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Stability and Mass Parabola in Integrated Nuclear Model

N.Ghahramany*, H.Sarafraza, E. Yazdankish

Physics Department, Shiraz University, Shiraz, Iran
*Corresponding Author: ghahramany@susc.ac.ir

Abstract Following our previous introduction of nuclear quark-like model, in this paper, a more precise formula is presented for nuclear binding energy in the context of modified Integrated Nuclear Model (INM). INM is based upon quark-like model with three basic assumptions from which the nuclear binding energy and magic numbers are easily obtained. INM is modified here to give the nuclear mass parabola and most stable nuclei in each nuclei group. Our findings are compared with Liquid Drop Model (LDM), unmodified Integrated Nuclear Model (INM) and experimental data for most stable nuclei.

Keywords Stability, Mass Parabola, Integrated Nuclear Model, Binding Energy

1. Introduction

Nuclear physicists have been struggling hard for so many years to present a simple and complete nuclear model from which the characteristics of nuclei can be explained and comprehended .The existing successful nuclear models can only explain certain characteristics of the nuclei and make no comments about other nuclear properties. For example Liquid Drop Model (LDM) was presented by Von Weizsacker [1] and then was extended by Bohr and Wheeler [2].This model give the nuclear binding energy formula in terms of A and Z [3] as follow:

$$B(A,Z) = a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_a(N - Z2A - 1 \pm \delta + \eta \quad (1)$$

In which five experimentally determined constants are introduced.

Liquid Drop Model (LDM) has been successful in the calculation of binding energy, mass parabola and most stable isobars. However this model fails to predict other properties of nuclei such as, the magic numbers and nuclear magnetic moments. On the other hand Nuclear Shell Model [4] which is based upon Schrodinger equation solution with selected potential such as rounded edge potential well , predict the magic numbers and nuclear magnetic moments by using spin-orbit couplings in a relatively complicated manners[5,6].

We have presented the Integrated Nuclear Model (INM) based upon nuclear quark-like model from which all magic numbers are easily obtained and a new magic number is predicted [7,8]. This model is also used to find the magnetic dipole moment of deuteron with greater precision [9]. Using INM, the nuclear binding energy formula was also obtained from quark-like model of nuclei [10] and is given as follow:

$$B(A,Z) = \left[A - \left(\frac{(N^2-Z^2)+\delta(N-Z)}{3Z} + 3\right)\right] \times \frac{M_N c^2}{\alpha} \quad A > 5 \quad (2)$$

In formula (2) $\delta(N-Z) = \begin{cases} 0, & N \neq Z \\ 1, & N = Z \end{cases}$

and $M_N c^2$ is the mass of nucleon instead of up-quark mass used in reference [10]. The coefficient α is a dimensionless constant defined to have a range from 90 to 100 [10].

Formula (2) provides the nuclear binding energy for most of the stable nuclei in term of only one coefficient namely α which is simpler than LDM with several coefficients. However this formula needs to be modified to give us the mass parabola in the same way as LDM.

In this paper the modified nuclear binding energy formula is presented in order to find the nuclear mass parabola and the stability of isobaric groups of nuclides. The determination of the mass parabola itself is an indication of the validity of the INM which is based upon the quark structure of the nuclei instead of nucleon structure.

2. Modified Nuclear Binding Energy Formula in INM

In the binding energy formula (2) presented in INM a coefficient α is introduced which varies between 90 to 100 for all stable nuclides with $A > 5$. The coefficient α may be called “nuclear stability coefficient”.

A careful investigation of the stability coefficient for many stable nuclides indicates the fact that α depends upon atomic number (Z) and mass number (A). In fact for isobar nuclides, the stability coefficient α is proportional to atomic number (Z) whereas for isotopic nuclides the stability coefficient α is inversely proportional to the mass number (A). Therefore

$$\alpha \propto \frac{Z}{A}$$

Further analysis of most of the stable nuclides allows proper modification of formula (2) as follow:

$$B(A,Z) = \left[A - \left(\frac{(N^2-Z^2)+\delta(N-Z)}{3(Z-k)} + 3\right)\right] \times \frac{A^{n+s} M_N c^2}{126(Z-k)} \qquad A > 5 \qquad (3)$$

In which the coefficient k, s and n are defined as

$$s = \begin{cases} 0.0003, & N,Z\ even \\ -0.0003, & N,Z\ odd \end{cases} \qquad k = \begin{cases} 2, & Z \le 118 \\ 0, & Z > 118 \end{cases} \qquad n = 0.87\ to\ 0.88$$

As can be seen, only one free coefficient namely, n exist that has a limited range of fine tuning and all other coefficients are known constants.

