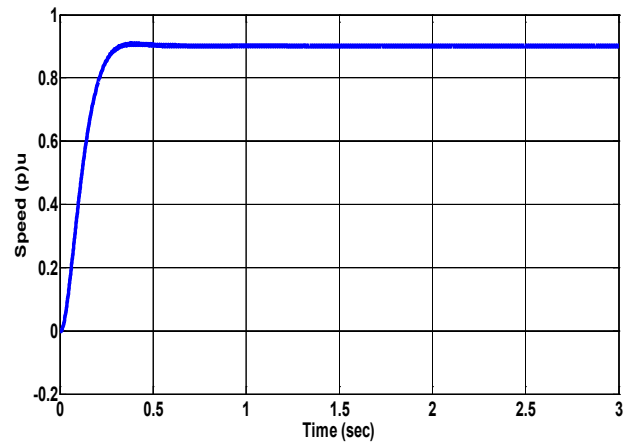


Figure 4(a). Transient speed response for constant load torque.

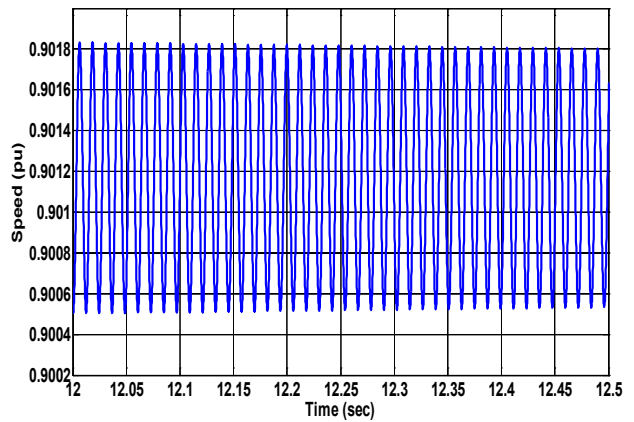


Figure 4(b). Steady state speed response for constant load torque.

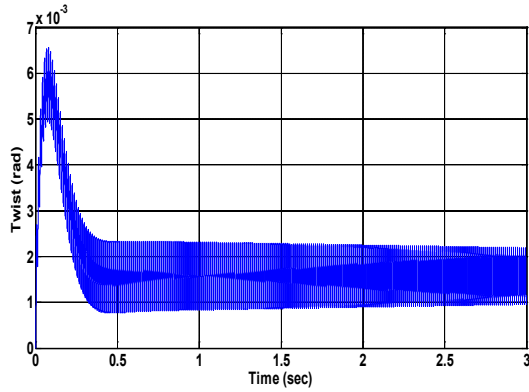


Figure 5(a). Transient shaft twist response for constant load torque.

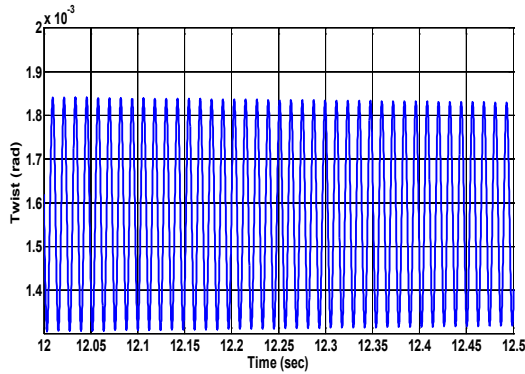


Figure 5(b). Steady state shaft twist response for constant load torque.

(ii) Performance of D.C. drive with constant voltage supply and pulsating load-torque

Voltage applied to armature: 200 V

Pulsating load torque:

$$T_{L0} = 0.75 * \text{full load torque}$$

$$T_{L1} = 0.25 * \text{full load torque}$$

$$\omega_L = 10 \text{ rad/sec}$$

In this case the load torque is assumed to be of pulsating in nature as shown in [Fig. 3]. The load torque comprises of sinusoidally varying component T_{L1} of frequency $\omega_L = 10 \text{ rad/sec}$, which is superimposed on a constant component T_{L0} .

Fig. 6(a) and 6(b) show the variation of armature current in transient and steady state, while Fig. 7(a) and Fig.7 (b) show the variation of motor speed in transient and steady state respectively. Similar variations for shaft twist are shown in Fig. 8 (a) and Fig. 8(b).

It is observed that transient as well as the steady-state wave form of armature current , motor speed and the twist also has one pulsating component at the frequency of pulsation of load-torque $\omega_L = 10 \text{ rad/sec}$ and another is oscillating at the frequency of torsional oscillations of the system $\beta_2 = 519.82 \text{ rad/sec}$, which is due to the elasticity of shaft. In this case also, the oscillations of frequency β_1 are not noticeable.

The average value of the twist, for the case under study, is equal to $1.5 * 10^{-3}$ radians and the resulting shear stress is equal to 19.125 kg/cm^2 . In this case also the shear stress is

under within the permissible limit.

