The old and new Concepts of Physics
An open-dialogue journal

www.uni.lodz.pl/concepts

TACHYONS AND PSEUDOTACHYONIC RELATIVITY

Luís Filipe T. Dias Ferreira
Colégio Valsassina
Av. Teixeira da Mota, Quinta das Teresinhas
1959-010 Lisboa, Portugal
email: luisdiasferreira@clix.pt

(Received 15 March 2005; accepted 18 May 2006)

Abstract

I propose a reinterpretation of "Lorentz tachyonic transformations" — for $|v| > c$ — and subsequently the fundaments of a Pseudotachyonic Relativity.

We'll see that some usually "absurd results" become comprehensible. Besides some brief reflections on issues of time inversion and causality, the main ideas and conclusions presented here are:

- a tachyonic reference frame may only be detected as its "associated frame", which has lower-than-light velocity ($\hat{v} = c^2/v$) and in which time appears in an inverted flow;
- we cannot directly detect tachyons but instead their pseudotachyonic "images".
- information cannot be communicated in a faster-than-light way.

DOI: 10.2478/v10005-007-0003-1

In a future article, I intend to show how these new transformations naturally conduce to a strictly relativistic conception of antimatter: in brief, **an antiparticle is no more then the subluminal "image" of a tachyonic homologous particle**.

1 Introduction

1.1 The nature of tachyons

The problems involved in the consideration of *tachyons*, the hypothetical particles that move faster than light, have often puzzled investigators. Several questions arise, mainly over issues of *causality* and search of experimental evidence for the *existence* of these bizarre particles. Tachyons are still generally regarded as unphysical because of the apparent paradoxes, destabilisations and violations they seem to implicate and since one supposes that they have never been observed in nature[4].

My general aim is to show that, in truth, tachyons are physical and that they have been observed in nature: in fact, they can only appear to us in the form of *subluminal* – also said *bradyonic* – *antiparticles*, and by this I take the liberty of meaning particles not only of opposite charge but also with *negative energy*. The so-called *causality problems*, related to time paradoxes, must be faced from a new point of view, the principle of special relativity remaining as a true law of nature. Moreover, other interesting issues arise regarding antiparticles and their interaction with other particles.

The extension of the subject, however, makes it advisable to divide its presentation in separate articles.

In this first paper, we will not talk directly about tachyons, but about *tachyonic frames*, giving a new insight to the subject by a simple reinterpretation of Lorentz transformations for $|v| > c$. Consider a tachyon or even a physical tachyonic action; if we coherently assume the possibility of tachyonic frames to exist, then we may always find one in regard to which the particle or the action have a lower-than-light velocity.

Now, we may easily conclude that it is quite difficult to build upon specific experiments (even ideal ones, for instance concerning light beams) the fundaments of a tachyonic Relativity. We can't but have faith in the mathematical aspects of the problem and in severe logical reasons and criteria. On the other hand, the several inconsistencies or paradoxes, clearly pointed out since Langevin, that arise from the direct application of the usual relativistic transformations to tachyonic velocities give special significance to a *reinterpretation* of these transformations. As a result, tachyonic transformations will appear as "pseudotachyonic" ones and antiparticles as bradyonic "images" of tachyons.

1.2 Tachyonic and pseudotachyonic frames

First of all, one must remark that there isn't any scientific reason for excluding *a priori* the possibility of tachyonic frames to exist. In fact, the fundaments of the Theory of Special Relativity – the principle of relativity and the postulate of the constancy of the speed of light – don't forbid this possibility. They were indeed established by Einstein himself without any restriction concerning the relative velocity of the reference frames[1]. The *equivalence* enounced by these principles is based on the idea of symmetry in the mutual relationship between frames and in the fact that velocity (except for c) is a relative characteristic or condition; and there seems to be no motive to break this symmetry between frames, whatever their relative velocity is. I propose, then, to extend these principles to **all** inertial reference frames, considering all of them equivalent – and this will be crucial to understand or to deal with problems involving tachyonic frames and events.

