

ON THE VACUUM STRESS-ENERGY TENSOR IN GENERAL RELATIVITY

R. Van Nieuwenhove
Institutt for Energiteknikk
P.O. Box 173, NO-1751 Halden Norway
e-mail: rudivn@hrp.no

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Abstract

It is shown that it is possible to interpret the Einstein field equation as a relation between the stress-energy tensor of matter and the stress- energy tensor of the vacuum.

1 Introduction

In General Relativity (GR), gravity is described in terms of curvature of the spacetime continuum, unlike other forces. While GR provides a very successful and elegant description of gravity, its geometrical nature also presents an obstacle to arrive at a quantum mechanical description of gravity. In [1], it was argued that gravitation is not a fundamental force and that it results from the modification of the vacuum energy density by the presence of a mass. A test particle moving in the resulting vacuum energy density gradient experiences a net effective force which can be identified with the gravitational force [1]. This point of view is explored here in a more rigorous way, resulting in a new non-geometrical interpretation of the Einstein field equation, thereby opening the way to a quantum description of gravity.

2 The vacuum stress-energy tensor

The vacuum stress-energy tensor T_{vac} can not be directly observed in contrast to, for instance, the position of a particle. Since we can not measure T_{vac} directly, we could try to observe how the vacuum (described by T_{vac}) influences the motion of matter. From a quantum field theoretical point of view, matter is in a constant interaction with the surrounding vacuum. When the vacuum would be perfectly isotropic, the effect of these interactions on the motion of a test particle would cancel out. If there exists gradients in the vacuum pressure (or energy density) a net force would be exerted on a test particle and by measuring the deviations on its path, one is able to measure in an indirect way T_{vac} . So, in the absence of other forces, the motion of a test particle would be described by an equation which involves only gradients in T_{vac} . If the vacuum would be perfectly homogeneous and isotropic, no conclusions about T_{vac} could be made at all since there would be nothing to measure. What could cause a spatial variation of T_{vac} ? The most obvious reason is the presence of a mass. From a quantum mechanical point of view, interactions exist between a particle and its surrounding vacuum. This modified vacuum will, through vacuum-vacuum interactions, also modify the vacuum at larger distances, up to infinity.

This can be visualised as follows: Due to the Heisenberg uncertainty relation, virtual particles can pop briefly into existence. If these virtual particles are charged, they have to come into pairs, such as for example electron-positron pairs. During their brief existence, these electrons and positrons can interact through the exchange of virtual photons with other charged virtual particles at another location. In this way, the vacuum interacts with itself. Instead of an electron-positron pair, we could also consider, for instance, a proton-antiproton pair. This proton-antiproton pair could interact with another virtual neutron-antineutron pair through the exchange of virtual pions. These interactions have a very short range (order 10^{-15} m) because the pion is so massive. Nevertheless, many such short-range vacuum-vacuum interactions can in the end lead to a long-range effective force just like an elastic deformation in a solid can be transmitted over large distances by virtue of small scale interatomic forces. So, there is no need for a massless virtual particle such as the graviton, to explain the long range of the gravitational force. In [1], it was claimed that it is just this effect which is responsible for gravitation. One could also say that it is not possible to distinguish the effect of gravitation and the effect of a non-uniform vacuum. Using these new concepts, it was possible to show that the missing mass problem can be solved without having to invoke strange forms of dark matter by assuming that the galaxies are located inside "bubbles" of slightly modified vacuum energy density [2].

In 1911, A. Einstein published a paper [3] in which he discussed the influence of gravitation on the propagation of light. In this paper, he showed that the speed of light, as observed by a distant observer, is slowed down by gravitation and by using Huygens principle, he was then able to deduce the deflection of light around the sun. Since the speed of light can be obtained by combining the electrical permittivity ϵ_0 and the magnetic permeability μ_0 of the vacuum according to $c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$, ϵ_0 and/or μ_0 must be considered changed by gravity to a distant observer. This is also the point of view put forward by Puthoff [4]. When Einstein later on developed his General Relativity (GR) theory, the deflection of (star)light around the sun was calculated in a completely different way and gravity was described within the context of a "curved spacetime geometry". Nevertheless, the initial thoughts by Einstein show that a different path could have been taken. While

the predictions of GR have been confirmed to great precision, the geometrical interpretation or formalism used has also been a stumble block for arriving at a quantum theory of gravitation.

The view that gravity results from changes in the vacuum quantum fluctuations was first expressed by Andrei Sakharov in 1967 [6]. Sakharov identified the action term of Einstein's geometrodynamics with the change in the action of quantum fluctuations when space is curved.