Now with this modified formula (3), one can find the mass parabolas and stability line.

First we write the nuclear mass as follow:

$$M(A,Z) = AM_N + (M_H - M_N)Z - \left[A - \left(\frac{(N^2-Z^2)+\delta(N-Z)}{3(Z-k)} + 3\right)\right] \times \frac{A^{n+s} M_N}{126(Z-k)}$$

$$= AM_N + (M_H - M_N)Z - \left[\frac{5}{3}A - \frac{A^2+\delta(N-Z)}{3(Z-k)} - 3\right] \times \frac{A^{n+s} M_N}{126(Z-k)} \qquad (4)$$

Differentiating $M(A,Z)$ with respect to Z and equating it to zero give us Z_A at which $M(A,Z)$ is minimum. After carrying out the standard calculation, we end up with an equation of power of three in Z such as:

$$aZ^3 + bZ - c = 0 \qquad (5)$$

Where

$$a = (M_H - M_N);\ b = \left[\left(\frac{5}{3}A - 3\right)\frac{A^{n+s} M_N}{126}\right];$$

$$c = \left[\left(\frac{A^2 + \delta(n-z)}{189}\right) A^{n+s} M_N\right]$$

Equation (5) is solved with Maple program and we get one real solution:

$$Z_A = k + \frac{1}{6}\frac{\left[\left[108c + 12\sqrt{3}\sqrt{\frac{4b^3+27c^2a}{a}}\right]a^2\right]^{1/3}}{a} - \frac{2b}{\left[\left[108c+12\sqrt{3}\sqrt{\frac{4b^3+27c^2a}{a}}\right]a^2\right]^{1/3}} \qquad (6)$$

Fig.(1) shows the mass parabolas for odd A ($A = 135$) and for even A ($A = 102$) in which Z_A is shown by down ward arrows. For odd A with $s = 0$ only one parabola is obtained; for even A isobars we get two parabolas.

As shown in fig.(1) for $A = 135$ only one stable isobar exist at $z = 56$ which is very close to our finding at $Z_A = 55.8$ but for even A nuclides depending upon the distance between two obtained parabolas, we get several stable even-even isobars. Here for $A = 102$ there are two stable isobars at $Z = 44$ and $Z = 46$ and for $Z = 44$ we get the most stable one, since this is closer to the calculated value namely, $Z_A = 44.3$. For LDM for $A = 135$, $Z_A = 55.7$ and for $A = 102$, $Z_A = 44.7$ [11]. It is seen that our calculated values of Z_A is closer to the experimental data as compared to the LDM.

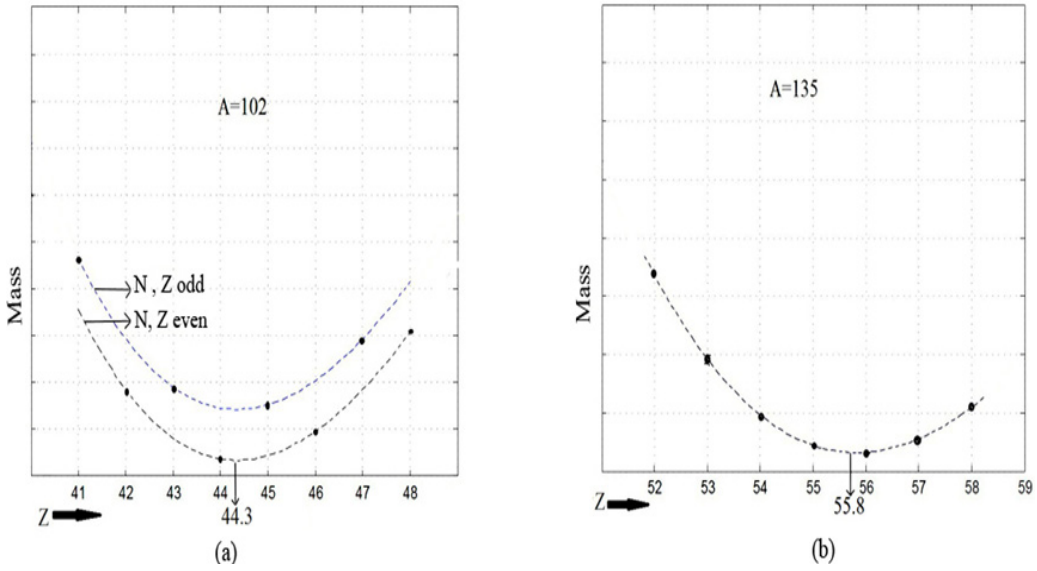

Figure 1. The isobars mass parabolas a) for even A nuclides b) for odd A nuclides

Using equation (6) , we plot Z_A verses N to obtain the stability line shown in figure (2).