The principle of equivalence immediatly means that we must not conceive a tachyonic frame as something quite peculiar, where strange things happen. On the contrary, it is as trivial as any bradyonic frame. Coordinates, for instance, physically mean exactly what they mean in our own reference frame or in any bradyonic one. It's very important to understand that **there is no fundamental difference in nature between our reference frame and a tachyonic one.** Equivalence also means that, logically, our reference frame is tachyonic relatively to that one; so, none is 'better' than the other. Bizarre results appear only when we try to **relate** both frames.

Consider the problem of relating the space-time coordinates of a certain event. This basic problem is resolved in classic terms through the well known Lorentz transformations. If we apply them straight to $|v| > c$, imaginary values appear, concerning the coordinates x and t. However, if we dare to assume the validity of these "Lorentz tachyonic transformations" – a procedure I justify further on –, the usual transformation rule for velocities,

$$u = \frac{u' + v}{1 + u'.v/c^2},$$

also remains valid and, extraordinarily, gives us real values for any value of v and u', taking into account its discontinuity point $u' = -c^2/v$. This happens because the imaginary unit of dx and dt is cancelled in their quotient. Yet, one may notice that, although the combined velocity is always real, some strange results come out, apparently opposite to intuition[6]. In truth, these results become quite comprehensible in the frame of the further presented theory – a subject I examine in Appendix B – and this is certainly a point to its credit.

Remark that, inversely, real values also appear for tachyonic velocities u' – if we accept their possibility – in a bradyonic frame with velocity v. As a mathematical function, the composition law $u = f(u', v)$ may be easily extended for u' outside the domain $(-c, c)$. Its graphic is an equilateral hyperbole, which, by the way, clearly displays a particularly worthy of notice fact: that is, the speed of light, c, **is not** a discontinuity point but instead a point of absolute continuity in the main branch of the hyperbole. Whether or not this extension is physically valid is a fundamental question, to which we may theoretically answer: yes, it is! – since we interpret the results in terms of the "pseudotachyonic transformations" I propose in the following paragraphs. This problem is the subject of Appendix B.

Add to these remarks, the composition rule for velocities allows the interchange of variables without modifying the result[6]: $f(u', v) = f(v, u')$. This physically means that the result for a bradyonic velocity u' in a tachyonic frame with velocity v is exactly the same then this tachyonic velocity v in a bradyonic frame with velocity u'. The point I really want to emphasize, in this preliminary approach, is that applying the above generalised transformation rule to *a tachyonic velocity in a subluminal frame* or applying it to *a subluminal velocity in a tachyonic frame* is equivalent! This is most satisfactory from a relativistic point of view and is one more argument to encourage us not to simply discard the possibility of tachyonic movements or reference frames.

Finally, allow me to signalise that the restriction usually *postulated* to the standard form of Lorentz transformations, and to Relativity itself, to the gap $(-c,\ c)$ is of a physical kind, not mathematical. However, this 'disagreement' between physical and mathematical arguments is, in my opinion, no less then troublesome. We should try to overcome it, and that's exactly what I propose in this paper.

We'll begin, now, by the resolution of the Lorentz transformations problem for $|\beta| > 1$ ($\beta = v/c$, v being the velocity of a tachyonic frame S" in relation to the reference frame S and in the direction of the xx axis). From a mathematical point of view, one must remark that in the deduction of the transformation formulae by Einstein, in his founding article[1], the restriction $|\beta| < 1$ has no relevance at all. We cannot say the same from the physical reasoning. In fact, the author accomplish his task having recourse to a beam of light emitted, reflected and received in the moving frame. In a first approach, it is quite evident that, if $v > c$, *in our judgement* the beam should never reach the point of reflection, if we suppose it put forward the frame origin, or would never be received if it's behind the origin. This is one of several symptoms, not exactly of the inexistence of tachyonic frames, but instead of the crucial fact that this kind of frames – and 'normal' events that take place in them – cannot be *directly detected* by a bradyonic one.

