

VACUUM MODIFIED GRAVITY AS AN EXPLANATION FOR FLAT GALAXY ROTATION CURVES

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Abstract

A theory is proposed which allows explaining the observed flat galaxy rotation curves, without needing to invoke dark matter. Whereas other theories have been proposed in the past which realize the same, the present theory rests on basic physical principles, in contrast to for instance the MOND theory. The key to arrive at this new theory is to consider from the start the energy density of the vacuum. The way to calculate the effect of the corresponding vacuum pressure on a mass has previously been laid down by Van Nieuwenhove (1992). We obtain a modification of Newton's law of gravitation with some peculiar properties such as the occurrence of regions of repulsive gravity. The theory can make detailed predictions about galaxy rotation curves and is also able to explain to the Pioneer anomaly.

Many physicists nowadays are convinced that some form of dark matter has to exist to explain the behavior of groups of galaxies or to explain the observed flat galaxy rotation curves. After many years of research however, this hypothetical dark matter [6] has not been found and some scientists start doubting its existence all together. In 1983 [4], the Modified Newtonian Dynamics (MOND) theory was proposed to explain the flat galaxy rotation curves without having to invoke dark matter. In 1996 [9], the author proposed another theory to explain the flat rotation curves. This theory was based on an alternative view in which gravity results from a distortion of the quantum vacuum through its interaction with a mass [8]. Using these concepts, it was shown how the existence of a large scale distortion of the vacuum energy [10] could explain the flat rotation curves. The required deviation of the background vacuum energy was found to be tiny (of the order of 10^{-6}). Nevertheless, it was shown that this resulted in a dramatic modification of the motion of stars within the vacuum bubble. The weakness of this theory was that no new predictions could be made to confirm the proposed theory. In this paper, an extended version of this original theory has been elaborated which allows to make some detailed predictions such that the theory becomes falsifiable.

1 Different aspects of vacuum energy

In the Einstein field equations [12], the cosmological constant appears on the left side in

$$R_{\mu\nu} - \frac{1}{2} \cdot R \cdot g_{\mu\nu} + \Lambda \cdot g_{\mu\nu} = \frac{8\pi G}{c^4} \cdot T_{\mu\nu}, \quad (1)$$

where R and g pertain to the structure of spacetime and T pertains to matter. The cosmological constant has the same effect as an intrinsic energy density of the vacuum (ρ_{vac}). However, it needs not to have a counterpart in quantum physics. It serves the function of an integration constant in that its presence does not violate the conservation properties of the Einsteinian tensor. Nevertheless, the relation between Λ and the energy density of the vacuum is often made. The equation of state for this vacuum is given by $p = -\rho$ in which p is the pressure. So, a positive energy density corresponds to a negative

pressure and this was the equation of state which was used in [8]. In 1998, [2], another form of energy, called quintessence (or fifth element), was proposed to explain the observations of an accelerating rate of cosmic expansion. Unlike Λ , quintessence can vary over space and has an equation of state given by $p = w\rho$ where w is less than $-1/3$. Whereas Λ is part of the geometric tensor (left side in equation (1)), quintessence enters on the right hand side of the equation, thought of as a scalar field. Whereas the effect of vacuum energy is normally evaluated by means of the Einstein Field Equations, a method has been proposed in [8] to evaluate its effect on the motion of matter by means of a simple classical description. The method consists in representing a mass (m) by a volume through the relation

$$V_m = \frac{m c^2}{\rho_V} \quad (2)$$

in which ρ_V corresponds to the (background) energy density of the vacuum. The next step consists in evaluating the force F on this mass by

$$F = V_m \cdot \frac{dp}{dr} \quad (3)$$

in which r is the radial coordinate and where we use the convention that F is positive when attractive. Equation (3) means that the mass experiences a net force when the pressure of the vacuum is not balanced out. So, in this theory, a truly constant vacuum energy density (such as described by Λ) can not induce a force, however large the energy density may be [8].

2 Vacuum modified gravity

Consider next a mass M and let us deduce the total force between this mass M and a (small) test mass m . Since the energy in the vacuum (at least the varying part) contributes to the gravitational force, we can write

$$F = \frac{G \cdot m \cdot (M + M_V)}{r_1^2}, \quad (4)$$

where M_V is given by

$$M_V = \frac{4\pi}{c^2} \cdot \int_0^{r_1} \rho(r) \cdot r^2 \cdot dr. \quad (5)$$

