The old and new Concepts of Physics
An open-dialogue journal

www.uni.lodz.pl/concepts

COMPLEMENTARY SPECIAL RELATIVITY THEORY AND SOME OF ITS APPLICATIONS

Pavol Valent
Okruzna 12, 93401 Levice, Slovakia
e-mail: Pavol.Valent@stonline.sk

(Received 3 October 2006; accepted 2 December 2006)

Abstract

This article introduces a mathematical model for the Complementary Special Relativity Theory (CSRT) in relation to Einstein's Special Relativity Theory (SRT). The CSRT is technically derived following criterion of the logical independence of the Einstein's postulates. The epithet "Complementary" was selected in order to distinguish the CSRT theory from alternative theories.

The mission of the CSRT is not to replace the Einstein's Special Relativity. Both of them must coexist and must have physical applications. Forasmuch as the CSRT postulates do not embody constant velocity of the light by the Einstein's way, the CSRT need not have 4-dimensional symmetry of the Lorentz and Poincare group.

Existence of the CSRT transverse Doppler effect formula and the time dilation formula for moving clocks, which are different from Einstein's, are crucial moments of the new theory. Moreover, the article demonstrates some physical applications of the CSRT on the John D. Anderson's discovery of a quasi

DOI: 10.2478/v10005-007-0027-6

acceleration of the Pioneer 10 spacecraft. It will be shown that the Pioneer 10 does not accelerate, but it moves with predicted velocity, and that the DSN (Deep Space Network) data remarkably well fit the predictions of the CSRT theory in range, Doppler, annual, and diurnal periodicity of the phenomenon.

1 Genesis of the Complementary Special Relativity Theory

The Einstein's SRT is an axiomatic theory built on two logically independent postulates. The postulates A1, A2 of an axiomatic system S[A1, A2] = (Theory T1) are by the criterion considered as independent if there exist models, interpretation or physical application for next two, complementary axiomatic systems: S[Non A1, A2] = (Theory T2) and S[A1, Non A2] = (Theory T3), respectively.
Let us study SRT from the aspects of the criterion. Einstein's postulates will be marked by symbols E1, E2.
E1: Light signals propagate in vacuum with the constant velocity c independent of the motion of its source.
E2: All inertial systems of reference are equivalent for formulation of physical rules.

All the experiments on the speed of light, due to two-way method used, just prove the fact that the light in a vacuum has a constant velocity c in every inertial reference frame F(x,y,z,t) which is attached to a detector, receiver or material object, able to register the speed of the light. (Michelson's result = MR). This is a characteristic property of vacuum.
The first Einstein's postulate can be formulated in the form of logical conjunction.
E1: In vacuum in an inertial reference frame F, which is rigidly attached to a receiver, the speed of light is always constant c (proposition a) and hereby the speed of light in an inertial reference frame F', which is attached to a moving emitter, is always the same constant c (proposition b). Both of F and F' are called Einstein frames.

E1 = $a \wedge b$. The second part of this conjunction is equivalent to the statement that the light spreads from a point source in the form of unique spherical wave front by constant speed c. It is the same proposition as that the one-way speed of the light is regardless of the relative motion of a receiver and an emitter always c. We stress that the one-way speed of the light can not be measured because of absence of the proper time interval. That is why the proposition b (isotropy of the one-way velocity) is just a postulate.

We derive the flat space-time theory T2 (CSRT) based on the propositions Non E1, E2.

Non E1: $\neg(a \wedge b) = \neg a \vee \neg b$

The proposition $a =$ MR is an experimentally verified , therefore regarded as a true statement:

(Non E1 and MR): $(\neg a \vee \neg b) \wedge a = a \wedge \neg b$

Wording:

In vacuum in an inertial reference frame F which is attached to a receiver, the speed of light is always constant c and hereby it is not true that the speed of light in the inertial reference frame F' which is attached to moving emitter is always the same constant c.

2 Complementary Special Relativity Theory

2.1 Postulates

To create a postulates of the new flat space-time theory demands axiomatically establish the one-way velocity postulate which will correspond to (Non E1 and MR) and E2 propositions.

2.1.1 One-way velocity postulate (OWVP)

The one-way velocity $\boldsymbol{u'}$ of the light (electromagnetic signal) receding from the source that is placed in origin O' of a moving inertial frame of reference F' to a point P which is at rest in frame F, emitted at the moment when F and F' coincide, is vector function dependent on α and $\boldsymbol{v}$, with the norm $|\boldsymbol{u'}| = u' = \sqrt{c^2 + v^2 - 2cvcos\alpha}$, where $\boldsymbol{v}$ is the relative velocity of F' as seen from F, $|\boldsymbol{v}| = v$, and α being the angle between the directions of OP and positive x-axis, see Fig. 1.

Let denote the postulates of the CSRT, in honor of Michelson and Morley, by symbols M1, M2 .

M1 $\equiv$ (Non E1 and MR) and OWVP

M2 $\equiv$ E2

Comment 1: The inertial reference frame F remains Einstein's, whereas the frame F' does not. It comes to this that the velocity of light is constant c in every inertial frame attached to any massive receiver or detector regardless of relative velocity between a source and receiver.

Afterwards we use this statement by clock synchronization in subsection 3.5, and by derivation of the law of the addition of velocities in section 6.

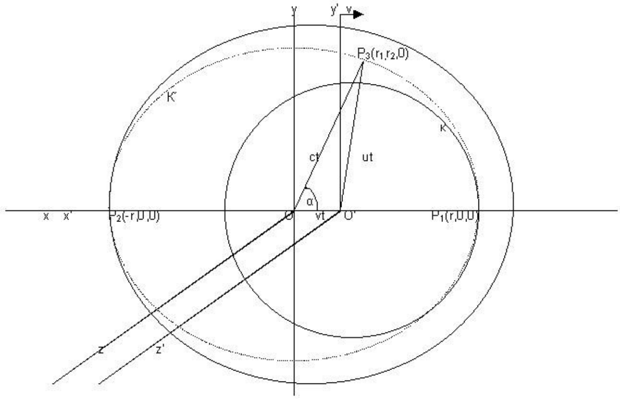

Figure 1: Points P_1, P_2, P_3 are at relative rest in the frame $F(0, x, y, z)$. The speed of light as seen in theframe F is c, in the frame F' is u.

Comment 2: In the axiomatic system S[M1,M2], regard to the postulate M1, we can not choose the light velocity as invariant quantity. The only possibility to choose invariant quantity for a space-time event, is the time coordinate. That is why we can use the Galilean transformation for x,y,z,t coordinates. We stress that this fact does not mean that the CSRT will be from the aspect of simultaneity, identical to the Newtonian space-time theory. For transmission of an information is in the CSRT used a light signal in the sense of the OWVP.

Comment 3: The constant c will represent the one-way and also the two-way speed of the light between the source and the receiver that are at relative rest.

Comment 4: The system E1, E2 is well applied in micro space (big velocities and small distances, high energy collisions). The system M1, M2 could be well applied in macro space (small velocities and big distances).

Comment 5: The author of this article does not know in which sub-

system in nature the axiomatic system E1, Non E2 could be applied. However, there exists some indication that it could be linked with the situation in black holes.
Comment 6: When we have spoken in an inertial system about velocity, we implicitly assumed the clock synchronization in any advance given method, e.g. by Einstein's method.