3 Reinterpretation of the Einstein field equation

The Einstein field equations [5]

$$R_{\alpha\beta} - \frac{1}{2} \cdot g_{\alpha\beta} R + \Lambda g_{\alpha\beta} = 8\pi G \cdot T_{\alpha\beta} \quad (1)$$

are invariant under a multiplication of the metric tensor by a constant scalar. This can easily be seen by writing the same spacetime interval using metric tensor g and another metric tensor g' :

$$ds^2 = g_{ij} dx^i dx^j = g'_{ij} dx'^i dx'^j \quad (2)$$

If $g'_{ij} = \alpha \cdot g_{ij}$ one must have that $dx'^i = \frac{1}{\sqrt{\alpha}} \cdot dx^i$ and $dx'^j = \frac{1}{\sqrt{\alpha}} \cdot dx^j$. So, multiplying the metric tensor by a constant is equivalent to a coordinate transformation and the Einstein field equations must be invariant under such a transformation. Under this transformation, each term in the Einstein field equations becomes multiplied by α . Next we consider α to be constant with the dimension of an energy density and we postulate that the stress-energy tensor of the vacuum is given by:

$$T_{\alpha\beta}^{vac} = \alpha \cdot g_{\alpha\beta} \quad (3)$$

We can thus replace all the metric tensors in the Einstein tensor and in the cosmological constant term by $T_{\alpha\beta}^{vac}$ so that we get a relation between the stress-energy tensor of matter and the stress-energy tensor of the vacuum. So this equation now shows how the vacuum properties are modified by the presence of a mass. As was mentioned in Section 1, only gradients in the vacuum pressure (or in a more general sense in the stress-energy tensor) can result in forces on a mass so that the cosmological constant term (now written as $\Lambda \cdot T_{\mu\nu}^{vac}$) has no

justification to be part of (1). Leaving out the cosmological constant term does of course not mean that we didn't take the vacuum into account since every $g_{\alpha\beta}$ has been replaced by $T_{\mu\nu}^{vac}$ in the Einstein tensor. Instead of a purely geometrical description of gravitation, we thus arrive at a more physical description, in which the movement of matter is completely dictated by the vacuum. In this non-geometrical description of gravitation, particles move along paths of minimal energy instead of along paths with extremal proper time. Consider now the g_{00} component of the Schwarzschild metric; $g_{00} = -(1 - \frac{2GM}{c^2 r})$. This term is very similar to the radial dependence of the vacuum energy density as proposed in [1], namely $\rho = \rho_0(1 - \frac{GM}{c^2 r})$, in which ρ_0 has the dimension of an energy density, and from which Newton's law could be recovered. Finally, if T_{vac} , given by eq. (3) is really the stress-energy tensor of the vacuum, it should also be a conserved quantity, meaning that its divergence should be zero. Since all the components of the covariant derivative of $g_{\mu\nu}$ are zero, this condition is indeed fulfilled.

4 Discussion

Since the proportionality factor α , in eq. (3), drops out of the equations, the amplitude of α does not matter. Therefore, the apparent enormous disparity (of order 120) of the value of the vacuum energy density between quantum mechanical estimations and observations (as based on the field equations with the cosmological term) is automatically resolved since its value drops out of the equations. In other words, the energy of a spacially uniform vacuum is not a source for gravitation. A separate term, like $\Lambda g_{\alpha\beta}$ should never be included into eq. (1) as mentioned before (based on quantum field considerations). Cosmological models including dark energy can not fit into the proposed reinterpretation of the field equations. Possible discrepancies between observations and an increasingly expanding universe and eq. (1) without the cosmological term, could be attributed either to i) a wrong interpretation of the measurements or ii) a too strong restriction of the properties on the overall assumed vacuum solution. Especially the assumption that space (or the vacuum) is uniform over very large scale lengths (larger than galaxies, clusters of galaxies, superclusters, ..), as is assumed in all cosmological models, could well

turn out to be erroneous. If this assumption is indeed erroneous, then also all the conclusions on dark matter and dark energy are wrong. Even when the absolute value of the vacuum energy density drops out of the equations (see above), one can also seriously question its so-called calculated enormous value. One usually forgets to mention that this estimation is based on simply summing up the energies of a large number of harmonic oscillators (a relativistic field can be viewed in this way) *without* taking into account all the vacuum-vacuum interactions mentioned previously. These interactions might well reduce the vacuum energy density to a very small or zero value.

5 Conclusion

It has been shown that it is possible to turn the Einstein field equation into an equation which relates the stress-energy tensor of matter to changes in the stress-energy tensor of the vacuum. Within this framework, strong arguments have been put forward to drop the cosmological term from the field equations. The stress-energy tensor of the vacuum is proportional to the metric tensor, though the proportionality factor never enters the solutions. This reinterpretation has a profound impact on cosmological models and opens the way to a quantum theory of gravitation as both sides of the Einstein field equations contain only non-geometrical terms.

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Comment on ON THE VACUUM STRESS-ENERGY TENSOR IN GENERAL RELATIVITY

Edward Kapuścik

Department of Physics, University of Łódź

Łódź, Poland

In my opinion, the paper by Rudi van Nieuwenhove is purely a speculation. The speculation however concerns an important physical problem which up to now is not solved. In view of that we should rather speak about a hypothesis which may be either proven to be true or to be false. Before this nobody has the right to reject the hypothesis.

R. van Nieuwenhove proposed a specific form for the energy-momentum tensor of the vacuum. That vacuum should be the universal reservoir of energy is rather plausible because otherwise we shall not know what is the primary source of energy.

