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The Use of Cost Functions for the Teaching of Natural Monopoly in Intermediate Microeconomics

Albert Ying-Je Huang

Department of Business Administration, Minghsin University of Science and Technology, Hsinchu 30401, Taiwan
*Corresponding Author: yjhuang@must.edu.tw

Abstract This study proposes a logarithmic long-run cost function and its relevant short-run cost lines which can be used to teach second-year or upper-level undergraduates the topic of natural monopoly in microeconomics courses. The long-run and short-run relationships among total cost, average cost, and marginal cost can be both mathematically and graphically illustrated. The Mean Value Theorem can be used to verify the condition of positive fixed costs. This study may make a contribution to intermediate-level microeconomics education by demonstrating how to combine basic calculus with simple graphs to facilitate the teaching of cost properties of natural monopoly. Some undergraduates' views on this method were collected and analyzed. The findings from the data analysis show that the method was generally well received by most of respondents in this study.

Keywords Natural Monopoly, Logarithmic Long-run Cost Function, Subadditivity, Economics Education, Intermediate Microeconomics

1. Introduction

The concept of natural monopoly can be traced back to the classic *Principles of Political Economy* originally published by John Stuart Mill in 1848 [1]. While the topic of natural monopoly has commonly been included in undergraduate-level economics textbooks, the textbook treatment of this topic is usually confined to graphical illustrations. For a first-year undergraduate with little knowledge of mathematical tools, a graphical illustration may be a proper approach to understanding the topic of natural monopoly at the elementary level. Nevertheless, for a second-year or upper-level undergraduate with some basic concepts of calculus, a mathematical analysis can be a powerful approach to learning this topic at the intermediate level. To my knowledge, proper mathematical examples based on "explicit" cost functions are not available in intermediate microeconomics textbooks. To make up for this deficiency, this study demonstrates how to use explicit cost functions to aid the teaching of the cost properties of natural monopoly at the intermediate level.

A natural monopolist can be defined as a single firm able to produce the desired output at a lower cost than "two or more firms" in a particular market [2-6]. Similarly, Perloff [7] defined a natural monopoly as a firm which "can produce the total output of the market at lower cost than several firms could." Formally, let $q_1, \ldots, q_n$ be the individual output of n firms such that industry output $Q = q_1 + \ldots + q_n$. The cost function C(.) is said to be subadditive at output level Q if the following inequality holds: $C(Q) < C(q_1) + \ldots + C(q_n)$. The above definition of natural monopoly indicates that the "subadditivity" of cost function is a necessary condition of the existence of natural monopoly [2,3,8]. Similarly, other textbooks such as [7,9] also define natural monopoly in this way.

In addition to the subadditivity of cost function, three cost properties of natural monopoly are commonly mentioned in undergraduate textbooks. First, the production technology of natural monopoly is characterized by "large fixed costs" and "small marginal costs" [7,10,11,12]. This situation often exists in some "public utilities" such as electricity providers, natural gas distributors, local cable companies, and local telephone companies [13]. For example, Landsburg [10] indicated that the fixed costs of developing new software could be very high, while the marginal costs of copying it onto disks or distributing it over the Internet would be very low. This cost property shall be specifically applied to the case of short run due to the existence of fixed costs.

Second, the property of "strong economies of scale" means that the average cost of natural monopoly declines over a wide range of output levels [4,5,9,13-18]. That is, the minimum efficient scale is larger than the market size [11,19]. Similarly, a natural monopolist has "increasing returns to scale" over the range of output to survive in the industry [12]. However, the long-run average cost (*LAC*) could decline even without economies of scale due to a price fall in an important input. The one-to-one relationship between "returns to scale" and "the slope of the *LAC* curve" can hold under the assumption of "fixed input prices" [16]. To focus on the case of natural monopoly, input prices are assumed to be fixed in this paper. Hence, the decline of average cost can be specifically attributed to the existence of increasing returns to scale. Third, its long-run marginal cost (*LMC*) lies below its long-run average cost so that a downward sloping *LAC* curve exists [9,19]. While those textbooks provide simple graphs to illustrate different cost properties of natural monopoly, they do not use any explicit cost functions to facilitate students' learning at the intermediate level.