2.2 Definition of Space Coordinates

The rectangular Cartesian coordinate system S (xyz) will be regarded as a conceptual framework. Using one Cartesian system, we can characterize any space point by three numbers, e.g. point P_1 by coordinates x_1, y_1, and z_1. Let $P_2(x_2, y_2, z_2)$ be another point. The coordinate differences between these two points are denoted by Δx, Δy, Δz . The distance Δs between the two points P_1 and P_2 is given by $(\Delta s)^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$, where $\Delta x = x_2$ - x_1, $\Delta y = y_2$ - y_1, $\Delta z = z_2$ - z_1.

We use without proof the well known proposition which declares that the same distance Δs should be calculated with respect to every other Cartesian coordinate system. This is the same proposition as that between the corresponding coordinates of any two Cartesian coordinate systems there exist orthogonal coordinate transformations, which keep the three-dimensional space interval invariant.

2.3 Law of Inertia

Bodies and photons when removed from interaction with other bodies will continue in their states of rest or straight-line uniform motion. A photon is in interaction with a body in the sense of the M1 postulate.

2.4 Inertial Frame of Reference

A frame of reference F(xyzt) is regarded as an inertial frame of reference if the law of inertia is valid within it.

2.5 Clock Synchronization

We draw the attention that the Einstein's synchronization will be conserved.Let us state that to define a synchronization means to define the time coordinate t. Let we have in an inertial frame of reference F(xyzt) any particle at rest and denote it as point P with a coordinate x,y,z , i.e. P(x,y,z,). Let C_o and C_p denote two standard clocks at rest at the origin O(0,0,0) and at the point P (x,y,z) , respectively. Consider that the signal is emitted from the origin O at the reading t_o of the clock C_o, and then reaches the point P at the reading t_p of the clock C_p. The clocks C_p and C_o are regarded as synchronization if and only if $t_p = t_o + r/c$, where $r = \sqrt{x^2 + y^2 + z^2}$ is the distance between the two points O and P. We can synchronize the two clocks by means of a signal propagating from P at the time t_p to O at the time t_{op}. Similarly, C_p and C_o are regarded as synchronization if and only if $t_{op} = t_p + r/c$, where c is the one-way speed of the signal in both directions: from O to P and from P to O, respectively. It will be shown in section 6 that the synchronization does not depend on velocity of the source which coincides with initial clock. It comes to this that we can synchronize stationary and moving frames, respectively, in the same manner and simultaneously.

Comment 7: The basic merit of this method of synchronization, from aspect of CSRT, we understand after we integrate it with transformation equations below. As we know, the proper time interval is not relevant to the definition of simultaneity. If we will compare a proper time interval of any space-time event in a stationary frame, and its corresponding proper time interval registered by a clock placed in a moving reference frame which is synchronized by the same method (or vice versa), we are sure that we use in both of the frames the same time units.

2.6 Transformation Equations from F' to F

Let the light signal is emitted from the source which is placed in origin O' of a moving inertial frame of reference F'(x',y',z',t') at the moment when F and F' coincide, i.e. $O' \equiv O$ (see Fig. 1). We can describe spreading of the light at the same form, as seeing in F and F', respectively:

In frame F: $x^2 + y^2 + z^2 = (ut)^2$, $u = c$,
In frame F': $x'^2 + y'^2 + z'^2 = (u't')^2$
The transformation equations from F' to F :
$x' = x - vt$, $y' = y$, $z' = z$, $u' = \sqrt{c^2 + v^2 - 2cv\cos\alpha}$, $t' = t$, $\cos\alpha = x/ct$

The time coordinates t and t' are equal (invariant quantity) and the compensation for this fact is the infinite set of spherical wave fronts as seeing from point of view of the frame F' and unique spherical wave front as seen in the frame F.
Comment 8:

One can think of the restoring of locality, with the help of the following assumption: Let emitter spreads "virtual" photons in all directions, with all possible velocities and all the matter have some sort of field which absorb only the photon having a velocity "c" in the reference frame associated with this matter. Once the absorption act happens, all the virtual photons disappear, so one photon can be absorbed only once.

2.7 Inverse Transformation Equations from F to F'

$x = x' + vt'$, $y = y'$, $z = z'$, $u = c$, $u = \sqrt{u'^2 - v^2 + 2cv\cos\alpha}$, $t' = t$, $\cos\alpha = (x' + vt')/ct'$

2.8 Frequency Transformation

In Fig. 2, a light source S, which is at rest in the frame F', has the constant velocity v as seen in frame F (i.e., the laboratory frame) in where the absorber is at rest at the point R. The θ and θ' are the angles between vectors $\boldsymbol{v}$ and $\boldsymbol{r_A}$ as well as $\boldsymbol{v}$ and $\boldsymbol{r_B}$, respectively. The $d\theta$ is the angle between $\boldsymbol{r_A}$ and $\boldsymbol{r_B}$.

F':

The source emits the n-th crest of light wave at the point A at the time t'_A . Then at the time $t'_B = t'_A + d\tau'$ the source reaches the point B , and emits the (n+dn)-th crest of light wave. Both, t'_A and t'_B are the two readings of the same clock connected rigidly with the source. Thus the difference $d\tau' = t'_B$ - t'_A is a proper time interval in F'.

The A' is the point in which the source is placed at the moment when the first light wave (n-th crest) is received by the absorber at

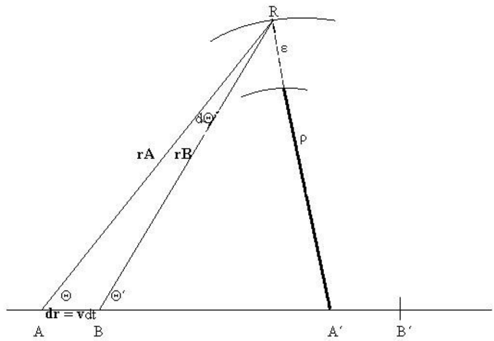

Figure 2: The n-th crest is emitted at the instance when the source is at point A, $(n + dn)$-th crest is emitted when it is at the point B. The A' is the point at which the source is placed at the moment when the first light wave is received by the absorber at R.

R, as well as B' is the point in which is the source placed at the moment when the last wave [(n + dn)-th crest] is received by the absorber at R. The distance between the first and the last light wave fronts is in general, from aspect of source (F'), changing in time. At the moment of the absorption of the first wave front we denote this distance [difference of radiuses of the two spherical wave fronts which are bound with n-th and (n +dn)-th crests , respectively], by ϵ.
Definition 1: The proper frequency $\nu' = dn/d\tau'$,

The first wave front reaches the absorber at R in time $|A'R|/u = t'_1 = t_1 = |\boldsymbol{r_A}|/\ c = r_A/$c. At this moment is the source at point A'. $|A'R| = (r_A/c)\sqrt{c^2 + v^2 - 2cv\cos\theta}$, and hereby the last spherical wave front is from the point A'(source) at the distance $\rho = [(r_A/c) - d\tau']\sqrt{c^2 + v^2 - 2cv\cos\theta'}$.

It follows, that all waves bound with the dn crests are inserted into the annulus with radius ϵ.

