The old and new Concepts of Physics
An open-dialogue journal

www.uni.lodz.pl/concepts

ACCELERATORS OF CHARGED PARTICLES AS NEW SOURCES OF THE HIGH MAGNETIC FIELDS

Stanisław Bednarek
Department of Physics and Applied Informatics,
University of Łódź
Łódź, Poland

(Received 13 July 2008; accepted 1 October 2008)

Abstract

This article has been focused on possible new applications of accelerators used for investigations of controlled nuclear fusion. These accelerators generate beams of charged particles with intensity of several tens of megaampers and duration of order of magnitude as nanoseconds. The computations performed in this article demonstrated that beams of particles from the accelerators could be sources of high magnetic fields with induction from 500 T to 1780 T, which have a practical application in the investigation of certain materials.

1 Introduction

Strong magnetic fields have important significance in many disciplines of physics and technology. The examples of applying high magnetic fields encompass, among others, investigation of magnetic materials, semiconductors and superconductors. Producing high magnetic fields with the use of coils runs into many problems, the most important among them is to ensure proper mechanic strength of coils and adequate speed of cooling them [1, 2]. During production of magnetic fields with induction higher than 50 T the problems are impossible to overcome and therefore higher magnetic fields are produced in the form of impulses only.

The highest magnetic fields with induction reaching 2500 T are produced using the method of explosive compression of an initial magnetic flux [3]. This method requires extremely difficult conditions to be met and has several drawbacks. The major problem is a destruction of samples and whole system for production the field [4]. A significant problem is also associated with performing measurements in conditions of shock wave propagation [5].

The difficulties presented above in a broad outline cause searching for new methods and possibilities of production of high magnetic fields useful for research. One of these methods is using a beam of electrons with high energy obtained from an accelerator which is applied in studies on elementary particles [7, 8]. Thus one can produce magnetic fields with induction of about 20 T and duration of order of 10^{-12} s in space with dimensions of several μm. Such fields allow investigation of remagnetization phenomena in some samples.

In this article the attention was paid to a possibility of applying another class of accelerators for producing high magnetic fields. These accelerators have very high intensity of beam of charged particles and are used for investigation of controlled nuclear fusion by inert method. On the basis of available technical data it was calculated the parameters of possible magnetic fields to be obtained by this class of accelerators. It turned out that in this way one can produce magnetic field with higher induction, longer duration and filling greater volume than those described in the cited articles [7, 8]. Due to this fact, these fields can be more useful and have wider application in research.

2 Magnetic field of ROKKO I system [9]

This system was constructed in Japan and is designed for performing nuclear fusion using inert method which relies on compression of nuclear fuel by the beam of accelerated electrons. The energy of these electrons amounts: $E = 5 - 7\,\text{MeV}$. The electrons are focused in the area with diameter of $d = 12\,\text{mm}$. Duration of a single impulse of these particles amounts $\Delta t = 10\,\text{ns}$ and its energy $E_1 = 4\,\text{MJ}$. Surface power density of electrons reaches $P_s = 100\,\text{TW/cm}^2$ [10]. The parameters given above enable to calculate induction of magnetic field B which could be obtained using this system.

In order to calculate the induction, the current intensity I corresponding to beam of particles with given parameters should have been determined previously. According to well known definition formula:

$$I = \frac{\Delta q}{\Delta t}. \tag{1}$$

In formula (1) Δq denotes the charge carried by the beam of particles during time of the impulse. Since the accelerated particles are electrons with elementary charge $e = 1.6 \cdot 10^{-19}\,\text{C}$, the charge Δq is expressed by the formula:

$$\Delta q = ne, \tag{2}$$

in which: n – number of electrons in the impulse (so-called bunch of particles). The number of electrons in the bunch can be determined knowing the energy of impulse and using known equations of relativistic dynamic. It allows to write the following equation:

$$E_1 = nE_0 \left(\frac{1}{\sqrt{1-\beta^2}} - 1 \right), \tag{3}$$

in which: E_0 – energy corresponding to the rest mass of electron ($E_0 = 0.511\,\text{MeV}$), $\beta = v/c$ – means ratio of accelerated electrons velocity to light velocity $c = 3 \cdot 10^8\,\text{m/s}$. Ratio β can be calculated knowing energy of the accelerated electron E. Equation of relativistic dynamics for energy has then the following form:

$$E = \frac{E_0}{\sqrt{1-\beta^2}}. \tag{4}$$

A transformation of the equation (4) gives the formula allowing to calculate β:

$$\beta = \sqrt{1 - \left(\frac{E_0}{E}\right)}. \tag{5}$$

For the energy of electrons $E = 7\,\mathrm{MeV}$ from the formula (5) one obtains $\beta = 0.9947$, whereas for $E = 4\,\mathrm{MeV}$ one obtains $\beta = 0.9918$. Using the equations (4) and (5) one gets the formula allowing the calculation of number of particles, which are included in the bunch:

$$n = \frac{E_1}{E - E_0}. \tag{6}$$

After taking into consideration that $1\,\mathrm{MeV} = 1.6 \cdot 10^{-13}\,\mathrm{J}$ and substitution this value to the formula (6) $E = 7\,\mathrm{MeV}$ one obtains $n = 3.85 \cdot 10^{18}$, whereas for $E = 5\,\mathrm{MeV}$ one obtains $n = 5.57 \cdot 10^{18}$. Using equations (1) and (2) we obtain the following formula which allows to calculate the current intensity during the impulse:

$$I = \frac{ne}{\Delta t}. \tag{7}$$

After substitution of the calculated values n and the remaining data to the formula (7) was obtained: $I = 6.16 \cdot 10^{7}\,\mathrm{A}$ for $n = 3.85 \cdot 10^{18}$ and $I = 8.91 \cdot 10^{7}\,\mathrm{A}$ for $n = 5.57 \cdot 10^{18}$. Duration of the impulse produced by the accelerator amounts $\Delta t = 10\,\mathrm{ns}$ and relative velocity of particles is β. Hence length l of the particles bunch is expressed by the following formula:

$$l = c\beta\Delta t. \tag{8}$$

After substitution of the calculated values β to formula (8) was obtained: $l = 2.9841\,\mathrm{m}$ for $\beta = 0.9947$ and $l = 2.9754\,\mathrm{m}$ for $\beta = 0.9918$. A diameter of the beam amounts $d = 12\,\mathrm{mm}$ as it was given previously.

It is assumed that sizes of the samples investigated in the magnetic field will amount to $a = 5\,\mathrm{mm}$, and centres of these samples will be placed in the distance $r = 10\,\mathrm{mm}$ from the axis of beam (Fig. 1). The calculated lengths of the particles bunches are significantly greater that the samples sizes, hence the bunch of moving particles can be

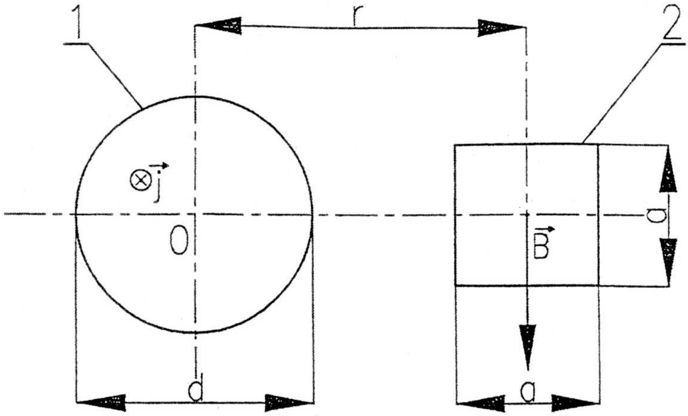

Figure 1: Reciprocal location of beam of electrons and sample in the ROKKO II system, presented in cross-section; 1 – beam of electrons, 2 – sample.

regarded with good approximation as the infinitely long conductor. This approximation will be used in calculations of induction of the magnetic field produced by the beam of particles in the sample area. According to this approximation, the value of induction of the magnetic field B in centre of the sample is expressed by the formula:

$$B = \frac{\mu_0 I}{2\pi r}, \tag{9}$$

in which: μ_0 – denotes the magnetic permeability of free space ($\mu_0 = 12.56 \cdot 10^{-7}\,\mathrm{Vs/(Am)}$. After substitution previously calculated values of l and the assumed value r to the formula (9) was obtained: $B = 1232\,\mathrm{T}$ for $I = 6.16 \cdot 10^7\,\mathrm{A}$ and $B = 1782\,\mathrm{T}$ for $I = 8.91 \cdot 107\,\mathrm{A}$.