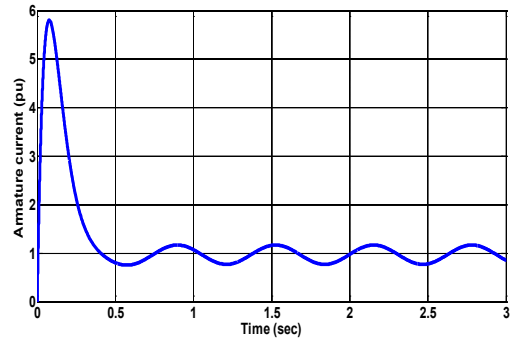


Figure 6(a). Transient armature current response for pulsating load torque

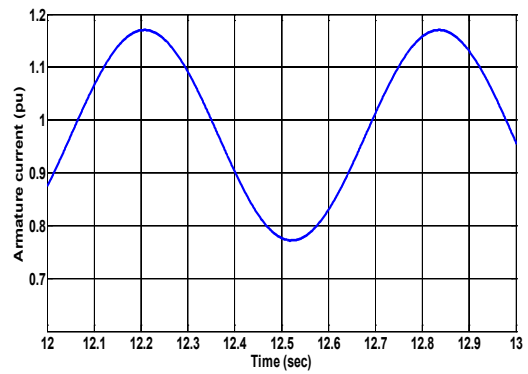


Figure 6(b). Steady state armature current response for pulsating load torque.

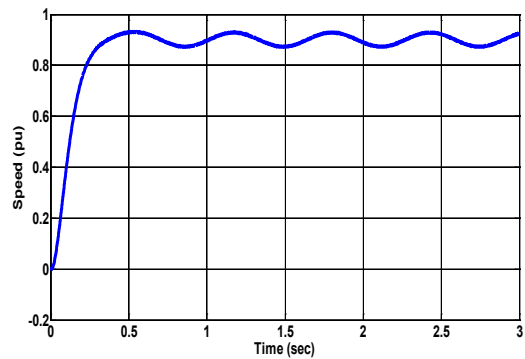


Figure 7(a). Transient speed response for pulsating load torque.

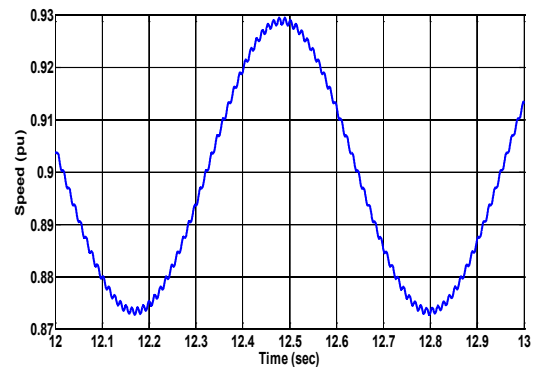


Figure 7(b). Steady state speed response for pulsating load torque.

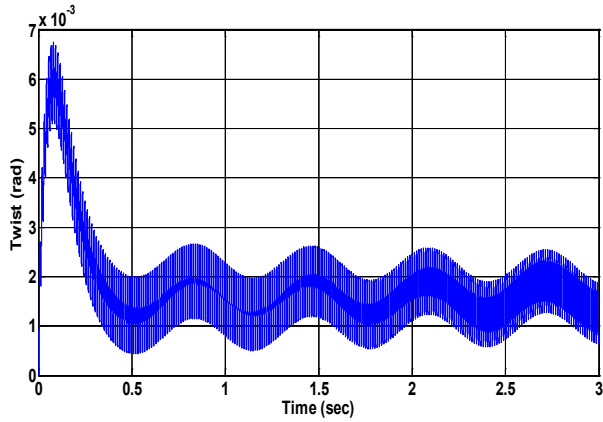


Figure 8(a). Transient twist response in shaft for pulsating load torque.

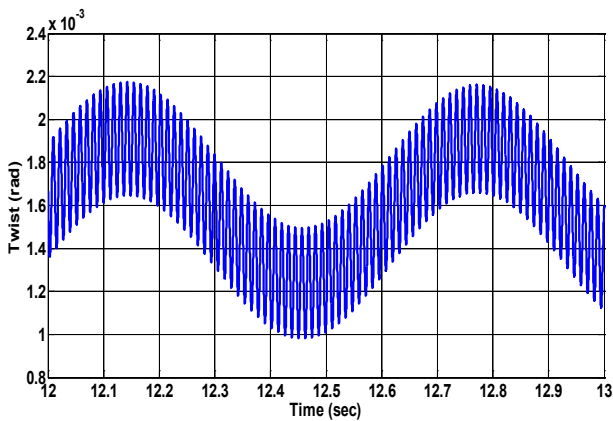


Figure 8(b). Steady-state twist response in shaft for pulsating load torque.

(B) Analysis under Resonance Conditions

In order to investigate the behavior of the dc drive under resonance conditions, the performance of the drive with elastic coupling and pulsating load torque is determined in two different cases. In the first case value ω_L is taken equal to β_1 and in the case ω_L is taken equal to β_2 . For a typical set of drive system data, the performance in terms of armature current, motor speed and twist in the shaft for the drive is determined in transient as well as steady state conditions. A study of these curves is given below:

(i) Performance of D.C. drive with load-torque pulsating at resonant frequency ($\omega_L = \beta_1 = 7.888014$)

For frequency of pulsating load torque $\omega_L = \beta_1$, Fig. 9(a) and 10(a) respectively show the transient variations of armature current and motor speed. To observe these variations more clearly, variations of armature current and motor speed for few cycles on an expanded scale are shown in Fig. 9(b) and Fig. 10(b) respectively. Transient variation for shaft twist in resonance condition is shown in Fig. 11.