$\epsilon = (r_A/c)\sqrt{c^2 + v^2 - 2cv\cos\theta}$ - $(r_A/c - d\tau')\sqrt{c^2 + v^2 - 2cv\cos\theta'}$
$(r_A/c)\sqrt{c^2 + v^2 - 2cv\cos\theta}$ - $(r_A/c - d\tau')\sqrt{c^2 + v^2 - 2cv\cos(\theta + d\theta)}$
$(r_A/c)[\sqrt{c^2 + v^2 - 2cv\cos\theta)}$ - $\sqrt{c^2 + v^2 - 2cv\cos(\theta + d\theta)}] +$

$+ \, d\tau' \sqrt{c^2 + v^2 - 2cv\cos(\theta + d\theta)}$

$\epsilon = (r_A/c)[\sqrt{c^2 + v^2 - 2cv\cos\theta} -$
$- \sqrt{c^2 + v^2 - 2cv(\cos\theta\cos d\theta - \sin\theta\sin d\theta)}] +$
$+ d\tau' \sqrt{c^2 + v^2 - 2cv(\cos\theta\cos d\theta - \sin\theta\sin d\theta)}$

F:

According to the transformation equations from F' to F, the receiver in R absorbs all the waves which belong to the dn crests in time $d\tau = \epsilon/c$, where $d\tau$ is also proper time interval in F.

Definition 2: The frequency as measured by the receiver at R is $\nu = dn/d\tau$

Now we can calculate the relationship between $d\tau'$ and $d\tau$.

Particular cases:

2.8.1 Doppler effect formulas for longitudinal motion

1) When θ =0, then $d\theta = 0$, $cos\theta = cosd\theta = 1$, and ϵ reduces to $\epsilon = d\tau' \sqrt{c^2 + v^2 - 2cv}$

The $(n + dn)$-th crest is an element of the last wave front, which reaches the receiver R in time

$$d\tau = \epsilon/c = d\tau' \sqrt{c^2 + v^2 - 2cv}/c \tag{1}$$

$1/d\tau = (1/d\tau') \sqrt{c^2/(c^2 + v^2 - 2cv)}$

After multiplying by dn, we get $dn/d\tau = (dn/d\tau')[c/(c - v)]$

$$\nu = \nu' \frac{c}{c - v} \tag{2}$$

2) When $\theta = \pi$, then $d\theta$ =0, $cos\pi = -1$, $cosd\theta = 1$,

ϵ reduces to $\epsilon = d\tau' \sqrt{c^2 + v^2 + 2cv}$

$d\tau = \epsilon/c = d\tau' \sqrt{c^2 + v^2 + 2cv}/c$

$dn/d\tau = (dn/d\tau')[c/(c + v)]$

$$\nu = \nu' \frac{c}{c + v} \tag{3}$$

2.8.2 Transverse Doppler effect

In order to calculate the ϵ, we need the value of $\cos d\theta$ and $\sin d\theta$ in case when the angle $\theta = \pi/2$.

Let $|\boldsymbol{r_B}| = r_B$, $|\boldsymbol{r_A}| = r_A$

According to the geometry shown in Fig. 2, we have $d\boldsymbol{r} = \boldsymbol{v}dt$, $\theta' = \theta + d\theta$, $\boldsymbol{r_B} = \boldsymbol{r_A}$ - $d\boldsymbol{r}$

$r_B^2 = (\boldsymbol{r_A}$ - $d\boldsymbol{r})^2 = r_A^2$ - $2\boldsymbol{r_A} \,.\, d\boldsymbol{r} =\ r_A^2$ - $2r_A cos\theta |d\boldsymbol{r}|$, or

$r_B = r_A$ - $vcos\theta dt$, where $|d\boldsymbol{r}| = dr = vdt$ is used,

and dt is the coordinate time interval in F corresponding to proper time interval $d\tau'$ in the moving frame F'.

From the rectangular triangle Δ ABR, the $\cos d\theta = r_A/r_B = r_A/r_A = 1$, it follows $\sin d\theta = 0$.

It means, the ϵ reduces to $d\tau'\sqrt{c^2+v^2}$.

then

$$d\tau = \epsilon/c = d\tau'\sqrt{c^2+v^2}/c \tag{4}$$

and

$$\frac{dn}{d\tau} = \frac{dn}{d\tau'}\frac{c}{\sqrt{c^2+v^2}}, \tag{5}$$

or

$$\nu = \nu'\sqrt{\frac{c^2}{c^2+v^2}} = \nu'\sqrt{1-\frac{v^2}{c^2+v^2}}. \tag{6}$$

A consequence of the existence of this formula is that in the CSTR it is possible to observe a light signal from a source moving transversally at speed c, while in STR it is not.

$$\lim_{v\to c}\sqrt{1-\frac{v^2}{c^2+v^2}} = \sqrt{\frac{1}{2}}. \tag{7}$$

From point of view of the CSRT, as well as from point of view of the SRT, all the experiments on the relativistic transverse Doppler shift are also time dilation experiments.

It is said in Ref[8] that Klein et al. verified the time dilation to 3.2x10^{-5} using Li^+ ions circulating at $0.064c$. The difference between SRT and CSRT predictions for transverse frequency transformation at the speed $v = 0.064c$ is 8.37x10^{-6}.

3 Time Dilation in Closed Path

From equation (4) we have

$$d\tau' = d\tau\sqrt{1-\frac{v^2}{c^2+v^2}}. \tag{8}$$

We see that the proper time interval $d\tau'$ in F' is smaller then corresponding proper time interval $d\tau$ in frame F. In other words, a moving clock runs slowly comparing to a series of rest clocks synchronized according to our definition of simultaneity

$$\tau' = \int d\tau \sqrt{1 - \frac{v^2}{c^2 + v^2}}. \tag{9}$$

Let the motion of the clock is not straight-line and let its velocity is $\boldsymbol{u}$. We assume that the magnitude of $\boldsymbol{u}$ is constant in time, i.e., $u = |\boldsymbol{u}| =$ constant. The proper time interval of the clock moving in the closed path is given by the integral:

$$\boldsymbol{\tau' = \oint d\tau \sqrt{1 - \frac{u^2}{c^2 + u^2}}}, \tag{10}$$

so that the equation above reduces to

$$\boldsymbol{\tau' = \sqrt{1 - \frac{u^2}{c^2 + u^2}} \oint d\tau = \tau \sqrt{1 - \frac{u^2}{c^2 + u^2}}}. \tag{11}$$

Introduction this time dilation formula to Hafele's and Keating's "Around-the-World Atomic Clocks" experiment [7] gives due to very small velocity of the flying atomic clock almost the same kinematic predictions as Einstein's STR. If we for a while omit the gravitational red shift, using Hafele's and Keating's comparison with situation in non rotating frame, plugging in numbers for a 48 hours round trip flight at constant speed at the equator, we would get -257 nsec and +154 nsec for the eastbound and westbound flights, respectively, while Einstein's STR predictions are -260 nsec and +156 nsec, respectively.

4 Law of the Addition of Velocities for Light Waves

Let space-time coordinates of a particle P as seen in the frame F be (x, y, z, t), and the corresponding coordinates in the frame F' be denoted by (x', y', z', t'). The transformation equations and their differentials produce:

$$\begin{array}{llll} x = x' + vt', & dx = dx' + vdt', & x' = x - vt, & dx' = dx - vdt, \\ y = y', & dy = dy', & y' = y, & dy' = dy, \\ z = z', & dz = dz', & z' = z, & dz' = dz, \\ t = t', & dt = dt', & t' = t, & dt' = dt. \end{array}$$

Velocities of the particle as seen in F and F', respectively, are given by

$\boldsymbol{u} = (dx/dt, dy/dt, dz/dt)$ and $\boldsymbol{u'} = (dx'/dt', dy'/dt', dz'/dt')$.