We may, however, deduce the standard Lorentz transformations by another path, the one normally used: the fundamental invariance of the Minkowski line element ds, given by the quadratic equation[2]

$$ds^2 = dx^2 + dy^2 + dz^2 + (i.c.dt)^2 .$$

We still use a light beam, in our 'mental experience', but we are no more compelled to consider its reflection; the beam just needs to reach a point P, which Cartesian coordinates are (x, y, z, t) and $(x'', y'' z'', t'')$ in each reference frame. Once again, the restriction $|\beta| < 1$ isn't of any relevance in the deduction of the formulae (except for a question related to signals plus or minus of a constant equal to ± 1, which causes no trouble because it can be resolved by a coherence criterion similar to the one I examine in Appendix A). Thus, we may assume that the standard form of Lorentz transformations must apply also to $|\beta| > 1$, ds remaining a fundamental invariant.

In a recapitulation, this basic assumption is mathematically justified by the fact that neither the deduction of the transformations, according to the principles of Relativity, nor the fundamental invariance of the Minkowski line element ds depend on the restriction $|\beta| < 1$. But, from a *physical point of view*, the assumption concerns a *reinterpretation* of Lorentz transformations – related to the fact, above-mentioned, that one cannot directly detect a tachyonic frame.

Certainly, $\sqrt{1-\beta^2}$ and the variables x'' and t'' assume imaginary values; it will be convenient to write the transformations formulae in the following form:

$$\begin{cases} x'' = \frac{x-\beta.c.t}{i.\alpha} = \frac{\beta.c.t-x}{\alpha}.i \\ y'' = y \\ z'' = z \\ t'' = \frac{t-x.\beta/c}{i.\alpha} \end{cases} \quad \text{in which } \alpha = \sqrt{\beta^2-1}. \qquad (1)$$

Of course, an immediate question arises: *what is the physicall meaning of the imaginary numbers for* x'' *and* t''*?* They clearly mean, I think, that the direct relationship between frames S and S" has no sense, is physically impossible. But then another question is: *how does Nature overcome this obstacle?*

We'll find an answer if we remark that in the space-time invariant ds,

$$ds^2 = dx^2 + dy^2 + dz^2 + (i.c.dt)^2\,,$$

its fourth term is $i.c.dt$, **not** dt. Concerning the space-time relativist conception, there is no abuse, then, in considering this term as the fundamental *fourth coordinate*; in fact, if we deal with problems in a tensorial approach, for instance with the space-time invariant in its generalised form, $ds^2 = \sum g_{ij}.dx^i.dx^j$, the fourth component of the space-time contravariant quadrivector is again $i.c.dt$. It's exactly the fact that its value is an imaginary number that distinguish it from the other ('spatial') coordinates. But notice that, for $|\beta| > 1$, according to (1), $i.c.t''$ results a real number, whilst x'' is an imaginary one. This is the fundament for the following interpretation: to achieve physical sense, the upper Lorentz transformations implicate an *interchange of coordinates:* x'' assumes the role of a 'time coordinate' whilst $s''_t = i.c.t''$ arises as a 'spatial coordinate'. Then, if we write

$$\begin{cases} |x^*| = |i.c.t''| \\ |i.c.t^*| = |x''|\,, \end{cases}$$

the modulus being justified by the quadratic form of the invariant ds, these new coordinates refer to a new but related frame, S*. In order to keep real values for all the measurable variables (time t included), respecting the principles of relativity, we shall therefore identify each point of space-time continuum not by the quadrivector (x'', y'', z'', ict'') but by its "image" (x^*, y'', z'', ict^*), according to the symmetrical conditions

$$\begin{cases} x^* = -i.c.t'' \\ i.c.t^* = -x'', \end{cases} \text{for } \beta > 1 \text{ and } \begin{cases} x^* = i.c.t'' \\ i.c.t^* = x'', \end{cases} \text{for } \beta < -1; \quad (2)$$

the option for the signal is resolved by a consistence criterion I examine in Appendix A. We will hence confine the analysis to $\beta > 1$; in this case, then,

$$\begin{cases} x^* = \frac{x.\beta - c.t}{\alpha} \\ y^* = y \\ z^* = z \\ t^* = \frac{x/c - \beta.t}{\alpha}. \end{cases} \quad (3)$$

