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PHYSICS STANDING ON BASELESS PLATFORM

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Abstract

This paper reports on three important discoveries. These are :

1. Discovery of flaw in the long running concept of representation of dimensional formula of a physical quantity,
2. Discovery of flaw in the long running concept of representation of derived unit of a physical quantity, and
3. Discovery of flaw in the long running procedure of solution of numerical problems in Physics.

With a view to getting rid of the aforesaid flaws, alternative suggestions have also been offered.

1 Introduction

Theoretical study of Physical Science is based on a lot of conventions. For a systematic study one is to follow the normal conventions which have already earned International recognition. Now what about those conventions of representation which have no resemblance with the well known mathematical laws ? It is a high time to think of such conventions and to get rid of them with alternative flawless replacements, if possible or to get rid of them for ever. This paper considers the long running concepts of representation of dimensional formula and derived unit of a physical quantity [6, 3, 5, 1]. It reports on the discovery of flaw in the existing mode of representation of dimensional formula as well as that of derived unit of a physical quantity. It also discloses the discovery of flaw in the long running procedure of solution of numerical problems in Physics [2, 4, 9]. To get rid of the aforesaid flaws, alternative suggestions have also been incorporated for immediate replacements.

The paper consists of six sections. In section 1, important definitions are offered. Section 2 deals with the well known laws of indices. In section 3, dimensional formula of a physical quantity has been considered and arguments are made in support of the discovery of flaw in the long running concept of representation of dimensional formula. Section 4 deals with derived unit and that the long running concept of derived unit is flawed has been logically justified in this section. In section 5, the long running concept in regard to the procedure of solution of numerical problems in Physics has been offered by considering a few numerical problems. Evidence in support of discovery of flaw in the said procedure of solution of numerical problems in Physics has also been provided in this section. In the discussion section 6, suggestions have been made with a view to opening out a way of getting rid of the aforesaid flaws discovered.

2 Definitions

Unit : Any physical quantity is measured in terms of small part of it. This small part (known as the standard of measurement) is called the unit of measurement of the quantity.

Fundamental unit : The units of length, mass and time (in the study of General Physics and Mechanics) are called fundamental units. They are so called because the units of all other physical quantities that appear in the study of General Physics and Mechanics can be derived from them.

Derived unit : The units of all physical quantities (such as area, volume, force, work etc.) derived from the fundamental units are called the derived units.

Dimensions : The dimensions of a physical quantity indicate the powers to which the fundamental units of length, mass and time must be raised so as to represent it. The dimensions of velocity are 1 in length, 0 in mass and -1 in time.

Dimensional formula : The dimensional formula of a physical quantity represents an expression showing its relation to the fundamental units of measurement. Thus if $[L]$ stands for the unit of length, $[M]$ for the unit of mass and $[T]$ for the unit of time, then the dimensional formula of velocity will be $[LT^{-1}]$.

Variable : The following definition of variable is from [8].A quantity which may assume different values in the course of a discussion is called a variable.Another definition of variable taken from [6] is as follows :

A variable is a symbol that may have any value within a given range.

3 Laws of indices

If 'a' and 'b' are two non-zero real numbers then for all rational values of m and n, the following laws of indices [7] exist in elementary algebra.

(i) $a^m.a^n = a^{m+n}$,

(ii) $a^m \div a^n = a^{m-n}$,

(iii) $(a^m)^n = a^{mn}$,

(iv) $a^0 = 1$,

(v) $a^{-n} = \dfrac{1}{a^n}$,

(vi) $(ab)^m = a^m b^m$,

(vii) $(\dfrac{a}{b})^m = \dfrac{a^m}{b^m}$.

4 Revisiting the concept of dimensional formula

The idea of dimensions originated with Fourier. The nature of physical quantities (but not their magnitude) may be expressed by a very convenient method [3]. A length must be expressed as a number of units of length. But if the magnitude of the quantity be excluded, its nature may still be indicated by writing it in some special form. For example, the physical quantity length is represented as [L] where [L] stands for the unit of length. Similarly the physical quantities mass and time are represented as [M] and [T] respectively where [M] denotes the unit of mass and [T] denotes the unit of time. Based on this kind of representation of the nature of the three fundamental quantities length, mass and time, dimensional formulae of all other derived quantities in the study of General Physics and Mechanics can be obtained in terms of L, M and T.