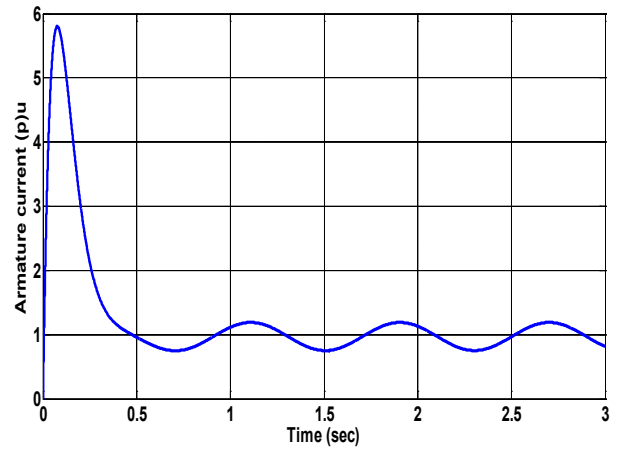


Figure 9(a). Transient armature current response in resonance condition ($\omega_L = \beta_1$).

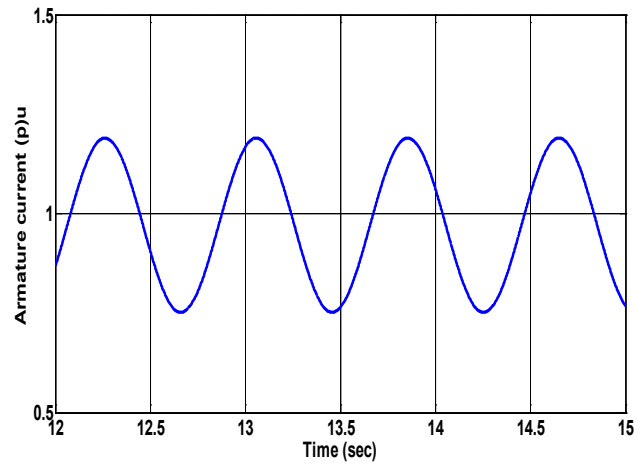


Figure 9(b). Armature current response in resonance condition ($\omega_L = \beta_1$) for few cycles on an expanded scale.

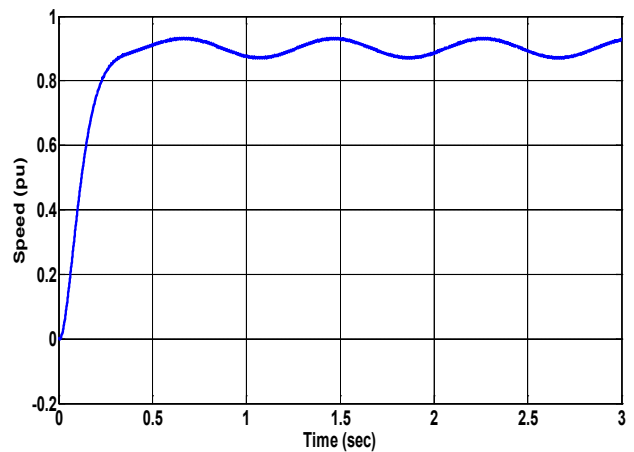


Figure 10(a). Transient motor speed response in resonance condition ($\omega_L = \beta_1$).

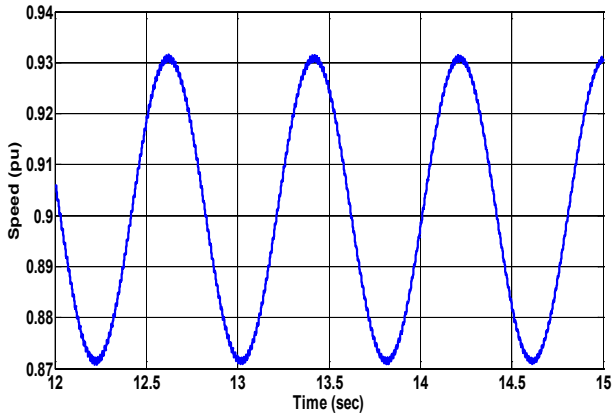


Figure 10(b). Motor speed response in resonance condition ($\omega_L = \beta_1$) for few cycles on expanded scale.

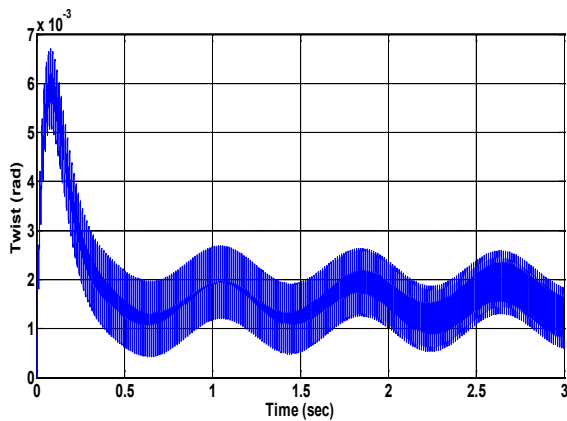


Figure 11(a). Transient response in shaft twist in resonance condition ($\omega_L = \beta_1$).

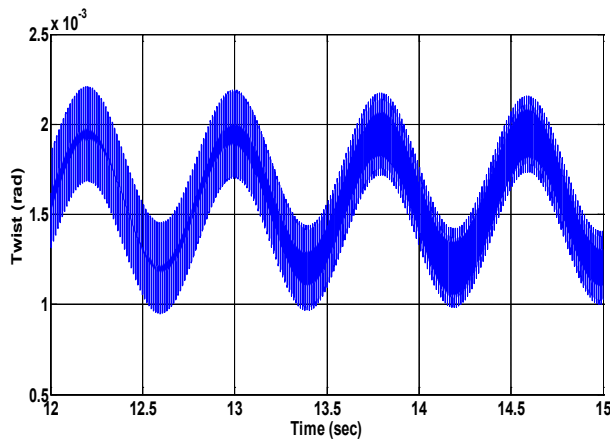


Figure 11(b). Transient response in shaft twist in resonance condition ($\omega_L = \beta_1$) few cycles on an expanded scale.

