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Strong Insertion of A Baire-one Function

Majid Mirmiran

Department of Mathematics, University of Isfahan, Isfahan , 81746-73441, Iran
mirmir@sci.ui.ac.ir

Abstract Necessary and sufficient conditions in terms of lower cut sets are given for the strong insertion of a Baire-one function between two comparable real-valued functions on the topological spaces that $\Lambda-$sets are $G_\delta-$sets.

Keywords Strong insertion, Strong binary relation, Baire-one function, $\Lambda-$sets, Lower cut set.

1 Introduction

A generalized class of closed sets was considered by Maki in 1986 [7]. He investigated the sets that can be represented as union of closed sets and called them $V-$sets. Complements of $V-$sets, i.e., sets that are intersection of open sets are called $\Lambda-$sets [7]. In addition,the insertion of a Baire- one function has also recently considered by the author in [9].

Results of Katětov [3] , [4] concerning binary relations and the concept of an indefinite lower cut set for a real-valued function, which is due to Brooks [2], are used in order to give necessary and sufficient conditions for the strong insertion of a Baire-one function between two comparable real-valued functions on the topological spaces that $\Lambda-$sets are $G_\delta-$sets.

A real-valued function f defined on a topological space X is called *Baire-one* if the preimage of every open subset of R is a $F_\sigma-$set in X.

If g and f are real-valued functions defined on a space X, we write $g \leq f$ in case $g(x) \leq f(x)$ for all x in X.

The following definitions are modifications of conditions considered in [5].

A property P defined relative to a real-valued function on a topological space is a B_1-*property* provided that any constant function has property P and provided that the sum of a function with property P and any Baire-one function also has property P. If P_1 and P_2 are B_1-properties, the following terminology is used: (i) A space X has the *weak* B_1-*insertion property for* (P_1, P_2) iff for any functions g and f on X such that $g \leq f, g$ has property P_1 and f has property P_2, then there exists a Baire-one function h such that $g \leq h \leq f$.
(ii) A space X has the *strong* B_1-*insertion property for* (P_1, P_2) iff for any functions g and f on X such that $g \leq f, g$ has property P_1 and f has property P_2, then there exists a Baire-one function h such that $g \leq h \leq f$ and such that if $g(x) < f(x)$ for any x in X, then $g(x) < h(x) < f(x)$.

In this paper, for a topological space that $\Lambda-$sets are $G_\delta-$sets, is given a sufficient condition for the weak B_1-insertion property. Also for a space with the weak B_1-insertion property, we give necessary and sufficient conditions for the space to have the strong B_1-insertion property. Several insertion theorems are obtained as corollaries of these results.

2 The main results

Before giving a sufficient condition for insertability of a Baire-one function, the necessary definitions and terminology are stated.

Definition. Let A be a subset of a topological space (X, τ). We define the subsets A^Λ and A^V as follows:
$A^\Lambda = \cap\{O : O \supseteq A, O \in (X, \tau)\}$ and $A^V = \cup\{F : F \subseteq A, F^c \in (X, \tau)\}$.
A^Λ is called *kernel* of A.

The following first two definitions are modifications of conditions considered in [3], [4].

Definition. If ρ is a binary relation in a set S then $\bar{\rho}$ is defined as follows: $x \ \bar{\rho} \ y$ if and only if $y \ \rho \ \nu$ implies $x \ \rho \ \nu$ and $u \ \rho \ x$ implies $u \ \rho \ y$ for any u and v in S.

Definition. A binary relation ρ in the power set $P(X)$ of a topological space X is called a *strong binary relation* in $P(X)$ in case ρ satisfies each of the following conditions:

1) If $A_i \ \rho \ B_j$ for any $i \in \{1, \ldots, m\}$ and for any $j \in \{1, \ldots, n\}$, then there exists a set C in $P(X)$ such that $A_i \ \rho \ C$ and $C \ \rho \ B_j$ for any $i \in \{1, \ldots, m\}$ and any $j \in \{1, \ldots, n\}$.

2) If $A \subseteq B$, then $A \ \bar{\rho} \ B$.

3) If $A \ \rho \ B$, then $A^\Lambda \subseteq B$ and $A \subseteq B^V$.