Inserting equation (5) into equation (4) and using equations (2, 3) and the equation of state $p = -\alpha\rho$ (with α positive) and equating the force given by equation (4) to the force given by equation (3) results in

$$\int_0^{r_1} \rho(r) \cdot r^2 \cdot dr = -a \cdot r^2 \frac{d\rho(r)}{dr} - b, \quad (6)$$

where

$$a = \frac{\alpha c^4}{4\pi G \rho_V} \quad (7)$$

and

$$b = \frac{c^2 M}{4\pi}. \quad (8)$$

After some lengthy algebra, one finds that

$$\rho(r) = \frac{1}{(r/\sqrt{a})} \cdot \left[p \cdot \cos\left(\frac{r}{\sqrt{a}}\right) + q \cdot \sin\left(\frac{r}{\sqrt{a}}\right) \right], \quad (9)$$

where

$$p = \frac{b}{a^{3/2}} \quad (10)$$

and q is an arbitrary constant. The gravitational force can then be expressed as (see (3))

$$F = -\alpha c^2 \cdot \frac{m}{\rho_V} \cdot \frac{d\rho(r)}{dr}. \quad (11)$$

Inserting (9) into equation (11), one obtains

$$F = -\alpha c^2 \cdot \frac{m}{\rho_V} \cdot \left\{ \frac{-\sqrt{a}}{r^2} \cdot \left[p \cdot \cos\left(\frac{r}{\sqrt{a}}\right) + q \cdot \sin\left(\frac{r}{\sqrt{a}}\right) \right] + \frac{1}{r} \cdot \left[-p \cdot \sin\left(\frac{r}{\sqrt{a}}\right) + q \cdot \cos\left(\frac{r}{\sqrt{a}}\right) \right] \right\}. \quad (12)$$

For small values of r ($r \ll \sqrt{a}$), this can be approximated by

$$F = \alpha c^2 \cdot \frac{m}{\rho_V} \cdot \frac{\sqrt{a} \cdot p}{r^2}. \quad (13)$$

It can easily be verified that this is nothing else than Newton's law (by inserting a and p). For larger values of r , the form of the gravitational force becomes however considerably more complicated. Assuming that q is not zero, it should have the same dimension as p and the most simple choice is to assume that $p = q$. This choice allows to describe also the relatively flat galaxy rotation curves. We can work at that

$$p = \frac{b}{a^{3/2}} = \frac{MG^{3/2}(4\pi)^{1/2}\rho_V^{3/2}}{\alpha^{3/2}c^4}. \quad (14)$$

Further we will define R (with a dimension of length) to be

$$R = \sqrt{a} = \frac{\alpha^{1/2}c^2}{\sqrt{4\pi G\rho_V}}. \quad (15)$$

Introducing further the dimensionless ratio $x = r/R$, we can write this force as

$$F = \frac{GmM}{R^2} \left[\frac{1}{x^2} [\cos(x) + \sin(x)] - \frac{1}{x} [-\sin(x) + \cos(x)]. \right] \quad (16)$$

So, by means of some very basic physical principles and a minimum of assumptions we have arrived at a modified Newtonian description of the gravitational force and propose to designate this approach by Vacuum Modified Gravity (VMG). At this stage, the value of R is not yet defined. This information will however be obtained from galaxy rotation curves, as explained before. It is interesting to see (equation (16)) that the force is composed of two parts. We therefore define the following functions

$$f_1(x) = \frac{1}{x^2} (\cos(x) + \sin(x)), \quad (17)$$

$$f_2(x) = \frac{1}{x} (-\sin(x) + \cos(x)), \quad (18)$$

$$f_t(x) = f_1(x) + f_2(x), \quad (19)$$

$$f_N(x) = \frac{1}{x^2}, \quad (20)$$

$$f_p(x) = \frac{1}{x} (\cos(x) + \sin(x)). \quad (21)$$

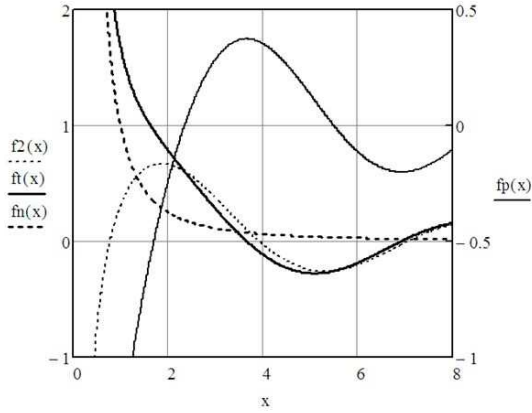


Figure 1: Radial dependencies of the normalized forces $f_2(x)$, $f_t(x)$, $f_n(x)$, as defined in equations 17-21. Note that x is given by $x = r/R$. The profile of the vacuum pressure is also shown (right scale). The behavior of the vacuum modified gravity force $f_t(x)$ is markedly different from the Newtonian gravity force $f_n(x)$ when x is not very small.

The function $f_t(x)$ is proportional to the total force and f_N is the Newtonian force. The function $f_p(x)$ is proportional to the vacuum pressure. The behavior of these functions (except of $f_1(x)$) is shown in Fig. 1. The function $f_1(x)$ still has a close resemblance to the Newtonian dependence given by $f_N(x)$ but the function $f_2(x)$ is however of a different character. In fact, one can see that this corresponds to the “bubble force” which was originally introduced in [10], without derivation. The total force $f_t(x)$ has an interesting behavior. At $x = 3.660$ the force changes from being attractive to being repulsive and becomes attractive again at $x = 6.925$. It oscillates then further from positive to negative up to infinity. The bubble force starts out negative (from $r = 0$) and becomes positive at $x = 0.785$ and switches back to negative at $x = 3.927$ and oscillates further up to infinity. So, because of the peculiar action of the vacuum, every mass will induce, at some large radius, regions of repulsive force.