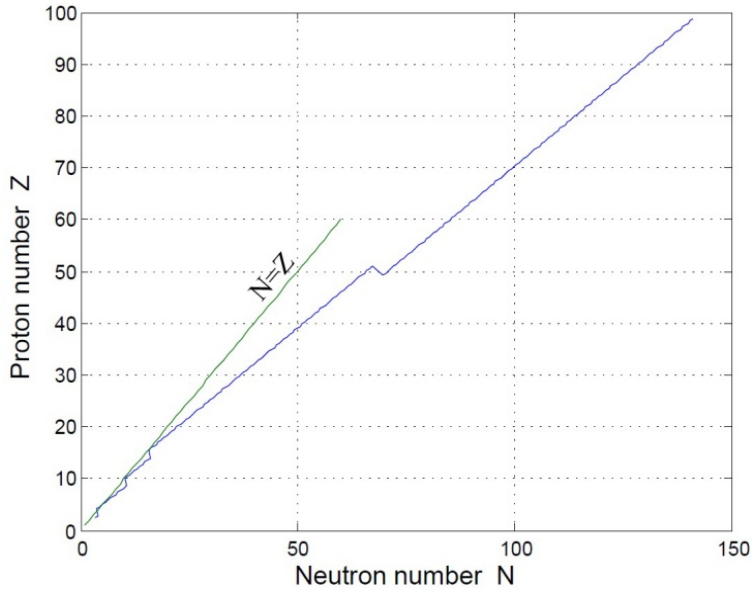

Figure 2. Stability line in modified INM

Finally in table (1) a comparison is made between our findings of nuclear binding energy and the results of other models and experimental data.

3. Conclusion

Determination of the stability mass parabola and the nuclear stability line in modified INM and comparison of the most stable nuclei obtained from this model, with the experimental data and LDM is an indication of the validity of the modified INM which is based upon quark-like model of nuclei. Also comparison of nuclear binding energy obtained from modified INM and experimental data, results in a closer match than LDM for most of the light, medium and heavy nuclei. Other characteristics of nuclides are being tested by INM in our research group and the results are promising.

Table 1. Comparison of nuclear binding energy with different model

Z	Nucleus	A	B(EXP) MeV	B(LDM) MeV	B(INM) MeV	B(Modified INM) MeV
1	H	1	0	-26.461		0
2	He	4	28.296	21.9452		28.27305
3	Li	6	31.994	27.64		31.7322
3	Li	7	39.244	38.3835	0	38.9815
4	Be	9	58.165	56.6316	28.27305	59.3114
5	B	10	64.751	63.0939	30.10864	65.2428
5	B	11	76.205	75.0627	33.58272	77.4225
6	C	12	92.162	87.749	54.71667	91.8896
6	C	13	97.108	93.629	65.03467	96.9228
7	N	14	104.659	99.6605	75.73481	104.2934
7	N	15	115.492	112.2803	92.19658	113.8012
8	O	16	127.619	123.7138	96.69506	127.5033
8	O	17	131.763	130.9744	104.82993 115.06522	130.1135
8	O	18	139.807	141.24997	127.94649 131.23772	138.5735
9	F	19	147.801	149.6775	139.15385	145.9720
10	Ne	20	160.645	160.15493	147.91676	158.8530
10	Ne	21	167.406	168.363	160.75488	167.4783
10	Ne	22	177.77	179.44476	167.29278	177.7011
11	Na	23	186.564	188.0092	178.76377	186.7211
12	Mg	24	198.257	196.68558	186.66229	197.1147
12	Mg	25	205.588	205.5993	198.70651	206.2926
12	Mg	26	216.681	217.2668	206.02692	214.5963
13	Al	27	224.952	224.1192	217.4098	222.5858
14	Si	28	236.537	233.089	225.38779	237.1105
14	Si	29	245.011	242.5576	236.6431	246.7711
14	Si	30	255.62	254.6751	244.7457	255.7162
15	P	31	262.917	260.9052	255.17021	264.1575
16	S	32	271.781	269.23215	261.40635	272.0444
16	S	33	280.442	279.1541	271.82458	280.8788
16	S	34	291.839	291.6321	280.5625	288.9729
17	Cl	35	298.21	297.29795	292.09649	297.3346
17	Cl	37	317.101	317.675	299.71709	313.3686
18	Ar	36	306.717	305.02832	317.5582	304.5650
18	Ar	40	343.811	346.7388	309.3663	343.6705
19	K	39	333.724	333.24958	343.29829	335.6762
19	K	41	351.619	354.4661	334.60819	352.6519
20	Ca	40	342.052	340.41858	350.20724 346.90367	342.5981
20	Ca	44	380.96	383.66084	379.8567	382.8547
21	Sc	45	387.848	390.6604	389.4375	391.2262
22	Ti	46	398.193	399.52474	398.22944	398.8393
22	Ti	47	407.073	408.53494	409.04261 420.01354	407.5979
22	Ti	48	418.7	419.9271	426.51645	419.2554
22	Ti	49	426.842	427.26614	437.60839 435.44378	427.4059
22	Ti	50	437.781	437.02429		436.8867
23	V	50	434.794	434.6673		436.8867

