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SETUP OF TURBULENCE MECHANICS ACCOUNTING FOR A PREFFERED ORIENTATION OF EDDY ROTATION

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Abstract

Some principal aspects of a modified setup of turbulence mechanics accounting for a preferred orientation of eddy rotation by distinguishing the velocity fluctuations at each flow field point (in addition to their magnitude and direction) by the curvature of their streamlines are discussed. The approach is formalized by including the velocity fluctuation streamline curvature radius into the set of arguments of the probability distribution specifying the applied statistical averaging procedure. It is shown that the decomposed presentation of the suggested turbulence mechanics, resulting from the simultaneous application of a sequence of averaging operations of the same type, enriches the setup by reflecting several aspects of the multi-scale turbulence structure (like cascading processes) immediately in terms of average fields.

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1 Introduction

The conventional turbulence mechanics (CTM) is based on the Reynolds concept of turbulence [1] as a form of fluids motion with fluctuating flow velocity. Richardson [2] particularizes the concept emphasizing the space-time structure of the turbulence formed by the motion of a hierarchy of rotating eddies of different scales with the cascading process of eddy generation. Kolmogorov [3] utilized the Richardson's concept as an argument to found the local homogeneity and isotropy of the statistical structure of the small-scale turbulence. The argumentation contradistinguishes the statistical structure of the small-scale turbulence from the structure of the large-scale turbulence. When the former is statistically homogenous and isotropic, then the latter, fed immediately by the average flow attributing eddies' rotation with a preferred orientation, is not. The continuum mechanics [4]-[6] explains the interaction type coupling the average flow and the turbulence constituent responsible for a preferred orientation of eddy rotation as an action of the antisymmetric constituent of stresses. However, the baseline assumption of the CTM about the symmetric turbulent (Reynolds) stress tensor [7] excludes this type of interaction.

An approach removing the abovementioned contradiction was suggested in [8]-[10] (referred henceforth as [8]). The approach (the formalism of which is explained in Section 2) is set up as a modification of the CTM to account for a preferred orientation of eddy rotation. The modification stems from the distinguishing the velocity fluctuations, besides their magnitude and direction, also by the curvature of the velocity fluctuation streamlines. The approach is formalized by the inclusion of the fluctuating velocity constituent streamline curvature radius into the set of arguments of the probability distribution which specifies the applied statistical averaging procedure. Due to the made averaging specification the turbulence becomes split into two interacting constituents differing by their degree of contribution to the preferred orientation of eddy rotation. The description of the non-orientated turbulence constituent is left to the competence of the CTM ignoring the preferred orientation of eddy rotation while the description of the orientated turbulence constituent would be formulated within the equation of moment of momentum (angular momentum). Founding on the common continuum mechanics principles

[4]-[6] it is shown that the interaction of the average flow with the turbulence constituent contributing to the preferred orientation of eddy rotation is reflected in the properties of the probability distribution becoming non-invariant in respect to permutation of its arguments - the components of velocity fluctuation. The situation exemplifies the general statement of the systemic description of stochastic systems [11] stressing the interrelation of the properties of the probability distribution and of the conditions under which the probability distribution is formed.

The suggested modification of the turbulence mechanics is connected to many formerly formulated approaches and conceptions. So, the inclusion of the equation of moment of momentum (angular momentum) into the motion description setup relates the suggested turbulence mechanics to approaches [12]-[15] evaluating the turbulence as a promising field of application of the moment hydrodynamics [16]-[20] as well as to a similar Mattioli's work [21] in 30s attempting to generalize semi-empirical methods of turbulent flow calculation intensively discussed at that time. The interpretation of the turbulence constituents contributing and not contributing to the preferred orientation of eddy rotation as the large-scale and the small-scale turbulence, respectively, relates the suggested turbulence description to the Richardson-Kolmogorov conception about the cascading turbulence, accounted for in the two-scale approximation. The same interpretation relates the suggested turbulence description to the methods of large-eddy-simulation (LES) in turbulence modeling [22]-[24]. The essence of the discussed modification of the turbulence mechanics setup relates it also to the structure-based turbulence models [25]-[28]. These models are aimed to modify the turbulence description accounting for the non-local turbulence structure properties in one-point turbulence models within the conventional specification of the probability distribution properties. Finally, the suggested approach, as a modification of the CTM, includes the CTM as well as all particular turbulence descriptions, like $K-\varepsilon$ and similar turbulence models [29], [30], formulated within the CTM. The applications of the suggested modification of the CTM to calculations of turbulent flows in plain channels, round tubes and between rotating cylinders [8], as well to discussion of different oceanographic problems [31]- [34] prove the theory [8] as a useful tool in turbulence research and in practical turbulent flow calculations.

