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CONSISTENT QUANTUM HISTORIES: TOWARDS A UNIVERSAL LANGUAGE OF PHYSICS

Wojciech P. Grygiel
Philosophical Department
The Pontifical Academy of Theology, Kraków

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Abstract

The consistent histories interpretation of quantum mechanics is a reformulation of the standard Copenhagen interpretation that aims at incorporating quantum probabilities as part of the axiomatic foundations of the theory. It is not only supposed to equip quantum mechanics with clear criteria of its own experimental verification but, first and foremost, to alleviate one of the stumbling blocks of the theory – the *measurement* problem. Since the consistent histories interpretation operates with a series of quantum events integrated into one quantum history, the measurement problem is naturally absorbed as one of the events that build up a history. The interpretation rests upon the two following assumptions, proposed already by J. von Neumann: (1) both the microscopic and macroscopic regimes are subject to the same set of quantum laws and (2) a projector operator that is assigned to each event within a history permits to transcribe the history into

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a set of propositions that relate the entire course of quantum events. Based on this, a universal language of physics is expected to emerge that will bring the quantum apparatus back to common sense propositional logic. The basic philosophical issue raised this study is whether one should justify quantum mechanics by means of what emerges from it, that is, the properties of the macroscopic world, or use the axioms of quantum mechanics to demonstrate the mechanisms how the macroscopic world comes about from the quantum regime.

1 Introduction

The conceptual gap between the common sense experience and the abstract formalism of quantum mechanics has been the source of many interpretational controversies within this theory since the time of its advent at the beginning of the 20^{th} century. The main objective of all interpretational efforts is to develop an appropriate framework of logically consistent discourse valid for both microscopic (quantum) and macroscopic (classical) regimes. In other words, the universality of such language should expand over the entire physics. However, the diversity of interpretations of quantum mechanics available today indicates that its universal language of interpretation is not at hand[1]. A world-renowned physicist, Roger Penrose, claims that the interpretational intricacies of quantum mechanics will be resolved within the context of a more general theory of *quantum gravity*[2]. This theory combines quantum mechanics with the general theory of relativity.

The key contributions of Johann von Neumann into quantum mechanics include the embedding of the theory within the abstract framework of Hilbert vector spaces and the development of a formal approach to the *measurement problem*[3]. This problem arises as a contrast between the description of a quantum system by means of a wave function comprising several superimposed single states with the result of measurement that yields one concrete value of a physical quantity. In order to alleviate these complications, von Neumann introduced the notion of *the reduction of the wave vector*, known otherwise as the "quantum leap". In the nomenclature used by Penrose, the process of the reduction of the wave vector is denoted as the procedure **R** in order to emphasize its radical irreversibility and discontinuity in distinction to the reversible and continuous unitary evolution **U** of the wave function according to the Schrödinger equation[4]. The discontinuity of this transition causes the radical incompatibility of procedures **U** and **R** and generates most conceptual and interpretational problems of quantum mechanics. The measurement problem finds no resolution within the context of the classical Copenhagen interpretation insofar as this interpretation does not account for the **U**/**R** leap and thus provides no context for experimental verification of quantum mechanics.

Von Neumann's consideration of the basic propositions of physics, referred to by him as *elementary predicates*, provides grounds to attempt the formulation of a more general language of description of physical phenomena. Roland Omnes explains that "a predicate is a statement to an observable A, that is a self-adjoint operator, and a set of real numbers Δ belonging to the spectrum of A. The predicate is expressed by the sentence 'the value of A is in Δ"'[5]. The association of mathematical objects with predicates is mediated through *projection operators*. The quantum histories constitute a step ahead from what was achieved by von Neumann and are expected to facilitate the formulation of the universal language of interpretation. Such language should ascertain the necessary link between the quantum formalism and the outcomes of physical experiments. Since the quantum formalism finds its ultimate source in the physical reality, the language of quantum histories permits the completion of the following circle: *real – formalism – real*. In addition to this, there are two main advantages of this interpretation. First of all, it is entirely objective in the sense that by its broadened context it absorbs the measurement problem as one of the ordinary events within a history. Thus, it does not need a conscious subject to validate quantum processes and can be applied even in distant galaxies. Secondly, it redefines quantum probabilities so that they refer to an entire history and are no longer stuck at the intersection of two incompatible quantum procedures. The probabilities are introduced as part of the axiomatic foundations of quantum mechanics with no necessary correlation to an experiment thereby making it a uniformly probabilistic theory. Robert Griffiths refers to the quantum histories interpretation as "Copenhagen done right".

The precise development of the consistent histories interpretation of quantum mechanics covers three stages. The very idea of histories was first proposed by Robert Griffiths in 1984[6]. By noticing that some histories may be nonsense (e.g., a version of the Young experiment set up to establish through which hole the particle has passed) he proposed the history consistency condition based on the criterion of additivity of the probabilities of histories in order to secure their meaningfulness. The second stage was accomplished by Roland Omnes (1988)[7] who provided clarification of the relation of probabilities with logic. It turned out that the classical Boolean logic holds for the so called families of consistent histories. Inasmuch as these two stages can be considered to be formal aspects of the consistent histories interpretation, the third stage worked out by Gell-Mann and Hartle (1991)[8] investigates its material aspect, namely, what constitutes the underlying principle in nature that assures the logical consistency of histories. The phenomenon of decoherence is responsible for the continuous emergence of the classical macroscopic world out of the quantum microscopic realm[9]. The formal part, that is, the structure of the language of quantum histories is the subject of this article. However, most of the philosophical concerns of the quantum histories interpretation spring up from the imposition of the so called consistency conditions upon the probabilities of histories. These conditions demand that quantum probabilities fulfill the *additivity* requirement that is proper to standard Boolean logic. The final question is as follows: does forcing the consistency conditions upon quantum probabilities make the interpretation consistent?