It is observed that transient variation in armature current and motor speed is composed of one component of frequency $\omega_L = \beta_1$, which is superposed on a constant component, while the second component is of frequency of the torsional oscillations β_2 . It is observed that the magnitude of the pulsation in motor speed goes on increasing with time.

Fig. 11 shows the transient variation in twist of shaft. It is

observed that magnitude of twist also goes on increasing with time. The twist attains a value of 0.06 in a short period of 3 seconds only. This is an undesirable situation as in this condition, twist will attain such a large value that the resulting shear stress may exceed the maximum permissible limit and mechanical failure of shaft will take place.

(ii) Performance of D.C. drive with load-torque pulsating at resonant frequency ($\omega_L = \beta_2 = 519.8234292$)

For frequency of pulsating load torque $\omega_L = \beta_2$, Fig. 12(a) and 13(a) respectively show the transient variations of armature current and motor speed. To observe these variations more clearly, variations of armature current and motor speed for few cycles on an expanded scale are shown in Fig. 12(b) and Fig. 13(b) respectively. Transient variations for shaft twist under resonance condition are shown in Fig. 14.

It is observed that in this case also, transient variation in armature current and motor speed is composed of one component of frequency $\omega_L = \beta_2$, which is superposed on a constant component, while the second component is of frequency β_1 . It is observed that this situation is also the resonance condition as the magnitude of the pulsation in motor speed and twist goes on increasing with time and attains large peaks.

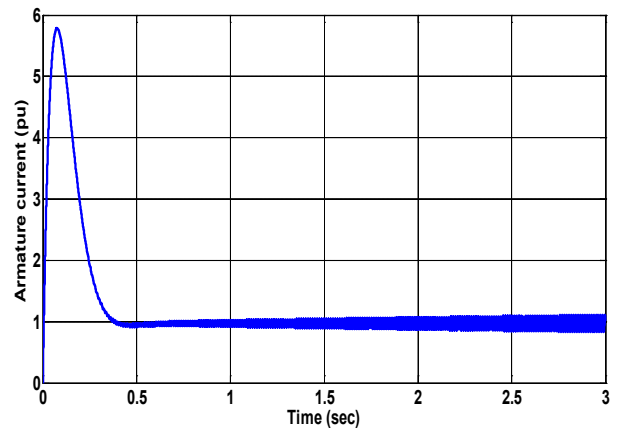


Figure 12(a). Transient armature current response in resonance condition ($\omega_L = \beta_2$).

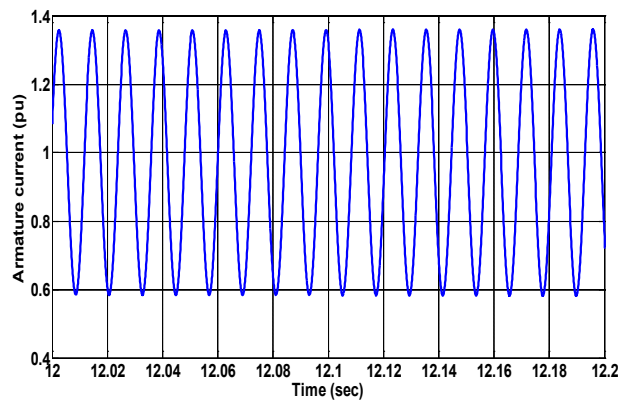


Figure 12(b). Armature current response in resonance condition ($\omega_L = \beta_2$) for few cycles on an expanded scale.

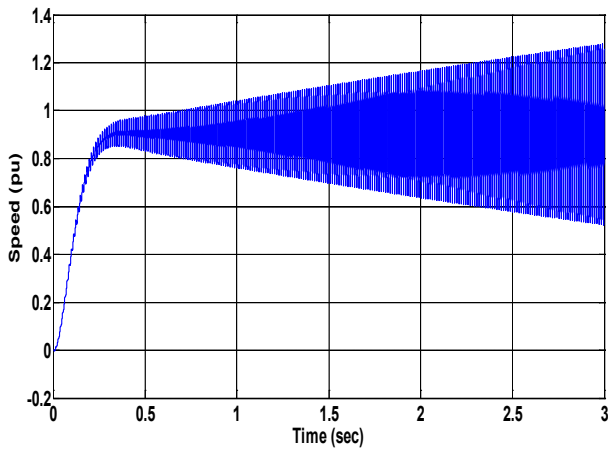


Figure 13(a). Transient motor speed response in resonance condition ($\omega_L = \beta_2$).

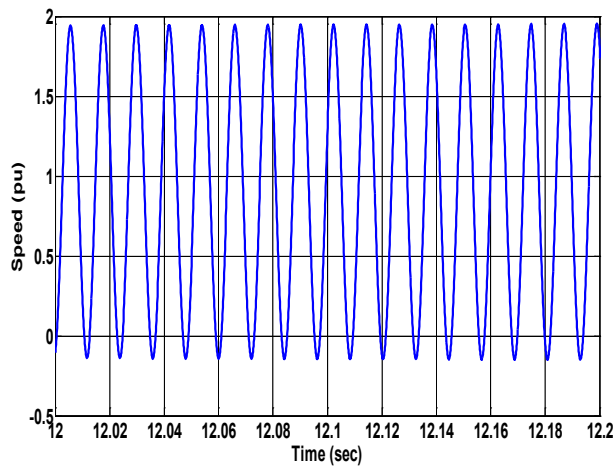


Figure 13(b). Motor speed response in resonance condition ($\omega_L = \beta_2$) for few cycles on expanded scale.