3 Galaxy rotation curves

Before applying the modified gravity equations to a galaxy, we need also to derive the equations inside the bulge of the galaxy. We will also neglect completely the mass in the disk and in the halo. So, we are dealing here with a highly simplified model. Its purpose is to demonstrate some qualitative features rather than to model a realistic galaxy. Let us denote the bulge radius by r_b and the normalized bulge radius by $x_b = r_b/R$. The total mass contained in the bulge is M . The mass inside a sphere of radius r_1 can then be written as

$$M_{r_1} = \frac{r_1^3}{r_b^3} \cdot M. \tag{22}$$

The equivalent mass of the vacuum energy, contained in a sphere of radius r_1 is given by equation (5). The equation to solve for $\rho(r)$ then becomes

$$\int_0^{r_1} \rho(r) \cdot r^2 \cdot dr = -a \cdot r^2 \frac{d\rho(r)}{dr} - \frac{q}{3} \cdot r_1^3 \tag{23}$$

in which a has the same meaning as before (see equation (7)) and q is given by

$$q = \frac{3Mc^2}{r_b^3 4\pi}. \tag{24}$$

To solve for $\rho(r)$, we expand $\rho(r)$ in powers of r , determine the corresponding coefficients by solving equation (23) and reassemble the function again in a concise form. Then we obtain:

$$\rho(r) = -q + \frac{h}{r/R} \cdot \sin\left(\frac{r}{R}\right) \tag{25}$$

in which $x = \frac{r}{\sqrt{a}} = \frac{r}{R}$ and h is an arbitrary constant. The resulting force is then given by:

$$F = \frac{gmM}{R^2} \cdot d \cdot \left[\frac{\cos(x)}{x} - \frac{\sin(x)}{x^2} \right]. \tag{26}$$

The free parameter d is finally obtained by requiring that the force from the region inside the bulge matches the force on the outside of the bulge at the boundary r_b . We then obtain:

$$d = \frac{\frac{1}{x_b} \cdot (\cos(x_b) + \sin(x_b)) + (\sin(x_b) - \cos(x_b))}{\frac{\sin(x_b)}{x_b} - \cos(x_b)}. \tag{27}$$

The relation between d and h is given by:

$$h = d \cdot \frac{Mc^2}{4\pi R^3}. \quad (28)$$

Equation (28) shows how the mass energy density (in the bulge) is related to the magnitude of the vacuum energy density. Equating the centripetal force to the gravitational force provides an equation for the rotation velocity:

$$V_{in} = \sqrt{d \cdot \left(\frac{\sin(x)}{x} - \cos(x) \right)} \quad (29)$$

in which V_{in} is the normalized velocity given by:

$$V_{in} = \frac{v}{\sqrt{\frac{GM}{R}}}. \quad (30)$$

Next, we consider the region outside of the bulge of the galaxy. Equating again the gravitational force (equation (16)) to the centripetal force results in the following velocity profile:

$$V_{out} = \sqrt{\frac{1}{x} \cdot (\cos(x) + \sin(x)) + (\sin(x) - \cos(x))} \quad (31)$$

in which

$$V_{out} = \frac{v_{out}}{\sqrt{\frac{GM}{R}}}. \quad (32)$$

It is instructive to consider the ratio between the magnitude of the vacuum energy perturbation (ρ_0) to the background vacuum energy density ρ_V . It is given by

$$\frac{\rho_0}{\rho_V} = \frac{MG}{\alpha Rc^2}. \quad (33)$$

It is interesting to see that this is the same relation (up to a constant factor) as already given in [10]. The radial dependence of the force (or acceleration) is shown in Fig. 2. The complete velocity profile is shown in Fig. 3. As can be seen from Fig. 3, we obtain a plateau region in velocity. No fitting parameters were obtained for obtaining

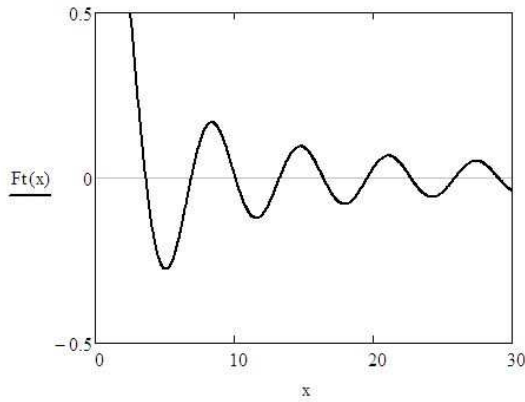


Figure 2: Radial dependence of the vacuum modified gravity force showing the remarkable behavior that the force oscillates between being attractive and being repulsive.