The concept of a lower indefinite cut set for a real-valued function was defined by Brooks [2] as follows:

Definition. If f is a real-valued function defined on a space X and if $\{x \in X : f(x) < \ell\} \subseteq A(f, \ell) \subseteq \{x \in X : f(x) \leq \ell\}$ for a real number ℓ, then $A(f, \ell)$ is a *lower indefinite cut set* in the domain of f at the level ℓ.

We now give the following main results:

Theorem 2.1. Let g and f be real-valued functions on the topological space X, that $\Lambda-$sets in X are $G_\delta-$sets, with $g \leq f$. If there exists a strong binary relation ρ on the power set of X and if there exist lower indefinite cut sets $A(f, t)$ and $A(g, t)$ in the domain of f and g at the level t for each rational number t such that if $t_1 < t_2$ then $A(f, t_1)\ \rho\ A(g, t_2)$, then there exists a Baire-one function h defined on X such that $g \leq h \leq f$.
Proof. Theorem 2.1, of [9].

If a space has the strong B_1-insertion property for (P_1, P_2), then it has the weak B_1-insertion property for (P_1, P_2).The following result uses lower cut sets and gives a necessary and sufficient condition for a space satisfies that weak B_1-insertion property to satisfy the strong B_1-insertion property.

Theorem 2.2. Let P_1 and P_2 be B_1-property and X be a space that satisfies the weak B_1-insertion property for (P_1, P_2). Also assume that g and f are functions on X such that $g \leq f, g$ has property P_1 and f has property P_2. The space X has the strong B_1-insertion property for (P_1, P_2) iff there exist lower cut sets $A(f-g, 2^{-n})$ and there exists a sequence $\{F_n\}$ of subsets of X such that (i) for each n, F_n and $A(f-g, 2^{-n})$ are completely separated by Baire-one functions, and (ii)$\{x \in X : (f-g)(x) > 0\} = \bigcup_{n=1}^{\infty} F_n$.
Proof. Suppose that there is a sequence $(A(f-g, 2^{-n}))$ of lower cut sets for $f - g$ and suppose that there is a sequence (F_n) of subsets of X such that

$$\{x \in X : (f-g)(x) > 0\} = \bigcup_{n=1}^{\infty} F_n$$

and such that for each n, there exists a Baire-one function k_n on X into $[0, 2^{-n}]$ with $k_n = 2^{-n}$ on F_n and $k_n = 0$ on $A(f-g, 2^{-n})$. The function k from X into $[0, 1/4]$ which is defined by

$$k(x) = 1/4 \sum_{n=1}^{\infty} k_n(x)$$

is a Baire-one function by the Cauchy condition and the properties of Baire-one functions, (1) $k^{-1}(0) = \{x \in X : (f-g)(x) = 0\}$ and (2) if $(f-g)(x) > 0$ then $k(x) < (f-g)(x)$: In order to verify (1), observe that if $(f-g)(x) = 0$, then $x \in A(f-g, 2^{-n})$ for each n and hence $k_n(x) = 0$ for each n. Thus $k(x) = 0$. Conversely, if $(f-g)(x) > 0$, then there exists an n such that $x \in F_n$ and hence $k_n(x) = 2^{-n}$. Thus $k(x) \neq 0$ and this verifies (1). Next, in order to establish (2), note that

$$\{x \in X : (f-g)(x) = 0\} = \bigcap_{n=1}^{\infty} A(f-g, 2^{-n})$$

and that $(A(f-g, 2^{-n}))$ is a decreasing sequence. Thus if $(f-g)(x) > 0$ then either $x \notin A(f-g, 1/2)$ or there exists a smallest n such that $x \notin A(f-g, 2^{-n})$ and $x \in A(f-g, 2^{-j})$ for $j = 1, \ldots, n-1$.
In the former case,

$$k(x) = 1/4 \sum_{n=1}^{\infty} k_n(x) \leq 1/4 \sum_{n=1}^{\infty} 2^{-n} < 1/2 \leq (f-g)(x),$$

and in the latter,

$$k(x) = 1/4 \sum_{j=n}^{\infty} k_j(x) \leq 1/4 \sum_{j=n}^{\infty} 2^{-j} < 2^{-n} \leq (f-g)(x).$$