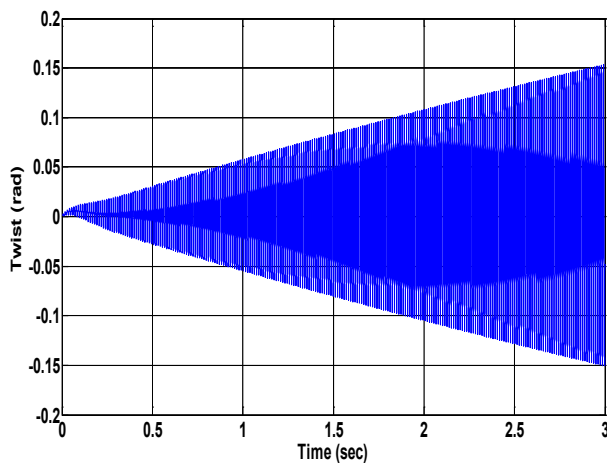


Figure 14(a). Transient Variation in shaft twist in resonance ($\omega_L = \beta_2$).

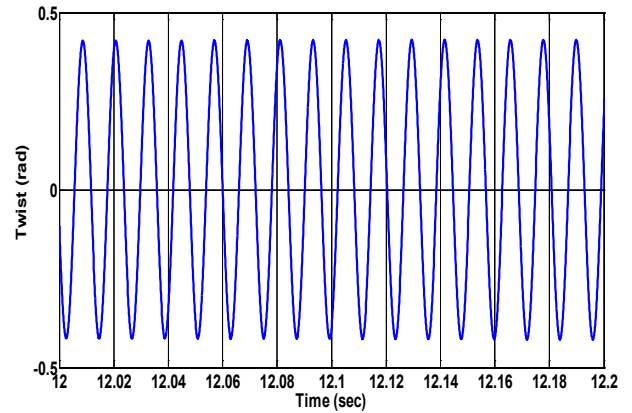


Figure 14(b). Transient Variation in shaft twist in resonance ($\omega_L = \beta_2$) few cycles on an expanded scale.

It is observed that resonance at $\omega_L = \beta_2$ is more severe compared to resonance in the previous case at $\omega_L = \beta_1$ as magnitudes of armature current, motor speed and motor shaft in this case are much higher for resonance at $\omega_L = \beta_2$.

It is observed that the twist for the case when $\omega_L = \beta_2$, attains a value of 0.15 radians in a short time of 3 seconds, whereas in the previous case of resonance at $\omega_L = \beta_1$, the value of twist was merely 0.06 radians after 3 seconds. It is obvious that for the case when $\omega_L = \beta_2$, the magnitude of twist increases at a much faster rate and the resulting value of shear stress exceeds the permissible value, which will result in the mechanical failure of the drive system.

It should be noted that value of frequency β_1 depends on value of electrical parameters i.e inductance L and resistance R of armature of motor, whereas value of frequency β_2 depends on mechanical parameters of drive such as moment of inertia J_1 and J_2 , damping B_1 and B_2 and elasticity of shaft C .

(C) Effect of Resonance on Twist and Mechanical Failure of Drive System

The shear stress produced in the shaft depends upon the shaft twist. In this section a comparative study of all the above cases discussed is done and the effect of twist on the shear stress is calculated.

The non-rigidity of shaft produces angular twist ($\theta_1 - \theta_2$). The value of twist determines the shear stress in the shaft as given below:

$$Q = G(\theta_1 - \theta_2) d / l \tag{9}$$

Where, Q = shear stress
 G = modulus of rigidity
 $\theta_1 - \theta_2$ = angular twist in shaft
 d = diameter of shaft
 l = length of shaft

The mechanical failure of the shaft may occur due to the following reasons:

(i) Failure of shaft due to excessive shear stress produced by large value of average twist under normal operating condition. It may also fail due to large values of instantaneous twist under resonance condition. The failure in the later case is termed as 'dynamic failure'.

(ii) Shaft may experience 'fatigue' due to pulsations in the values of twist. The failure due to fatigue depends upon the magnitude and frequency of the alternating component of twist.

The Ultimate shear stress is 3700-4500 kg/cm² as per Indian Standards Code: 1570-1961 for C-14 steel.

A comparison of twist and shear stress produced in normal condition ($\omega_L=0$ or $\omega_L=10$), as well as resonance condition ($\omega_L=\beta_1$ or $\omega_L=\beta_2$), as discussed in sections 3(A) and 3(B) respectively, has been summarized in Table 1.

Table 1. Comparison of Twist and Shear Stress in normal and resonance conditions at $t=15$ sec

Variation	$\omega_L=0$ (rad/sec)	$\omega_L=10$ (rad/sec)	$\omega_L=\beta_1=7.88$ (rad/sec)	$\omega_L=\beta_2=519.82$ (rad/sec)
Twist (rad)	$1.5*10^{-3}$	$1.5*10^{-3}$	0.11	0.365
Shear Stress (kg/cm ²)	19.125	19.125	1402.5	4653.75

A comparison of values shown in the Table-1 show that under normal condition of operation (when resonance does not occur). The average value of twist is calculated to be $1.5*10^{-3}$ radians and the shear stress corresponding to this value of twist is 19.125 kg/cm². Under normal condition, the shear stress produced in the shaft is very small.