the flat velocity region. It just follows from consistently taking into account the vacuum energy density. From the calculated velocity profile, one can observe a number of interesting features. The velocity does not remain constant up to infinity (as in the MOND model) but goes to zero at (approximately) $x = 3.66$. The plateau velocity comes out to be

$$V_{pt} = \sqrt{\frac{GM}{R}} \cdot 1.28. \quad (34)$$

Inside the bulge region, the velocity profile is very close to that obtained by using Newton's law. From Fig. 2 one sees that the force oscillates between positive and negative when moving further out, as discussed earlier. As a result, matter which was originally present in between the zero crossings will unavoidably move towards the location of these zero crossings and accumulate there. The present analysis concentrates only on the field structure of one galaxy. In the presence of several galaxies, the wavy vacuum perturbations will overlap and interfere, creating complicated patterns. This could possibly explain the fact that galaxies form a bubble-like or foamy distribution in space with galaxies lying along the surfaces of the bubbles (called voids) and nothing in the centres of the bubbles [3]. The attentive

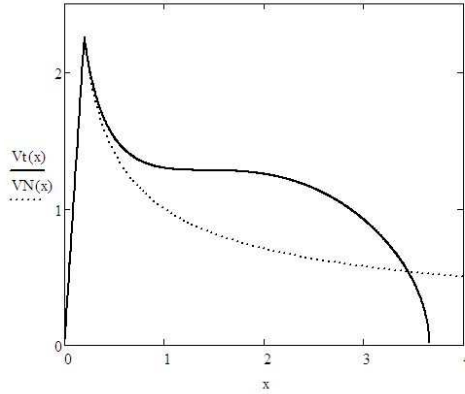


Figure 3: Predicted normalized velocity profile $V_t(x)$ for an idealized galaxy (consisting of a central bulge only) as based on vacuum modified gravity and comparison to the Keplerian velocity profile $V_N(x)$ where x is given by $x = r/R$. While a region of constant velocity (V_{pl}) is observed, the velocity drops to zero at $x = 3.66$.

reader will have noticed that equation (34) is in contradiction with the Tully Fisher relation [7], unless one requires that

$$R = \beta \cdot \sqrt{M}. \tag{35}$$

In this case one obtains that $V_{pl}^4 \sim M$.

Using galaxy data, as provided in [5], one finds that the value β should be about $1.22 m/\sqrt{kg}$, though it is not our aim here to pinpoint the most exact value. Let us now consider as an example our own galaxy. We assume a mass of 4×10^{41} kg and using equation (35) one finds that $R = 25$ kpc and by means of equation (35) we find $V_{pl} = 238$ km/s. Based on the value of R (25 kpc), the velocity profile is predicted to fall to zero at $r = 3.66 \times 25$ kpc = 91.5 kpc. The velocity is however expected to start dropping significantly at 60 kpc. The above mentioned values could of course be different when assuming a more realistic mass distribution. Inserting equation (35) into equation (15), one finds that:

$$\rho_V = \frac{\alpha c^4}{4\pi\beta^2 MG}, \tag{36}$$

which shows how the background vacuum energy density is related to the total mass of the galaxy. So, this shows also that ρ_V not be a constant of nature. In our initial derivations however, we have assumed V to be independent of position, while the position dependent part was contained in $\rho(r)$. Of course, we would still get (about) the same result as long as the gradients in ρ_V are much smaller than the gradients in $\rho(r)$. In fact, one could assume a whole hierarchy of scale lengths with smaller scale length perturbations superimposed on larger scale length perturbations. Our own solar system could represent a very small perturbation on top of the vacuum deviation (global ρ_V) of our galaxy as a whole. In this respect, it is found that the Pioneer anomaly can be reproduced perfectly when assuming a value of β given by $\beta = 0.2m/\sqrt{kg}$. Calculating the acceleration (from equation (16)) and subtracting from it the acceleration as obtained by Newton's law, one obtains an additional, almost constant, sunward acceleration of $8 \times 10^{-10} m/s^2$. Of course, β has been chosen here to fit the observation and this value is different from the one used previously ($1.22 m/\sqrt{kg}$) to describe the galaxy rotation curves. Nevertheless, it is remarkable that reducing the scale length by 8 orders of magnitude (galaxy to solar system), β remains of the same order. For the case of our solar system we further find (equation (33)) that $\frac{\rho_0}{\rho_V} = 1.6 \times 10^{-11}$. So, the Pioneer anomaly can be explained by a minute perturbation of the background vacuum. According to equation (35), very large mass distributions will lead to oscillating force fields having a very large scale length. This could result in voids with extremely large dimensions.

4 Relativistic vacuum modified gravity

In general relativity, the line element summarizing the Schwarzschild geometry is given by ($c \neq 1$):

$$ds^2 = - \left(1 - \frac{2GM}{c^2 r}\right) \cdot (cdt)^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} \cdot (dr)^2 + r^2 \cdot (d\theta^2 + \sin^2\theta \cdot d\phi^2). \quad (37)$$

To obtain the relativistic expression for vacuum modified gravity, one has to replace the potential $-GM/r$ by the potential given below:

$$\Phi = -\frac{MG}{r} \cdot \left(\cos\left(\frac{r}{R}\right) + \sin\left(\frac{r}{R}\right)\right), \quad (38)$$

in which R is given by equation (35).