Relation between the norms of the velocities $\boldsymbol{u'}$ and $\boldsymbol{u}$ can be expressed in the 3-vector form:

$|\boldsymbol{u'}|^{\mathbf{2}} = (dx'/dt')^2 + (dy'/dt')^2 + (dz'/dt')^2 = [(dx - vdt)/dt]^2 + (dy/dt)^2 + (dz/dt)^2 = (dx/dt)^2 + (dy/dt)^2 + (dz/dt)^2$ - $2(dx/dt)v + v^2 = |\boldsymbol{u}|^{\mathbf{2}} + |\boldsymbol{v}|^{\mathbf{2}}$ - $2(dx/dt)v = |\boldsymbol{u}|^{\mathbf{2}} + |\boldsymbol{v}|^{\mathbf{2}}$ - $2uvcos\alpha$ $|\boldsymbol{u}|^{\mathbf{2}} + |\boldsymbol{v}|^{\mathbf{2}}$ - $2\boldsymbol{u}.\boldsymbol{v}$.

If α being the angle between the directions of the vector $\boldsymbol{u}$ and positive x-axis, so $cos\alpha = dx/udt$.

Now we will suppose that the particle P is a light wave.

a) Let the wave P moves to a perspective receiver which is at relative rest in the frame F. Due to postulates M1 and M2, the velocity of light equals c in the frame which is attached to a receiver: $|\boldsymbol{u}| = c$. Forasmuch as $|\boldsymbol{v}| = v$, and c and v are constant scalars, we can write:

$$u' = \sqrt{c^2 + v^2 - 2cvcos\alpha}. \tag{12}$$

The u' is the speed of the light wave P as seen in the frame F'.

b) If the particle P is a light wave which moves to a perspective receiver which is at relative rest in frame F', then $|\boldsymbol{u'}| = c$, and the speed u of the light wave P as seen in the frame F is

$$u = \sqrt{c^2 + v^2 - 2cvcos\alpha'} = \sqrt{c^2 + v^2 + 2cvcos\alpha}, \tag{13}$$

where $\alpha' = \pi - \alpha$.

5 Aberration Formula

Aberration of light may be described in Fig. 3. The point A is the location of a celestial body. An observer in the point B is at rest with respect to the celestial body. Assume that a receiver R moves with a velocity v relative to B. The spatial positions of B and R are assumed to coincide with each other at a given time when the same

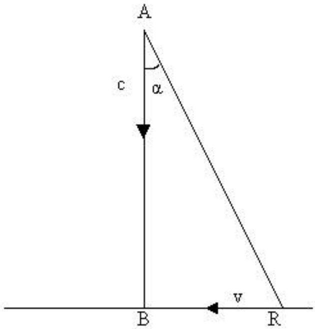

Figure 3: Classical aberration angle α. The point A is the celestial body, R is a receiver moving toward point B. Velocity v is perpendicular to the light ray from A to B.

light ray from the celestial body is received by the two observers in the directions of A $\rightarrow$ B and A $\rightarrow$ R , respectively. The directions of A $\rightarrow$ B and A $\rightarrow$ R we will call conformable with classical theory the real and the sight line directions of the light ray, respectively. The angle α between them being the aberration angle of light. Let the time interval in which an observer overcomes distance from R to B is τ. Let us consider in frame F attached to A and B. Regard to assumption that they are at relative rest, the velocity of light in the real direction is c. Then our annual aberration formula is

$$\tan\alpha = \frac{v\tau}{c\tau} = \frac{v}{c}. \tag{14}$$

Putting the orbital velocity of the earth $v = 29.75$ km/sec into this equation, we get the maximum value of the annual aberration angle

$$\alpha = 20\text{''}.47 \tag{15}$$

which equals to classical aberration constant or to the first order approximation of the STR aberration. To make this explicit, let us use the dn and $d\tau$ technic. The sight line direction in frame F' is given by $n-th$ and $(n+dn)-th$ crests which consequently pass through ocular of an observer located at moving receiver R. As seen in Fig. 4, we obtain the distance between the two crests C_n and $C_{(n+dn)}$ in the frame F when we change the τ into $d\tau$. The distance $|C_n C_{(n+dn)}|$ equals to $d\tau\sqrt{c^2+v^2}$. Since a distance is in the CSRT invariant, the $d\tau\sqrt{c^2+v^2}$ is also the distance of the $n-th$ and $(n+dn)-th$ crests in frame F'.

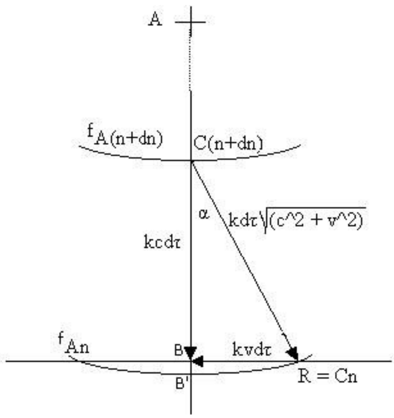

Figure 4: While the n-th crest C_n passes over ocular of receiver R, the $(n+dn)$-th crest is at point $C(n+dn)$ at the light ray from A to B.

It follows that the observer moving perpendicularly with respect to the light ray measures out the proper time interval $d\tau' = d\tau\sqrt{c^2+v^2}/$c, and gains corresponding frequency formula to the transverse Doppler formula derived in section (3.8.1), equation (5):

$$\nu' = \nu\sqrt{\frac{c^2}{c^2+v^2}} = \nu\sqrt{1-\frac{v^2}{c^2+v^2}} \tag{16}$$

Both of them are called red-shift of a spectral line.

6 Red-shift of light from Galaxies

Forasmuch as the CSTR is not reciprocal theory, the relationship between aberration and Doppler effect described in previous section needs precision. We know by now that the aberration does not depend on the distance between a celestial body A and the point B. On the other side, the frequency $\nu' = dn/d\tau'$, measured by receiver R on the same axis depends on remoteness of a celestial body.
In Fig. 4, the n-th and the (n+dn)-th crests, designated as C_n and $C_{(n+dn)}$, respectively, are elements of corresponding spherical wave fronts f_{An} and $f_{A(n+dn)}$ with center at the celestial body A, and with corresponding radiuses r_{An} and $r_{A(n+dn)}$. By analogy to the section 3.8, we denote the difference of this radiuses by $\epsilon = r_{An} - r_{A(n+dn)} = cd\tau$.
We can see that the distance $|C_{(n+dn)}B| = kcd\tau \leq \epsilon = |C_{(n+dn)}B'|$, where $\sqrt{1 - v^2/(c^2+v^2)} \leq k \leq 1$. We get the left bound of the interval in case when the $C_{(n+dn)}$ crest is coincident with the celestial body A. Then $kd\tau\sqrt{c^2+v^2} = cd\tau$. It follows $k = c/\sqrt{c^2+v^2} = \sqrt{1 - v^2/(c^2+v^2)}$

It comes to this, that the proper time interval during which the receiver R absorbs the dn crests is $d\tau' = kd\tau\sqrt{c^2+v^2}/$c.

Then

$$\nu' = \frac{1}{k}\nu\sqrt{\frac{c^2}{c^2+v^2}} = \frac{1}{k}\nu\sqrt{1 - \frac{v^2}{c^2+v^2}}, \tag{17}$$

where ν is the proper frequency of the light from the source placed in the celestial body A. We get the formula (16) above on condition that $k = 1$, i.e. in limit case when $r_{An} \rightarrow \infty$. It follows that the wave-length of light from farther stationary galaxies is longer then from nearby galaxies. Furthermore if a galaxy, conformable with expanding model of universe, is receding from the observer located at the point B by velocity u on the axis BA, then the distance between the n-th and the (n+dn)-th spherical wave fronts increases to $(c + u)d\tau$. This contributes to following expansion of the waves-length of light from receding galaxies. In this case the coefficient $k > 1$.