In Section 2 the turbulence is split into two constituents by applying one single averaging operation. In Section 3 the turbulence is treated from a more universal point of view [35] according to which instead of one single averaging operator a sequence of averaging operators of the same type is applied. Unlike the recent methods of multi-scale decomposition of the turbulent velocity field [36]-[38], developed as methods for the turbulence numerical simulation, the approach in [35] allows to reflect some aspects of cascading turbulence immediately in terms of average description. The approach, if applied to the setup of turbulence mechanics discussed in Section 2, does not only widen its theoretical capacity, but, as it is shown in Section 4, reveals also some substantial restrictions imposed on the formulation of closure assumptions.

2 The modified setup of the turbulence mechanics accounting for the preferred orientation of eddy rotation

The formalism of the theory elaborated in [8] stems from inclusion of the curvature radius $\mathbf{R}$ of $\mathbf{v}^{'}$ streamline ($\mathbf{v}^{'}$ is the fluctuation of the flow velocity $\mathbf{v}$, $\mathbf{R} = |\partial \mathbf{e}/\partial s|^{-2}\, \partial \mathbf{e}/\partial s$, $\mathbf{e} = \mathbf{v}^{'}/v^{'}$, s is the length of the $\mathbf{v}^{'}$ streamline curve) into the set of arguments of the probability distribution. The inclusion suggests the following changes in the turbulence mechanics setup.

Using identity $v^{'2} = \left(\mathbf{v}^{'} \times \mathbf{R}\right) \cdot \left(\mathbf{v}^{'} \times \mathbf{R}/R2\right)$, where, $R = |\mathbf{R}|$, we have for the turbulence energy $K = \frac{1}{2}\left\langle v^{'2} \right\rangle$ (angular brackets denote statistical averaging in the sense specified above) the presentation

$$K = K^{\Omega} + K0. \tag{1}$$

In (1) $K^{\Omega} = \frac{1}{2}\mathbf{M} \cdot \mathbf{\Omega}$, where

$$\mathbf{M} = \left\langle \mathbf{v}^{'} \times \mathbf{R} \right\rangle,$$

$$\mathbf{\Omega} = \left\langle \mathbf{v}^{'} \times \mathbf{R}/R2 \right\rangle,$$

and $K0 = \frac{1}{2}\left\langle \mathbf{M}^{'}\cdot\mathbf{\Omega}^{'}\right\rangle$, where $\mathbf{M}^{'} = \mathbf{v}^{'}\times\mathbf{R}-\mathbf{M}$ and $\mathbf{\Omega}^{'} = \mathbf{v}^{'}\times\mathbf{R}/R2-\mathbf{\Omega}$. Let's note that $\mathbf{M}$ has the physical sense of the density per unit mass of moment of momentum (angular momentum, spin) and $\mathbf{\Omega}$ has the sense of angular velocity of rotation defined as independent from the vorticity of the average velocity field. It is natural to connect the quantities $\mathbf{M}$ and $\mathbf{\Omega}$ by the relation $\mathbf{M} = J\mathbf{\Omega}$, where J is the effective moment of inertia while the quantity $\ell = \sqrt{J}$ determines the characteristic spatial scale of the turbulence constituent contributing to the preferred orientation of eddy rotation. (Here and henceforth it is assumed that J is a scalar.) It is natural to interpret the condition $\mathbf{M}\neq 0$ (or $\mathbf{\Omega}\neq 0$) as evidencing the presence of a preferred orientation of eddy rotation. Within the interpretation the energy K^{Ω} is interpreted as relating to eddies having a preferred rotation orientation while $K0$ is the energy of the turbulence constituent with eddies of no preferred rotation orientation. Let's note that when $\mathbf{R}$ is not included into the set of arguments of the probability distribution or when $\mathbf{v}^{'}$ and $\mathbf{R}$ determine as statistically independent then $\mathbf{M}$ and $\mathbf{\Omega}$ vanish.