3 Inhomogenity of magnetic field

The magnetic field produced by the considered system is not homogenous in the sample space. Therefore a description of its inho-

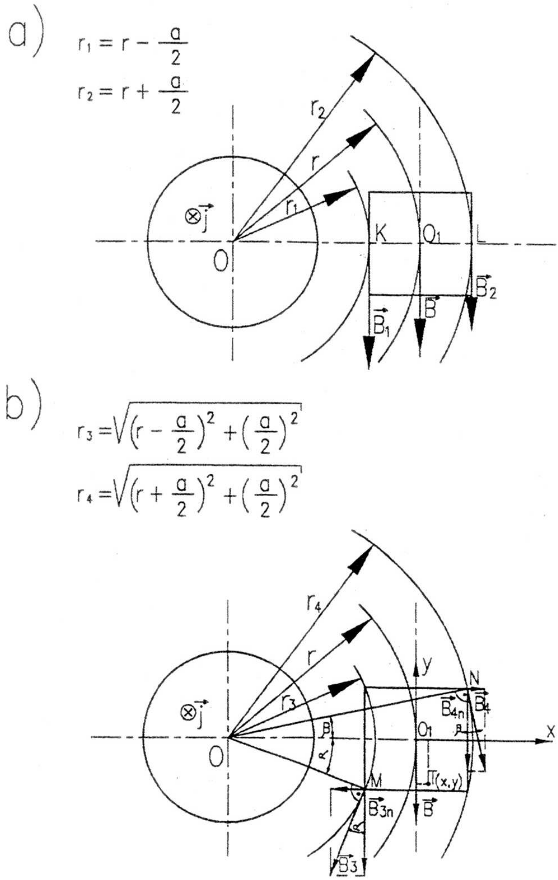

Figure 2: Space distribution of magnetic field in selected points of the sample: a) in horizontal plane passing through the system axis, b) in plane perpendicular to the system axis in diagonal of the sample.

mogenity in this space is very important for the application of this field. The point of reference for inhomogenity description will be the induction B of the magnetic field in the sample centre O_1 (Fig. 2). According to the assumed symbols, a value of the induction B is expressed by the formula (9). In the points K, L of the sample, which are the nearest and the furthest from the axis of the beam in horizontal plane passing through the system axis, the values of the induction of magnetic field B_1 and B_2 are expressed by the following formulas:

$$B_1 = \frac{\mu_0 I}{2\pi\left(r - \frac{a}{2}\right)}, \tag{10}$$

$$B_2 = \frac{\mu_0 I}{2\pi\left(r + \frac{a}{2}\right)}. \tag{11}$$

As measures of inhomogenity of magnetic field in the plane passing through the system axis there were assumed the following expressions:

$$\Delta B_1 = \frac{B_1 - B}{B} \cdot 100\,\%, \tag{12}$$

$$\Delta B_2 = \frac{B_2 - B}{B} \cdot 100\,\%. \tag{13}$$

After substitution the formulas (9, 10, 11) to (12) and (13) was derived:

$$\Delta B_1 = \frac{a}{2r - a} \cdot 100\,\%, \tag{14}$$

$$\Delta B_2 = -\frac{a}{2r + a} \cdot 100\,\%. \tag{15}$$

For earlier accepted values $r = 10\,\mathrm{mm}$ and $a = 5\,\mathrm{mm}$ from formulas (13) and (14) was calculated: $\Delta B_1 = 33\,\%$ and $\Delta B_2 = -20\%$. Inhomogenity of the produced magnetic field does not depend on intensity of current of the beam and depends only on geometrical parameters of the system.

It should be noted that the induction of magnetic field practically remains constant alongside any division crossing through the sample and parallel to axis of the beam. It is due to the fact that the length of the bunch of electrons is significantly greater than the sample sizes and the assumption, accepted on this basis, that the bunch can be considered as an infinitely long conductor. Space distribution of values of the magnetic field induction in the horizontal plane passing through the system axis can be determined using the formula (9). A diagram of inhomogenity of magnetic field in this plane is presented in Fig. 3a.