2 The logical structure of quantum mechanics

Sampling spaces in quantum mechanics

The precise definition of sampling spaces is of critical importance for the exactness of the entire analysis of consistent histories. Griffiths relates this issue in great detail in his book entitled: "Consistent Quantum Theory"[10]. A sampling space is the entire (complete) ensemble of mutually exclusive possibilities – events – one and only one of which actually occurs (or is true) at a given instant. The main idea in Griffith's exposition is that the notion of a sampling space pertains in a very specific way to the quantum regime. In the Hilbert space formed by a complete set of eigenvectors of a certain Herimitian operator, the sum of projection operators for all directions sums up to the unitary operator. In Griffiths' own words: "Any decomposition of the identity of a quantum Hilbert space $\mathcal{H}$ can be thought of as a quantum sample space of mutually exclusive properties associated with the projectors"[11]. In other words, the entire spectrum of the operator $\hat{\mathbf{A}}$ constitutes a sampling space where an event is understood as the possession by the system under study of a certain value of the observable associated with the operator $\hat{\mathbf{A}}$ selected from among all the eigenvalues from the spectrum of the given operator. The hermiticity of the operator assures the orthogonality and hence the mutual exclusiveness (additivity) of the events – properties. Moreover, by summing up all probabilities of obtaining the respective eigenvalues and keeping in mind that $p_i = |c_i|^2 = c_i^* c_i = \langle \mathbf{\Psi} \mid \mathbf{i} \rangle \langle \mathbf{i} \mid \mathbf{\Psi} \rangle = \langle \mathbf{\Psi} | \hat{\mathbf{P}}_i | \mathbf{\Psi} \rangle$, we obtain:

$$\sum_{i=1}^{N} p_i = \sum_{i=1}^{N} \langle \mathbf{\Psi} \mid \mathbf{i} \rangle \langle \mathbf{i} \mid \mathbf{\Psi} \rangle = \langle \Psi | \sum_{i=1}^{N} |\mathbf{i}\rangle \langle \mathbf{i} \mid \Psi \rangle = \langle \Psi \mid \Psi \rangle = 1. \quad (1)$$

Naturally, this equation holds for a Hilbert space that is normalized. Also, the square $|c_i|^2$ guarantees that $p_i > 0$. Thus the quantum sampling space constructed upon the Hilbert space formed by the eigenvectors of a Hermitian operator fulfills the three axioms of classical probability, namely, *positivity*, *normalization* and *additivity*. This makes the space fitting for the convenient use of probability calculus.

The spectral theorem and the language of quantum logic

The formulation of the consistent histories interpretation hinges upon two cardinal ideas put forward by Johann von Neumann. The first one is the use of uniform quantum apparatus for both classical and quantum regimes. The second one, that is central for the formulation of the language of quantum logic, rests upon the assignment of a *quantum subspace* and a *projection operator* to a quantum mechanical property in much the same way as a physical property of a classical system is associated with a subset of points in the phase space. In turn, a projection operator can be associated with a predicate relating that "the value of an observable A described by an appropriate operator falls within the range Δ". This strategy shows in brief how one might be able to re-write the mathematical formalism entirely in terms of propositions stating the possession of a system under study of a certain value (or a range of values) of a physical property.

At the outset of the exposition of the rules of quantum logic, two important properties of projection operators in general must be introduced:

1. $\hat{\mathbf{P}}_{\mathbf{i}} = \hat{\mathbf{P}}_{\mathbf{i}}^{\mathbf{2}}$, because:

$$\hat{\mathbf{P}}_{\mathbf{i}}^{\mathbf{2}} |\Psi\rangle = |\mathbf{i}\rangle \langle \mathbf{i} \mid \mathbf{i} \rangle \langle \mathbf{i} \mid \Psi \rangle = |\mathbf{i}\rangle \langle \mathbf{i} \mid \Psi \rangle = \hat{\mathbf{P}}_i |\Psi\rangle . \quad (2)$$

This seems rational for a projection of a projection on the same direction no longer causes any alteration.

2. A projection operator is a self-adjoint operator and possesses two eigenvalues: **0** i **1**.

$$\hat{\mathbf{P}}_j\,|\mathbf{i}\rangle = |\mathbf{j}\rangle\,\langle\mathbf{j}\mid\mathbf{i}\rangle = 0\,|\mathbf{j}\rangle \text{ for i} \neq \text{j and } |\mathbf{i}\rangle\,\langle\mathbf{i}\mid\mathbf{i}\rangle = 1\,|\mathbf{i}\rangle \text{ for i=j} \quad (3)$$

The second property can be easily used to assign logical values to propositions asserting whether a system under study has a particular value of a physical property. In other words, one can attribute the truth value to statements such as: "a particle is in an eigenstate described by vector $|\mathbf{i}\rangle$" or, equivalently ,,the observable A, that describes the corresponding property of a particle under study, has the value λ_i that is an eigenvalue of the operator $\hat{\mathbf{A}}$". Thus, a definite mathematical object, such as a projection operator, can be associated with a predicate. However, the full correlation between projection operators and respective statements on the possession of physical properties finds its justification in *the spectral theorem*[12]:

$$\hat{\mathbf{A}} = \sum_{i=1}^{N} \lambda i\,|\mathbf{i}\rangle\,\langle\mathbf{i}|. \quad (4)$$

This equation enables the decomposition of a Hermitian operator into components that are projection operators on the directions determined by the eigenvectors of the operator $\hat{\mathbf{A}}$. Consequently, each projector within the above sum answers the question whether the value of the observable associated with the operator $\hat{\mathbf{A}}$ is equal to the eigenvalue for a particular direction in the Hilbert space. The following projection operator, defined as a sum of single projection operators covering certain range of the values of $\lambda_i \in \Delta$:

$$\hat{\mathbf{P}}_{\Delta} = \sum_{i\in\Delta} |\mathbf{i}\rangle\,\langle\mathbf{i}| \quad (5)$$

correlates with the proposition that "the value of the observable associated with the Hermitian operator A falls within the range Δ". This involves the Hilbert subspace H_{Δ}. Finally, the following axioms of the language of quantum logic can be derived[13]:

(1) The unitary operator $\hat{\mathbf{I}}$ that projects onto the entire Hilbert space corresponds with the proposition whose truth value is always 1.

(2) The operator $\hat{\mathbf{I}} - \hat{\mathbf{P}}_{\Delta}$ corresponds to the proposition that is the negation of $\hat{\mathbf{P}}_{\Delta}$.

(3) If the truth of the proposition P_1 implies the truth of the proposition P_2 then the subspace into which the operator $\hat{\mathbf{P}}_2$ projects is contained in the subspace into which the operator $\hat{\mathbf{P}}_1$ projects.