As learning can be more effective by walking through algebraic examples, this study may make a contribution to the undergraduate education of intermediate microeconomics by applying an explicit cost function to the teaching of natural monopoly topics. Two advantages of this study shall be further mentioned as follows. First, second-year or upper-level undergraduates do not need to waste their time learning the cost properties of natural monopoly by the same graphical approach as first-year undergraduates do. Second, these post-freshman undergraduates can foster their practical abilities in the use of mathematical tools and learn how to use basic calculus, which they have already studied, to enhance their understanding of cost properties of natural monopoly in intermediate microeconomics. In this study, I propose a logarithmic long-run total cost (*LTC*) function and its relevant linear short-run total cost (*STC*) functions. Then, the cost properties of natural monopoly can be easily verified by merely using simple mathematical techniques. Both the mathematical analysis and graphical illustration are shown in section II. To evaluate the usefulness of this method, the response of students is analyzed in section III. Finally, conclusions are drawn and limitations are indicated in section IV.

2. The Model

To articulate the mathematical model of this study, I first specify some notations as follows. Output level is denoted by q. Without loss of generality, $q_i < q_j$ is assumed, where q_i and q_j are the output levels making the *LTC* equal to the *i*th short-run total cost (STC_i) and the *j*th short-run total cost (STC_j) respectively. In the *i*th short-run, total cost shall be equal to the sum of fixed cost and variable cost ($STC_i = FC_i + VC_i$). This study specifically uses a logarithmic long-run total cost function given by $LTC = ln(aq+1)$, where a is a positive constant. Obviously, the value of the logarithmic *LTC* will be zero as the output level is zero. At the output level q_i, the tangent slope of *LTC* can be given by $LMC(q=q_i) = a/(aq_i+1)$.

According to the envelope relationship between long-run and short-run cost functions, this study simply uses the tangent line of the logarithmic *LTC* at output q_i to be the relevant linear short-run total cost STC_i. At point E_i with the coordinate (q_i, $a/(aq_i+1)$) on the *LTC* "envelope", we can derive the relevant STC_i line by using the coordinate and tangent slope at point E_i. That is,

$$\frac{STC_i - \ln(aq_i+1)}{q-q_i} = \frac{a}{aq_i+1} \tag{1}$$

Hence, the *i*th linear *STC* function can be shown as the sum of fixed cost and variable cost. That is,

$$STC_i = FC_i + SMC_i\, q \tag{2}$$

where

$$FC_i = \ln(aq_i+1) - \frac{aq_i}{aq_i+1} \tag{3}$$

and

$$SMC_i = \frac{a}{aq_i+1} \tag{4}$$

Figure 1 depicts the relationship of *LTC* and *STC*, whereas Figure 2 illustrates the relationships of *LAC*, *LMC*, *SAC*, and *SMC*. Note that *SMC* is the tangent slope of STC. According to (4), $SMC_i > SMC_j$ holds for any output $q_i < q_j$. This means that the tangent slope of STC_i is always higher than that of STC_j. Therefore, as shown in Figure 2, the *i*th short-run marginal cost (SMC_i) is always higher than the *j*th short-run marginal cost (SMC_j). Being an envelope curve, the logarithmic long-run total cost function can be derived from a set of linear short-run total cost functions. Note that the fixed cost is a cost that does not vary with the level of output in the short run [9,16]. In Appendix 1, we can easily verify that fixed cost FC_i in (3) shall be positive by applying the Mean Value Theorem. That is, the following condition will hold for any positive output q.

$$\ln(aq+1) - \frac{aq}{aq+1} > 0 \tag{5}$$

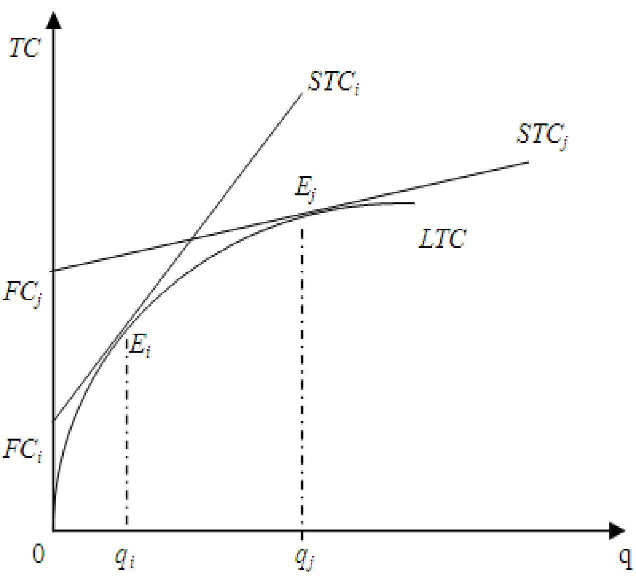

Figure 1. Linear short-run total cost functions and their logarithmic long-run total cost "envelope" for the case of natural monopoly