Under resonance condition, when $\omega_L=\beta_1$, the value of twist is calculated to be 0.11 rad. and the resulting shear stress is 1402.5 kg/cm². This is a large value compared to the twist obtained in normal operating condition.. Though this value is in permissible range for the material, but it can produce fatigue as twist is pulsating with in very large values.

Also, when $\omega_L=\beta_2$, the value of twist is calculated to be 0.365 radians and corresponding value of shear stress is 4653.75 kg/cm². It is observed that in this case, a very large value of shear stress is produced in the shaft which exceeds the permissible range as per Indian standard code: 1570-1961 for C-14 steel. This excessive shear stress is enough to cause damage the shaft and the mechanical failure of whole drive system.

4. Need of Controller for Protection of System under Resonance

The performance of the dc drive under resonance condition analyzed shows that during resonance, the magnitude of speed, armature current, and twist goes on increasing. Under resonance condition, the increasing values of twist produce excessive shear stress in the system. If these

parameters are left unchecked, they may cause damage to the entire drive system.

To protect the system from such an incident, some protective measure is required. For this purpose, design of a controller has been suggested. It is desired that the controller must sense the presence of resonance so that the drive system is protected well before the shear stress exceeds the permissible limits. Therefore, the controller must act a little earlier than the instant at which mechanical failure takes place. For the case under study, when $\omega_L=\beta_2$, shear stress produced by the twist exceeds the maximum allowable limit at the value of twist of 0.36 radians. Therefore, for the case under study, the controller is made to operate at the value of twist equal to 0.2 radians.

4(A). MATLAB/SIMULINK MODEL FOR NORMAL OPERATION

For the case under study, the drive system is described by equations (1) to (4). These equations are simulated to obtain a simulink model of the drive as shown in Appendix-C. In this model the drive is supplied through a constant voltage source.

4(B). MATLAB / SIMULINK MODEL WITH RESONANCE CONTROLLER

The model of Appendix-C is modified to control the operation under resonance conditions. Complete Matlab/simulink model of dc drive system with resonance controller is shown in Appendix-D. A controlled voltage source has been used in controller of Appendix-D, instead of constant voltage source as used in case of model of Appendix-C.

The simulink model with resonance controller is composed of three components (a) Saturation block (b) Embedded MATLAB function block and (c) Triggered subsystem. Each component has its own function in the control process.

Saturation block is used to sense the resonance condition. The limit of the saturation block is set in such a way that when the value of twist exceeds a predefined limit, the system is identified as operating in resonance condition. For the system under study, the predefined value of the twist for the saturation block is taken as 0.2 radians.

Embedded Matlab function gives conditional output as per the requirement. In this case, its output is 1 or high, when the input to this block is greater than 0.2 radians. Otherwise the output of this functional block is zero. Thus when resonance condition arises, saturation block senses the resonance condition. The output of the saturation block is given as input to the Embedded Matlab function block, which generates the response accordingly. According to the response of embedded Matlab block, the operation of the triggered subsystem is controlled.

For the case under study, when value of twist exceeds the value 0.2, the Embedded Matlab block generates a high signal. This high signal controls the triggered subsystem and output of the triggered subsystem becomes zero. Thus the power supply is made zero, and the drive system will automatically be switched off. In this way the input voltage

to the drive system is controlled in presence of resonance, and the drive system is protected from destructive effects of the resonance condition.

5. Conclusion

The performance of a D.C. drive fed with a constant voltage source, with elastic coupling and pulsating load torque is presented in transient and steady state, both in normal as well as resonance conditions using Matlab/Simulink software as shown in Appendix-B. From the analyses presented, following conclusions are drawn:

(i) For a constant load-torque, the armature current and motor speed and twist in transient as well as steady state have pulsations of frequencies β_1 and β_2 as shown in Figs. 3(a), (b), 4(a), (b) and 5(a), (b) respectively. The value of the twist is small and the shear stress produced is well under permissible limits.

(ii). Analysis of the drive for a periodically varying load-torque of a frequency, other than the resonance frequency, taken as 10 rad/sec is presented. The armature current, motor speed and twist in transient as well as steady state have pulsations of frequencies equal to the frequency of load-torque pulsations ω_L , β_1 and β_2 as shown in Figs. 6(a), (b), 7(a), (b) and 8(a), (b) respectively. The value of twist and the resulting shear stress is almost same as for the case (i) of constant load torque. In this case also the shear stress is under the permissible limit.

(iii). Performance in resonance condition, for a periodically varying load-torque varying at a frequency ω_L equal to resonance frequency β_1 is obtained. Transient variations of armature current, motor speed and twist are shown in Figs. 9(a), (b), 10(a), (b) and Fig. 11 respectively. Similarly performance in resonance condition is also studied for the case when ω_L equal to resonance frequency β_2 , transient variations of armature current, motor speed and twist are shown in Figs. 12 (a), (b), 13(a), (b) and Fig. 14 respectively.