The splitting of the turbulence energy into two energy constituents in (1) founds a three-level description of turbulent flows. The average flow is described by the Reynolds equation, the orientated turbulence constituent is described by the equation for the moment of momentum $\mathbf{M}$ and the unorientated turbulence constituent is described by the equation for energy $K0$. The CTM excludes the preferred orientation of eddy rotation therefore the CTM is specified within the suggested treatment as describing the turbulence constituent with energy $K0$. In particular, the assumptions of $K-\varepsilon$ and similar models, formulated within the CTM, can be applied to the description of this turbulence constituent if the total turbulence energy K is replaced with $K0$ in the current notation.

So as (a) the non-triviality of $\mathbf{M}$ and $\mathbf{\Omega}$ presumes the inclusion of $\mathbf{R}$ into the set of arguments of the probability distribution specifying the statistical averaging procedure applied; (b) the interaction between the average flow and the turbulence constituent with $\mathbf{M}$, $\mathbf{\Omega}$, and K^{Ω} cannot be excluded; and (c) this interaction type is explained within the continuum mechanics [4]-[6] as the act of antisymmetric constituent of stresses, then the inclusion of $\mathbf{R}$ into the arguments of the probability distribution must be accompanied with the non-invariance of the probability distribution in respect to permutation of its arguments $v_j^{'}$ and $v_i^{'}$. This non-invariance is immanent for ascribing the turbulent (Reynolds) stress tensor $\sigma_{ij} = -\rho \left\langle v_j^{'} v_i^{'} \right\rangle$ (ρ is medium density, $v_j^{'}$ and $v_i^{'}$ are components of $\mathbf{v}^{'}$ and the Latin indices obtain values 1, 2, 3) with the asymmetry enabling the interaction. The asymmetry of the turbulent stress tensor vanishes together with $\mathbf{M}$, $\mathbf{\Omega}$, and K^{Ω} if the applied probability distribution is specified in the way adopted in the CTM or if the preferred orientation of eddy rotation is absent. Let us emphasize that the symmetry of the turbulent stress tensor in the latter case (as a physical assertion) does not follow from the mathematically trivial commutativity of $v_j^{'}$ and $v_i^{'}$ in the product $v_j^{'} v_i^{'}$ but from the invariance of the applied probability distribution in respect to permutation of its arguments $v_j^{'}$ and $v_i^{'}$.

The treatment of the symmetry properties of the turbulent stress tensor within the discussed modification of the CTM outlines a natural, though often veiled, aspect of the description of any object behaving stochastically - the interrelation between the average and the statistical characterizations of the object's state. The interrelation means that the probability distribution characterizing the statistical properties of a stochastic object does not only determine the average situation but in turn obtains its specific properties under the conditions formulated in terms of average quantities. As applied to the turbulence problem, the average quantities determining the conditions of formation of the probability distribution properties should satisfy the general conservation laws of momentum, moment of momentum and energy i.e. they should be specified as determining the medium state fixed in terms of turbulence mechanics.

3 Decomposed presentation of the turbulence mechanics accounting for a preferred orientation of eddy rotation

The setup of any average turbulence description can be generalized by simultaneous application of a sequence of averaging operators $\mathcal{P}[n]$ ($n = 0, 1, ..., N, N+1$) instead of just one averaging operator in the common approach. The method [11], [31] assumes that the sequence of averaging operators is ordered by the rule

$$\mathcal{P}[n]\,\mathcal{P}[m] = \mathcal{P}[m]\,\mathcal{P}[n] = \mathcal{P}[n]\,, \tag{2}$$

where $m \leq n$, while $\mathcal{P}[0] \equiv 1$, $\mathcal{P}[N] \equiv \langle\rangle$ and $\mathcal{P}[N+1] \equiv 0$. The rule (2) states the validity of the Reynolds averaging rules for each $\mathcal{P}[n]$.

