

DISCRETE STATES OF CONTINUOUS ELECTRICALLY CHARGED MATTER

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Abstract

By introducing in the Euler equation non mechanical (mesic) force densities equal to the products of corresponding potentials and gradient of the matter density, and supposing that the dynamical process has a diffusion character, the equation of continuity with internal sources (both positive and negative) is obtained. These sources are proportional to the algebraical sum of electrostatic, kinetic and proper energy densities, the constant of proportionality being the inverse value of the diffusion constant $D = \hbar/2m$, taken with negative sign. The stationary state is obtained if the solution of the continuity equation holds in any point of the object, which is possible only for discrete values of the constant E with dimension of energy, entering in the equation of continuity. The equation of continuity can be transformed to the Bohm equation with the quantum potential, which may be solved by reducing it to the corresponding Schrödinger equation. Thus the spatial

distribution of the density of matter is given by the square of the Schrödinger wave function. Once the density distribution function is known, the velocity \vec{u} in the diffusion process can be calculated and the process is deterministic. Superimposed to the diffusion process is the classical motion with velocity \vec{v} , for which the equation of continuity without internal sources of matter holds.

1 Introduction

From the voluminous number of publications concerning the possible ways of obtaining the Schrödinger equation and its interpretations (see e.g. Max Jammer [1]) we have selected three as the most important for our purpose.

In 1952, taking the solution of the one particle, time-dependent Schrödinger equation

$$-\frac{\hbar^2}{2m}\Delta\Psi + V(\vec{x})\psi = j\hbar\frac{\partial\Psi}{\partial t}, \quad (1)$$

in the form

$$\Psi = R \exp(jS/\hbar) \quad R = \mu^{1/2}, \quad (2)$$

Bohm [2] obtained the equation

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V(x) - D^2 m \left(\frac{\Delta\mu}{\mu} - \frac{(\nabla\mu)^2}{2\mu^2} \right) = 0 \quad \left(D = \frac{\hbar}{2m} \right). \quad (3)$$

The last member on the left side is the Bohm quantum potential

$$U(\vec{x}) = -\frac{\hbar^2}{4m} \left[\frac{\Delta\mu}{\mu} - \frac{(\nabla\mu)^2}{2\mu^2} \right]. \quad (4)$$

If μ is substituted by R^2 , the Bohm quantum potential (4) can be written in the form

$$U(\vec{x}) = -D^2 2m \frac{\Delta R}{R} = -\frac{\hbar^2}{2m} \frac{\Delta R}{R}. \quad (5)$$

The gradient of the Bohm quantum potential is

$$\nabla U(\vec{x}) = -\frac{\hbar^2}{4m} \nabla \left[\frac{\Delta\mu}{\mu} - \frac{(\nabla\mu)^2}{2\mu^2} \right] = -\frac{\hbar^2}{2m} \nabla \left(\frac{\Delta R}{R} \right). \quad (6)$$

Bohm sets the velocity $\nabla S/m = \vec{v}$ and $\partial S/\partial t = E/m$. $V(\vec{x})$ is the electrical potential energy, so that the equation of motion (3) gets the form

$$E + m \frac{v^2}{2} + V(\vec{x}) - D^2 m \left(\frac{\Delta\mu}{\mu} - \frac{(\nabla\mu)^2}{2\mu^2} \right) = 0. \quad (7)$$

The equation of continuity for the motion of a particle with velocity \vec{v} is the second equation obtained by Bohm [2] in the form

$$\frac{\partial \mu}{\partial t} + \nabla \left(\mu \frac{\nabla S}{m} \right) \equiv \frac{\partial \mu}{\partial t} + \nabla(\mu \vec{v}) = 0. \quad (8)$$

Solution of the equations (7) and (8) can be obtained by the inverse procedure shown by Bohm [2], consisting in the transformation of these non linear equations in the correspondent Schrödinger equation, and in reconstruction from its solution both $S(\vec{x}, t)$ and $\mu(\vec{x}, t)$.

Bohm [2] interpreted equation (7) as a Hamilton-Jacobi equation of a particle moving under influence of electrical and, newly by him introduced, quantum potential (5).