Thus $0 \leq k \leq f-g$ and if $(f-g)(x) > 0$ then $(f-g)(x) > k(x) > 0$. Let $g_1 = g+(1/4)k$ and $f_1 = f-(1/4)k$. Then $g \leq g_1 \leq f_1 \leq f$ and if $g(x) < f(x)$ then

$$g(x) < g_1(x) < f_1(x) < f(x).$$

Since P_1 and P_2 are B_1-properties, then g_1 has property P_1 and f_1 has property P_2. Since by hypothesis X has the weak B_1-insertion property for (P_1, P_2) , then there exists a Baire-one function h such that $g_1 \leq h \leq f_1$. Thus $g \leq h \leq f$ and if $g(x) < f(x)$ then $g(x) < h(x) < f(x)$. Therefore X has the strong B_1-insertion property for (P_1, P_2). (The technique of this proof is by Lane [5].)

Conversely, assume that X satisfies the strong B_1-insertion for (P_1, P_2). Let g and f be functions on X satisfying P_1 and P_2 respectively such that $g \leq f$. Thus there exists a Baire-one function h such that $g \leq h \leq f$ and such that if $g(x) < f(x)$ for any x in X, then $g(x) < h(x) < f(x)$. We follow an idea contained in Powderly [10]. Now consider the functions 0 and $f - h$.0 satisfies property P_1 and $f - h$ satisfies property P_2. Thus there exists a Baire-one function h_1 such that $0 \leq h_1 \leq f - h$ and if $0 < (f-h)(x)$ for any x in X, then $0 < h_1(x) < (f-h)(x)$. We next show that

$$\{x \in X : (f-g)(x) > 0\} = \{x \in X : h_1(x) > 0\}.$$

If x is such that $(f-g)(x) > 0$, then $g(x) < f(x)$. Therefore $g(x) < h(x) < f(x)$. Thus $f(x) - h(x) > 0$ or $(f-h)(x) > 0$. Hence $h_1(x) > 0$. On the other hand, if $h_1(x) > 0$, then since $(f-h) \geq h_1$ and $f - g \geq f - h$, therefore $(f-g)(x) > 0$. For each n, let

$$A(f-g, 2^{-n}) = \{x \in X : (f-g)(x) \leq 2^{-n}\},$$

$\mathrm{F}_n = \{x \in X : h_1(x) \geq 2^{-n+1}\}$*and*
$k_n = \sup\{\inf\{h_1, 2^{-n+1}\}, 2^{-n}\} - 2^{-n}$.
Since$\{$x$\in X : (f-g)(x) > 0\} = \{x \in X : h_1(x) > 0\}$, it follows that

$$\{x \in X : (f-g)(x) > 0\} = \bigcup_{n=1}^{\infty} F_n.$$

We next show that k_n is a Baire-one function which completely separates F_n and $A(f-g, 2^{-n})$. From its definition and by the properties of Baire-one functions, it is clear that k_n is a Baire-one function. Let $x \in F_n$. Then, from the definition of $k_n, k_n(x) = 2^{-n}$. If $x \in A(f-g, 2^{-n})$, then since $h_1 \leq f - h \leq f - g, h_1(x) \leq 2^{-n}$. Thus $k_n(x) = 0$, according to the definition of k_n. Hence k_n completely separates F_n and $A(f-g, 2^{-n})$.

Theorem 2.3. Let P_1 and P_2 be B_1-properties and assume that the space X satisfied the weak B_1-insertion property for (P_1, P_2). The space X satisfies the strong B_1-insertion property for (P_1, P_2) iff X satisfies the strong B_1-insertion property for (P_1, B_1) and for (B_1, P_2).
Proof. Assume that X satisfies the strong B_1-insertion property for (P_1, B_1) and for (B_1, P_2). If g and f are functions on X such that $g \leq f, g$ satisfies property P_1, and f satisfies property P_2, then since X satisfies the weak B_1-insertion property for (P_1, P_2) there is a Baire-one function k such that $g \leq k \leq f$. Also, by hypothesis there exist Baire-one functions h_1 and h_2 such that $g \leq h_1 \leq k$ and if $g(x) < k(x)$ then $g(x) < h_1(x) < k(x)$ and such that $k \leq h_2 \leq f$ and if $k(x) < f(x)$ then $k(x) < h_2(x) < f(x)$. If a function h is defined by $h(x) = (h_2(x) + h_1(x))/2$, then h is a Baire-one function, $g \leq h \leq f$, and if $g(x) < f(x)$ then $g(x) < h(x) < f(x)$. Hence X satisfies the strong B_1-insertion property for (P_1, P_2). The converse is obvious since any Baire-one function must satisfy both properties P_1 and P_2. (The technique of this proof is by Lane [6].)