Application of the sequence of averaging operators to an arbitrary variate a results in the following expansion

$$a = \sum_{n=1}^{N+1} a[n]' \tag{3}$$

in which $a[n]' = a[n-1] - a[n]$, where $a[n] = \mathcal{P}[n]\,a$ and $a[N+1]' \equiv \langle a\rangle$, define the variability constituent of variate a corresponding to n. (The variability expressed by $a[n]'$ decreases if n increases.) Expansion (3) generalizes the common expansion of variates into the sum of average and fluctuating constituents. For arbitrary variates a and b we have $\left\langle a[n]'\,b[n]'\right\rangle = 0$ for $n \neq m$, i.e. only the variability constituents of a and b corresponding to the same n prove correlated and therefore

$$\langle ab\rangle = \sum_{n=1}^{N+1} \left\langle a[n]'\,b[n]'\right\rangle .$$

As applied to the turbulence energy $K = \frac{1}{2}\left\langle v'^2\right\rangle$ the method results in the following presentation of K,

$$K = \sum_{n=1}^{N} K^n, \tag{4}$$

where $K^n = \frac{1}{2}\left\langle v[n]'^2\right\rangle$. The balance equation for K split into the set of balance equations for K^n, represented as

$$\rho\frac{\partial}{\partial t}K^n = h^n_{j,j} + \sum_{m=n+1}^{N+1} Q^{m/n} - \sum_{m=0}^{n-1} Q^{n/m} + \rho q^n, \tag{5}$$

corresponds to decomposition (4). In (5)

$$h_j^n = -\rho \sum_{m=n}^{N+1} \left\langle v_j [m]^{'} K^{v[n]^{'}} [m]^{'} \right\rangle + \sum_{m=0}^{n-1} h_j^{n/m},$$

where $K^{v[n]^{'}} = \frac{1}{2} v [n]^{'2}$, $h_j^{n/m} = \left\langle \sigma_{ij}^{[m]} [n]^{'} v_i [n]^{'} \right\rangle$ in which $\sigma_{ij}^{[m]} = -\rho \left(v_j [m]^{'} v_i [m]^{'} \right) [m]$ if $m \succeq 1$ and $\sigma_{ij}^{[0]}$ is the molecular stress tensor) denote flux vectors for energies K^n. The quantities $Q^{m/n} = \left\langle \sigma_{ij}^{[n]} [m]^{'} v_{i,j} [m]^{'} \right\rangle$ and $Q^{n/m} = \left\langle \sigma_{ij}^{[n]} [m]^{'} v_{i,j} [m]^{'} \right\rangle$ describe the interaction between K^n and K^m for $1 \leq m, n \leq N$, the interaction of energies K^n with the energy of the average flow for $m = N + 1$ and the molecular dissipation of energies K^n for $m = 0$. Quantities $q^n = \left\langle f_i [n]^{'} v_i [n]^{'} \right\rangle$, where f_i, denote the components of the body force acting on the medium, describe the effect of external fields on K^n. The index following comma in the subscript denotes differentiation by the respective space coordinate and the Einstein summation is assumed. Let us note the similar properties of set (5) and of expansion (4). Due to this similarity, in order to merge several adjacent energy levels in the energy representation (4) into a single level it would be necessary just to sum up the corresponding balance equations in (5). The resulting single equation would have exactly the same structure as the initial equations under summation. Owing to this property all decompositions founded on different sets of averaging operators deduced from series $\mathcal{P}[n]$ can be readily switched to each other. The property expresses the invariance of the energy-decomposed presentation in respect to the specification of the set of averaging operators. The invariance expresses the fact that the decomposition can only redistribute the turbulence energy between different energy constituents but not change its total value.