(4) The conjunction (product) of the two propositions P_1 and P_2 is represented by the product of the corresponding operators $\hat{\mathbf{P}}_1\hat{\mathbf{P}}_2$ provided that this product is also a projection operator (see the next section for the detailed discussion of this issue).

(5) The alternative (summation) of the two propositions P_1 and P_2 is represented by the sum of the corresponding operators $\hat{\mathbf{P}}_1+\hat{\mathbf{P}}_2$ provided that this sum is also a projection operator.

Non-commuting quantum operators and the specificity of quantum logic

The sampling spaces, namely, the Hilbert subspaces that are used to represent physical properties in quantum formalism are spanned by the sets of the eigenvectors of the corresponding Hermitian operators [14]. Since quantum mechanical operators are matrices (matrix mechanics of Heisenberg) and matrices multiply non-commutatively, there are certain restrictions as to the freedom of multiple definition of physical properties upon one subset of the Hilbert space and the only operators that permit such definitions are operators that commute

The fact that two commuting operators have the same set of eigenvectors $|\mathbf{i}\rangle$ has far reaching consequences. In the language of probability calculus, this means that these two operators share their sampling spaces. In short, they are *compatible*[15]. As a result, the same projection operator $|\mathbf{i}\rangle\langle\mathbf{i}|$ permits the formulation of a predicate on the *simultaneous* possession of the values of observables corresponding to the operators $\hat{\mathbf{G}}$ and $\hat{\mathbf{H}}$. In other words, it permits to ascertain the truth value of the following conjunction: “the value of the observable G, described by the Hermitian operator $\hat{\mathbf{G}}$ is $\lambda_{G,i}$” <u>and</u> “the value of the observable H, described by the Hermitian operator $\hat{\mathbf{H}}$ is $\lambda_{H,i}$”. On the other hand, if operators $\hat{\mathbf{G}}$ and $\hat{\mathbf{H}}$ do not commute, both the Hilbert spaces that correspond to them as well as any conjunctions of propositions mediated by corresponding projection operators are *incompatible*. At this point, it is easy to notice the correlation of this result with the Heisenberg uncertainty principle. It states that the simultaneous measurement of the values of observables associated with non-commuting operators with infinite precision is not possible.

For instance, this situation occurs in the simultaneous measurement of particle's momentum and velocity.

The exclusiveness of the sampling spaces and the resulting lack of common eigenvectors for these two operators imply that one cannot point out a projection operator that answers the cumulative question whether the observables represented by these operators possess definite values simultaneously. The answer to such question can be given neither in the positive nor in the negative suggesting that such proposition would be entirely *meaningless*. This situation is novel as compared to classical physics where no restrictions exist as to the simultaneous measurement of any two observable. *Meaninglessness* (incompatibility) of propositions is a feature that pertains exclusively to the regime of quantum logic. It emerges as a result of compartmentalization of the quantum mechanical sampling space, that is, the Hilbert space.

3 Quantum histories

Definition of quantum histories

The construction of quantum histories draws its analogy from the simple experiment of a throw of a two-sided coin. Whether one deals with classical or quantum histories, one needs to specify clearly the pertinent sampling space[16]. The simplest sampling space (event algebra) for a single throw of a two-sided coin consists of two events: a head (H) or a tail (T). For instance, the stochastic experiment of throwing a coin three times generates the different sequence of simple events such as: HTT, TTH, HTH, HHH etc. Each of these sequences represents a separate *history*. A complete set of such histories that contains all possible outcomes of triple-throw events constitutes a new *sampling space*. Similarly, in quantum mechanics one can construct a quantum history for any quantum process by following a selected observable as a function of time as depicted in the following diagram:

$$t_1 ... t_2 ... t_3 ... t_4 ... t_5 ... t_n$$

$$A_1 ... A_2 ... A_3 ... A_4 ... A_5 ... A_{n.}$$

According to the postulate of Johann von Neumann, each observable A_i at the time instant t_1 can be assigned a projection operator

$\hat{\mathbf{P}}_i$:

$$\hat{\mathbf{P}}_1 \quad \hat{\mathbf{P}}_2 \quad \hat{\mathbf{P}}_3 \quad \hat{\mathbf{P}}_4 \quad \hat{\mathbf{P}}_5 \quad \hat{\mathbf{P}}_n.$$

Since each of these projection operators may be used to state the truth value of the proposition relating the possession of a certain value of a given observable at a different instant, this series of operators is equivalent to the series of propositions that account for the time characteristics of the entire quantum process and thus forms a *quantum history*. Robert Griffiths' definition of a quantum history runs as follows: "a quantum history of a physical system is a sequence of quantum events at successive times, where a quantum event at a particular time can be any quantum property of the system in question". A quantum history can be also likened to a quantum process recorded on a video tape where successive time instances t_i correspond to a series of scenes captured at each t_i. Like in the simple case of a throw of a two-sided coin, two sampling spaces must be clearly distinguished in the discussion of quantum histories. The first consist of the projection operators in the Hilbert subspace spanned by the eigenvectors of the operator $\hat{\mathbf{A}}_i$ at a given time point t_i. The second sampling space contains all possible quantum histories in the time range from t_1 to t_n indicating that a full quantum history is a single event in such sampling space as it has been the case in a triple-throw event of a coin (HHT, TTH, etc.).

The exposition of the formal content of the consistent histories interpretation begins by noticing that since the operator $\hat{\mathbf{A}}_i$corresponds to the observable A_i at any time point t_i and its eigenvectors span a Hilbert subspace $\mathcal{H}_i$, there exists a product Hilbert space $\mathcal{H}$ given as a tensor product of its components:

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes ... \otimes \mathcal{H}_n. \tag{6}$$

Any quantum history is an event that belongs to the Hilbert space (sampling space) $\mathcal{H}$. The projection operator for each time instant stating in the language of quantum logic that 'the value of A_i is in Δ' is defined as follows:

$$\hat{\mathbf{P}}i = \sum_{j\in\Delta} |\mathbf{i},\mathbf{j}\rangle \langle\mathbf{j},\mathbf{i}| \,, \tag{7}$$

where the vector $|\mathbf{i},\mathbf{j}\rangle$ is the eigenvector of $\hat{\mathbf{A}}_i$:

$$\hat{\mathbf{A}}_i \, |\mathbf{i},\mathbf{j}\rangle = \lambda_{i,j} \, |\mathbf{i},\mathbf{j}\rangle \,. \tag{8}$$