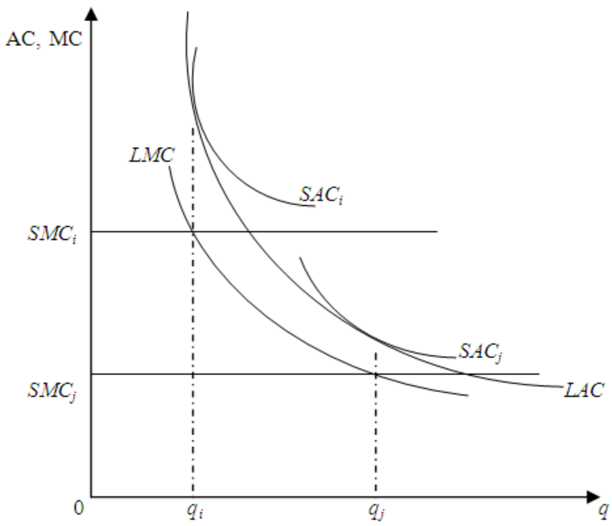

Figure 2. Short-run average cost functions and their long-run average cost "envelope" for the case of natural monopoly

The logarithmic *LTC* function specified in this study satisfies the subadditivity of cost function for defining the natural monopoly. Let industry output $Q = q_1 + \ldots + q_n$, where $q_1, \ldots, q_n$ is the individual output of n producers. We can easily verify that a single firm can produce industry output at lower cost than any combination of two or more firms in the market. That is,

$$\ln(aQ+1) < \ln(aq_1+1) + \ldots + \ln(aq_n+1) \quad (6)$$

The subadditivity implied in (6) can be verified as follows:

$\ln(aq_1+1)+\ldots+\ln(aq_n+1) = \ln[(aq_1+1)\ldots(aq_n+1)]$

$= \ln[1+a(q_1+\ldots+q_n)+a^2(q_1 q_2+\ldots+q_{n-1} q_n)+\ldots+a^n(q_1\ldots q_n)]$

$> \ln[1+a(q_1+\ldots+q_n)]$

$= \ln(aQ+1)$

Based on the prerequisite of positive fixed-cost as shown in (5), we can further verify three cost properties of natural monopoly mentioned in the Introduction section of this study as follows:

1. Production technology with "large fixed costs" and "small marginal costs" in the short run:

We can see that $FC_j > FC_i$ always holds for any $q_j > q_i$, since we can differentiate (3) to obtain

$$\frac{dFC}{dq} = \frac{a^2 q}{(aq+1)^2} > 0. \quad (7)$$

Hence, as shown in (7), fixed cost *FC* must be higher at a larger output level *q*. By contrast, marginal cost *SMC* will be lower at a larger *q*, since the following condition (8) can be obtained by differentiating (4).

$$\frac{dSMC}{dq} = \frac{-a^2}{(aq+1)^2} < 0 \quad (8)$$

2. Strong economies of scale:

The average cost of natural monopoly is declining over the entire range of output levels in the long run. Given (5), we can verify that *LAC* is downward sloping as follows:

$$\frac{dLAC}{dq} = \left[\frac{aq}{aq+1} - \ln(aq+1)\right]\frac{1}{q^2} < 0 \quad (9)$$

3. *LMC* lying below its *LAC*:

Given that *LAC* is downward sloping, we can obtain that

$$LMC - LAC = \left[\frac{aq}{aq+1} - \ln(aq+1)\right]\frac{1}{q} < 0$$

for any $a, q > 0$ (10)

This means that the declining *LMC* is lower than the declining *LAC*. As shown in Figure 2, the declining *LAC* is always higher than the declining *LMC* for any output level *q* due to *LAC-LMC* >0. Moreover, both *LMC* and *LAC* will approximate to zero as output grows to infinity.