Let us consider the magnetic field in the plane perpendicular to the system axis and passing through the centre of the sample, (Fig. 3b). Using the descriptions applied in Fig. 2b, the values of vertical components of magnetic induction B_{3n} and B_{4n} in points M, N which lie on diagonal of the sample in the mentioned plane can be expressed by the following formulas:

$$B_{3n} = \frac{\mu_0 I \cos\alpha}{2\pi\sqrt{\left(r - \frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2}}, \tag{16}$$

$$B_{4n} = \frac{\mu_0 I \cos\alpha}{2\pi\sqrt{\left(r + \frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2}}, \tag{17}$$

in which:

$$\cos\alpha = \frac{r - \frac{a}{2}}{\sqrt{\left(r - \frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2}}, \tag{18}$$

$$\cos\beta = \frac{r + \frac{a}{2}}{\sqrt{\left(r + \frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2}}. \tag{19}$$

Similarly to previous considerations, the expressions which are the measure of inhomogenity of magnetic field on a diagonal of the sample were accepted:

$$\Delta B_3 = \frac{B_3 - B}{B} \cdot 100\,\%, \tag{20}$$

$$\Delta B_4 = \frac{B_4 - B}{B} \cdot 100\,\%. \tag{21}$$

After substitution the formulas (9, 16, 17, 18, 19) to formulas (20) and (21) and transformation the following formulas were derived:

$$\Delta B_3 = \frac{2a(r - a)}{(2r - a)^2 + a^2} \cdot 100\,\%, \tag{22}$$

$$\Delta B_4 = -\frac{2a(r + a)}{(2r + a)^2 + a^2} \cdot 100\,\%. \tag{23}$$

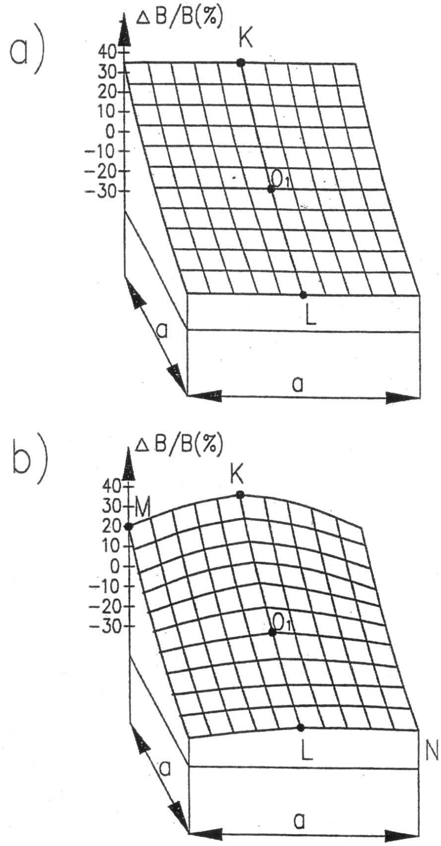

Figure 3: Inhomogenity of magnetic field $\Delta B/B$ inside sample a) in horizontal plane passing through the system axis, b) in plane perpendicular to the system axis and passing through the sample centre.

After substitution, in turn, $r = 10\,\mathrm{mm}$ and $a = 5\,\mathrm{mm}$ to the formulas (22) and (23), it was calculated: $\Delta B_3 = 20\,\%$ and $\Delta B_4 = -23\,\%$.

The value of B_n component of the induction of magnetic field in any point $T(x, y)$, (see Fig. 3b) of the plane perpendicular to the axis of the system and passing through the sample centre is expressed by the formula:

$$B_n = \frac{\mu_0 I(r + x)}{2\pi\left[(r + x)^2 + y^2\right]}, \tag{24}$$

in which: x, y denote coordinates of the point T in relation to the sample centre O_1. Using the formulas (24) and (9) as well as following the same way like described earlier, it was determined the inhomogenity of the magnetic field on the fragment of the discussed plane limited by the sample. The graph of this inhomogenity is shown in Fig. 3b.