We'll say that *S**, the "image" reference frame to which the quadrivector (x^*, y^*, z^*, ict^*) is referred to, and *S"* are **associated frames** – a relationship that must be independent from *S*. The precedent reasoning means that a tachyonic frame *S"* (and any event that takes place in it) can only 'manifest' itself, 'appear' or be detected in 'our' frame *S* as its associated one, *S**.

Remark that the fundamental invariance of the space-time interval ds remains in these (we may call them so) "Lorentz pseudotachyonic transformations". This result is implicit in the reasoning conducing to the symmetrical conditions (2); and, as a matter of fact,

$$\begin{aligned} &(ds^*)^2 = (dx^*)^2 + (dy^*)^2 + (dz^*)^2 - (c.dt^*)^2 = \\ &= (dy)^2 + (dz)^2 + \\ &+ \tfrac{1}{\alpha^2}.\left[(dx)^2.\beta^2 + c^2.(dt)^2 - 2v.dx.dt - (dx)^2 - v^2.(dt)^2 + 2v.dx.dt\right] = \\ &= (dy)^2 + (dz)^2 + \tfrac{1}{\alpha^2}.\left[(dx)^2.\left(\beta^2 - 1\right) - c^2.(dt)^2.\left(\beta^2 - 1\right)\right] = \\ &= (dy)^2 + (dz)^2 + \tfrac{\alpha^2}{\alpha^2}.\left[(dx)^2 - c^2.(dt)^2\right] = \\ &= (dx)^2 + (dy)^2 + (dz)^2 - (c.dt)^2 = (ds)^2. \end{aligned}$$

As mentioned before, this *mathematical solution* – in avoiding imaginary values for the measurable variables of space and time – is also a consistent *physical solution* to the problem of tachyonic reference frames. These new proposed transformations and others (derived from them or related to them) shall describe not exactly **tachyons** – which cannot be directly detected – but their **"image"** in a subluminal reference system; and this "image" is the only way we can *"see"* them.

Now, concerning the new frame S^*, we'll also say that it is a **pseudotachyonic frame**, since its velocity $\hat{v}$, in relation to S, is (as we'll see in section 2) bradyonic: $\hat{v} = c^2/v$. Attention, however! Despite this fact, S^* must not be taken by an ordinary bradyonic frame, because in it time flows in an opposite sense – and this is a remarkable feature! As a matter of fact, if we consider the usual bradyonic frame S' which velocity is $u = c^2/v$, equal to that of S^* (resulting $\beta_u = u/c = 1/\beta$), we may easily conclude that, transforming the quadrivector (x, y, z, ict) into both frames,

$$x^* = x', \quad y^* = y', \quad z^* = z', \quad t^* = -t'.$$

This ordinary bradyonic frame, S', will be named **paraframe** of S^*. It is very useful for reasoning and some comparisons.

Curiously, the inverse pseudotachyonic transformation (from S^* to S) is absolutely identical to the first:

$$\begin{cases} x = \frac{x^*.\beta - c.t^*}{\alpha} \\ t = \frac{x^*/c - \beta.t^*}{\alpha}. \end{cases} \tag{3.a}$$

In fact, the invariance of expressions for inverse transformations is a general rule in Pseudotachyonic Relativity. Going further, in a while, we'll understand the mathematical and physical justifications for this statement.