5 Conclusions

A modified, relativistic theory of gravitation has been proposed in which the vacuum energy density plays a key role. At each point in space, Newton's law holds locally if one takes into account the additional energy in the vacuum contained in an enclosed sphere up to that point. Globally, one obtains however a different dependence of force on distance and the exact equation for this has been derived. It is found that any mass will, at some determined regions, induce repulsive forces. This modification of Newton's law has a free scale parameter R which depends on the environment (or overall mass distribution) in which the mass is located. It has been shown that this theory (designated by Vacuum Modified Gravity) naturally explains the observed flat galaxy rotation curves without the need to invoke dark matter. To be consistent with the Tully Fisher relation, it was necessary to impose the condition $R = \beta\sqrt{M}$. To describe correctly the galaxy rotation curves, it was found that $\beta = 1.22m/\sqrt{kg}$. Unlike the MOND theory, the present theory predicts that the rotation curves eventually drop off sharply to zero velocity. The proposed theory also explains the Pioneer anomaly, as well as the foamy distribution of galaxies. In addition, large voids are explained as a result of large mass distributions creating large regions of repulsive force (antigravity).

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Comment on
**VACUUM MODIFIED GRAVITY AS AN
EXPLANATION FOR FLAT GALAXY
ROTATION CURVES**

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R. Van Nieuwenhove in his paper "*Vacuum Modified Gravity as an explanation for flat galaxy rotation curves*" develops a theory (actually, a model rather than a theory) that aims to solve many problems of contemporary astrophysics such as: 1) flat rotation curves of spiral galaxies (without recourse to the dark matter hypothesis), 2) the not yet understood Pioneer anomaly, 3) cosmic voids, etc.

The Author states he comes to his model starting from several simple principles and contrasts it with the arbitrariness of other MOND-like models. In fact, the Author's argumentation is very weak and arbitrary, as well. I'm afraid that the main result of his work – the modified force of gravitational interaction of two point masses is wrong.

The Author refers the reader to his earlier works where he developed the basis for his current paper. I looked through only one of them which he refers to most frequently: "*Quantum Gravity: a Hypothesis*" published in *Europhysics Letters*. To me its content is

nonsensical. I don't like the current paper, as well, in which the Author uses mysterious and vague notions such as "*distortion of the quantum vacuum through its interaction with a mass*". I have read Weinberg's 'Quantum Theory of Fields' and never encountered anything like that.

All results the Author obtains are based on equation (6) describing the interaction force between mass M and a test mass m in the ambient space of the "varying part" of quantum vacuum described by the equation of state $p = -\alpha\rho$. He assumes that interaction with the vacuum should contribute to the total effective force between m and M . The Author argues that the absolute value F of this force should read (I rewrite equation (6) to its equivalent form)

$$F = \frac{Gm}{r^2} \left(M + \frac{4\pi}{c^2} \int_0^r \rho(x)x^2 dx \right) = \frac{mc^2}{\rho_V} \frac{dp}{dr},$$

where $\frac{mc^2}{\rho_V}$ is a formal quantity having dimension of volume which is associated with mass m (it appeared 'Deus ex machina' with no physical justification, it is "*equivalent volume*" of mass m , a "*bubble*" of the particle's stuff inside a "*turbulent vacuum*", as the Author describes it in his "*Quantum Gravity: a Hypothesis*"), ρ_V is, as the Author states, the (background) vacuum energy density. This is simultaneously the equation for unknown function $\rho(r)$. The general solution of this equation (or Eq. (6)), the Author correctly finds, is (9).

The first problem with this equation I perceive, is that the varying part of the energy density of vacuum ρ depends on the chosen pair of particles m and M . I do not understand how ρ would change for a system composed of many interacting particles (and the author did not explain this). Would it be a kind of superposition of ρ 's of all pairs or what? Of course, the force should be a vector field, thus the effective force, and thus also the overall ρ , should depend on the spatial configuration of the point masses. This in turn implies that ρ should depend on geometry of the mass distribution in spatially extended objects. The Author should necessarily solve this problem, otherwise, it is impossible to describe gravity of extended objects such as galaxies (from this standpoint, I claim, in contrary to the Author's opinion, that he did not solved the problem of flat rotation

curves of spiral galaxies, which are spatially extended objects). In other words, what would be the counterpart of Poisson equation for the gravitational potential in the Author's model of gravitation?

To see its physical content, the above equation may be interpreted as the radial component of the Newtonian gravitational force exerted on a test mass m both by a point mass M located in the center of symmetry, and by a spherically symmetric mass distribution $\rho(r)/c^2$ surrounding M . This force is the same as the buoyant force exerted by the gradient of pressure of the 'quantum vacuum' on the volume $\frac{mc^2}{\rho_V}$.