The projection operator into the Hilbert space $\boldsymbol{H}$, proper to a single quantum history, describes a separate event in this space :

$$\hat{\mathbf{P}}_{=}\hat{\mathbf{P}}_1 \cdot \hat{\mathbf{P}}_2 \cdot \hat{\mathbf{P}}_3 \cdot \ldots \cdot \hat{\mathbf{P}}_n. \tag{9}$$

The quantum condition – families of quantum histories

In classical mechanics, each value of an observable is obtained deterministically with the probability equal to one. However, in quantum mechanics an observable A_i may assume several values at each time instant t_i, established by the solution of the eigenvalue equation for the corresponding operator $\hat{\mathbf{A}}_i$(see the above section). While an observation of a property (observable) in classical mechanics yields one history ("one video"), the quantum mechanical paradigm involves a series of different hypothetical paths since the state of the quantum system at each time point is given by the superposition of a number of pure states. The measurement performed on such a system yields an output value selected out of the spectrum of the eigenvalues of $\hat{\mathbf{A}}_i$ with their respective probabilities. In other words, the probabilistic character of quantum mechanics in the consistent histories interpretation is reflected by the need to use not a single quantum history but the so called *families of histories*. The full "quantum video" is then a combination of a number of single "videos". Also, it is worthwhile to point out that this interpretation does not alter the status of time for, like in the standard Copenhagen regime, time continues to function as a parameter and not as an observable (there is no time operator).

The notion of the family of histories was introduced by Robert Griffiths in 1984[17]. The subsequent presentation follows Roland Omnes' exposition. Both of them achieve the same goal but Omnes' presentational strategy seems more transparent. First of all, the spectrum of the operator $\hat{\mathbf{A}}_i$ is divided into a series of exclusive subsets, denoted with the index k: $\left\{\Delta_j^{(k)}\right\}$[18]. According to the exclusivity condition:

$$\Delta_i^{(k)} \cap \Delta_i^{(k')} = 0 \text{ for } k \neq k'. \tag{10}$$

In such a formulation, the proposition asserting that ,,the value of the observable represented by the operator $\hat{\mathbf{A}}_i(t_i)$ falls within $\Delta_i^{(k)}$"

is tied up with the projector $\hat{\mathbf{P}}_i^{(k)}(t_i)$:

$$\mathbf{P}_i^{(k)} = \sum_{j \in \{k\}} |\mathbf{i}, \mathbf{j}\rangle \langle \mathbf{j}, \mathbf{i}| \,. \tag{11}$$

A single quantum history that belongs to the family of histories is described by the following succession of projectors:

$$\hat{\mathbf{P}}_1^{(k_1)}(\mathrm{t}_1),\ \hat{\mathbf{P}}_2^{(k_2)}(t_2),\ \hat{\mathbf{P}}_3^{(k_3)}(t_3),\ \ldots, \hat{\mathbf{P}}_n^{(k_n)}(t_n).$$

For each time point t_i, these projectors fulfill the following conditions:

(1) *exclusiveness*: $\hat{\mathbf{P}}_i^{(k)}(t_i)\hat{\mathbf{P}}_i^{(l)}(t_i) = 0$ for the values of $\mathrm{k} \neq \mathrm{l}$ because $\Delta_i^{(k)}$and $\Delta_i^{(l)}$are exclusive,

(2) *completeness*: $\sum_k \hat{\mathbf{P}}_i^{(k)}(t_i) = I$, because the sum of all ranges $\Delta_i^{(k)}$covers the entire spectrum of the operator $\hat{\mathbf{A}}_i$.

By denoting the quantum histories with the index a, a projection operator $\hat{\mathbf{c}}_a$ proper to a particular history within the family can be defined as a product of single time point projection operators $\hat{\mathbf{P}}_i^{(k)}$ in the descending time order:

$$\hat{\mathbf{c}}_a = \hat{\mathbf{P}}_n^{(k_n)}(\mathrm{t}_n)\cdot \ldots \cdot \hat{\mathbf{P}}_3^{(k_3)}(t_3) \cdot \hat{\mathbf{P}}_2^{(k_2)}(t_2) \cdot \hat{\mathbf{P}}_1^{(k_1)}(t_1). \tag{12}$$

The above conditions for the exclusiveness and completeness of the sampling spaces formed by projection operators $\hat{\mathbf{P}}_i^{(k)}$at given time instants t_i suggest that each full quantum history can be treated as a separate event in a new sampling space that consists of all the histories within the family (similarly to the triple throws of a coin: HHT, THT etc.). Consequently, it is clear that by summing up all projectors corresponding to the quantum histories within the given family we obtain:

$$\sum_a \hat{\mathbf{C}}_a = I. \tag{13}$$

The family of quantum histories constitutes a complete logical framework for a given quantum process. The main advantage for the use of the quantum histories interpretation over the standard Copenhagen is that they provide an account of the entire history of a given quantum process regardless whether any measurements are performed. While in the Copenhagen regime the information on the state of a system investigated could be gained only at the time of a measurement, the histories generate a thorough record of the physical properties of the system at each time instant as a quantum process heads from the beginning to its completion.