Twenty six years before Bohm, E. Madelung [3] derived practically the same equations as Bohm, but the function $m\mu$ has been interpreted by him as the actual matter distribution.

Quite different is the derivation of the Schrödinger equation by Nelson [4] in 1966. Whereas Madelung and Bohm derived their equations from the Schrödinger equation, Nelson started from the idea that a charged particle executes some kind of Brownian motion in presence of external forces. Using the stochastic mechanics, he introduced two velocities, one called by him the flow velocity \vec{v} , for which holds the equation of continuity in the form

$$\frac{\partial \mu}{\partial t} + \text{div}(\vec{v}\mu) = 0 \quad (9)$$

and the other called by him the osmotic velocity \vec{u} , satisfying the equation of continuity

$$\text{div}(\vec{u}\mu) - \frac{\hbar}{2m} \Delta \mu = 0, \quad (10)$$

for a continuum in a stationary state containing inner sources of matter.

For these velocities he derived two differential equations

$$\frac{\partial \vec{u}}{\partial t} = -\frac{\hbar}{2m} \nabla \text{div} \vec{v} - \nabla(\vec{v}\vec{u}), \quad (11)$$

$$\frac{\partial \vec{v}}{\partial t} = \frac{\vec{F}}{m} - (\vec{v} \nabla) \vec{v} + (\vec{u} \nabla) \vec{u} + \frac{\hbar}{2m} \Delta \vec{u}. \quad (12)$$

The relation between the velocity \vec{u} and the matter density Nelson obtained from the theory of Brownian motion as

$$\vec{u} = \frac{\hbar}{2m} \frac{\nabla \mu}{\mu}, \quad (13)$$

from which (10) follows immediately.

The velocity \vec{u} is by definition (13) a gradient. If \vec{v} is taken as a gradient too, then from equations (11) and (12) one gets the Bohm equations (7) and (8), which can be reduced to the Schrödinger equation.

Thus Nelson obtained the Schrödinger equation as a result of a special type of a priori made assumptions, but again, his equations concern motion of a particle, not of the continuum. Both, the Bohm and Nelson interpretations of their equations are deterministic. Bohm speaks of so called "hidden" parameters.

2 The Euler equation with internal sources of matter

Let us return to the Bohm equation (3), which can be written in the form

$$-D\Delta m\mu + \frac{1}{D}\mu \left[\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V(\vec{x}) + D^2m \frac{(\nabla \mu)^2}{2\mu^2} \right] = 0, \quad (3b)$$

reminding of the diffusion equation with internal sources of matter. Returning to the Madelung interpretation and inspired also by the work of Nelson, we shall try to find what assumptions are to be made to enable the derivation of the Schrödinger equation starting with the Euler equation for the electrically charged continuum acted on by mechanical and non-mechanical (mesic) forces.

Let us consider a material continuum (object) constituted by electrically charged but mutually electrically non-interacting medium in which the matter density is given by $m\mu(\vec{x}, t)$ where m is the total mass of the object and $\mu(\vec{x}, t)$ is the distribution function normalized to one.

The Euler equation for the volume element δV moving with a velocity \vec{v} under the influence of external forces with volume densities

\vec{f}_{ext} has the form

$$\frac{d}{dt} \int \int \int_{\delta V} m\mu \vec{v} dV = \vec{f}_{ext} \delta V. \quad (14)$$

The left side can be expanded

$$\frac{d}{dt} \int \int \int_{\delta V} m\mu \vec{v} dV = \int \int \int_{\delta V} \left(m\mu \frac{d\vec{v}}{dt} + \vec{v} \frac{dm\mu}{dt} \right) dV + \int \int \int_{\delta V} m\mu \vec{v} \frac{d(dV)}{dt}. \quad (15)$$