It is not clear to me why masses m and M , that come symmetrically into the ordinary Newtonian interaction of two point masses, are treated on different footing in the other part? The form of the above equation, together with symmetry arguments (any component of it should be proportional to the product mM), imply that ρ should be proportional to M (note, that if this equation was true, then $\rho(r)$ would change its value after replacing m and M with each other). The Author should make this point clear. I agree with the Author's conclusion that ρ in this equation must be proportional to M . However, I cannot agree that symmetry considerations imply that $q = p$ in equation (9) (the Author did not show that $q = p$). What the arguments imply is that $p = 0$. In order to see this, first subtract the above equation from that with m and M interchanged, then we obtain for $M \neq m$

$$\frac{G}{r^2} \left(\frac{4\pi}{c^2} \int_0^r \rho(x)x^2 dx \right) = -\alpha \frac{c^2}{\rho_V} \frac{d\rho}{dr}$$

(we used the relation $p(r) = -\alpha\rho(r)$; the Author uses p to denote two different things which may be misleading). On differentiating we get $r^2\rho''(r) + 2r\rho'(r) + \frac{r^2}{h^2}\rho(r) = 0$ with $h^2 = \frac{\alpha c^4}{4\pi G\rho_V}$. The general solution reads $\rho(r) = \frac{1}{r} (c_1 \sin \frac{r}{h} + c_2 \cos \frac{r}{h})$. By substituting this general solution to the integral just above, we infer that $c_2 = 0$ (this corresponds to the Author's nonzero p), hence $\rho(r) = \frac{c_1}{r} \sin \frac{r}{h}$ and c_1 (which corresponds to the Author's nonzero $q = p$) cannot be determined as one would expect, since the vacuum density should not change when m and M are interchanged with each other (otherwise ρ would depend on the direction in space determined by positions

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of these two masses, which is again the point I raised above). This argumentation undermines the main conclusion of the paper that at small distances the gravitational interaction between point masses reduces to the Newton's inverse square law, since now, using equation (11), which says that $F \propto -\rho'(r)$, we have $F \sim \frac{c_1}{3h^3}r$ close to $r = 0$ and not $\propto \frac{1}{r^2}$ as for Newton's gravitation.

I expect that the Author will show and convince the Editor that I am not right and that his paper is worth of publishing in NCP. At the moment I refrain from carrying on evaluation of the further parts of this paper.

As for now I strongly discourage the Editor from accepting this paper for publication in NCP.

Authors' response

I do not really understand why the referee is of the opinion that the content of my previous paper "*Quantum Gravity: a Hypothesis*" is nonsensical. After all, it has been published in a refereed journal and Newton's law was perfectly reproduced. The referee finds the concept of "distortion of the quantum vacuum through its interaction with a mass" mysterious and vague. However, such concepts belong to common (accepted) physics. There are numerous papers and books in which aspects of this concept are described ("vacuum polarization" for instance is a well-known effect and the Casimir force can also be considered). A quick search by Google using the search-terms "interaction with the vacuum" and "quantum" results in 5630 sites. I just list of few arbitrary ones:

<http://technology.newscientist.com/article/mg15821375.200-lets-play-quantum-chess.html>

<http://www.iop.org/EJ/abstract/0953-4075/21/17/014>

<http://aps.arxiv.org/abs/hep-ph/0702145>

http://prola.aps.org/abstract/PRA/v54/i2/p1686_1

<http://www.iop.org/EJ/abstract/0305-4470/25/4/031>

The concept of representing a mass by a "bubble" inside the "turbulent vacuum" can for instance be found in the book "*Die Struktur des Vakuums - Ein Dialog über das nichts*" from Johann Rafelski and Berndt Müller, Verlag Harri Deutsch, 1985. The main objection of the referee is related to the absence of symmetry in the equations between m and M . The origin of this is that the referee has missed my point of using the term "test mass" for the mass m . So, all my derivations are based on the assumption that M is much larger than m . To be absolutely clear on this, I just quote the definition of test mass found on the English Wikipedia site: <http://en.wikipedia.org/wiki/Test-mass>: "*In physical theories, a test particle is an idealized model of an object whose physical properties (usually mass, charge, or size) are assumed to be negligible except for the property being studied, which is considered to be insufficient to alter the behavior of the rest of the system.* The concept of a test particle often simplifies problems, and can provide a good approximation for physical phenomena". The ρ in my equations corresponds to the (perturbed) vacuum energy density induced by the large mass

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M (ONLY) and not by the small test mass m . So, the vacuum perturbation induced by the test mass m does not enter into the equations and this causes the asymmetry. I have chosen this approach because it simplifies of course the equations. In addition, when considering the galaxy rotation curves, one can analyze these in terms of small test particles encircling the centre of the much more massive galaxy. In manipulating the equations (such as interchanging m and M) one should also be aware of the following: The radial coordinate “ r ” is assumed to have its origin at the centre of mass of the large mass M . This is of importance when evaluating for instance the derivative term $\frac{d\rho}{dr}$. Next, I would like to point out that the main argument used by the referee to advice a rejection of the paper is wrong. Considering first the situation in which m is the test mass (Eq. (1)), followed by the situation in which M is the test mass (Eq. (2)).