7 Acceleration of the Pioneer 10/11 Spacecraft

John D. Anderson et al. state (see references:[1],-,[6],[10]) that some "mysterious" acceleration $a_p = (8 \pm 3)$ x 10^{-8} cm/s^2 (see ref

[1], page 2858) slows down motion of the spacecraft receding from the Sun on hyperbolic orbit. There is no standard explanation of this effect. Author of this article believes that particularly the diurnal and annual periodicity indicate that the acceleration is unaccountable by standard theories. In order to show the origin of the average acceleration a_p, we suppose that the Earth is placed on the abscissa Sun-spacecraft in order for a while to neglect the Earth orbital and revolutionary movements. We will consider this event at first from point of view of the SRT, and second from point of view of the CSRT. The time necessary to reach the distance s from the Earth to the spacecraft (E$\mapsto$S), and back from the spacecraft to the Earth (S$\mapsto$E) can be described like this:

7.0.3 Range

SRT:

E$\mapsto$S: Let's send the radio-signal from the Earth to the spacecraft S at the moment when its distance from the Earth is s. Let the speed of the spacecraft in regard to the Sun is v. Let denote the time, in which the radio-signal reaches the spacecraft in inertial system Earth, by t_1. The speed of the radio-signal regard to the Sun and Earth is c. Then $t_1 c = s + vt_1$, $t_1 = s/(c - v)$.

S$\mapsto$E: The radio-signal spans the distance $s + vs/(c - v)$ in time

$t_2 = [s + sv/(c - v)]/c = s/(c - v)$

$$t_1 + t_2 = 2s/(c - v). \tag{18}$$

CSRT:

E$\mapsto$S: by the law of addition of velocities $t'_1(c+v) = s + t'_1 v$, $t'_1 = s/c$. When the signal reaches the spacecraft, the distance between Earth and spacecraft is $s + sv/c$.

S$\mapsto$E: $t'_2 = (s + sv/c)/c$.

The round trip of the radio signal takes time

$$t'_1 + t'_2 = \frac{2s}{c} + \frac{sv}{c^2}. \tag{19}$$

The signal returns to the Earth sooner than the STR predicts in difference

$$\Delta t = (t_1+t_2)-(t_1'+t_2') = \frac{2s}{c-v} - \frac{2s}{c} - \frac{sv}{c^2} = \frac{sv(c+v)}{c^2(c-v)} = \frac{tv^2(c+v)}{c^2(c-v)}, \quad (20)$$

where $s = tv$ is used.

7.0.4 Seeming clock acceleration a_t

Let we consider this phenomenon from John D. Anderson's et al. point of view [Ref [1] p.2859] as a seeming tracking station's average unmodeled clock acceleration a_t. Forasmuch as the annual and diurnal variations of the term are sinusoidal, we can neglect the Earth orbital and circumvolution motions.
The instantaneous drift in the clock's rate for the distance $s = vt$ would be

$$\frac{tv^2(c+v)}{c^2(c-v)} \frac{d}{dt}\left[\frac{tv^2(c+v)}{c^2(c-v)}\right] dt = \left[\frac{v^2(c+v)}{c^2(c-v)}\right]^2 tdt \quad (21)$$

and its integral time

$$\int_0^t \left[\frac{v^2(c+v)}{c^2(c-v)}\right]^2 tdt = \frac{1}{2}\left[\frac{v^2(c+v)}{c^2(c-v)}\right]^2 t^2 = \frac{1}{2}|a_t|t^2, \quad (22)$$

i.e.

$$\left[\frac{v^2(c+v)}{c^2(c-v)}\right]^2 = |a_t|, \quad (23)$$

from where for $v = 12.2$ km/s we get $a_t =$ - 2,7354x10$^{(-18)} s/s^2$, and using identity $a_t = a_p/c$,
we get $a_p =$ - 8,2x10$^{(-13)} km/s^2$ (compare to Ref[1] - p. 2858 and p.2859, respectively).

7.0.5 Annual periodicity

The annual and diurnal periodicity (Ref[2], Figure 1, plot B, p.7) of the imaginary acceleration a_p is in accordance with CSRT. Let us consider, for example, the Earth orbital movement. The unmodeled acceleration a_p is related to the real speed of the radio signal u = $\sqrt{c^2 + v_{rel}^2 - 2cv_{rel}cos\alpha}$ emitted from Earth to Pioneer 10, and vice versa from Pioneer to Earth. It depends on instant relative velocity v_{rel} between the Earth and the spacecraft, and on the expression "$\cos\alpha$". Forasmuch as the optic angle of Earth orbit diameter 2AU from point of view of receding spacecraft draws nearer to zero, maximum of the absolute value of the angle α with rising distance between the Earth and the spacecraft draws nearer to π, so the annual unmodeled variation along the average value of a_p wanes with rising distance.

8 One-Way Velocity Experiment

We can demonstrate consequences of the one-way velocity postulates (CSRT as a test theory). Let we have a standard clock in the place of the source. The source S is sending out electromagnetic signal with frequency ν into two parallel courses. In the first course to the stationary receiver SR, while in the second course to the moving receiver MR. We will register the number of crests (not frequency) which will be registered by the stationary receiver as well as by the moving receiver. Let us start to emit the electromagnetic signal at the moment when the moving receiver (receding from S) is in regard to the source at the distance a, and the distance between the source S, and the stationary receiver is s, $a < s$. We denote this time by t_0= 0.

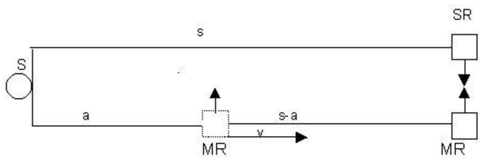

Figure 5: The source S emits a splitting light into two parallel courses at the instance when the receiver MR receding from S is at the distance a.

As is depicted in Fig. 5, a breakdown switch will switch off the both counters on the stationary receiver as well as on the moving receiver at the moment when the moving receiver comes to the same distance from the source as the stationary one keeps.
The number of the registered crests on both receivers is identical when we consider in framework of SRT, and it is various when we consider in framework of CSRT.
SRT:
The first crest reaches the stationary receiver SR in the time $t_1 = s/c$. The last one in the time $t_{la} = (s-a)/v$. The time interval, as measured in inertial "system Source" during which the stationary receiver receives crests is $\Delta t_{SR} = (s-a)\ /\ v$ - $s/c = (sc-ac-sv)/vc$. The SR records $N_{rec} = \nu\ (sc-ac-sv)/vc$ crests. The first crest reaches the moving receiver MR in the time $t_1 = a/(c-v)$. The last one in time $t_{la} = (s-a)/v$. The coordinate time interval, during which the moving receiver receives the crests is
$\Delta t_{MR} = (s-a)/v$ - $a/(c-v) = (sc-ac-sv)/[v(c-v)]$.
Here we use the corresponding proper time interval and the relativistic Doppler formula:

$$N_{rec} = \nu \Delta t'_{MR} \sqrt{\frac{c-v}{c+v}} = \nu \frac{sc-ac-sv}{v(c-v)} \sqrt{1-\frac{v^2}{c^2}} \sqrt{\frac{c-v}{c+v}}, \qquad (24)$$

$$N_{rec} = \nu \frac{(sc-ac-sv)}{v(c-v)} \frac{(c-v)}{c} = \nu \frac{sc-ac-sv}{vc} \qquad (25)$$

crests.
CSRT:
The number of crests received by the stationary receiver SR is the same as in SRT:

$$N_{rec} = \nu \frac{sc-ac-sv}{vc} \qquad (26)$$

crests. The moving receiver records the first crest in time $t_1 =$ a/c, the last one in time $t_{la} = (s-a)/v$. The time interval, during which the moving receiver records the crests is $\Delta t_{MR} = (sc-ac-av)/vc$. In the CSRT is the coordinate time interval equal to proper time interval. It follows that the number of the crests recorded by MR is

$$N_{rec} = \frac{(sc - ac - av)}{vc}\frac{\nu c}{c+v} = \frac{\nu(sc - ac - av)}{v(c+v)}. \qquad (27)$$