2.1 Applications

Definition. A real-valued function f defined on a space X is called *upper semi-Baire-one* (resp. *lower semi-Baire-one*) if $f^{-1}(-\infty, t)$ (resp. $f^{-1}(t, +\infty)$) is a $F_\sigma-$set for any real number t.

The abbreviations usc, lsc, usB_1 and lsB_1 are used for upper semicontinuous, lower semicontinuous, upper semi-Baire-one, and lower semi-Baire-one, respectively.

Example : Let X be linearly ordered by a relation $\leq$.Take as open subsets for a topology on X all sets of the form $\{x \in X : x < a\}$, for $a \in X$, then X is a topological space that $\Lambda-$sets are $G_\delta-$sets.

Note : If X is a T_1 space, the results of this paper concerning insertion of Baire-one functions are trivial: Every subset of X is a $V-$set since it is a union of singletons.In a space where every $V-$set is an $F_\sigma-$set, every subset of X is both an $F_\sigma-$set and a $G_\delta-$set.For such a space,every real-valued function on X is a Baire-one function.Also, if X is a T_1 space, for any subset A of X, A^Λ and A^V are both equal to A.

Remark 1. [3], [4]. A space X has the weak $c-$insertion property for (usc, lsc) iff X is normal.

Before stating the consequences of Theorems 2.1, 2.2, and 2.3 we suppose that X is a topological space that $\Lambda-$sets are $G_\delta-$sets.

Corollary 3.1. For each pair of disjoint $G_\delta-$sets G_1, G_2, there are two $F_\sigma-$sets F_1 and F_2 such that $G_1 \subseteq F_1$, $G_2 \subseteq F_2$ and $F_1 \cap F_2 = \emptyset$ iff X has the weak B_1-insertion property for (usB_1, lsB_1).
Proof. Corollary 3.1, of [9].

Before stating the consequences of Theorem 2.2, we state and prove the necessary lemmas.

Lemma 3.2. The following conditions on the space X are equivalent:

(i) Every two disjoint $G_\delta-$sets of X can be separated by $F_\sigma-$sets of X.

(ii) If G is a $G_\delta-$set of X which is contained in a $F_\sigma-$set F, then there exists a $F_\sigma-$set H such that $G \subseteq H \subseteq H^\Lambda \subseteq F$.
Proof. Lemma 3.2, of [9].

Lemma 3.3. Suppose that X is the topological space s.t. we can separate every two disjoint $G_\delta-$sets by $F_\sigma-$sets. If G_1 and G_2 are two disjoint $G_\delta-$sets of X, then there exists a Baire-one function h on X into $[0,1]$ s.t. $h(G_1) = \{0\}$ and $h(G_2) = \{1\}$.
Proof. Lemma 3.3, of [9].

Lemma 3.4. Suppose that X is the topological space s.t. we can separate every two disjoint $G_\delta-$sets by $F_\sigma-$sets. If G_1 and G_2 are two disjoint $G_\delta-$sets of X and G_1 is a countable intersection of $F_\sigma-$sets, then there exists a Baire-one function h on X into $[0,1]$ s.t. $h^{-1}(0) = G_1$ and $h(G_2) = \{1\}$.
Proof. Suppose that $G_1 = \bigcap_{n=1}^\infty F_n$, where F_n is a $F_\sigma-$set of X. We can suppose that $F_n \cap G_2 = \emptyset$, otherwise we can substitute F_n by $F_n \setminus G_2$. By Lemma 3.3, for every $n \in N$, there exists a Baire-one function h_n on X into $[0,1]$ s.t. $h_n(G_1) = \{0\}$ and $h_n(X \setminus F_n) = \{1\}$. We set $h(x) = \sum_{n=1}^\infty 2^{-n} h_n(x)$.

Since the above series is uniformly convergent, it follows that h is a Baire-one function from X to $[0,1]$. Since for every $n \in N, G_2 \subseteq X \setminus F_n$, therefore $h_n(G_2) = \{1\}$ and consequently $h(G_2) = \{1\}$. Since $h_n(G_1) = \{0\}$, hence $h(G_1) = \{0\}$. It suffices to show that if $x \notin G_1$, then $h(x) \neq 0$.