4 Magnetic field of PBFA II system [9]

This system is also designed for controlled nuclear fusion by inert method and was constructed in United States [11]. In this system electrons are accelerated to the energy of $E = 4\,\mathrm{MeV}$. A duration of a single impulse amounts $\Delta t = 70\,\mathrm{ns}$. Current intensity corresponding to the beam of electrons amounts to $I = 25\,\mathrm{MA}$. This beam is focused on a capsule of nuclear fuel with a diameter of $d = 8\,\mathrm{mm}$. For this diameter of the beam one can assume, like it was assumed earlier, that a size of the sample is $a = 5\,\mathrm{mm}$ and its centre is in a distance of $r = 10\,\mathrm{mm}$ from the beam axis. In the same distance it will be calculated the value of the induction B of magnetic field produced by the beam.

In order to evaluate the length of bunch of particles and to assess a possibility of applying the approximation of infinitely long conductor, one requires to calculate relative velocity β of accelerated particles. The formula (5) can be used for these calculations. In the considered system electrons are accelerated, hence $E_0 = 0.511\,\mathrm{MeV}$, and from the formula (5) $\beta = 0.9837$. After substitution of the calculated value of β and $\Delta t = 70\,\mathrm{ns}$ to the formula (8), it was calculated a length of bunch of particles $l = 20.66\,\mathrm{m}$. This value fulfils very well the condition $l \gg r$ and it allows to apply the approximation of infinitely long conductor. Induction of magnetic field in the sample centre is calculated using the formula (9). After substitution $I = 25\,\mathrm{MA}$ and $r = 10\,\mathrm{mm}$, was obtained $B = 500\,\mathrm{T}$. Inhomogenity of the produced magnetic field is the same like in the case of the system considered earlier, (see Fig. 3a, b).

5 Conclusions

Producing high magnetic fields with the induction of several hundreds T or more takes place with the use of explosive compression of initial magnetic flux [3, 4]. This method requires special safety conditions and its major drawback is a destruction of samples and a system generating the field. As evidenced in the literature, using a beam of elementary particles from an accelerator, commonly applied in a domain of elementary particles investigations, has also significant disadvantages [7, 8]. This method allows to achieve induction of the field not exceeding several tens teslas. A duration of the impulse is limited to picoseconds and sizes of the investigated samples can amount only several micrometers.

Calculations performed in this article show that applying accelerators used in controlled nuclear fusion by inert method for producing high magnetic fields has remarkable advantages. It allows producing the impulse magnetic fields with induction from several hundreds teslas to over one thousand teslas. Both the sample and the system generating field are not destroyed and the impulses of the field can be easy repeated which allows many times to investigate of the same sample in so extremely high magnetic field. It was till impossible. A duration of impulses of the field is about 10^3 times longer than a duration obtained with the using of accelerators designed for investigations in breach of elementary particles. Similarly, sizes of samples are 10^3 times greater.

An important advantage is also the fact that using accelerators, applied in investigations of controlled nuclear fusion by inert method, for producing strong magnetic fields do not require additional investment outlays, furthermore it is almost free of charge since it could be performed as an alternative activity during carrying out the main investigation of the controlled nuclear fusion. Above considerations, altogether, justify a view that a proposed application of the accelerators may open new possibilities of investigations in physics of strong magnetic fields.

References

[1] M. Motokawa, H. Noijri, Y. Tokunaga, Physica B **155**, 96 (1989).

[2] H. Kolm, A. Freeman, Sci. Amer. **212**, 66 (1965).

[3] A. D. Sacharov, Usp. Fiz. Nauk **88**, 725 (1966).

[4] F. Bitter, Sci. Amer. **213**, 65 (1965).

[5] H. Noijri, et. al., Physica B **201**, 579 (1994).

[6] B. E. Kane, et. al., Rev. Sci. Instr. **68** 3843 (1997).

[7] C. H. Back, H. C. Siegman, Jour. Mag. Mag. Mater. **200**, 774 (1999).

[8] H. C. Siegman, Eur. Phys. News **6**, 24 (2000).

[9] W. Scharf, Accelerators of charged particles, application in science and technology, (PWN Warsaw 1998), (in Polish), p. 455.

[10] S. Nakai, et. al., Laser and Particle Beams **1**, 29 (1983).

[11] J. P. Van-Devender, *Light ion beam fusion*, in.: R. J. Briggs, A. Toepfer (ed.) Proc. of the Fifth Int. Conf. on High-power Particle Beams, Sept. 12–14, 1983, (Lawrence Livermore Nat. Lab. Conf.-830911, Livermore 1983), p. 17.