On the other side his hypothesis is quite arbitrary and closely connected with the already existing Einstein theory of gravity. What will happen to the hypothesis if sometime in the future the Einstein theory will fail? Does it mean that the van Nieuwenhove hypothesis has no independent physical meaning?

That Einstein theory may not be so universal, as it is thought, may follow from the following consideration.

In [1] we have shown that all equations of physics follow from the

simple scheme

$$K_{\beta\mu}^{\alpha\nu}\nabla_\nu\Psi_\alpha(x) = \Phi_{\mu,\beta}(x), \quad (1)$$

$$\nabla_\mu\Pi_\gamma^\mu(x) = \Omega_\rho(x), \quad (2)$$

where four collection of fields are used. Here $x = (x^0, x^1, x^2, x^3)$ are the usual spacetime coordinates, ∇_ν denotes some kind of differentiation with respect to the ν -th coordinate, $K_{\beta\mu}^{\alpha\nu}$ are numerical factors (different for each type of a theory) and the summation over repeated indices is understood.

The first collection of basic fields, denoted by $\Psi_\alpha(x)$ (α stands for all indices used in the given theory), describes all spacetime properties of the considered physical system including, first of all, the symmetries of the system. The second collection of fields, denoted by $\Phi_{\mu,\beta}(x)$, determines the evolution of basic fields $\Psi_\alpha(x)$ with respect to the coordinates indexed by μ and the set of indices β may be different from the set of indices α if some constraints are present in the considered system. The third collection of fields, denoted by $\Omega_\rho(x)$, describes the influence of the external environment on the studied system. Again, the set of indices ρ may be different from both the sets α and β . The fourth collection of fields, denoted by $\Pi_\gamma^\mu(x)$, clearly defines the balance equations present in the system.

The above sketched scheme must be completed by some sort of constitutive relations which, according to specific properties of the studied system, additionally relate the fields in (1) and (2).

Applying such scheme to gravity we have to identify the basic fields $\Psi_\alpha(x)$ with the components of some fourth order tensor $R_{\mu\nu\lambda}^\alpha(x)$ which eventually may or may not be interpreted as the curvature tensor of spacetime. Choosing the kinematical factors in (1) as

$$K_{\alpha,\omega,\rho,\zeta,\mu}^{\beta,\eta,\lambda,\epsilon,\nu} = \delta_\alpha^\beta\delta_\omega^\eta(\delta_\rho^\lambda\delta_\zeta^\epsilon\delta_\mu^\nu + \delta_\zeta^\lambda\delta_\mu^\epsilon\delta_\rho^\nu + \delta_\mu^\lambda\delta_\rho^\epsilon\delta_\zeta^\nu), \quad (3)$$

(here all indices are spacetime indices) from (1) we get the equation

$$R_{\mu\nu\lambda;\rho}^\alpha(x) + R_{\mu\lambda\rho;\nu}^\alpha(x) + R_{\mu\rho\lambda;\nu}^\alpha(x) = J_{\mu\nu\lambda\rho}^\alpha(x), \quad (4)$$

where $J_{\mu\nu\lambda\rho}^\alpha(x)$ are components of some fifth order tensor. Giving the tensor $R_{\mu\nu\lambda}^\alpha(x)$ all the symmetries of the curvature tensor and

taking $J_{\mu\nu\lambda\rho}^\alpha(x) = 0$, eqs. (3) are the famous Bianchi identities. In such a case we may indeed interpret $R_{\mu\nu\lambda}^\alpha(x)$ as the curvature tensor of our spacetime.

The only equation of the Einstein theory which has the form of eq.(2) is the energy-momentum conservation law

$$T_{\mu;\nu}^\nu(x) = f_\mu(x), \quad (5)$$

where $f_\mu(x)$ is the density of the external force which acts on the system. Clearly, in the case of the whole Universe $f_\mu(x) = 0$ and the energy-momentum tensor $T_{\mu\nu}(x)$ may be obtained as the solution of the equation

$$T_{\mu;\nu}^\nu(x) = 0. \quad (6)$$

One of the solution of this equation is the van Nieuwenhove case

$$T_{\mu\nu}(x) = \alpha g_{\mu\nu}(x). \quad (7)$$

Such solution is always the part of any other solution of the inhomogeneous equation (5). In this sense it is a part of any energy-momentum tensor of any physical system. This universality property may justify the van Nieuwenhove hypothesis.

In the above approach the standard Einstein equation

$$R_{\mu\nu}(x) - \frac{1}{2}g_{\mu\nu}(x)R(x) = \frac{8\pi G}{c^4}T_{\mu\nu}(x) \quad (8)$$

turns out to be a constitutive relation which relates the Ricci tensor $R_{\mu\nu}(x)$ obtained from the fundamental tensor $R_{\mu\nu\lambda}^\alpha(x)$ to the energy-momentum tensor $T_{\mu\nu}(x)$. We regret it very much for such a degradation of the Einstein equation from the status of a fundamental equation to the status of a constitutive relation. In the framework of any theory the constitutive relations may be quite different (as an example, see electrodynamics) and therefore also in the theory of gravity there may be quite different from the Einstein one relations between $R_{\mu\nu\lambda}^\alpha(x)$ and $T_{\mu\nu}(x)$.

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