In the last member one can use the relation

$$\frac{d(dV)}{dt} = dV \operatorname{div} \vec{v}. \quad (16)$$

Because δV can be made arbitrary small, one can replace dV by δV and divide by δV . So we get (14) in the form

$$m\mu \frac{d\vec{v}}{dt} + \vec{v} \frac{dm\mu}{dt} + \vec{v} m\mu \operatorname{div} \vec{v} = \vec{f}_{ext} \quad (17)$$

or in extended form

$$\begin{aligned} & m\mu \left(\frac{\partial \vec{v}}{\partial t} + (\vec{v} \nabla) \vec{v} \right) + \vec{v} \left(\frac{\partial m\mu}{\partial t} + \vec{v} \nabla m\mu + m\mu \operatorname{div} \vec{v} \right) = \\ & = m\mu \left(\frac{\partial \vec{v}}{\partial t} + (\vec{v} \nabla) \vec{v} \right) + \vec{v} \left[\frac{\partial m\mu}{\partial t} + \operatorname{div}(m\mu \vec{v}) \right] = \vec{f}_{ext}. \end{aligned} \quad (18)$$

The inertial force density expressed by the second term has the character of friction and disappears for any value of \vec{v} owing to the validity of the equation of continuity

$$\frac{\partial m\mu}{\partial t} + \operatorname{div}(m\mu \vec{v}) = dm\mu/dt = 0. \quad (19)$$

Now, the question is, whether the supposed validity of the equation of continuity implies disappearance of the second member on the left side of (18) or whether in contrary the necessity of its disappearance dictated by the experience implies the validity of the equation of continuity.

Suppose, that the right side of the equation of continuity differs from zero, what is the case, when some internal sources or sinks of matter exist in the object. This may occur, if some forces with mesic character act on the matter. Such kind of forces are considered in the relativistic mechanics and let us suppose, that they exist also in the non relativistic mechanics.

3 Mechanical and non mechanical (mesic) external forces

In a stationary electric field with potential $\varphi(\vec{x})$ the electric force density acting on a volume element δV with electric charge density ρ is $\vec{F}_e = -\delta V \rho \nabla \varphi$. We can make this expression more symmetrical by taking $\nabla(\rho\varphi) = \rho \nabla \varphi + \varphi \nabla \rho$ and putting

$$\vec{f}_{ext} = \vec{f}_e + \vec{f}_e^m; \quad (\vec{f}_e = -\rho \nabla \varphi; \quad \vec{f}_e^m = \varphi \nabla \rho). \quad (20)$$

Our next assumption is, that the **electrically charged matter diffuses and that the charge density** $\rho = e\mu$, where e is the total charge of the object.

Then from the diffusion law

$$m\mu \vec{u} = -D \nabla m\mu \quad D[cm^2 s^{-1}], \quad (21)$$

one gets

$$\rho \vec{u} = -D \nabla \rho, \quad (22)$$

so that

$$\vec{f}_e^m = \varphi \nabla \rho = \varphi e \nabla \mu = -\vec{u} D^{-1} e \mu \varphi. \quad (23)$$

This force density has a non mechanical character (a non relativistic analogy of so called mesic forces known in the relativistic mechanics of incoherent matter (see e.g. C. Moller [5])), whose action leads to creation or disappearance of the matter.

At this stage let us accept the postulate, that **by the same procedure, which was used for the force density \vec{f}_e^m , similar force density can be deduced from any potential**. Thus for example from the velocity potential $u^2/2$ we obtain the mesic force density

$$\vec{f}_{Ekin}^m = -\vec{u} D^{-1} m\mu \frac{u^2}{2}. \quad (24)$$

The sum of the mesic force densities (23) and (24) with second member on the left side of (18) (\vec{v} replaced by \vec{u}) gives the total mesic force density

$$\vec{f}_t^m = \vec{u} \left[\frac{\partial m\mu}{\partial t} + \text{div}(m\mu\vec{u}) + D^{-1} \left(e\mu\varphi + m\mu \frac{u^2}{2} \right) \right] \quad (25)$$

or, if $\text{div}(m\mu\vec{u})$ is replaced with the aid of (21) by $-D\Delta m\mu$.

$$\vec{f}_t^m = \vec{u} \left[\frac{\partial m\mu}{\partial t} - D\Delta m\mu + D^{-1} \left(e\mu\varphi + m\mu \frac{u^2}{2} \right) \right]. \quad (26)$$