Table (contd.)

Z	Nucleus	A	B(EXP) MeV	B(LDM) MeV	B(INM) MeV	B(Modified INM) MeV
23	V	51		445.416		445.7475
24	Cr	52	445.845	455.5536	446.8546	455.6228
24	Cr	53	456.349	463.39155	455.28655	464.0847
25	Mn	55	464.289	481.1957	462.20892	482.4365
26	Fe	54	482.075	468.80385	482.30496	471.9696
26	Fe	56	471.763	490.5518	470.09324 494.95007	492.0997
27	Co	59	492.258	516.2997	516.9666	517.1329
28	Ni	58	517.313	502.61932	508.02694	506.0674
28	Ni	60	506.459	524.93075	529.02083	526.6298
29	Cu	63	526.846	550.7451	550.87284	551.6894
29	Cu	65	551.385	569.4077	572.5846	568.8508
30	Zn	64	569.212 559.098	558.6975	562.37022 578.59789	558.7607
30	Zn	66	578.136	576.4769	594.627	577.7257
30	Zn	68	595.387	596.72888	600.3837	595.8342
31	Ga	69	601.996	603.8851	616.54873	602.4001
31	Ga	71	618.951	620.99784	611.89844	618.9171
32	Ge	70	610.521 628.686	612.65537	628.65957	611.0470
32	Ge	72	645.665	631.17806	645.29982	628.2117
32	Ge	74	652.564	647.61204	650.43133	644.4527
33	As	75	679.99 696.866	655.4886	679.32321	652.5537
34	Se	78	686.321	682.06496	696.67873	679.6684
34	Se	80	704.37	697.35903	683.54105	695.4010
35	Br	79	714.274	689.2439	701.12258	687.4491
35	Br	81	721.737	705.81675	712.55477	703.7878
36	Kr	82	732.258	715.7677	721.61748	714.8280
36	Kr	83	749.235	722.76424	730.68841	723.0077
36	Kr	84	739.283	731.78274	748.85597	730.9761
36	Kr	86	757.856	746.12903	742.09493	747.9164
37	Rb	85	748.928	739.5233	760.81913	739.0302
37	Rb	87	757.356	755.05569	745.0301	756.5981
38	Sr	86	768.469	748.7453	754.44401	748.3195
38	Sr	87	775.538	756.09715	772.1865	756.6489
38	Sr	88	783.893	765.42723	774.67303	768.2167
39	Y	89	791.087	772.4887	784.95789	776.0138
40	Zr	90	799.722	781.019	794.88848	783.4294
40	Zr	91	814.677	788.70082	796.30213	791.9112
40	Zr	92	805.765	798.32021	815.99203	800.2144
40	Zr	94		814.04797	806.51142	814.4118
41	Nb	93		804.7359		805.9039

Table (contd.)