Probability of quantum histories

The evolution of a quantum system described by the deterministic Schrödinger equation depends on the initial state of the system. In other words, one can predict the future state of a system based on its initial condition. Likewise, the course of each quantum history depends on the process of the preparation of the system. Since quantum histories are meant to handle not only isolated hypothetical systems but real laboratory situations, they must be apt to account for the complexity of states of large measuring devices composed of huge number of atoms. In particular, this issue is of major concern for systems coupled with the environment. Quantum mechanics has developed means to treat systems whose complexity precludes the direct determination of the system's wave function. The initial mixed state of such complex systems is described by the density matrix $\hat{\rho}$ [19]. For a mixed state where the system under study is found in the state $|\psi_i\rangle$ with probability p_i, the density matrix is given by the following equation:

$$\hat{\rho} = \sum_i p_i \,|\psi_i\rangle\, \langle\psi_i|. \tag{14}$$

The notion of the density matrix will be used to compute the probability of a quantum history. Before this is accomplished, a certain adjustment needs to be made. The formalism of quantum mechanics is more commonly set up within the Schrödinger picture where wave functions evolve in time and operators remain time independent. However, the specificity of quantum histories, namely, their reliance on the time dependence of projection operators makes it more suitable to switch to the so called Heisenberg picture where the operators evolve in time and wave functions stay time independent[20]. This is achieved by means of the unitary operator of time evolution. The mathematical details of the derivation will be omitted here. The final expression for the probability p_i of obtaining the value λ_i of the observable in a measurement performed on the system in a superposition state assumes the following form in the Heisenberg picture:

$$p_i = \left\| \hat{\mathbf{P}}_i(t) \left| \mathbf{\Psi}_0 \right\rangle \right\|^2 , \tag{15}$$

where $\hat{\mathbf{P}}_i(t)$is the corresponding projection operator and $|\mathbf{\Psi_o}\rangle$ is the wave vector that describes the state of the system at the beginning (t $= 0$). This expression is useful for the purposes of quantum histories due to the clear division into the part that sets the initial conditions and the part that is responsible for the time evolution of the system investigated.

By way of analogy, the above expression for the probability of finding the system in a certain state can be re-written to define the probability of a quantum history described by the projection operator $\hat{\mathbf{c}}_a$:

$$p_a = \left\| \hat{\mathbf{c}}_a \left| \mathbf{\Psi}_0 \right\rangle \right\|^2 . \tag{16}$$

This expression can be intuitively comprehended by assuming that each observable A_i is a position or a momentum. In Omnes' own words: "the action of the operator $\hat{\mathbf{c}}_a$ on a wave function is then found to select the Feynman paths in phase space crossing the different windows $\Delta_i^{(k)}$ at the different times t_i"[21]. In order to obtain the form of the expression suitable for the derivation of the consistency condition of quantum histories, a series of rearrangements needs to be implemented[22]. First of all, it is clear that:

$$p_a = \left\| \hat{\mathbf{c}}_a \left| \mathbf{\Psi}_0 \right\rangle \right\|^2 = \left\langle \mathbf{\Psi}_0 \right| \hat{\mathbf{c}}_{\mathrm{a}}^* \hat{\mathbf{c}}_a \left| \mathbf{\Psi}_0 \right\rangle$$

The next step involves the introduction of a unitary operator composed of the complete sum of projection operators in the Hilbert space on the left hand side of the above equation:

$$p_a = \left\| \hat{\mathbf{c}}_a \left| \mathbf{\Psi}_0 \right\rangle \right\|^2 = \left\langle \mathbf{\Psi}_0 \right| \sum_{i=1}^{N} \left| \mathbf{i} \right\rangle \left\langle \mathbf{i} \right| \hat{\mathbf{c}}_a^* \hat{\mathbf{c}}_a \left| \mathbf{\Psi}_0 \right\rangle = \sum_{i=1}^{N} \left\langle \mathbf{\Psi}_0 \mid \mathbf{i} \right\rangle \left\langle \mathbf{i} \right| \hat{\mathbf{c}}_a^* \hat{\mathbf{c}}_a \left| \mathbf{\Psi}_0 \right\rangle =$$

$$= \sum_{i=1}^{N} \left\langle \mathbf{i} \right| \hat{\mathbf{c}}_a^* \hat{\mathbf{c}}_a \left| \mathbf{\Psi}_0 \right\rangle \left\langle \mathbf{\Psi}_0 \mid \mathbf{i} \right\rangle = \sum_{i=1}^{N} \left\langle \mathbf{i} \right| \hat{\mathbf{c}}_a^* \hat{\mathbf{c}}_a \hat{\mathbf{P}}_{\Psi_0} \left| \mathbf{i} \right\rangle = \mathrm{Tr}\ (\hat{\mathbf{c}}_a^* \hat{\mathbf{c}}_a \hat{\mathbf{P}}_{\Psi_0}).$$

Using the invariance of the trace of a product of matrices with respect to the cyclical permutation within the product one can write:

$$p_a = \mathrm{Tr}\ (\hat{\mathbf{c}}_a \hat{\mathbf{P}}_{\Psi_0} \hat{\mathbf{c}}_a^*) \tag{17}$$

The projection operator, $\hat{\mathbf{P}}_{\Psi_0}$,that appears in the above equation, describes the state of an isolated quantum system at the beginning of the quantum process investigated for which a quantum history will be constructed. However, projection operators associated with a pure quantum state are replaced with density matrices for mixed states proper to complex macroscopic devices such as a measuring apparatus composed of a large number of atoms. By substituting the projector operator $\hat{\mathbf{P}}_{\Psi_0}$with the corresponding density matrix $\hat{\rho}_0$ for a mixed state, the expression for the probability of a quantum history assumes the final form[23]:

$$p_a = \mathrm{Tr}\ (\hat{\mathbf{c}}_a \hat{\rho}_0 \hat{\mathbf{c}}_a^*). \tag{18}$$

The fundamental achievement of the consistent histories interpretation in regards to probabilities lies the fact that the probability of a history is unrelated to the act of measurement. Moreover, the above expression for the probability of a quantum history encompasses the whole of the quantum process indicating that stochastic methods can be uniformly applied to its entirety. This a considerable benefit in comparison to the Copenhagen interpretation where probabilities occur only at the connection of two incompatible quantum procedures **U** and **R**. There arises one more question that is addressed neither by Griffiths nor by Omnes. It regards the physical sense of the status of quantum probabilities. The quantum probability is directly related to an angle in the Hilbert space formed by two vectors. The cosine of this angle gives the probability of transition between the two states described by these vectors[24]. Although the strict Born interpretation of the quantum history probability p_a as a square of the amplitude of a wave function is preserved as evidenced by Eq. 16, its relation to an angle in the Hilbert space is no longer obvious for the probability of a history refers to the summation over a series of states occurring sequentially in time and not to a single state described by a vector. This suggests that the consistent histories interpretation blurs the original meaning of quantum probability proper to the Copenhagen regime.

The consistency condition for quantum histories

Although Equation 18 permits the computation of probabilities for any quantum history, these probabilities have proper meaning only within well defined sampling spaces. This definition is achieved by the imposition of the so called history *consistency* conditions that select histories with definite meaning. Only these histories may occur in reality as opposed to those that are meaningless (unreal).