The difference between CSRT and SRT predictions is

$$\nu\frac{(sc - ac - av)}{v(c+v)} - \nu\frac{(sc - ac - sv)}{vc} = \nu\frac{sv}{c(c+v)}. \qquad (28)$$

Notice that the difference does not depend on "a" but only on "s", and that it is linearly dependent on the distance "s". This formula is in relation with the Anderson's discovery that the difference: observed Doppler velocity of the Pioneer 10 minus model Doppler velocity linearly increases with distance, ([1], p. 2859).
The confrontation of the recorded numbers of crests gives two alternatives. Either the records on MR and SR are equal, then the one-way speed of the electromagnetic signal is c, or there exists a difference in the sense described above, what will represent that the one-way speed is $(c + v)$. Of course, the same result will be obtain also in the case when we would not separate emitted signal into two parallel beams. The (modified) experiment could be easy and replicate performed by NASA using some spacecraft receding from Sun as a source of electromagnetic signal, the International Space Station and a shuttle as a stationary receiver and moving receiver, respectively.

9 Acknowledgments

Author thanks Ladislav Emanuel Roth, Jet Propulsion Laboratory - NASA, for his recognition of the eventual relationship of the CSRT with the DSN data of the Pioneer 10/11 spacecraft, and for information support. Author acknowledges Zbigniew Oziewicz, Cuautitlan Izcalli, Mexico/Wroclaw, for appreciation during the International Conference : Ideas of Albert Abraham Michelson in Mathematical Physics (Mathematical Conference Center at Bedlewo, Poland, August 2002), and Yuan Zhong Zhang, Institute of Theoretical Physics, Academia Sinica, China for his private comments .

References

[1] John D. Anderson , Philip A.Laing, Eunice L.Lau, Anthony S.Liu, Michael Martin Nieto, Slava G. Turyshev, "Indication, from Pioneer

10/11, Galileo, and Ulysses Data, of Apparent Anomalous, Weak, Long-Range Acceleration", Physical Review Letters - Volume 81, Number 14, 5 October 1998

[2] John D Anderson, Philip A.Laing, Eunice L.Lau, Anthony S.Liu, Michael Martin Nieto, Slava G. Turyshev, "Study of the Anomalous Acceleration of Pioneer 10 and 11", arXiv:gr-qc/0104064 v4

[3] Anderson J. et al. reply (to the comment of Murphy on "Indication, from Pioneer 10/11, Galileo, and Ulysses Data, of an Apparent Anomalous, Weak, Long -Range Acceleration"), arXiv:gr-qc/99O6113v1

[4] John D. Anderson, Philip A.Laing, Eunice L.Lau, Michael Martin Nieto, Slava G. Turyshev, "The Search for a Standard Explanation of the Pioneer Anomaly", arXiv:gr-qc/0107022v2

[5] John D. Anderson, Michael Martin Nieto, Slava G. Turyshev, "A Mission to Test the Pioneer Anomaly", arXive:gr-qc/0205059

[6] Jose de Jesus Cruz Guzman and Zbigniew Oziewicz, " Frolicher-Nijenhuis Algebra and Non Inertial Deformation of Maxwell Equations", Bulletin De La Societe des Sciences et des Lettres De Lodz, Poland, Volume LII, Serie: Recherchers Sur Les Deformations, Volume XXXVIII (Lodz 2002)

[7] J.C.Hafele and R.E.Keating, Science 177 (1972)166, idem 177 (1972) 168

[8] Roger W. McGowan, David M.Giltner, Scott J.Sternberg, and Siu Au Lee, "New Measurement of the Relativistic Doppler Shift in Neon", Physical Review Letters, Volume 70, 18 January 1993)

[9] Yuval Ne'eman, "Abraham Albert Michelson and His Cloud - Some History and Epistemological Analysis", Bulletin De La Société des Sciences et des Lettres De Łódź, Poland, Volume LII, Série: Recherchers Sur Les Déformations, Volume XXXVIII (Łódź 2002)

[10] Asher Peres, "The speed of light need not be constant", arXiv:gr-qc/0210066

[11] Slava G.Turishev, John D.Anderson, Philip A.Laing,

Eunice L.Lau, Anthony S.Liu and Michael Martin Nieto, "The Apparent Anomalous, Weak, Long-Range Acceleration of Pioneer 10 and 11", Presented at XXXIV-th Rencontres de Moriond Meeting on gravitational Waves and Experimental Gravity. Les Arcs, Savoi, France (January 23-30 , 1999)

[12] Pavol Valent, "On Insurpassability of Einsteins Theories of Time", Obzory matematiky, fyziky a informatiky 55/1999 Katedra Fyziky FPV UKF, Tr. A.Hlinku 1, 949 74 Nitra, Slovakia

[13] Pavol Valent, "The Acceleration of the Pioneer 10/11 Spacecraft and the New Electrodynamics Derived from the New Alternative Special Theory of Relativity", Bulletin De La Société des Sciences et des Lettres De Łódź, Poland, Volume LII, Série: Recherchers Sur Les Déformations, Volume XXXVIII (Łódź 2002)

[14] Pavol Valent, "Einstein's Special Theory of Relativity and Its Two Complementary Theories", Bulletin De La Société des Sciences et des Lettres De Łódź, Poland, Volume LIV, Série: Recherchers Sur Les Déformations, Volume XLIV (Łódź 2004)

[15] Pavol Valent, "Explaining Pioneer 10 Anomalous Acceleration: Complementary Special Relativity Theory", Galilean Electrodynamics, Volume 17, Number 3, May/June 2006,Published by Space Time Analyses, Ltd.,Arlington MA, USA

[16] Robert M.Wald, "General Relativity", The University of Chicago Press, 1984

[17] Clifford M.Will, "Theory and Experiment in Gravitational Physics", Cambridge University Press, 1993

[18] Yuan Zhong Zhang, "Special Relativity and Its Experimental Foundations", World Scientific Publishing Co. Pte. Ltd., Singapore, 1998

Comment

The old and new Concepts of Physics
An open-dialogue journal

www.uni.lodz.pl/concepts

Comment on TEST OF THE COMPLEMENTARY SPECIAL RELATIVITY THEORY

David Cyganski
Worcester Polytechnic Institute
100 Institute Road, Worcester, MA, USA
e-mail: cyganski@wpi.edu

Abstract

Valent's Complementary Special Relativity Theory (CSRT) can be easily tested with available experimental data. The Doppler shift effect on two photon absorption of an ion beam strongly distinguishes the predictions of CSRT and Special Relativity Theory (SRT). CSRT is rejected by this test while the prediction of SRT conforms with the experimental outcome.