Finally, from (3) we deduce the next formulae:

$$\begin{cases} \alpha.\Delta x^* = \beta.\Delta x - c.\Delta t \\ \alpha.\Delta t^* = \frac{1}{c}.\Delta x - \beta.\Delta t. \end{cases} \tag{4}$$

So,

$$\Delta t = 0 : \quad \Delta x = \tfrac{\alpha}{\beta}.\Delta x^*, \quad \text{or} \quad \Delta x = \sqrt{1-(1/\beta)^2}.\Delta x^*; \tag{5}$$

this reveals an *identical length contraction* to the one verified relatively to *S'*, the paraframe of *S**. On the other hand,

$$\Delta x^* = 0 : \quad \Delta t = -\frac{\beta}{\alpha}.\Delta t^*, \quad \text{or} \quad \Delta t = -\frac{1}{\sqrt{1-(1/\beta)^2}}.\Delta t^*; \qquad (6)$$

here, we get exactly a *symmetrical time expansion* compared to the one verified relatively to *S'*. These results suggest, once again, that an *inversion of time* is occurring as far as pseudotachyonic frames are involved. The next subsection confirms this remarkable fact.

1.3 The experiment of a light beam

Consider the ideal experiment of a light beam sent in the positive sense of the xx axis, from one extremity (**A**) of a bar to the other (**B**) and then reflected in a mirror to **A**. An analysis of *common sense* leads to an irresolvable paradox: if the frame *S"* in which the bar and the mirror stand still has a faster-than-light velocity, the beam will never reach the mirror in **B** and, of course, will never be reflected. This violates the equivalence of the systems 'at rest' and 'in motion' and so it's unacceptable. We must be aware that this reasoning shows, in fact, the limits of common sense, this sort of 'quiet evidence' we should never take for granted. As in many other occasions, our difficulty to resolve a problem lies in the prejudices of our common sense, in which noose we often put our head in.

In truth, the paradox is simply one of the reasons why a tachyonic frame *(S")* cannot be detected as itself – with a faster-than-light velocity – but only as its subluminal associated one *(S*)*. Therefore, the resolution of this problem must be achieved trough the recourse to *S**, in which we'll consider the "images" of the bar and the mirror steady. The amazing result reveals a *time inversion* in all the process at the frame S; things happen in S as if the pseudotachyonic frame *S* 'was coming back in time'*.

In fact, let us consider the bar immobile in the associated frame *S**, being $A \equiv O^*$ [see figure 1, related to the frame *S*, for $\beta = 2$].

a) Emission (in **A**):

$$\begin{cases} x_1^* = 0 \\ t_1^* = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 0 \\ t_1 = 0 \end{cases} ;$$

b) Reflection (in **B**):

$$\left\{ \begin{array}{l} x_2^* = r^* \\ t_2^* = \frac{r*}{c} \end{array} \right. \Rightarrow \left\{ \begin{array}{l} x_2 = r^*.\sqrt{\frac{\beta-1}{\beta+1}} \\ \\ t_2 = -\frac{r^*}{c}.\sqrt{\frac{\beta-1}{\beta+1}} \end{array} \right. \quad ;$$

the second result means that there is a *time inversion* in the detection of the beam in S; the frame S^* *'is coming back in time'*, in the frame S. This comes to conclude that the photon "image" (an *antiphoton*) behaves like *'a photon that goes back in time'* (which, in fact, must be true to any kind of antiparticle).

Remark that the origin of S^*, at the instant t_2, A_2, has an x coordinate given by $x_{A_2} = c.t_2/\beta$; consequently, $r = \Delta x_2 = x_2 - x_{A_2} = \frac{\alpha}{\beta}.r^*$, and this represents the contraction of the bar in S.

c) Reception (in **C**):

$$\left\{ \begin{array}{l} x_3^* = 0 \\ t_3^* = \frac{2r^*}{c} \end{array} \right. \Rightarrow \left\{ \begin{array}{l} x_3 = -\frac{2r^*}{\alpha} \\ t_3 = -\frac{2r^*}{\alpha}.\frac{\beta}{c} \end{array} \right. \Rightarrow x_3 = \hat{v}.t_3;$$

this result has an obvious interpretation, since (as we'll see in the next section) $\hat{v} = c^2/v$ is the velocity of the frame S^* in relation to S. We may note, however, that the *inversion of time* remains in the detection of the beam in S.