Now if $x \notin G_1$, since $G_1 = \bigcap_{n=1}^\infty F_n$, therefore there exists $n_0 \in N$ s.t. $x \notin F_{n_0}$, hence $h_{n_0}(x) = 1$, i.e., $h(x) > 0$. Therefore $h^{-1}(0) = G_1$.

Lemma 3.5. Suppose that X is the topological space s.t. we can separate every two disjoint $G_\delta-$sets by $F_\sigma-$sets. The following conditions are equivalent:

(i) For every two disjoint $G_\delta-$sets G_1 and G_2, there exists a Baire-one function h on X into $[0,1]$ s.t. $h^{-1}(0) = G_1$ and $h^{-1}(1) = G_2$.

(ii) Every $G_\delta-$set is a countable intersection of $F_\sigma-$set.

(iii) Every $F_\sigma-$set is a countable union of $G_\delta-$set.
Proof. (i) $\Rightarrow$ (ii). Suppose that G is a $G_\delta-$sets. Since $\emptyset$ is a $G_\delta-$set, by (i) there exists a Baire-one function h on X into $[0,1]$ s.t. $h^{-1}(0) = G$. Set $F_n = \{x \in X : h(x) < \frac{1}{n}\}$. Then for every $n \in N$, F_n is a $F_\sigma-$set and $\bigcap_{n=1}^\infty F_n = \{x \in X : h(x) = 0\} = G$.

(ii) $\Rightarrow$ (i). Suppose that G_1 and G_2 are two disjoint $G_\delta-$sets. By Lemma 3.4, there exists a Baire-one function f on X into $[0,1]$ s.t. $f^{-1}(0) = G_1$ and $f(G_2) = \{1\}$. Set $F = \{x \in X : f(x) < \frac{1}{2}\}$, $G = \{x \in X : f(x) = \frac{1}{2}\}$, and $H = \{x \in X : f(x) > \frac{1}{2}\}$. Then $F \cup G$ and $H \cup G$ are two $G_\delta-$sets and $(F \cup G) \cap G_2 = \emptyset$. By Lemma 3.4, there exists a Baire-one function g on X

into $[\frac{1}{2}, 1]$ s.t. $g^{-1}(1) = G_2$ and $g(F \cup G) = \{\frac{1}{2}\}$. Define h by $h(x) = f(x)$ for $x \in F \cup G$, and $h(x) = g(x)$ for $x \in H \cup G$. h is well-defined and a Baire-one function, since $(F \cup G) \cap (H \cup G) = G$ and for every $x \in G$ we have $f(x) = g(x) = \frac{1}{2}$. Furthermore, $(F \cup G) \cup (H \cup G) = X$, hence h defined on X and maps to $[0, 1]$. Also, we have $h^{-1}(0) = G_1$ and $h^{-1}(1) = G_2$.

(ii) $\Leftrightarrow$ (iii) By De Morgan law and noting that the complement of every $G_\delta-$set is a $F_\sigma-$set and complement of every $F_\sigma-$set is a $G_\delta-$set, the equivalence is hold.

Remark 2. [8] . A space X has the strong $c-$insertion property for (usc, lsc) iff X is perfectly normal.

Corollary 3.6. For every two disjoint $G_\delta-$sets G_1 and G_2, there exists a Baire-one function h on X into $[0, 1]$ s.t. $h^{-1}(0) = G_1$ and $h^{-1}(1) = G_2$ iff X has the strong B_1-insertion property for (usB_1, lsB_1).