Let henceforth all averaging operators $\mathcal{P}[n]$ be specified as suggested in Section 2. In this case, similar to the turbulence energy in (4), we shall have for the moment of momentum $\mathbf{M}$

$$\mathbf{M} = \sum_{n=n^*}^{N} \mathbf{M}^n,$$

where $\mathbf{M}^n = \langle \mathbf{M}^{[n]} \rangle$ in which $\mathbf{M}^{[n]} = (\mathbf{v}[n]' \times \mathbf{R}[n]')[n]$. The kinematic characteristic of motion $\mathbf{\Omega}^{[n]} = (\mathbf{v}[n]' \times \mathbf{R}[n]' / R[n]'^2)[n]$ corresponds to each moment of momentum $\mathbf{M}^{[n]}$, while $\mathbf{M}^{[n]}$ together with $\mathbf{\Omega}^{[n]}$ defines the effective moment of inertia J^n ($\mathbf{M}^{[n]} = J^n \mathbf{\Omega}^{[n]}$). The defined J^n provides the corresponding to n motion field variability constituent with spatial scale $\ell^n = \sqrt{J^n}$. It is natural to expect that $\ell^N > \ell^{N-1} > ... > \ell^1$.

Due to the non-vanishing $\mathbf{M}^{[n]}$ each K^n in (4) decomposes into a sum of energies K^{np},

$$K^n = \sum_{p=n}^{N+1} K^{np}, \tag{6}$$

in which $K^{np} = \frac{1}{2} \left\langle \mathbf{M}^{v[n]'}[p]' \cdot \mathbf{\Omega}^{v[n]'}[p]' \right\rangle$ for $p > n$ and $K^{nn} = K - \sum_{p=n+1}^{N+1} K^{np}$. Unlike the energy presentation in (4), interpreted as an expansion of turbulence energy into a sum of energies reflecting the contribution to K of different motion variability levels characterized by spatial scales ℓ^n, expansion (6) specifies K^n as reflecting the summary effect of motions of different order. The motion order reflected by K^{np} decreases if p decreases.

The energy representations (4) and (6) hint to the presence of two types of energy cascades in turbulent media - the cascading processes by the motion variability and the cascading processes by the motion order. The difference between these types of cascading processes does not exclude their co-action. So, using identity $\mathbf{v}[m]' = \mathbf{R}[m]' \times \mathbf{\Omega}^{v[m]'}$, where $\mathbf{\Omega}^{v[n]'} = \mathbf{v}[n]' \times \mathbf{R}[n]' / R[n]'^2$, the quantities $Q^{m/n}$ and $Q^{n/m}$ in (5) become decomposed as

$$Q^{m/n} = \sum_{p=n}^{N+1} Q^{mp/np} \text{ and } Q^{n/m} = \sum_{p=n}^{N+1} Q^{np/mp}, \tag{7}$$

in which for $p > n$

$$Q^{mp/np} = e_{kij} \left\langle \widetilde{\sigma}_{ij}^{m/n}[p]' \, \Omega_k^{[m]}[p]' \right\rangle + \left\langle \widetilde{m}_{ks}^{m/n}[p]' \, \Omega_{k,s}^{[m]}[p]' \right\rangle,$$

where e_{kij} are components of the Levi-Civita tensor, $\widetilde{\sigma}_{ij}^{m/n} = \sigma_{is}^{[n]}[m]' R_{j,s}[m]'$ and $\widetilde{m}_{ks}^{m/n} = e_{kij} \sigma_{is}^{[n]}[m]' R_j[m]'$, while

$Q^{mp/nn} = Q^{m/n} - \sum_{p=n+1}^{N+1} Q^{mp/np}$ (and analogously for $Q^{np/mp}$). Expressions (7) declare that the interaction between K^m and K^n can occur only between their energetic sublevels of the same order (i.e. corresponding to the same p).