For example the dimensional formula of velocity is $[LT^{-1}]$, that of force is $[MLT^{-2}]$ etc..

The overall procedure involved in finding dimensional formula of a physical quantity involves the applications of the well known LAWS OF INDICES in elementary algebra. The question that is of paramount importance at this stage is : How far is the use of the LAWS OF INDICES in obtaining dimensional formula of a physical quantity justified ? It may be noted that the laws of indices as mentioned in section 2 are applicable only under certain conditions, viz. when a and b are two non-zero real numbers and m and n have got rational values. The running concept of dimension clearly shows that none of L, M and T corresponds to any number but they are exclusively associated with the units of length, mass and time respectively and because of this fact the concept of application of the laws of indices while obtaining dimensional formula must be flawed.

Thus there exists a flaw in the long running mode of representation of dimensional formula of a physical quantity.

5 Revisiting the concept of derived unit

The concept involved in finding the derived unit of a physical quantity [6, 3, 5, 1] also makes use of the well-known laws of indices [7] of elementary algebra. For example, the SI unit of Velocity is $\frac{m}{s}$ or ms^{-1}, the SI unit of angular momentum is $kg.\frac{m.m}{s}$ or kgm^2s^{-1} and so on.

It may further be recalled that, as stated in section 2, the laws of indices are valid only when 'a' and 'b' are two non-zero real numbers and m and n have got rational values. Since none of the fundamental units in a particular system of unit corresponds to non-zero real number, the laws of indices will never be applicable in case of expressing derived unit of a physical quantity. Thus it seems that the long running concept of finding derived unit of a physical quantity is entirely flawed.

6 Revisiting the long running procedure of solving numerical problems in physics

While studying Physics, one very often has to go through statements like 'a body of mass m', 'a particle moving with velocity V' etc. .

A question that arises in the aforesaid context is whether such statements are ambiguous or not. If the statement 'a body of mass m' is considered, then what about m? Is it the numerical value of the mass only ? If not, does it represent the numerical value of the mass along with its unit ? Looked at this way it can be easily concluded that the statement 'a body of mass m' is very much misleading and hence ambiguous one. The aforesaid ambiguity will never arise if one takes care of the following quoted lines from [1]:

For purposes of calculation it is the measure of the magnitude that is of importance, and, to avoid a tedious prolixity of statement, such an expression as "a velocity V" will often be used in the sense "a velocity whose measure is V units of velocity."

It thus follows from above that in the statement "a velocity V", V is exclusively the numerical value of velocity in a particular system of unit only. Similarly in the statement "a body of mass m", m stands only for the numerical value of mass of the body in a particular system of unit and so on.

Furthermore, it is because of the aforesaid actual meaning of the long running ambiguous statements that in any mathematical equation involving physical quantities, the physical quantities involved must exclusively stand for variables and hence it is only the numerical value (not the unit) that must be associated with each of the physical quantities remaining present in the equation. For example, it is well known that the force P acting on a body of mass m moving with uniform acceleration f is given by $P = mf$. The equation $P = mf$ involves three physical quantities, viz. mass (m), acceleration (f) and force (P). Here m, f and P stand only for the numerical value of mass, acceleration and force in a particular system of unit respectively.

A further reason in support of this fact is as follows : The equation $P = mf$ results from Newton's 2^{nd} law of motion according to which $P \propto f$. Remembering that force (P) varies as acceleration (f) it may be said that P and f stand for two variables and each of them can take up numerical value only.

On account of the aforesaid ambiguity present in long running statements in Physics, the procedure of solution of numerical problems in Physics seems to be flawed. It has been demonstrated with the help of the following examples.

Example 1 : The following worked out solution of a problem is from [2].

Problem : Two protons in a nucleus of U^{238} are 6.0×10^{-15} metre apart. What is their mutual electric potential energy ?

Solution :

$$\begin{aligned} U &= \frac{1}{4\pi\epsilon_0} \; \frac{q_1 q_2}{r} \\ &= \frac{(9.0 \times 10^{10} N - m^2/Coul^2)(1.6 \times 10^{-19} Coul)^2}{6.0 \times 10^{-15} metre} \\ &= 3.8 \times 10^{-14} Joule \\ &= 2.4 \times 10^5 eV. \end{aligned}$$

It can be readily seen that the numerical values of the physical quantities along with their units have been substituted in computing the P.E. U in this problem making use of the formula

$$U = \frac{1}{4\pi\epsilon_0} \; \frac{q_1 q_2}{r}$$

and hence the concept is flawed.