$$\frac{Gm}{r^2} \cdot \left(M + \frac{4\pi}{c^2} \cdot \int_0^{r_1} \rho(r) \cdot r^2 \cdot dr \right) = -\frac{mc^2}{\rho_V} \cdot \alpha \frac{d\rho}{dr} \quad (1)$$

$$\frac{GM}{r^2} \cdot \left(m + \frac{4\pi}{c^2} \cdot \int_0^{r_1} \rho'(r) \cdot r^2 \cdot dr \right) = -\frac{Mc^2}{\rho_V} \cdot \alpha \frac{d\rho'}{dr'} \quad (2)$$

Subtracting (2) from (1) leads to

$$\begin{aligned} \frac{Gm}{r^2} \cdot \frac{4\pi}{c^2} \cdot \int_0^{r_1} \rho(r) \cdot r^2 \cdot dr - \frac{GM}{r^2} \cdot \frac{4\pi}{c^2} \cdot \int_0^{r_1} \rho'(r) \cdot r^2 \cdot dr = \\ = -\frac{mc^2}{\rho_V} \cdot \alpha \frac{d\rho}{dr} + \frac{Mc^2}{\rho_V} \cdot \alpha \frac{d\rho'}{dr'} \end{aligned} \quad (3)$$

I deliberately introduced the primes (on ρ) to make clear that the ρ in (Eq.1) is NOT the same as the ρ in (Eq.2): Their magnitudes are different (because respectively generated by M and m) and also their distribution is different (centred around M in Eq.(1) and centred around m in Eq.(2)). The prime on the “ r ” in Eq.(2) is introduced to make clear that the origin of the radial coordinate is now at the largest mass m instead of at M (in Eq.(1)). Without further knowledge of ρ , this can not be simplified further. It is also different from the equation derived by the referee (Eq.(4)) of which it is unclear to me

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how this was derived. Even omitting the primes in Eq.(3) does not lead to Eq.(4).

$$\frac{G}{r^2} \cdot \left(\frac{4\pi}{c^2} \cdot \int_0^{r_1} \rho(x) \cdot x^2 \cdot dx \right) = -\frac{c^2}{\rho V} \cdot \alpha \frac{d\rho}{dr} \quad (4)$$

If one assumes that ρ and $\frac{d\rho}{dr}$ are proportional to the generating (large) masses (M for Eq.(1) and m for Eq.(2)), all terms in Eq.(3) are proportional to the product mM (as it should be) and Eq.(3) just reduces to zero = zero, which is of course not in contradiction with any law. Since Eq.(4) used by the referee is wrong, all his subsequent deductions are invalid. I do however agree with the comment that I did not show that $q = p$ (in Eq.(9) of my paper), and I do not agree that $p = 0$ (as the referee claims). Only later, after submitting my paper, I realized that something was missing in my argumentation. The choice $q = p$, though not proven, seemed to be the most natural (and simple) at that time. If my paper is accepted, I will of course include a comment about this. Therefore, I hope that to have demonstrated that my paper contains significant new physics and that it is worth publishing in NCP.

R. Van Nieuwenhove

**Comment on
VACUUM MODIFIED GRAVITY AS AN
EXPLANATION FOR FLAT GALAXY
ROTATION CURVES**

Lukasz Bratek

H. Niewodniczański Institute of Nuclear Physics, PAN

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R. Van Nieuwenhove in his paper "*Vacuum Modified Gravity as an explanation for flat galaxy rotation curves*" develops a theory (actually, a model rather than a theory) that aims to solve many problems of contemporary astrophysics such as: 1) flat rotation curves of spiral galaxies (without recourse to the dark matter hypothesis), 2) the not yet understood Pioneer anomaly, 3) cosmic voids, etc.

The Author states he comes to his model starting from several simple principles and contrasts it with the arbitrariness of other MOND-like models. In fact, the Author's argumentation is very weak and arbitrary, as well. I'm afraid that the main result of his work – the modified force of gravitational interaction of two point masses is wrong.

The Author refers the reader to his earlier works where he developed the basis for his current paper. I looked through only one of them which he refers to most frequently: "*Quantum Gravity: a Hypothesis*" published in *Europhysics Letters*. To me its content is

nonsensical. I don't like the current paper, as well, in which the Author uses mysterious and vague notions such as "*distortion of the quantum vacuum through its interaction with a mass*". I have read Weinberg's 'Quantum Theory of Fields' and never encountered anything like that.

All results the Author obtains are based on equation (6) describing the interaction force between mass M and a test mass m in the ambient space of the "varying part" of quantum vacuum described by the equation of state $p = -\alpha\rho$. He assumes that interaction with the vacuum should contribute to the total effective force between m and M . The Author argues that the absolute value F of this force should read (I rewrite equation (6) to its equivalent form)

$$F = \frac{Gm}{r^2} \left(M + \frac{4\pi}{c^2} \int_0^r \rho(x)x^2 dx \right) = \frac{mc^2}{\rho_V} \frac{dp}{dr},$$

where $\frac{mc^2}{\rho_V}$ is a formal quantity having dimension of volume which is associated with mass m (it appeared 'Deus ex machina' with no physical justification, it is "*equivalent volume*" of mass m , a "*bubble*" of the particle's stuff inside a "*turbulent vacuum*", as the Author describes it in his "*Quantum Gravity: a Hypothesis*"), ρ_V is, as the Author states, the (background) vacuum energy density. This is simultaneously the equation for unknown function $\rho(r)$. The general solution of this equation (or Eq. (6)), the Author correctly finds, is (9).