In both the above cases, the armature current, motor speed and the shaft twist in transient state go on increasing with time and attain large peaks. The variations of twist for these cases, in transient condition are shown in Figs. (11 and 14) respectively. It is observed that the twist attains large values within very small time. A comparative study between the resonance conditions of the two cases, reveals that resonance at $\omega_L = \beta_2$ is more severe than resonance at $\omega_L = \beta_1$. It is to be noted that value of β_2 depends upon the mechanical parameters of the drive like moments of inertia J_1 , J_2 and elasticity of shaft C , whereas value of β_1 depends on electrical parameters like inductance L and resistance R of armature of motor. A comparison of values of twist and the resulting shear stress is given in Table-1. The excessively large magnitude of twist, specially for the case when $\omega_L = \beta_2$, causes a large value of shear stress, which leads to mechanical failure of the shaft and the whole drive system. Moreover, the high frequency of reversal of twist causes

fatigue in the shaft and it leads to the dynamic failure of the system.

(iv) As discussed above, the resonance at $\omega_L = \beta_2$ is more severe, and the twist attains such large values that mechanical failure of shaft takes place due to excessive shear stress produced. Therefore care should be taken that the value of β_2 does not match the frequency of load torque ω_L . As the frequency of load torque is a system requirement and not always the designer's choice, the only way to avoid resonance is a suitable selection of mechanical parameters of drive, i.e moment of inertia J_1 , J_2 and elasticity of shaft C . The value of elasticity of shaft depends on the length and diameter of the shaft.

(v) In order to provide safety to the drive system in resonance condition, a controller has been suggested using Matlab/Simulink software as shown in Appendix-C. The controller is designed such that it acts a little earlier than the instant at which mechanical failure takes place. Thus the drive system is protected from destructive effects of the resonance conditions. The Comparison between two resonance conditions at t=15sec. has been given in Table 2.

Table 2. Comparison between two resonance conditions At t=15sec

Variation		Normal Condition		Resonance condition	
		$\omega_L=0$	$\omega_L=10$	$\omega_L=\beta_1=7.88$	$\omega_L=\beta_2=519.8$
Current (p.u)	Min	0.899	1.1	1.02	0.56
	Max	0.901	1.099	1.22	1.232
	ripple	0.001	0.005	0.10	0.335
Speed (p.u)	Min	0.909	0.94	0.67	0.01
	Max	0.913	0.935	1.21	1.813
	ripple	0.002	0.0025	0.27	0.906
Twist (rad)		$1.5 \cdot 10^{-3}$	$1.5 \cdot 10^{-3}$	0.11	0.365
Shear Stress (kg/cm ²)		19.125	19.125	1402.5	4653.75

The ultimate shear stress is 3700-4500kg/cm² (Indian Standard:1570-1961 for C-14 steel)

The paper covers various aspects of analysis of D.C. motor drives taking into account the effects of mechanical factors associated with drives. This paper analyzed the systems performance under resonance condition. However, there are still some problems on which further work is suggested.

The above study of the performance of drive under resonance condition can be repeated for Induction Motor drives. Since torsional oscillations are always present in a system because these vibrations are caused due mechanical parameters of the system. In A.C. induction motor drive supply frequency is also present in the system. So there are also some oscillations due to various frequencies are present. A resonance condition may arise when two or more frequencies become equal. The performance of the induction motor drive system under resonance condition is yet to be analyzed.

Appendix-A

System Parameters:

- DC supply voltage, $V = 200V$ (1pu)
- Full load current, $I_n = 6.3 A$ (1pu)
- Rated speed = 1000 rpm (1pu)
- Armature resistance, $R = 4\Omega$
- Armature inductance, $L = 0.16 H$
- Moment of Inertia, $J_1 = 0.05 Kg\cdot m^2$
- Damping coefficient, $B_1 = 0.008 Nm/rad/s$
- Torsional stiffness of shaft, $C = 6750 Nm/rad$
- Moment of inertia of load, $J_2 = 0.05 Kg\cdot m^2$
- Damping coefficient for load, $B_2 = 0.008 Nm/rad/s$
- Constant component of load torque, $T_{10} = 0.75 pu$
- Pulsating component of load torque, $T_{11} = 0.25 pu$
- Length of the shaft, $l = 1m$
- Diameter of the shaft, $d = 0.03m$
- Modulus of rigidity, $G = 0.85 \cdot 10^{10} kg\cdot m^2$
- θ_1, θ_2 : angular displacement of shaft at motor and load ends respectively, rad.