1 Longitudinal and Transverse Doppler Shifts

In the paper *Complementary Special Relativity Theory and Some of Its Applications,* [1] Valent offers a theoretical alternative to special relativity, complementary special relativity (CSRT). Here, we will show that this theory is rejected by experimental data in favor of SRT.

Valent derives his theory's form of the Doppler shift equations in sections 2.8.1 and 2.8.2 of his paper. We have confirmed by several personal communications with the author, the correct form of the Longitudinal Doppler effect equations, according to his theory, for the specific case of a moving observer radially approaching or receding from a laboratory source emitting frequency f. For velocity v (as measured in the laboratory frame), which is positive for observer motion towards the fixed source, the frequency as measured by the moving observer is given by f' with

$$f' = f\left(\frac{1}{1-\frac{v}{c}}\right) \tag{1}$$

A derivation of the Doppler effect for Einstein's special relativity theory (SRT) can be found in [2] on pages 19-20 for general approach geometries. For the case, again, of the source fixed in the laboratory frame with frequency f and the receiver in the moving frame, from Eq. 15 on page 19 we have on substitution of the appropriate angle, the comparable formula for the longitudinal relativistic Doppler effect for a radially approching receiver:

$$f' = f\,\frac{\left(1+\frac{v}{c}\right)}{\sqrt{1-\frac{v^2}{c^2}}}. \tag{2}$$

Now, as Valent points out, the equations of CSRT and SRT agree in first order, hence the results of tests dominated by the first order component of the Doppler effect and based upon subrelativistic speeds do not distinguish these predictions sufficiently for critical comparison. However, as will be shown below, experimental tests exist for which the second and higher order effects are dominant. We shall select such an experiment from the literature and apply it as a discriminating test.

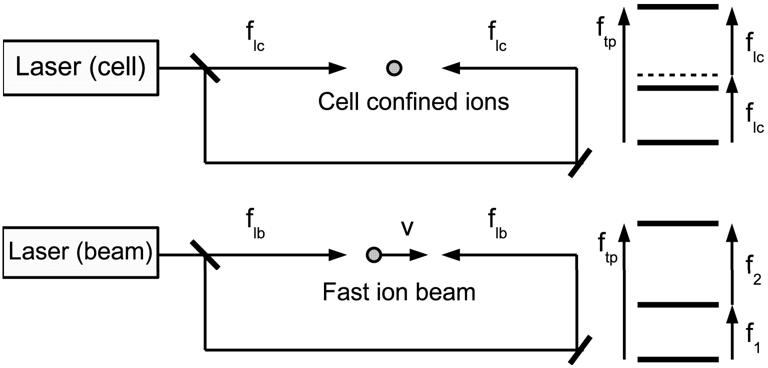

Figure 1: TPA scattering geometry and associated neon ion energy level diagrams.

2 Two Photon Absorption Test

We will base our comparison of CSRT and SRT on the experimental result found in a paper by Kaivola et al. [3] which documents one of the most precise experimental tests of the relativistic Doppler shift to have been conducted. This test improved the measurement of the coefficient of the second order term of the Doppler shift from an accuracy of 3% obtained by Mossbauer tests, to the level of 0.004%.

The Two Photon Absorption of neon ions in the laboratory frame and that of a fast beam of neon ions is exploited in a scheme that results in direct measurement of the second order term and which is independent of direct measurement of the velocity of the ions. In each case, a single laser is directed to pass through the neon gas from two counter-propagating directions as shown in Fig. 1. The energy level difference, which separates the initial and final excitation levels, that is to be supplied by a two photon process is associated with a frequency f_{tp}.

In the case of the gas confined to a cell, each impinging photon is of the same frequency, f_{lc}, hence absorption requires

$$2f_{lc} = f_{tp} \tag{3}$$

to fulfill the basic two photon excitation constraint. A weak non-resonant process is exploited, in which the first photon excites the ion to an intermediate state belonging to a manifold of non-resonant states. This allows a single laser to used for this process. The laser is feedback locked to the associated weak absorption peak and hence forced to fulfill Eq. 3.

In the ion beam case, the laser will be tuned to excite a particular intermediate resonance which is readily located since the strong resonant absorption enhancement is of order 10^9. The resonant TPA frequencies are measured in the laboratory frame and determined to have wavenumbers of 16816.66634 cm^{-1} and 16937.3862 cm^{-1}, which given $c = 2.99792458 \cdot 10^{10} cm/s$ correspond to frequencies of

$$f_1 = 5.04150973743446 \cdot 10^{14}\,\mathrm{Hz} \tag{4}$$

and

$$f_2 = 5.07770064099328 \cdot 10^{14}\,\mathrm{Hz}. \tag{5}$$

From this we also learn that

$$f_{tp} = f_1 + f_2 = 1.01192103784277 \cdot 10^{15}\,\mathrm{Hz} \tag{6}$$

and hence

$$f_{lc} = 5.05960518921387 \cdot 10^{14}\,\mathrm{Hz}. \tag{7}$$

For the resonant two photon process to occur in the neon beam, the neon ion must see the frequencies, f_1 and f_2 in its moving frame. Thanks to these two constraints we may solve for the single exciting laser frequency, f_{lb}, directed at the beam and the beam velocity, jointly, for which this resonant absorption can occur. In the experiment, the beam exciting laser is also feedback locked to the frequency that generates the peak resonant absorption. The difference between the two absorption-peak locked lasers, $f_{lc} - f_{lb}$ will then be compared to the theoretical values which can be computed via SRT and CSRT.

2.1 SRT Analysis

We will first analyze this experiment by applying Einstein's relativistic Doppler effects. In the laboratory reference frame, the ion beam is illuminated by counter-propagating beams each with frequency f_{lb}. The ions see two Doppler shifted laser beams with frequencies

$$f_{+} = f_{lb}\,\frac{\left(1+\frac{v}{c}\right)}{\sqrt{1-\frac{v^2}{c^2}}} \tag{8}$$

$$f_{-} = f_{lb}\,\frac{\left(1-\frac{v}{c}\right)}{\sqrt{1-\frac{v^2}{c^2}}} \tag{9}$$

as they approach and recede from the two source beam directions, respectively.

As previously indicated, resonant absorption will only take place when the conditions

$$f_{-} = f_1 = 5.04150973743 \cdot 10^{14} \tag{10}$$

$$f_{+} = f_2 = 5.07770064099 \cdot 10^{14} \tag{11}$$

are met simultaneously. On substituting Eqs. 8 and 9 into Eqs. 10 and 11, and then solving these equations simultaneously for the excitation laser frequency and ion velocity that uniquely satisfy these constraints we find

$$f_{lb} \quad = \quad 5.05957283032 \cdot 10^{14} \tag{12}$$

$$v \quad = \quad 3.57645529694414 \cdot 10^{-3} c \tag{13}$$

Thus on comparing the predicted feedback-locked frequencies of the lasers driving the ions confined in the cell and driving the beam, we find:

$$f_{lc} - f_{lb} = 3.235890 \cdot 10^{9} = 3235.890\,\mathrm{MHz} \tag{14}$$

The outcome of the laboratory measurement of this frequency difference resulted in

$$3235.94 \pm 0.14\,\mathrm{MHz} \tag{15}$$

Hence this experiment does not contradict the prediction of SRT.