The first problem with this equation I perceive, is that the varying part of the energy density of vacuum ρ depends on the chosen pair of particles m and M . I do not understand how ρ would change for a system composed of many interacting particles (and the author did not explain this). Would it be a kind of superposition of ρ 's of all pairs or what? Of course, the force should be a vector field, thus the effective force, and thus also the overall ρ , should depend on the spatial configuration of the point masses. This in turn implies that ρ should depend on geometry of the mass distribution in spatially extended objects. The Author should necessarily solve this problem, otherwise, it is impossible to describe gravity of extended objects such as galaxies (from this standpoint, I claim, in contrary to the Author's opinion, that he did not solved the problem of flat rotation

curves of spiral galaxies, which are spatially extended objects). In other words, what would be the counterpart of Poisson equation for the gravitational potential in the Author's model of gravitation?

To see its physical content, the above equation may be interpreted as the radial component of the Newtonian gravitational force exerted on a test mass m both by a point mass M located in the center of symmetry, and by a spherically symmetric mass distribution $\rho(r)/c^2$ surrounding M . This force is the same as the buoyant force exerted by the gradient of pressure of the 'quantum vacuum' on the volume $\frac{mc^2}{\rho_V}$.

It is not clear to me why masses m and M , that come symmetrically into the ordinary Newtonian interaction of two point masses, are treated on different footing in the other part? The form of the above equation, together with symmetry arguments (any component of it should be proportional to the product mM), imply that ρ should be proportional to M (note, that if this equation was true, then $\rho(r)$ would change its value after replacing m and M with each other). The Author should make this point clear. I agree with the Author's conclusion that ρ in this equation must be proportional to M . However, I cannot agree that symmetry considerations imply that $q = p$ in equation (9) (the Author did not show that $q = p$). What the arguments imply is that $p = 0$. In order to see this, first subtract the above equation from that with m and M interchanged, then we obtain for $M \neq m$

$$\frac{G}{r^2} \left(\frac{4\pi}{c^2} \int_0^r \rho(x)x^2 dx \right) = -\alpha \frac{c^2}{\rho_V} \frac{d\rho}{dr}$$

(we used the relation $p(r) = -\alpha\rho(r)$; the Author uses p to denote two different things which may be misleading). On differentiating we get $r^2\rho''(r) + 2r\rho'(r) + \frac{r^2}{h^2}\rho(r) = 0$ with $h^2 = \frac{\alpha c^4}{4\pi G\rho_V}$. The general solution reads $\rho(r) = \frac{1}{r} (c_1 \sin \frac{r}{h} + c_2 \cos \frac{r}{h})$. By substituting this general solution to the integral just above, we infer that $c_2 = 0$ (this corresponds to the Author's nonzero p), hence $\rho(r) = \frac{c_1}{r} \sin \frac{r}{h}$ and c_1 (which corresponds to the Author's nonzero $q = p$) cannot be determined as one would expect, since the vacuum density should not change when m and M are interchanged with each other (otherwise ρ would depend on the direction in space determined by positions

Comment

of these two masses, which is again the point I raised above). This argumentation undermines the main conclusion of the paper that at small distances the gravitational interaction between point masses reduces to the Newton's inverse square law, since now, using equation (11), which says that $F \propto -\rho'(r)$, we have $F \sim \frac{c_1}{3h^3}r$ close to $r = 0$ and not $\propto \frac{1}{r^2}$ as for Newton's gravitation.

I expect that the Author will show and convince the Editor that I am not right and that his paper is worth of publishing in NCP. At the moment I refrain from carrying on evaluation of the further parts of this paper.

As for now I strongly discourage the Editor from accepting this paper for publication in NCP.

Authors' response

I do not really understand why the referee is of the opinion that the content of my previous paper "*Quantum Gravity: a Hypothesis*" is nonsensical. After all, it has been published in a refereed journal and Newton's law was perfectly reproduced. The referee finds the concept of "distortion of the quantum vacuum through its interaction with a mass" mysterious and vague. However, such concepts belong to common (accepted) physics. There are numerous papers and books in which aspects of this concept are described ("vacuum polarization" for instance is a well-known effect and the Casimir force can also be considered). A quick search by Google using the search-terms "interaction with the vacuum" and "quantum" results in 5630 sites. I just list of few arbitrary ones:

<http://technology.newscientist.com/article/mg15821375.200-lets-play-quantum-chess.html>

<http://www.iop.org/EJ/abstract/0953-4075/21/17/014>

<http://aps.arxiv.org/abs/hep-ph/0702145>

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