Appendix-B

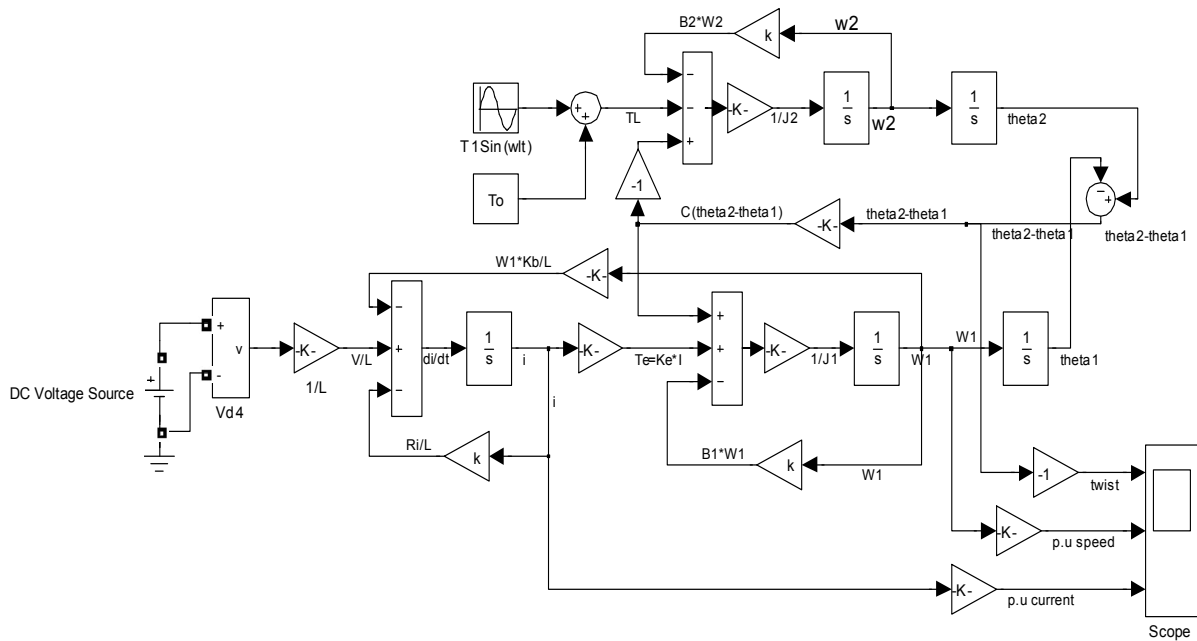
$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -C/J_1 & -B_1/J_1 & C/J_1 & 0 & K/J_1 \\ 0 & 0 & 0 & 1 & 0 \\ C/J_2 & 0 & -C/J_2 & -B_2/J_2 & 0 \\ 0 & -K/L & 0 & 0 & -R/L \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 1/L \\ 0 & 0 & 0 & -1/J_2 & 0 \end{bmatrix}^T$$

$$x = [\theta_1 \quad \dot{\theta}_1 \quad \theta_2 \quad \dot{\theta}_2 \quad i]^T$$

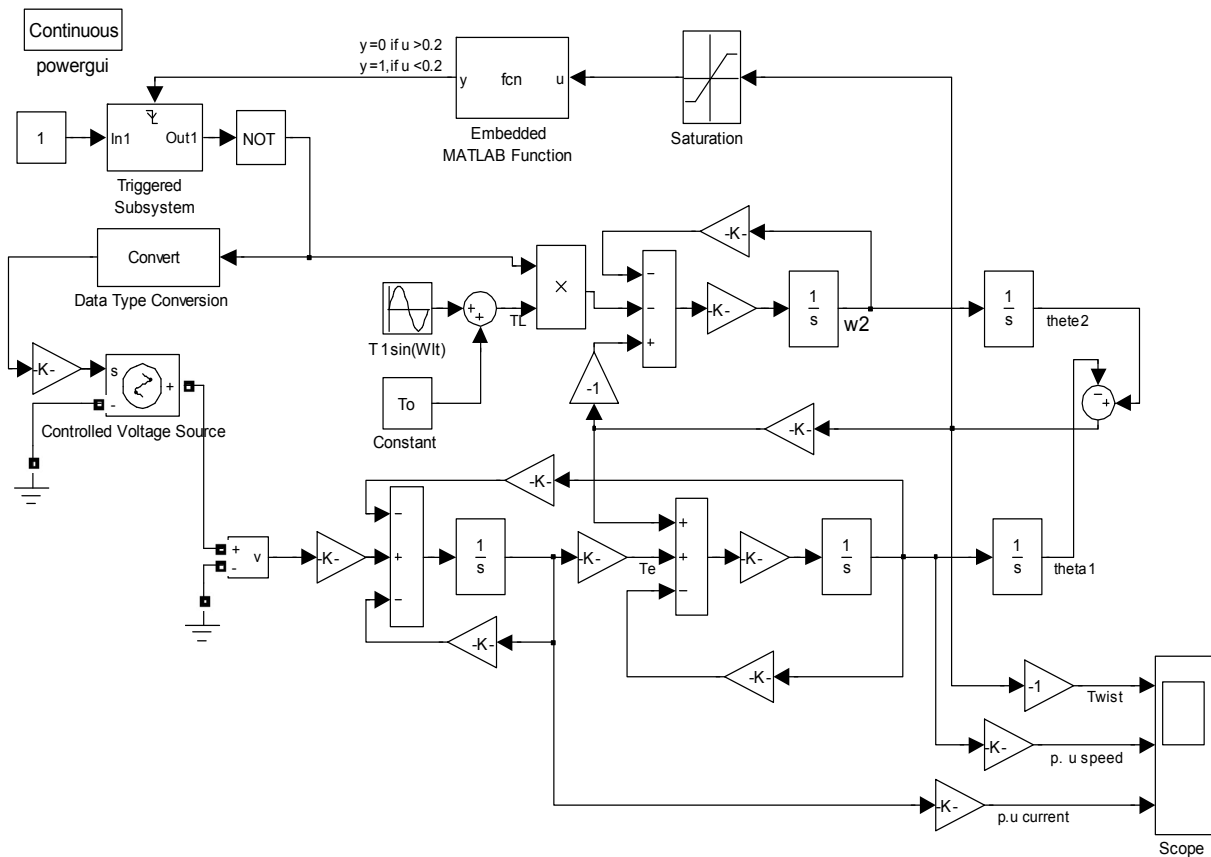
$$u = \begin{bmatrix} V \\ T_L(t) \end{bmatrix}$$

Appendix-C



(Matlab/simulink model of dc drive for normal operation)

Appendix-D



(Complete Matlab/simulink model of dc drive system with resonance controller)

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