4 Conditions for the stationary states

Let us take the partial time derivation in (26) in the form $\frac{\partial m\mu}{\partial t} = D^{-1}\mu E$, where E has the dimension of energy. All terms in brackets in (25) and (26) have dimensions gs^{-1} , and the stationary state will be reached, when $\vec{f}_t^m = 0$. This occurs for any value of \vec{u} , if

$$-D\Delta m\mu + D^{-1}\mu \left(e\varphi + m \frac{u^2}{2} + E \right) = 0. \quad (27)$$

This equation has the form of the equation of continuity with internal sources of matter

$$Q = -D^{-1}\mu \left(e\varphi + m \frac{u^2}{2} + E \right). \quad (28)$$

The first member of (27) represents the escape of matter through the boundary of a volume element, the second the sources (or sinks) of matter compensating the escape, so that the quantity of matter in the volume element at any point of the object remains constant.

The velocity \vec{u} before the brackets in equation (26) may be replaced by \vec{u} from (21) and we get another expression for the condition for a stationary case

$$\vec{f}_t^m = - \left[\frac{E}{m} + \frac{e}{m}\varphi + \frac{u^2}{2} - D^2 \frac{\Delta\mu}{\mu} \right] \nabla m\mu = 0. \quad (29)$$

This force density must again disappear in a stationary case for any value of $\nabla\mu$. This gives the condition for a stationary state in the form

$$E + e\varphi + m \frac{u^2}{2} - D^2 m \frac{\Delta\mu}{\mu} = 0 \quad (30)$$

or, if $\frac{u^2}{2}$ is substituted from (21)

$$\frac{u^2}{2} = D^2 \frac{(\nabla\mu)^2}{2\mu^2}, \quad (31)$$

one gets the condition for the stationary state in the form

$$E + e\varphi - D^2 m \left[\frac{\Delta\mu}{\mu} - \frac{(\nabla\mu)^2}{2\mu^2} \right] = 0. \quad (32)$$

The equation of motion is obtained as gradient of the last equation

$$\nabla \left\{ e\varphi - D^2 m \left[\frac{\Delta\mu}{\mu} - \frac{(\nabla\mu)^2}{2\mu^2} \right] \right\} = 0. \quad (33)$$

If the diffusion coefficient D is set equal to $\frac{\hbar}{2m}$, then the last term turns to the known Bohm [2] quantum potential

$$U(x) = -\frac{\hbar^2}{4m} \left[\frac{\Delta\mu}{\mu} - \frac{(\nabla\mu)^2}{2\mu^2} \right], \quad (34)$$

which emerges here in a quite natural way, if the existence of non mechanical forces is accepted.

The equation (29) is however one part of the total gradient of the energy densities in the stationary state

$$\vec{f}_t + \vec{f}_t^m = \nabla \left[E\mu + e\mu\varphi + m\mu \frac{u^2}{2} - D^2 m\mu \frac{\Delta\mu}{\mu} \right] = 0. \quad (35)$$

The sum of inertial force density and of the external force densities gives

$$\vec{f}_t = m\mu \nabla \left[\frac{u^2}{2} + \frac{e}{m}\varphi - D^2 \left(\frac{\Delta\mu}{\mu} \right) \right] = 0 \quad (36)$$

and must be zero, because in a stationary case according (29) $\vec{f}_t^m = 0$. If divided by $m\mu$ one obtains the equation of motion of the continuum, which differs from the classical one only by the last term.

5 Discrete states of the continuous electrically charged matter

Suppose, that we have an object filled with continuous, non mutually interacting, electrically charged substance, subject to external forces from an electrical potential φ and from two velocity potentials $v^2/2$ and $u^2/2$ ($rot \vec{v} = rot \vec{u} = 0$). We shall call in accordance with E. Nelson [4] the velocity \vec{v} the flow velocity and the velocity \vec{u} the osmotic velocity. For the motion with velocity \vec{v} the equation of continuity holds in the form (9), as in a continuum without internal sources of matter, whereas the motion with velocity \vec{u} is connected with appearance of mass sources given by (28) and \vec{u} obeys the diffusion equation (21).