3. Discussion

To evaluate students' views on the method demonstrated in this study, students' responses to a questionnaire were collected in 2013. All of the respondents were requested to read instruction on this method before they began to answer any question in the questionnaire. The instruction shows each respondent how a logarithmic long-run cost curve and its short-run cost lines can be used to verify the three cost properties of natural monopoly in intermediate microeconomics. The Likert items in the questionnaire are shown in Appendix 2. Each level on the five-point Likert-type scale is assigned an integer lying between 1 and 5. To further apply statistics to the data analysis, Likert-type scales are usually assumed to be intervals with equal space.

In this study the views of 20 respondents who were post-freshman undergraduates in the Department of Economics at NTHU (National Tsing Hua University, Taiwan) were finally collected and then analyzed. In all, 80% of these respondents were juniors, while the remainder were sophomores and seniors. All of the respondents had taken a course on intermediate microeconomics.

The seven findings from the descriptive statistics listed in Table 1 are summarized as follows:

1. Most of the respondents agreed or strongly agreed that there is a dearth of algebraic examples for the topic of natural monopoly in textbooks on intermediate microeconomics. That is, $P(X_1 \geq 4) = 55\%$.
2. Most of the respondents agreed or strongly agreed that combining graphs with algebraic examples will have a better learning effect than merely showing graphs in economics textbooks. That is, $P(X_2 \geq 4) = 65\%$.
3. Most of the respondents agreed that the method demonstrated in this study is useful in helping them understand the cost properties of natural monopoly. That is, $P(X_3 \geq 4) = 55\%$.

4. Less than half of the respondents thought that the method demonstrated in this study is difficult for them to understand. That is, $P(X_4 \leq 2)=45\%$.
5. Only 20% of the respondents thought that their level of understanding of intermediate microeconomics is poor or fair. That is, $P(X_5 \leq 2)=20\%$.
6. Only 35% of the respondents thought that their mathematical abilities are fair. That is, $P(X_6 \leq 2)=35\%$.
7. A total of 35% of the respondents agreed or strongly agreed that this method can improve their practical abilities in using mathematical tools. Only 25% of the respondents disagreed with this statement. That is, $P(X_7 \geq 4)=35\%$ and $P(X_7 \leq 2)=25\%$.

In addition to the findings from the descriptive statistics presented in Table 1, four findings from the Pearson correlation shown in Table 2 are indicated as follows:

1. The significant positive correlations between X_2 and X_3, between X_3 and X_4, as well as between X_2 and X_4 indicate that the learning effect of combining graphs and algebra, the improvement in the understanding of cost properties of natural monopoly, and the degree of difficulty of understanding this method are positively correlated with one another.
2. The significant positive correlation between X_5 and X_6 means that the self-evaluation of mathematical ability is positively correlated with the self- evaluated level of understanding of intermediate microeconomics.
3. The significant positive correlation between X_3 and X_7 means that the improvement in the understanding of cost properties of natural monopoly is positively correlated with improvement in the ability of using mathematical tools by this method.
4. The significant positive correlation between X_6 and X_7 means that the self-evaluation of mathematical ability is positively correlated with the improvement in the ability of using mathematical tools by this method.

Table 1. The descriptive statistics for each variable in the questionnaire

Variable	Mean	Median	StDev	Probability distribution of variable X_i					
				1	2	3	4	5	Total
X_1	3.55	4	0.686	0%	5%	40%	50%	5%	100%
X_2	3.85	4	1.089	0%	15%	20%	30%	35%	100%
X_3	3.15	4	1.137	15%	10%	20%	55%	0%	100%
X_4	2.70	3	1.031	10%	35%	35%	15%	5%	100%
X_5	3.10	3	0.852	5%	15%	45%	35%	0%	100%
X_6	2.80	3	0.696	0%	35%	50%	15%	0%	100%
X_7	3.20	3	0.951	0%	25%	40%	25%	10%	100%