In order to clarify the history consistency condition, one needs to recall the careful distinction between the two types of sampling spaces for a triple throw of a coin: (1) a simple {T,R} event algebra for a single throw and (2) a set of histories for a triple throw forming a new sampling space {THH, HTT, HHH, ...}. Similar division takes place in case of quantum histories and both sampling spaces have to fulfill the conditions of classical probability. The sampling space for a single time instant t_i (the analog of (1)) that is constructed upon the Hilbert subspace formed by the eigenvectors of the Hermitian operator $\hat{\mathbf{A}}_i$ fulfills the three axioms of classical probability, namely, *positivity*, *normalization* and *additivity*. However, in order to assure the meaningfulness of probabilities of quantum histories p_a (Eq. 18), one needs to impose the same conditions of classical probability for the quantum analog of the sampling space (2), that is, for the sampling space where an entire quantum history constitutes a single event within the family of quantum histories. In particular, the condition of additivity warrants the consistency of quantum histories. In other words, this condition demands that each history be an exclusive event within the family of histories. The mathematical derivation of all these conditions can be found in the original papers and books by Griffiths and Omnes.

There are two formulations of the consistency condition proposed by Griffiths, and Gell-Mann and Hartle (abbreviated as the GMH condition). Two quantum histories described by history operators $\hat{\mathbf{c}}_a$ and $\hat{\mathbf{c}}_b$, respectively, that belong to a given family of histories are consistent if:

(1) $Re\mathrm{Tr}\ (\hat{\mathbf{c}}_a \hat{\rho}_0 \hat{\mathbf{c}}_b^*) = 0$, for the Griffiths condition[25];
(2) $\mathrm{Tr}\ (\hat{\mathbf{c}}_a \hat{\rho}_0 \hat{\mathbf{c}}_b^*) = 0$, for the Gell-Mann and Hartle (GMH) condition [26].

The comparison of these two equations suggests that in the GMH consistency condition both the real and imaginary parts of the trace disappear. The GMH condition is obviously stronger and it implies the Griffiths condition and its fulfillment assures the validity of logic. The GMH condition is sufficient for the consistency of quantum histories. As far as the physical significance is concerned, this condition suggests that there is no overlap between any of the histories within a family that is consistent – they are mutually exclusive. Also, the issue of complementary gains clearer meaning within the paradigm of consistent histories. In Roland Omnes' own words: "Two families of histories (two logical frameworks) will be called complementary when they are both consistent though mutually incompatible". More precisely this means that "two families of histories are complementary when they are both consistent and there is no consistent family including both of them, i.e., containing all the properties occurring in either of them"[27].

The last question in the discussion of the consistent histories interpretation of quantum mechanics is whether one can visualize such histories. For instance, let us imagine a thick rope composed of a large number of threads intertwined with each other. Although these threads are packed close together, they never cross or merge. In a way, they are mutually exclusive, that is, consistent.

The final and most vital question to be posed is why should we bother doing all of this? The answer is: to close the *real – formalism – real* circle and come back with the quantum apparatus back to ordinary common sense propositional logic. In short, the purpose is to be able "to talk classical about quantum". As Robert Griffiths states, "here is where the conservatism comes in - one strictly limits the domain of discourse, the set of things which can sensibly be said about a quantum system, in an appropriate way"[28]. He then continues: "What the histories approach does is to specify that a proper domain of logical discourse in quantum theory is limited to a set of compatible propositions corresponding to subspaces with projectors that commute with each other"[29]. Families of consistent histories are these that are mutually exclusive, namely, they fulfill the additivity requirement whereby they constitute suitable frameworks for discourse on the properties of a quantum system based on the rules of propositional logic. Consequently, two frameworks are compatible if the projectors in one commute with the projectors in the other, otherwise they are not compatible.

A case of nonsense histories

The full-fledged formalism of the consistent histories interpretation of quantum mechanics is complex and the possibilities of its intuitive representation seem to be rather limited. However, it turns out that a demonstration of relatively simple experimental case is illuminating. There exists a version of the famous Young experiment where one attempts to resolve the question which pinhole the particle has gone though. The analysis follows that given by Roland Omnes[30]. $\phi_j^{(1)}$ stands for the phase of the wave function of a particle impinging at point j of the screen after crossing the right arm of the interferometer while $\phi_j^{(2)}$describes the passage through the second arm. The passage of the particle through the right arm of the interferometer is described by the history a and through the left by the history b. The history that corresponds to the assertion that the particle has exclusively gone through either right or the left arm is $c = a$ or b. The probabilities for these histories are[31]: $p(a) = A$, $p(b) = A$ and $p(c) = \left\{\exp\left(i\phi_j^{(1)}\right) + \exp\left(i\phi_j^{(2)}\right)\right\}^2$ where A is a certain constant. The consistency condition for the histories a and b demands that they are exclusive events, namely, the condition of their additivity is fulfilled: $p(a)+ p(b) = p(c)$. The condition assumes the following form: $\cos\left(\phi_j^{(1)} - \phi_j^{(2)}\right) = 0$ indicating that the phase difference should be independent of the location j on the screen to assure history consistency. This contradicts the physical fact that the phase difference strongly depends on the location of j. Thus quantum histories serve as a tool to demonstrate the meaningless of the version of the Young experiment attempting to resolve which pinhole the particle has passed through.

4 Conclusions

The article has demonstrated the conceptual way that leads to the formulation of the consistent histories interpretation of quantum mechanics. It has been shown that this interpretation places itself within the stream of contemporary interpretational efforts that are limited to the smaller or greater amendment of the standard Copenhagen interpretation proposed already in the 1920's. The founder of the consistent histories interpretation, Robert Griffiths asserts that the interpretation is just “Copenhagen done right”. The article has proven that Griffiths' doing “Copenhagen right” entails considerable reconstruction of the theory especially in regards to the understanding of quantum probability although the basic embedment within the Hilbert spaces remains unchanged. The concluding analysis of the consistent histories will be carried out according to the four levels of interpretation suggested by William Stoeger[32] as well as according to several additional interpretational criteria suggested by Roland Omnes and Chris Isham.