Under these assumptions the equation of motion in the stationary state will sound

$$m\nabla \frac{v^2}{2} + e\nabla\varphi - D^2m\nabla \left(\frac{\Delta\mu}{\mu} - \frac{(\nabla\mu)^2}{2\mu^2} \right) = 0. \quad (37)$$

This equation differs from the equation of motion (33) only by the first member, which is the force due to $d\vec{v}/dt$ (in the stationary state). The last member is comparable with the first two if the terms in the brackets are sufficiently large, which can be the case in the range of atomic dimensions.

The condition (27) for a stationary state reads now

$$D\Delta m\mu - D^{-1}\mu \left(e\varphi + m\frac{v^2 + u^2}{2} + E \right) = 0, \quad (38)$$

in which the source density

$$Q = -D^{-1}\mu \left(e\varphi + m\frac{v^2 + u^2}{2} + E \right). \quad (39)$$

By analogy, the equation (29) gets the form

$$E + e\varphi + \frac{mv^2}{2} - D^2m \left[\frac{\Delta\mu}{\mu} - \frac{(\nabla\mu)^2}{2\mu^2} \right] = 0, \quad (40)$$

reducible to (3b).

6 Discussion

In our case, we started from the standard procedure used in derivation of the equations of motion of a material continuum for motion with flow velocity \vec{v} and osmotic velocity \vec{u} , but without omission of the force densities of non mechanical character. We defined further the general way how to express the densities of supposed non mechanical forces by the potentials and postulated the validity of the diffusion law between the osmotic (diffusion) velocity \vec{u} and the matter density.

The most important in our interpretation is the equation of continuity (38) with internal sources of matter as the condition for the stationary states. To obtain the source densities of matter we had to introduce a special kind of non mechanical forces. The equation (38) must hold everywhere in the continuous object. To provoke a change of a stationary state, it is sufficient to evoke a perturbation in a limited part of the object, for example by an external potential, which would violate the validity of this equation. One may assume that this will create an local increase (or decrease) of the density of the internal sources in that part, the perturbation will spread abroad over the whole object and eventually a new stationary state will be reached or the system may return to the initial state. This naturally should be connected with energy exchange between the external perturbation field and the fields inside the object.

The transient period between two stationary states depends on the deviation of the value of dm/dt from zero caused by the perturbation. This period can be very short and so even unobservable, but in any case the transition will be a continuous process. In this respect our system is deterministic and the probability of finding the system in a stationary state or the probability of its transition to another stationary state is not of inherent nature. In this respect we stand on the position of Einstein in his controversy with Bohr (see Vigier [6]).

Because the distribution function μ is given by the solution of the corresponding Schrödinger equation, the velocity \vec{u} as a function of co-ordinates at any place of the object at the initial moment is known. So the initial conditions of the transient period caused by some perturbation are given and the transient states of the system should be in principle calculable.

There is however one fundamental difficulty in the attempt to apply this model directly to the atom structure. This difficulty consists in the necessity to consider the electrically charged matter as a non mutually interacting (incoherent) medium, it is, to neglect the interaction between a given volume element with the electrically charged surroundings. This difficulty was mentioned already by E. Madelung [3] and is naturally absent in the one particle interpretations by Bohm and Nelson.

Though our model looks very attractive for the elucidation of effects, where a small perturbation caused for example by the electromagnetic light wave results in the transition in a new state connected with much larger exchange of energy, it is still far from to account for many other quantum effects. Therefore our considerations till now are limited to bounded discrete states of the electrically charged continuum in time independent electric fields.