2.2 CSRT Analysis

We now analyze this experiment by applying Valent's complementary relativistic Doppler effects. The ions see two Doppler shifted laser beams with frequencies

$$f_{+} = f_{lb}\left(\frac{1}{1-\frac{v}{c}}\right) \tag{16}$$

$$f_{-} = f_{lb}\left(\frac{1}{1+\frac{v}{c}}\right) \tag{17}$$

Solving Eqs. 16 and 17 simultaneously for the excitation laser frequency and ion velocity that uniquely satisfy these constraints under CSRT we find

$$f_{lb} = 5.05954047163950 \cdot 10^{14} \tag{18}$$

$$v = 3.57645529694414 \cdot 10^{-3} c \tag{19}$$

In comparison with the SRT analysis, the CSRT predicted feedback-locked frequencies of the lasers driving the cell and beam ions now differs by twice the previous prediction.

$$f_{lc} - f_{lb} = 6471.757437\,\mathrm{MHz} \tag{20}$$

The laboratory measurement of this frequency difference 3235.94 ± 0.14 MHz strongly disagrees with this prediction.

3 Conclusions

As Valent points out in his paper, the predictions of SRT and CSRT agree at first order, but diverge in the second and higher order terms. The two photon absorption experiment allows precise measurement of second order and higher Doppler shift effects without a strong obscuring first order effect and hence is the ideal test of theories that differ at higher order. Expanding the two Doppler factors appearing in Eqs. 1 and 2 as a series in $\beta = \frac{v}{c}$ one obtains

$$\text{CSRT Factor:} \quad 1 + \beta + \beta^2 + \beta^3 + \beta^4 + \beta^5 + \cdots \tag{21}$$

$$\text{SRT Factor:} \quad 1 + \beta + \frac{1}{2}\beta^2 + \frac{1}{2}\beta^3 + \frac{3}{8}\beta^4 + \frac{3}{8}\beta^5 + \cdots \tag{22}$$

As can be seen, for longitudinal Doppler shift this second order disagreement of the theories is particularly strong, hence the degree of discrimination between the theories provided by the two photon experimental test.

This author would like to acknowledge the assistance of the author of the paper under review, in the form of several personal communications about his new prediction of the longitudinal Doppler effect for a moving receiver, which resulted in a definitively affirmed application of his CSRT in this review.

References

[1] *Complementary Special Relativity Theory and Some of Its Applications*, P. Valent, Concepts of Physics, this issue.

[2] *The Theory of Relativity,* W. Pauli, Dover Publications, Inc., 1958.

[3] *Measurement of the Relativistic Doppler Shift in Neon*, Matti Kaivola, Over Poulsen, Erling Riis, Siu Au Lee Physical Review Letters, Vol. 54, No. 4, January 28, 1985, pp. 255-258.

Reply to referee's comments

Kaivola's Test of the Complementary Special Relativity Theory

1 Foreword

In order to find missing link between measured frequencies f_1, f_2 and f_{lb}, respectively, Kaivola (and Cyganski) suppose that the ions see the two Doppler shifted laser beams with frequencies (see: *Measurement of the Relativistic Doppler Shift in Neon*, Matti Kaivola, Over Poulsen, Erling Riis, Siu Au Lee; Physical Review Letters, Vol. 54, No. 4, January 28, 1985, pp. 255-258):

$$f_1 = \nu_1 c = f_{lb} \frac{\frac{c-v}{c}}{\sqrt{1-\frac{v^2}{c^2}}}, \tag{1}$$

$$f_2 = \nu_1 c = f_{lb} \frac{\frac{c+v}{c}}{\sqrt{1-\frac{v^2}{c^2}}}. \tag{2}$$

However, we must realize that in both of the relativistic theories, SRT and CSRT, the frequencies f_1 and f_2 are the Doppler transformed frequencies emitted by moving ions and measured by detectors in a laboratory frame, towards which the ions are receding and approaching, respectively. It does not matter whether the frequencies f_1 and f_2 are obtained in the laboratory frame by resonance or not, they are not equal to frequency produced by ions in a fast beam.

The moving ions really see the f_{lb} frequency by the way as is depicted in equations (1), (2) on the right hand side. To add the left hand sides is from point of view of the SRT and CSRT unacceptable. In the section "CSRT Analysis" below we show rigorous relativistic path how to apply the CSRT on Kaivola's experiment.

2 SRT(Kaivola - Cyganski) analysis

Let us for a while solve Equations (1) and (2) simultaneously for the excitation laser frequency f_{lb} and the ion velocity v. Kaivola (Cyganski) finds:

$$f_{lb} = \sqrt{f_1 f_2}. \tag{3}$$

Dividing Eq.(1) by Eq.(2) we get formula for ion velocity v

$$\frac{f_1}{f_2} = \frac{c-v}{c+v}, \tag{4}$$

$v = 0,003576455296944 14c = 107219432,439800361929612 cm/s$.
The Kaivola's SRT theoretical second order Doppler shift prediction then is:

$$f_{lc} - f_{lb} = \frac{f_1 + f_2}{2} - \sqrt{f_1 f_2} = 3235.89 MHz. \tag{5}$$

3 CSRT analysis

First we compute the frequency emitted by ions in the fast beam in their rest frame. The one-way velocity of the light emitted from ion receding and approaching detector is $(c+v)$ and $(c-v)$, respectively. So we can easily calculate the ion frequency f_{ion}:

$$\begin{aligned} f_{ion} &= \nu_1(c+v) = 16816,66634 \times 30086465232,4398 \\ &= 505954047163950 Hz, \end{aligned}$$

$$\begin{aligned} f_{ion} &= \nu_2(c-v) = 16937,3862 \times 29872026367,56019 \\ &= 505954047163950 Hz. \end{aligned}$$

If we divide Eq.(6) by Eq.(7), we get the same formula for ion velocity v as in SRT analysis.

$$\frac{\nu_1}{\nu_2} = \frac{f_1}{f_2} = \frac{c-v}{c+v}, \tag{6}$$

$v = 0,00357645529694414c = 107219432,439800361929612 cm/s.$
If we multiply Eq.(6) by Eq.(7) we find relation between frequencies f_{ion} and f_{lb}.

$$f_{ion}^2 = \nu_1\nu_2(c^2-v^2) = \nu_1\nu_2(c^2-v^2)\frac{c^2}{c^2} = f_1 f_2 \frac{c^2-v^2}{c^2} = f_1 f_2 \sqrt{1-\frac{v^2}{c^2}}, \tag{7}$$

$$f_{ion} = \sqrt{f_1 f_2}\sqrt{1 - \frac{v^2}{c^2}}. \tag{8}$$

Since the frequency of the ions in the fast beam depends only on the f_{lb} and on the velocity v regard to the laser source, i.e. regard to the laboratory frame, we find that the expression $\sqrt{f_1 f_2}$ must be f_{lb}.

$$f_{ion} = f_{lb}\sqrt{1 - \frac{v^2}{c^2}}, \tag{9}$$

$$f_{lb} = f_{ion}\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = f_{ion}\gamma. \tag{10}$$

So the CSRT second order Doppler shift prediction is

$$\begin{aligned} f_{lc} - f_{lb} &= \frac{f_1 + f_2}{2} - f_{ion}\gamma = \\ &= 505960518921387Hz - 505957283032320Hz = \\ &= 3235.89MHz. \end{aligned}$$

This is the same result as Kaivola et al. have predicted in their experiment.

4 Conclusions

From point of view of the SRT, the Kaivola's experiment is called "time dilatation experiment". From point of view of the CSRT we can call it simply as second order Doppler shift experiment.

Pavol Valent