4 Formulation of closure assumptions for the decomposed description of turbulence

The turbulence energy K and moment of momentum $\mathbf{M}$ are not the only quantities subjected to the determined in the Section 3 decomposition procedure. For example, turbulent stress tensor σ_{ij} and dissipation function $\varepsilon = \mu^{\mathrm{mol}} \left\langle v'_{i,j} v'_{i,j} \right\rangle$ (μ^{mol} is coefficient of molecular viscosity) would become also decomposed as

$$\sigma_{ij} = \sum_{n=1}^{N} \sigma_{ij}^{n} \text{ and } \varepsilon = \sum_{n=1}^{N} \varepsilon^{n}, \tag{8}$$

where

$$\sigma_{ij}^{n} = -\frac{\rho}{2} \left\langle v_j \left[n\right]' v_i \left[n\right]' \right\rangle$$

and

$$\varepsilon^{n} = Q^{n/0} = \mu^{\mathrm{mol}} \left\langle v_{i,j} \left[n\right]' v_{i,j} \left[n\right]' \right\rangle .$$

It is evident that the closure assumptions formulated for σ_{ij}, σ_{ij}^n and for ε and ε^n must not violate conditions (8). It appears possible if closure assumptions for σ_{ij} and σ_{ij}^n as well as for ε and ε^n are formulated in a similar way. So, stating for the shear components of the symmetric constituent of the stress tensor τ_{ij} that $\tau_{ij} = \mu u_{(i,j)}$, where μ is the turbulent shear viscosity coefficient and $u_{(i,j)} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$, and, stating simultaneously for τ_{ij} that $\tau_{ij}^n = \mu^n u_{(i,j)}$, where μ^n is the turbulent shear viscosity coefficient corresponding to τ_{ij}^n, according to (8) we have

$$\sum_{n=1}^{N} \mu^{n} = \mu \ . \tag{9}$$

Sum (9) declares the invariance of the turbulent shear viscosity with respect to a specific realization of the decomposition procedure.

Consider now the dual vector to antisymmetric constituent of the stress tensor, $\boldsymbol{\sigma}$. (The components of $\boldsymbol{\sigma}$ are determined through the components of the turbulent stress tensor σ_{ij} as $\sigma_k = e_{kij}\sigma_{ij}$.) According to [8] $\boldsymbol{\sigma}$ is connected with the angular velocity of relative rotation $\boldsymbol{\Omega} - \boldsymbol{\omega}$, where $\boldsymbol{\omega}$ is the vorticity of the average velocity field, as

$$\boldsymbol{\sigma} = 4\gamma\left(\boldsymbol{\Omega} - \boldsymbol{\omega}\right), \tag{10}$$

where γ is the coefficient of rotational viscosity, characterizing the shear in relative rotation. The decomposition of the turbulent stress tensor in (8), if specified to its antisymmetric constituent, can be expressed in terms of the dual vector to the antisymmetric constituent of the stress tensor $\boldsymbol{\sigma}$ as

$$\boldsymbol{\sigma} = \sum_{n=1}^{N} \boldsymbol{\sigma}^n, \tag{11}$$

where $\boldsymbol{\sigma}^n = 4\gamma^n\left(\boldsymbol{\Omega}^n - \boldsymbol{\omega}\right)$ in which γ^n denote coefficients of rotational viscosity corresponding to $\boldsymbol{\sigma}^n$. Representing sum (11) in form

$$\sum_{n=1}^{N}\left(\frac{\gamma}{J} - \frac{\gamma^n}{J^n}\right)\left(\mathbf{M}^n - J^n\boldsymbol{\omega}\right) = 0,$$

where J expresses through J^n as $J = \sum_{n=1}^{N} J^n$, we conclude that, due to $\mathbf{M}^n - J^n\boldsymbol{\omega} \neq 0$,

$$\frac{\gamma}{J} = \frac{\gamma^n}{J^n} = \widetilde{\gamma} = const(n). \tag{12}$$

Condition (12) declares the independence of the reduced coefficient $\widetilde{\gamma}$ from n and therefore determines $\widetilde{\gamma}$ as not depending on the specifics of the applied decomposition. It can be shown in a similar way that all other coefficients of the decomposed presentation of theory [8] can also be reduced to the coefficients, which do not depend on the specifics of the applied decomposition. The only quantities that would require a specification within the decomposed description are the shear viscosity coefficients μ^n and the moments of inertia or corresponding to the latter spatial scales $\ell^n = \sqrt{J^n}$.