Z	Nucleus	A	B(EXP) MeV	B(LDM) MeV	B(INM) MeV	B(Modified INM) MeV
42	Mo	92	796.508	793.09932	795.94785	795.4930
42	Mo	95	821.625	820.5965	817.88079	821.6087
42	Mo	96	830.779	830.48545	827.97895	829.9908
42	Mo	98	846.243	846.8345	848.34648	846.2422
44	Ru	99	852.255	851.80259	857.39063	852.8166
44	Ru	100	861.928	861.94355	858.94508	861.2846
44	Ru	101	868.73	869.4767	869.40782	869.5955
44	Ru	102	874.844	878.87415	870.67943	875.7178
44	Ru	104	893.083	894.40381	891.54317	893.5445
45	Rh	103	884.163	885.0934	880.17114	885.1863
46	Pd	104	892.82	892.71256	889.22905	892.3174
46	Pd	105	899.914	900.53543	899.97871	900.7118
46	Pd	106	909.474	910.189	910.81159	908.9562
46	Pd	108	925.239	926.29915	922.81499	924.9800
46	Pd	110	940.207	941.11778	944.64934	940.3585
47	Ag	107	915.263	915.80492	919.77497	916.1749
47	Ag	109	931.727	932.86781	932.11781	932.5995
48	Cd	110	940.646	940.7986	940.98553	939.9041
48	Cd	111	947.622	948.22807	952.19342	948.0861
48	Cd	112	957.016	957.45559	953.35906	956.1212
48	Cd	113	963.556	964.25463	964.54063	964.0060
48	Cd	114	972.599	972.84985	975.82258	971.7369
49	In	113	963.094	963.41231	962.12798	963.5100
49	In	115	979.404	979.7077	984.90679	979.5262
50	Sn	116	988.684	987.89353	983.20639	986.9973
50	Sn	118	1004.955	1003.8319	1006.25447	1005.2137
50	Sn	120	1020.546	1018.5973	1018.68817	1020.0715
51	Sb	121	1026.325	1025.6927	1027.54763	1027.0924
51	Sb	123	1042.097	1040.1801	1040.04681	1043.4808
52	Te	126	1066.369	1063.5988	1061.3598	1066.7824
52	Te	128	1081.439	1076.7653	1085.47929	1082.4686
52	Te	130	1095.941	1088.937	1097.80741	1097.6144
53	I	127	1072.577	1070.8771	1069.98274	1073.9530
54	Xe	129	1087.651	1085.9613	1090.45272	1089.1120
54	Xe	131	1103.512	1100.1915	1103.42321	1105.2131
54	Xe	132	1112.448	1107.9009	1115.85749	1113.0778
55	Cs	133	1118.528	1115.3434	1124.19407	1117.6565
56	Ba	137	1149.681	1144.0675	1145.90412	1148.4568
56	Ba	138	1158.293	1151.5421	1158.66304	1158.9851

Table (contd.)

Z	Nucleus	A	B(EXP) MeV	B(LDM) MeV	B(INM) MeV	B(Modified INM) MeV
57	La	138	1155.774	1151.2653	1153.85554	1155.7274
57	La	139	1164.551	1159.105	1166.73445	1136.5646
58	Ce	140	1172.692	1167.8856	1174.39252	1170.7473
58	Ce	142	1185.29	1181.675	1187.94784	1186.3789
59	Pr	141	1177.919	1174.1085	1181.66244	1177.7014
60	Nd	142	1185.142	1181.5574	1188.56759	1184.4452
60	Nd	143	1191.266	1188.3505	1189.38183	1192.5988
60	Nd	144	1199.083	1196.7746	1202.61474	1200.6467
60	Nd	146	1212.403	1211.0565	1216.51726	1213.3904
62	Sm	152	1253.104	1253.5745	1258.49943	1252.2646
62	Sm	154	1266.94	1266.5934	1272.46549	1267.1604
63	Eu	151	1244.141	1245.4718	1250.92515	1243.8002
63	Eu	153	1258.998	1260.0164	1252.0625	1259.4052
64	Gd	156	1281.598	1281.9731	1286.04737	1281.7931
64	Gd	158	1295.896	1295.468	1300.35239	1296.8405
64	Gd	160	1309.29	1308.1556	1314.5448	1308.1365
65	Tb	159	1302.027	1301.9809	1306.82008	1300.7987
66	Dy	162	1324.106	1323.6067	1327.56172	1322.9576
66	Dy	163	1330.377	1329.4064	1327.81106	1330.3749
66	Dy	164	1338.035	1336.7617	1342.0879	1337.6889
67	Ho	165	1344.256	1343.3234	1348.27368	1345.0398
68	Er	166	1351.572	1351.0262	1354.12255	1352.1740
68	Er	167	1358.008	1357.0604	1354.45782	1359.0660
68	Er	168	1365.779	1364.6314	1368.95728	1367.0472
69	Tm	169	1371.352	1370.6194	1374.66657	1370.7159
70	Yb	172	1392.764	1391.7813	1395.18194	1392.6360
70	Yb	173	1399.131	1397.6674	1395.41458	1399.9972
70	Yb	174	1406.595	1405.069	1410.24421	1407.2110
71	Lu	175	1412.106	1411.0893	1415.66989	1410.8362
71	Lu	176	1418.394	1416.9087	1415.85338	1418.0879
72	Hf	177	1425.185	1424.3349	1421.10223	1425.2612
72	Hf	178	1432.811	1431.9414	1436.13156	1431.4239
72	Hf	180	1446.297	1444.9382	1451.43158	1446.5547
73	Ta	181	1452.24	1450.9735	1456.56802	1453.2859
74	W	182	1459.335	1458.1089	1461.40522	1459.8504
74	W	183	1465.525	1464.0784	1461.62301	1464.6875
74	W	184	1472.937	1471.526	1476.98029	1471.2362
75	Re	185	1478.341	1477.0087	1481.67469	1477.8280
75	Re	187	1491.877	1490.2904	1497.36935	1492.8904