Example 2 : The following worked out solution of a problem is from [4] .

Problem : The force on a particle of mass $10g$ is $(\vec{i}\,10 + \vec{j}\,5)$ N. If it starts from rest, what would be its position at time $t = 5s$?

Solution : We have $F_x = 10\ N$ giving

$$a_x = \frac{F_x}{m} = \frac{10\ N}{0.01\ Kg} = 1000\ m/s^2.$$

As this is a case of constant acceleration in x-direction,

$$x = u_x t + \frac{1}{2} a_x t^2 = \frac{1}{2} \times 1000\ m/s^2 \times (5\ s)^2$$
$$= 12500\ m.$$

Similarly, $a_y = \dfrac{F_y}{m} = \dfrac{5\ N}{0.01\ Kg} = 500\ m/s^2.$

And $y = 6250\ m.$

Thus, the position of the particle at $t = 5s$ is $\vec{r} = (\vec{i}\,12500 + \vec{j}\,6250)\ m.$

It may be noted that in the formula $a_x = \dfrac{F_x}{m}$ or $a_y = \dfrac{F_y}{m}$, each of a_x, a_y, F_x, F_y and m stands for the numerical value (not the unit) in a particular system of unit only. But making use of the unit along with numerical value in the formula above shows that the said concept is flawed. Further the unit ‘m’ is associated only with the modulus of $\vec{r}$, i.e. with $|\vec{r}|$. Thus the unit ‘m’ shown in the expression of $\vec{r}$ in the above solution must also be incorrect.

Furthermore, since none of the fundamental units corresponds to a non-zero real number, the laws of indices will not be applicable in case of the fundamental units. Thus the multiplication as well as division of one fundamental unit by itself must be forbidden in compliance with the fundamental criteria based on which the laws of indices are applicable. This fact has not yet been reflected in the procedure of solution of the following problem in physics.

Example 3 : The following worked out solution of a problem is taken from [9] .

Problem : Express 50 mi/hr in feet per second.

Solution :

$$50\ mi/hr = \frac{50\ \frac{\cancel{mi}}{\cancel{hr}} \times 5280\ \frac{ft}{\cancel{mi}}}{3600\ \frac{sec}{\cancel{hr}}}$$
$$= 73\frac{1}{3}\ \frac{ft}{sec}$$

In the procedure of solution of the aforesaid problem (Example 3), there is a clear indication of cancellation of similar kind of unit. But it may be noted that such a cancellation of unit is never permissible as far as the fundamental conditions of validity of the laws of indices are concerned. Hence there exists a flaw in the aforesaid long running concept of solution of numerical problems in physics [2, 4, 9].

7 Discussion

This paper questions the validity of the long running concepts of Dimensional formula and derived unit of a physical quantity [6, 3, 5, 1]. It also considers the procedure of solution of numerical problems in Physics available in existing texts [2, 4, 9]. The overall study leads to the discovery of flaw in the long running concepts of derived unit, dimensional formula as well as in the long running procedure of solution of numerical problems in Physics.

With a view to getting rid of the aforesaid flaws, the following suggestions are being offered for immediate implementation :

(i) The concept of dimensional formula and hence the method of dimensional analysis should be withdrawn for ever since the procedure involved in finding dimensional formula of a physical quantity is not at all in compliance with the fundamental conditions needed for the application of the laws of indices in elementary algebra.

(ii) The long running concept used for the representation of derived unit of a physical quantity should be withdrawn for ever. Instead of saying that velocity of a particle is 10 cm/s or 10 cms^{-1}, it would be wise to say a particle has 10 C. G. S. unit of velocity. Similarly, the statement momentum of a body is 100 $kgms^{-1}$, should be replaced by 'a body has 100 S.I. unit of momentum' etc..

(iii) To get rid of the flaw existing in the long running procedure of solution of numerical problems, the following points are to be kept in mind.

(a) Only the numerical values of physical quantities in a particular system of unit are to be substituted in the working formula involving a number of physical quantities.

(b) Cancellation of units as has been indicated in Example No. 3, should never be allowed.