The first level of an interpretation given by Stoeger regards the physical theory itself. What is most basic in a theory such as classical physics and quantum mechanics is the definition of the state space. The consistent histories interpretation retains the use of Hilbert spaces to describe quantum states by means of projection operators. However, in standard Copenhagen regime, the quantum probability is clearly associated with the corresponding quantum state insofar as it specifies the probability of obtaining a single value of an observable from among all these that contribute to the linear combination of respective pure states before the onset of the wave vector collapse. This close relation between a state and a probability no longer holds in the consistent histories interpretation. Quantum histories as sequences of events – properties are in themselves nothing revolutionary because they always happen regardless of whether someone wishes to assign any formal meaning to them or no. However, since in the consistent histories interpretation one assigns probabilities to histories and not to single states, the connection between probability and state is blurred. Although such probability preserves the form of the square of an amplitude of a wave function as evidenced by Equation 16, its distinct correlation with an angle between two vectors in the Hilbert space cannot be visualized. Whether one could generalize this and assert that some hypothetical “angle” between two histories exists, is highly disputable. The consistent histories interpretation of quantum mechanics does not alter the quantum mechanical structure of space-time for like in the standard Copenhagen interpretation time remains a parameter with no corresponding quantum operator. Together with time and probabilities, the quantum mechanical space-time both in the Copenhagen and the consistent histories interpretation takes up the following form: $\Re^3$ x **T** x **p**.

The principal gain of such re-definition of probabilities becomes evident at Stoeger's second interpretational level. It reflects the coherence of an interpretation within the confines of the theory with special emphasis whether the interpretation secures proper conditions of its experimental verification. According to Roland Omnes, this is precisely what the standard Copenhagen does not do and that is why it is in need of correction. As it has been explained, the probability of a quantum history integrates in itself the entire series of properties of the system expressed by means of projection operators beginning at the very heart of the quantum world where the given quantum process commences all the way through the single datum obtained by a macroscopic measuring device. This supplies a consistent tool of linking the quantum formalism with an experiment whereby quantum mechanics receives internal criteria of its experimental verification. Moreover, the definition of probabilities based over an entire quantum history makes them an intrinsic part of the theory. It is a notable improvement on the part of this interpretation for the probabilities are no longer stuck at the intersection of two incompatible quantum procedures: the deterministic evolution of the state wave function according to the Schrödinger equation and the indeterministic collapse of the wave function in a measurement process. As a matter of fact, the consistent histories interpretation liquidates this incompatibility by making the evolution of the wave function as well as the measurement problem part of a universal description within one quantum history. In short, this is how this interpretation proposes to eliminate the main obstacle to the internal consistency of quantum mechanics that is, the measurement problem.

The achievement of this internal consistency eliminates the contents of the third level of interpretation suggested by Stoeger. It includes any auxiliary theories needed to make the quantum picture complete. Such efforts were necessary in case of the standard Copenhagen interpretation in the form of the measurement theory. Since the consistent histories interpretation 'absorbs' the measurement problem, it treats it within its own formalism and does not need assistance of any external theory. However, this situation will change in case of the possible future discussion of the theory of decoherence. Decoherence explains the continuity between the micro (quantum) and the macro (classical) regimes and thus provides a fundament *in re* for the uniform application of the universal method of interpretation by means of consistent histories that 'clamps' both these regimes. This issue remains beyond the scope of this article but it yields intriguing material for further analysis.

The fourth level of interpretation of a physical theory in Stoeger's hierarchy is philosophical in the proper sense of the word for it focuses on ontological and epistemological considerations. The division of attitudes towards quantum mechanics into antirealist and realist proposed by Chris Isham falls within this category. The exact qualification of the consistent histories interpretation within Isham's categories can be established first by noticing that Roland Omnes declares himself to be a dedicated realist who maintains that the objectively existing reality is the only object of scientific investigations. Secondly, the idea of Johann Neumann that the set of quantum laws should be applicable both for the microscopic and macroscopic realms underpins the consistent histories interpretation. Combined with the fact that the interpretation aims to provide a universal language of quantum discourse, it suggest that the existence of a distinct reality that underlies the quantum mechanism must be at least tacitly acknowledged. Indeed, Roland Omnes devotes some attention to the objectification of quantum theory. Objectification means that a datum obtained in an experiment on a quantum system must uniquely emerge from the microscopic regime. This is achieved via decoherence. Omnes states that "the notion of fact had no sense in pure quantum mechanics before the discovery of the decoherence effect". In the realist context of consistent histories, quantum mechanics is made immune to the subjective influence of a conscious observer and can be evenly applied to processes where measuring devices are not present (at distant galaxies). In other words, the bothersome assertion that it is the observer who kills the cat in the Schrödinger's Cat paradox looses its credibility.

The reality of complex quantum states described by the linear combination of simple wave vectors implies that nature is the source of another kind of logic governed by rules that are distinct in comparison to the standard Boolean formalism. This logic introduces a new category of propositions that is unknown within the classical regime, namely, these that are *meaningless (incompatible)*. This peculiar property finds its origin in the quantum mechanical complementarity. The ultimate aim of the consistent histories interpretation of quantum mechanics is the development of the universal logical framework of discourse about quantum processes. This involves the completion of the circle: *real – quantum formalism – real* that is supposed to assure the return from the level of the formalism back to the common sense perception. Thus the lost bridge between quantum abstractness and reality is reinstated. From the formal point of view, "ending up" in real means imposing the so called *consistency conditions* upon the quantum histories and their probabilities. By subjecting these probabilities to the classical regime of their additivity, one selects families of quantum histories that are mutually exclusive, that is, they fulfill the conditions of standard propositional logic and accordingly fit the common sense. Interestingly enough, Dowker and Kent have demonstrated the insufficiency of the consistency conditions in the selection of the real histories[33]. Yet, there arises a puzzling question at this point. The probabilities of quantum histories are founded upon the entire history of a given quantum process both in the quantum and classical frameworks. This 'classicization' of logic is acceptable for the classical part but it remains unclear what warrants its extension deep into the quantum realm ruled by a different set of logical principles. The following quote from a new book of a Nobel laureate in physics, Robert B. Laughlin, expresses these concerns somewhat provokingly and yet fittingly as a closing statement of this article:

Unhappily, the otherwordliness of quantum mechanics is a convenient justification for indulging in even more otherworldly "interpretations" of it that miss the forest for the trees. The convoluted nature of these arguments infatuates the undergraduates but annoys the rest of us because they boil down in the end to attempts to describe quantum mechanics in terms of behavior that emerges from it, rather the other way around. They are, in other words, symptoms of a failed world view. One tries to be nice about this, but the temptations to be mean are sometimes irresistible[34].