Proof. Since for every two disjoint $G_\delta-$sets G_1 and G_2, there exists a Baire-one function h on X into $[0, 1]$ s.t. $h^{-1}(0) = G_1$ and $h^{-1}(1) = G_2$, define $F_1 = \{x \in X : h(x) < \frac{1}{2}\}$ and $F_2 = \{x \in X : h(x) > \frac{1}{2}\}$. Then F_1 and F_2 are two disjoint $F_\sigma-$sets that contain G_1 and G_2, respectively. This means that, we can separate every two disjoint $G_\delta-$sets by $F_\sigma-$sets. Hence by Corollary 3.1, X has the weak B_1-insertion property for (usB_1, lsB_1). Now, assume that g and f are functions on X such that $g \leq f$, g is usB_1 and f is lsB_1. Since $f - g$ is lsB_1, therefore the lower cut set $A(f - g, 2^{-n}) = \{x \in X : (f - g)(x) \leq 2^{-n}\}$ is a $G_\delta-$set. By Lemma 3.5, we can choose a sequence $\{G_n\}$ of $G_\delta-$sets s.t. $\{x \in X : (f - g)(x) > 0\} = \bigcup_{n=1}^{\infty} G_n$ and for every $n \in N$, G_n and $A(f - g, 2^{-n})$ are disjoint. By Lemma 3.3, G_n and $A(f - g, 2^{-n})$ can be completely separated by Baire-one functions. Hence by Theorem 2.2, X has the strong B_1-insertion property for (usB_1, lsB_1).

On the other hand, suppose that G_1 and G_2 are two disjoint $G_\delta-$sets. Since $G_1 \cap G_2 = \emptyset$, hence $G_2 \subseteq G_1^c$. Set $g = \chi_{G_2}$ and $f = \chi_{G_1^c}$. Then f is lsB_1 and g is usB_1 and furthermore $g \leq f$. By hypothesis, there exists a Baire-one function h on X s.t. $g \leq h \leq f$ and whenever $g(x) < f(x)$ we have $g(x) < h(x) < f(x)$. By definitions of f and g, we have $h^{-1}(1) = G_2 \cap G_1^c = G_2$ and $h^{-1}(0) = G_1 \cap G_2^c = G_1$.

Remark 3. [11]. A space X has the weak $c-$insertion property for (lsc, usc) iff X is extremally disconnected.

Corollary 3.7. For every F of $F_\sigma-$set, F^Λ is a $F_\sigma-$set iff X has the weak B_1-insertion property for (lsB_1, usB_1).

Proof. Corollary 3.6, of [9].

Remark 4. [1]. A space X has the strong $c-$insertion property for (lsc, usc) iff each open subset of X is closed.

Corollary 3.8. Every $F_\sigma-$set is a $G_\delta-$set iff X has the strong B_1-insertion property for (lsB_1, usB_1).

Proof. By hypothesis, for every F of $F_\sigma-$set, we have $F^\Lambda = F$ is a $F_\sigma-$set. Hence by Corollary 3.7, X has the weak B_1-insertion property for (lsB_1, usB_1). Now, assume that g and f are functions on X such that $g \leq f$, g is lsB_1 and f is B_1. Set $A(f - g, 2^{-n}) = \{x \in X : (f - g)(x) < 2^{-n}\}$. Then, since $f - g$ is usB_1, we can say that $A(f - g, 2^{-n})$ is a $F_\sigma-$set. By hypothesis, $A(f - g, 2^{-n})$ is a $G_\delta-$set. Set $F_n = X \backslash A(f - g, 2^{-n})$. Then F_n is a $F_\sigma-$set. This means that F_n and $A(f - g, 2^{-n})$ are disjoint $F_\sigma-$sets and also are two disjoint $G_\delta-$sets. Therefore F_n and $A(f - g, 2^{-n})$ can be completely separated by Baire-one functions. Now, we have $\bigcup_{n=1}^{\infty} F_n = \{x \in X : (f - g)(x) > 0\}$. By Theorem 2.2, X has the strong B_1-insertion property for (lsB_1, B_1). By an analogous argument, we can prove that X has the strong B_1-insertion property for (B_1, usB_1). Hence, by Theorem 2.3, X has the strong B_1-insertion property for (lsB_1, usB_1).

On the other hand, suppose that X has the strong B_1-insertion property for (lsB_1, usB_1). Also, suppose that F is a $F_\sigma-$set. Set $f = 1$ and $g = \chi_F$. Then f is usB_1, g is lsB_1 and $g \leq f$. By hypothesis, there exists a Baire-one function h on X s.t. $g \leq h \leq f$ and whenever $g(x) < f(x)$, we have $g(x) < h(x) < f(x)$. It is clear that $h(F) = \{1\}$ and for $x \in X \setminus F$ we have $0 < h(x) < 1$. Since h is a Baire-one function, therefore $\{x \in X : h(x) \geq 1\} = F$ is a $G_\delta-$set, i.e., F is a $G_\delta-$set.

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