Flawless solution procedure of the examples considered earlier will be as follows :

Example 1 : The formula to be used is

$$U = \frac{1}{4\pi\epsilon_0} \; \frac{q_1 q_2}{r}.$$

Considering all quantities in S.I. units we have,

$$\begin{aligned} U &= \frac{(9.0 \times 10^{10}) \times (1.6 \times 10^{-19})^2}{6.0 \times 10^{-15}} \\ &= 3.8 \times 10^{-14}. \end{aligned}$$

Therefore, the required potential energy is 3.8×10^{-14} S. I. units.

Example 2 : $a_x = \dfrac{F_x}{m}.$

Considering all quantities in S.I. units we have,

$$a_x = \frac{10}{0.01} = 1000.$$

Therefore, x-component of acceleration is 1000 S. I. units.
Again,

$$x = u_x t + \frac{1}{2} a_x t^2.$$

Considering all quantities in S. I. units we have,

$$x = \frac{1}{2} \times 1000 \times (5)^2 = 12500.$$

Therefore, distance moved along x-axis during 5 s is 12500 m.

Further, $a_y = \dfrac{F_y}{m}$.

Considering all quantities in S.I. units we have,

$$a_y = \frac{5}{0.01} = 500.$$

Therefore, y-component of acceleration is 500 S.I. units.
Also,

$$y = u_y t + \frac{1}{2} a_y t^2.$$

Using all quantities in S. I. units we have,

$$y = \frac{1}{2} \times 500 \times (5)^2 = 6250.$$

Therefore, distance moved along y-axis during 5 s is 6250 m.
Thus, the position of the particle at $t = 5s$ is

$$\vec{r} = (\vec{i}\, 12500 + \vec{j}\, 6250).$$

Example 3 : In 1hr, distance moved = 50 mi.
i.e. in 60×60 s, distance moved $= 50 \times 1760 \times 3$ ft.
Therefore,

$$1\ s \ldots\ldots\ldots\ldots = \frac{50 \times 1760 \times 3}{60 \times 60} ft. = \frac{220}{3} ft.$$
$$= 73\frac{1}{3} ft.$$

Therefore, in feet per second, the required result is $73\frac{1}{3}$.

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Comment

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Comment on PHYSICS STANDING ON BASELESS PLATFORM

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I think that some comments on the Bhattacharjee paper are necessary because the problem discussed by the Author is not so trivial as it might seem. Dealing with physical quantities we not only must properly take into account their physical dimensionality but also we must remember the physical meaning of these quantities. For example, the x, y and z components of the positions of material points all are measured in the units of length and therefore we formally can add them but what is the physical meaning of the sum of such different components?

Mathematics solved this problem for us by introducing many dimensional spaces with component wise addition. It is trivial to extend this idea to the set of all physical quantities which appear in the given physical problem. Different physical quantities, even those with the same dimension but different meaning, will appear as different components of a vector in the space of physical quantities (SPQ). Defining the addition as component by component operation we have a linear space in which addition never leads to physically meaningless expressions like $p+q$, where p is momentum and q some coordinate.

The SPQ is well-defined provided the system of units was chosen at the beginning. A change of the system of units is then represented by a map

$$(SPQ) \rightarrow (SPQ)',$$

where $(SPQ)'$ is the space of physical quantities expressed in the new system of units. The vectors of $(SPQ)'$ are obtained from the vectors of (SPQ) by a diagonal matrix whose diagonal elements are the conversion factors from the old units to the new ones.

Physical quantities changes also by changing the inertial reference frames in which physical phenomena are described. A passage from one inertial reference frame to another such frame requires some transformation rules for the physical quantities and we arrive to maps of the type

$$(SPQ)_{Frame1} \rightarrow (SPQ)_{Frame2}$$

implemented by non-diagonal matrices in the case when all physical quantities transform linearly.

All relations between physical quantities like

$$E = \frac{p^2}{2m},$$

or

$$p = mv$$

are realized as maps (in general, nonlinear) in the same (SPQ).

The above described structure of (SPQ) is especially essential in quantum physics where physical quantities are represented by self-adjoint operators. In the standard formulation of quantum physics there is no internal restriction which forbids addition of operators which correspond to physical quantities with different physical dimension or physical quantities with the same dimension but different physical meaning.