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[22] Since Omnes gives only an abbreviated description of the derivation of the final expression for the probability of a quantum history p_a (pages 145-147), the derivation presented below was carried through by the author of this thesis.

[23] Roland Omnes gives a more detailed justification of this replacement (pg. 147-148 of his *Understanding of Quantum Mechanics*) but due to its convoluted character, it will not be presented in this thesis.

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Comment by

Michael Heller
Philosophical Department
Philosophical Faculty PAT, Cracow
and Vatican Observatory

The paper by Wojciech P. Grygiel clearly presents the main idea of the consistent history interpretation of quantum mechanics, its advantages and some of its shortcomings. I will take the reading of this paper as an occasion to ponder on the question: why an interpretation of quantum mechanics at all?

It is natural that anything we do not understand calls for understanding, i.e., for an interpretation of some sort. In spite of its empirical successes and enormous mathematical elegance, quantum mechanics is a mysterious theory. Only if we agree that to understand in physics means to be accustomed to, we can claim that we understand this physical theory. We are accustomed to its various mathematical formulations, and we know very well how, with the help of them, make calculations leading (with enormous precision) to empirical predictions. But as soon as we try to translate what is going on into our mental images and our language, we are in trouble. And if we make an effort to sort it out, we are interpreting quantum mechanics.

We should not forget that early interpretations of quantum mechanics took their origin from the dispute whether it is a complete or incomplete physical theory. Einstein believed that it is a complete theory, and based his judgment on counterintuitive conclusions following from it. Bohr, on the contrary, was strongly inclined to believe that the world "in itself" is quantum and probabilistic and we better stick to the empirical predictions of the quantum theory without trying to enforce upon it our simplistic categories. Today *we know* that quantum mechanics is essentially an incomplete theory: it is but a "quantum mechanical sector" of a more general theory which should be a quantum gravity theory or even a quantum gravity theory coupled with a theory unifying all physical forces. And the evidence accumulates that this new theory, when finally formulated, will not rehabilitate our classical world view. The present search for such a theory is based on modifying either general relativity, the best theory of gravity we have, or quantum mechanics depending on preferences of a given author. However, it should be guessed that both these theories would truly be generalized to become "components" (perhaps nonlinearly mixed with each other) of the future "theory of everything". A discovery of this theory will entail a major conceptual revolution. Concepts do not evolve in isolation from the rest of physics. They undergo mutations and unexpected jumps only when involved in solving problems. No question can be correctly formulated until it is answered. When we solve problems, we answer questions, and only then we are able to see what we are looking for. General relativity and quantum mechanics should emerge from the final theory as its limiting cases. The chances are null that the present conceptual texture presupposed by general relativity and quantum mechanics could be accommodated by the structure of the future fundamental theory. Conceptual network of this theory will be vastly different from patterns of our every day thinking and speaking. We can suspect that such concepts as: nonlocality, entanglement, indeterminism, probability, and so on, are but shadows of their future generalized counterparts.

If the present quantum mechanics is incomplete then it is cut off from a larger structure, more or less as the top of an iceberg is cut off, for our eyes, from what is hidden in the water. It is quite natural that the strange behavior of the visible part cannot be fully understood without taking into account the behavior of the rest. To be sure, in the case of quantum mechanics we cannot hope, that its "invisible part" would behave in the classical manner. Even in its Bohmian version, the quantum world must be believed to possess strong nonlocal (nonclassical) properties. It often happens that if we contemplate a totality from the perspective of its part, we see only a projection of the whole into the plane of our horizon. Such a projection naturally produces discontinuities. I would agree with Roger Penrose that the state vector reduction (called also the wave function collapse) is a quantum gravity effect in the sense that it results from projecting a larger structure into its substructure that we call quantum mechanics.

How such a future theory could look like? Let us allow our imagination to wander along pathways drawn by present theories and models. In spite of their very different mathematical structures both general relativity and quantum mechanics can be formulated in terms of suitable algebras: general relativity in terms of the algebra $C^\infty(M)$ of smooth functions on a space-time manifold M, and quantum mechanics in terms of a $C*$-algebra. This allows us to expect that the future quantum gravity theory will also be expressed in an algebraic language. Hopf algebras or some other noncommutative algebras are currently the best candidates, but some their generalizations could not be excluded. Let us denote this "quantum gravity algebra" by A. If A is "reduced" to $C^\infty(M)$, it should reproduce general relativity; if A is "reduced" to a suitable $C*$-algebra C, it should reproduce quantum mechanics. In both these cases, the term "reproduce" can have different meaning. For instance, it could be understood as projecting on a certain space, representing in a certain space, restricting to the center, averaging, going to a limit... If A is a noncommutative algebra then, to obtain general relativity, we should expect to restrict it to its center (since $C^{\infty}(M)$) is a commutative algebra), and to obtain quantum mechanics, in its usual formulation, we should expect to represent A in a Hilbert space. It is hardly an exaggeration to say that the present quantum mechanics is but a theory of performing measurements of quantum magnitudes. It would be nice to show that the A-theory (quantum gravity), when narrowed to its measurement sector, will reproduce our present quantum mechanics. In such a situation we could expect that the A-theory provides some generalized dynamics which, when looked upon from the C perspective, looks like the usual unitary Heisenberg evolution. However, in the theory of measurement the $C^{\infty}(M)$ perspective must be involved as well. The state vector reduction could be the effect of an incompatibility of these two perspectives[1].

If in the above remarks there is a shade of truth, the consistent history interpretation of quantum mechanics is but another attempt to capture the whole structure from a partial perspective. It is from our point of view that we can speak about "otherworldliness of quantum mechanics", but in fact both our world and quantum mechanics are but shadows of the more fundamental reality. And all our struggles with interpreting quantum mechanics should not be regarded as "symptoms of failed world view", but rather as a claim for much broader vistas.

Castel Gandolfo, 20th of May, 2006.

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