7 Appendix

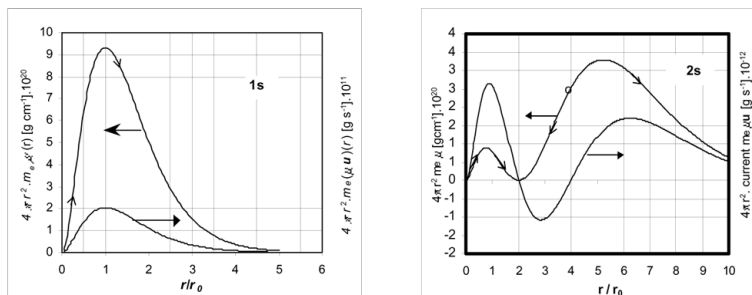


Figure 1: The matter distribution $4\pi r^2 m_e \rho(r)[gcm^{-3}]$ (left side scale) and fluxes of matter $4\pi r^2 m_e \bar{u}(r)[gs^{-1}]$ (right side scale) for states 1s and 2s of a continuum analogue to the hydrogen atom ($m_e = 9,109 \cdot 10^{-28} \text{ g}$, $e = 4,802 \cdot 10^{-10} \text{ CGSE}$, $\hbar = 1,055 \cdot 10^{-27} \text{ ergs}$, $r_0 = 0,5296 \cdot 10^{-8} \text{ cm}$, $D = 0,579 \text{ cm}^2 \text{ s}^{-1}$, $E_{1s} = -2.178 \text{ erg}$, $E_{2s} = -0.564 \text{ erg}$).

For two spherically symmetrical discrete states 1s and 2s of continuous electron matter the matter density and the matter current

density as well as the densities of the sources of matter integrated over the angles φ and θ are presented in graphical form. The first graph shows the matter distributions as functions of the distance from the center of the point potential source. The Bohr radius was used as the distance unit. The distributions are normalized to the mass of one electron. The second curves in the graph are the fluxes of matter. In the 1s state the flux is positive everywhere (in the direction of the radius), in the state 2s it changes the sign and is negative for $2 \leq r/r_0 \leq 4$. The arrows on the matter distribution curves correspond to the direction of the flow of matter.

The total sources of matter Q as well as the partial sources of matter Q_V, Q_{Ekin} and Q_E corresponding to the individual members on the right side of (28) are presented on the second graph. One sees, that there are regions, where the sources are positive, it is, where the rest matter is created and regions with negative sources (sinks), where the rest matter is absorbed. The total source of matter Q is the sum of individuals partial sources, due to the potential and kinetic energy densities and to the energy density corresponding to the proper value E . If integrated over the whole space, the total source of matter gives zero, so that the system is autonomous, with no matter exchange with the surroundings.

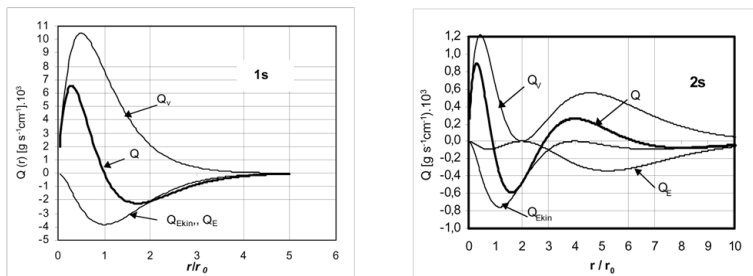


Figure 2: Partial Q_V, Q_{Ekin} and Q_E sources of matter created by the potential, kinetic and proper value energy densities and their algebraic sums Q , multiplied by $4\pi r^2$ in gs^{-1} for the states 1s and 2s.

References

- [1] Jammer, *The Philosophy of Quantum Mechanics*, J. Wiley and Sons, New York, (1974).
- [2] D. Bohm, *A Suggested Interpretation of the Quantum Theory in Terms of Hidden Variables*, I. Phys. Rev. 85,166 (1952).
- [3] E. Madelung, *Quantentheorie in Hydrodynamischer Form*, Zeitschr. für Physik 40,322,(1926).
- [4] E. Nelson, *Derivation of the Schrödinger Equation from Newtonian Mechanics*, Phys.Rev. 150, 1079 (1966).
- [5] C. Moller, *The Theory of Relativity*, Clarendon Press, Oxford 1972
- [6] J. P. Vigièr, *Einsteins Materialism and Modern Tests of Quantum Mechanics*, Annalen der Physik, 7. Folge, Band 45, 61 (1988).

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Moreover, author admits that the consideration of microscopic quantum system as a charged continuum has a considerable weakness in the necessity to switch of selfinteraction within the medium. Generalization to many body system would be completely obscure. Also the concept of the "continuous" energy exchange process lead-

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