Table 2. Pearson Correlation of each variable in the questionnaire

Correlation Coefficient	X_1	X_2	X_3	X_4	X_5	X_6	X_7
X_1							
X_2	0.327 (0.159)						
X_3	0.159 (0.504)	**0.572**** **(0.008)**					
X_4	0.022 (0.926)	**0.473*** **(0.035)**	**0.579**** **(0.007)**				
X_5	-0.099 (0.678)	-0.210 (0.375)	0.147 (0.537)	0.276 (0.240)			
X_6	-0.088 (0.712)	-0.181 (0.446)	0.040 (0.867)	-0.088 (0.712)	**0.479*** **(0.033)**		
X_7	0.226 (0.339)	0.335 (0.149)	**0.457*** **(0.043)**	0.225 (0.339)	0.234 (0.321)	**0.461*** **(0.041)**	

The p-value appears in parentheses. ** indicates significant at 1% and * at 5% level.

4. Conclusion

In addition to graphic illustrations of natural monopoly as commonly given in microeconomics textbooks, this study further uses a logarithmic long-run cost curve and its short-run cost lines to facilitate the teaching of cost properties of natural monopoly at the intermediate level. The specification of logarithmic long-run cost function can satisfy the subadditivity for the case of natural monopoly. This study is also helpful for students to understand the "envelope" relationship between the long-run and short-run cost functions. Three cost properties of natural monopoly such as "large fixed costs/small marginal costs", "strong economies of scale", and "*LMC* lying below *LAC*" can be both mathematically verified and graphically illustrated by using the explicit cost functions specified in this study. Moreover, the Mean Value Theorem can be used to verify the prerequisite for the existence of positive fixed costs. Given that the fixed cost is positive in the short run, we can easily verify the long-run cost properties of natural monopoly. This study may make a contribution to intermediate-level microeconomics education by demonstrating how to combine basic calculus with simple graphs to help the post-freshman undergraduates learn the cost properties of natural monopoly.

In addition to theoretical explanation, 20 undergraduates' views on this method were collected and analyzed. While the validity of the statistical results is limited by the scope of the sample, the findings from the data analysis show that the method in this study was generally well received by most of the respondents. The first limitation is that the sample size is small, with only 20 respondents. Secondly, all respondents are undergraduates in NTHU which is one of the top universities in Taiwan. Their performance levels and mathematical abilities are likely higher than the average level in Taiwan. While the usefulness of this method is favorably supported by the NTHU respondents with good abilities, the generalization of the empirical results in this study to all groups of undergraduates with different performance levels and mathematical abilities needs to be further examined. Future research shall investigate the comparison of the learning effect of this method among undergraduates of different performance levels and mathematical abilities.

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Appendix 1

The positive fixed cost (FC_i>0) can be verified as follows: Let $x=aq_i$, and $FC_i=f(x)=ln(x+1)-x/(x+1)$, where $f(x)$ is continuous on a closed interval $[0, x]$, and is differentiable on the open interval $(0, x)$. The condition $f'(x)>0$ holds for any $x>0$. According to the "Mean Value Theorem" [20], there is at least one point c in $(0, x)$ at which $f(x)-f(0) = f'(c)(x-0)$. Since $f(0)=0$ and $f'(c) > 0$, we can obtain that $FC_i = f'(c)x > 0$ for any $c \in (0, x)$.

Appendix 2 The Likert Items in the Questionnaire

Variable	Statement and Response Scale
X_1	There is a dearth of algebraic examples based on explicit cost functions for the topic of natural monopoly in intermediate microeconomics textbooks.
	(Strongly disagree) 1-2-3-4-5 (Strongly agree)
X_2	Combining graphs with algebraic examples will have a better learning effect than merely showing graphs in economics textbooks.
	(Strongly disagree) 1-2-3-4-5 (Strongly agree)
X_3	Is the demonstration of cost function in this study useful in helping you understand the cost properties of natural monopoly?
	(Very useless) 1-2-3-4-5 (Very useful)
X_4	The degree of difficulty for you to understand this method
	(Very difficult) 1-2-3-4-5 (Very easy)
X_5	What do you think of your level of understanding of intermediate microeconomics?
	(Poor) 1-2-3-4-5 (Excellent)
X_6	What do you think of your mathematical ability?
	(Poor) 1-2-3-4-5 (Excellent)
X_7	This method can improve your practical abilities in using mathematical tools.
	(Strongly disagree) 1-2-3-4-5 (Strongly agree)

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