5 Conclusion

The discussed turbulence mechanics modifies the CTM to account for the preferred orientation of eddy rotation. The modification is founded on the velocity fluctuations distinguishing by the curvature of the velocity fluctuation streamline and on the application of a sequance of averaging operators of same type. The modification enriches the average turbulence description on the classical mechanical background and embraces features from several formerly formulated turbulence descriptions and/or conceptions. It also enlarges the capacity of the turbulence mechanics in the discussion of various turbulence problems as well as in practical turbulent flow calculations.

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Modification of conventional turbulence mechanics is considered in the present paper. The author takes into account the preferred orientation of eddy rotation in the infinitesimal surrounding of each flow field point, i.e., he replaces one point averaging of the flow field at each flow field point by the velocity field ensemble averaging in the infinitesimal surrounding of each flow field point. Four problems that can be solved making use of the discussed modified theory are listed in the end of Section 5. The paper ends by list of references which includes 38 items over long time period (1894 - 2006). It is clear that the presented work is logical continuation of J. Heinloo's research in the field of turbulence (Refs. [8-11, 31-35].

The paper is interesting and intriguing, it deals with old and new ideas on Nature and therefore it corresponds exactly in the scope of the journal "Concepts of Physics". I recommend to publish the paper in your journal.

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Comment on SETUP OF TURBULENCE MECHANICS ACCOUNTING FOR A PREFFERED ORIENTATION OF EDDY ROTATION

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The paper addresses one of the potential extensions of the mathematical description of the turbulence phenomena. The approach is applicable in cases when the motion is substantially anisotropic in the sense that it has a preferred direction of (local or global) rotation. The author has introduced term "rotationally anisotropic turbulence" for such motions (e.g., [31]).

The central idea consists in introducing a specific equation describing temporal evolution of a sort of local angular momentum. Doing so presumes that the turbulent field can be split into two nontrivial parts: (statistically) isotropic motions and motions having a preferred direction of rotation. This partition resembles the analogous splitting of velocity into the mean and the fluctuating component.

The author has demonstrated earlier that such separation, if carried out systematically and combined with reasonable closure hypotheses, leads to a more general set of consistent equations for the parameters of such turbulent fields [8,9]. The idea itself has been discussed during many decades. It has been formulated in terms of categories used in this paper as early as in the end of the 1970s (see references in [9]). The resulting equations are reduced to the classical ones if there is no preferred orientation of eddies in the field. Usage of the approach of rotationally anisotropic turbulence has proved to be useful in several geophysical applications where a preferred direction of rotation naturally occurs in (quasi-) two-dimensional cases owing to the Earth's rotation (e.g.,[34].

The body of the paper addresses the problem of constructing of equations for description of motions with different (temporal or spatial) scales within the given turbulent flow as well as building reasonable closure assumptions for these equations. The analysis is correct and consistent.

The conclusions drawn in Section V, however, are weakly connected with the main body and are to some extent confusing. While the first conclusion is evidently true, it is still necessary to mention that there exist many descriptions of turbulent flows that account for the interaction of flow with external fields. Nothing in the body of the paper is said about negative viscosity, thus the reader has to guess in which of the references this problem is treated. It is also true that the proposed modification of the turbulence theory leads to the necessity to account for asymmetry of turbulent stress tensors, a property that usually is disallowed in the classical approaches to fluid dynamics. The third and fourth conclusions may be correct but their meaning remains vague.

The paper thus presents a nice exercise which is useful in many respects. Instead of making use of the obtained results, however, it ends with some quite general and not particularly well justified claims. Although the approach as such has a clear prospective in the theory of turbulence in specific types of media, its potential has been severely underexploited.

Since the theory contains some features that may be interpreted as contradicting with the assumptions of classical approach, it is not unexpected that it has not been become popular within three decades of its existence. It is not sufficient to demonstrate that a few selected examples can be treated in a simple or more consistent manner within the new theory. (Such exercises are of course of good use provided the relevant conclusions are rational). Instead, one has to show that the theory clearly improves the classical one in general, or is able to describe effects that have been overlooked or wrongly described by the existing approaches. We are looking forward for such a breakthrough.