Table (contd.)

Z	Nucleus	A	B(EXP) MeV	B(LDM) MeV	B(INM) MeV	B(Modified INM) MeV
76	Os	189	1505.007	1503.2513	1502.04693	1503.9947
76	Os	190	1512.799	1510.5523	1517.73611	1510.5094
76	Os	192	1526.116	1523.036	1533.53906	1525.0828
77	Ir	191	1518.088	1516.0377	1522.14121	1517.1743
77	Ir	193	1532.058	1529.0561	1538.08144	1531.9626
78	Pt	194	1539.577	1536.145	1542.34012	1538.6738
78	Pt	195	1545.682	1541.8699	1542.38144	1546.0526
78	Pt	196	1553.604	1549.0352	1558.40616	1553.3518
79	Au	197	1559.386	1554.5095	1562.51729	1560.1086
80	Hg	198	1566.489	1561.0472	1566.36429	1566.7061
80	Hg	199	1573.153	1566.9765	1566.48393	1574.1315
80	Hg	200	1581.181	1574.3316	1582.67354	1581.4803
80	Hg	201	1587.411	1579.9487	1582.63325	1588.7515
80	Hg	202	1595.165	1586.9882	1598.99688	1595.9441
81	Tl	203	1600.87	1592.4387	1602.80964	1602.7903
81	Tl	205	1615.072	1604.989	1619.26432	1617.1500
82	Pb	206	1622.325	1611.9829	1622.92633	1624.0399
82	Pb	207	1629.063	1617.5005	1622.8084	1631.2074
82	Pb	208	1636.431	1624.4233	1639.51406	1638.2975
83	Bi	209	1640.23	1629.8385	1643.0243	1640.8417
90	Th	232	1766.687	1769.1834	1764.19153	1767.0470
92	U	235	1783.864	1785.8988	1786.7108	1782.1712
92	U	238	1801.69	1804.1704	1803.59006	1802.6499

REFERENCES

[1] C.F. von Weizsacker, Z. Physik 96, 431 (1935).

[2] N. Bohr and J. A. Wheeler, Phys. Rev. 56, 426 (1939).

[3] H. A. Bethe, Rev. Mod. Phys. 8, 82 (1936).

[4] W. Elsasser, J. phys. Radium 4,549 (1933).

[5] Maria Goeppert-Mayer,Phys.Rev.75:1969-1970(1949).

[6] O.Haxel, J.D.H.Jensen, H.E.Suess,Z.Phys.128,295(1950).

[7] N.Ghahramani,H.Hora,G.H.Miley,M.Ghanaatian,M.Hooshmand,K.Philberth,F.Osman, Phys.Essays 21,200(2008).

[8] N.Ghahramani,M.Ghanaatian,M.Hooshmand,Iranian Physical Journal,vol.1,no.2(2007).

[9] N. Ghahramany and E. Yazdankish. Communication in theoretical physics, Vol.59, No. 5 (2013).

[10] N.Ghahramany,Sh.Gharaati,andM.Ghanaatian,New Approach to Nuclear Binding Energy in Integrated Nuclear Model, Physics of Particles and Nuclei Letters, Vol. 8, No. 2, pp. 97–106(2011).

[11] W. E. Meyerhof, Elements of Nuclear Physics, McGraw-Hill Series in Fundamentals of Phys.(1967).