As pointed out before, the resolution of this problem is accomplished at the cost of a phenomenon at the first sight bizarre: the experiment appears to the observer in S *in an inverted time flow:* the reception of the light beam in S^* corresponds to the emission of a light beam in S (in time $t_3 < t_1$); and the emission in S^* corresponds to the reception in S (in time t_1).

It's important to observe that this example is not very difficult to understand, or to intellectually accept, because we are dealing here with a reversible phenomenon. More general analysis include *irreversible phenomena* but can also be understood, revealing remarkable consequences, mainly the predictable *inversion of the second law of Thermodynamics* as far as antimatter is concerned.

2 Composition of velocities

In relation to the frame S^*, we may easily derive the equations for the composition of velocities from the transformation table (3).

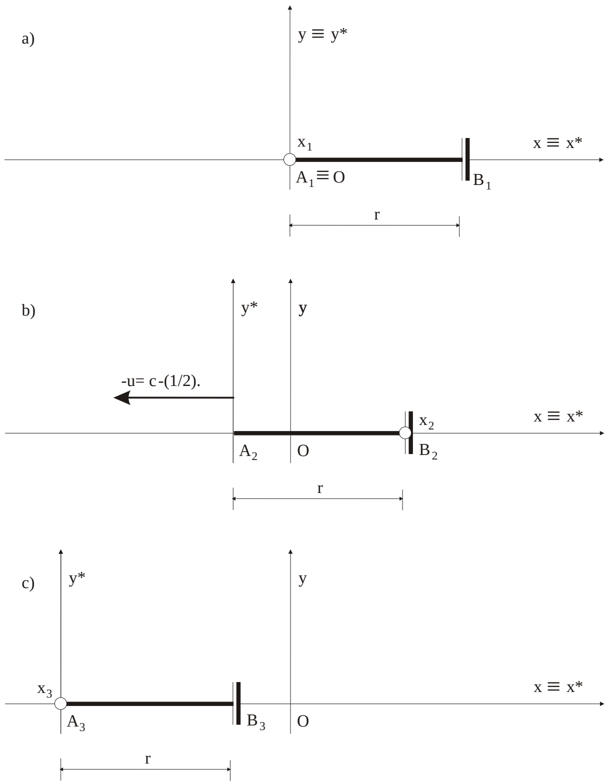

Figure 1: The experiment of a light beam

Taking into account that

$$\mathbf{u}_x^* = \frac{dx^*}{dt^*} = \frac{dx^*}{dt} \Big/ \frac{dt^*}{dt}; \quad \mathbf{u}_y^* = \frac{dy^*}{dt} \Big/ \frac{dt^*}{dt}; \quad \mathbf{u}_z^* = \frac{dz^*}{dt} \Big/ \frac{dt^*}{dt},$$

and also that

$$\frac{dt^*}{dt} = \frac{u_x - v}{\alpha.c},$$

the resulting equations are the following:

$$\begin{cases} u_x^* = \frac{u_x.\beta - c}{u_x - v}.c \\ u_y^* = \frac{u_y.\alpha}{u_x - v}.c \\ u_z^* = \frac{u_z.\alpha}{u_x - v}.c. \end{cases} \tag{7}$$

Remark that, if $\mathbf{u}'$ is the transformed velocity from $\mathbf{u}$ to *S'*, the above-mentioned S^* paraframe, then

$$\mathbf{u}_x^* = -\mathbf{u}_x'; \quad \mathbf{u}_y^* = -\mathbf{u}_y'; \quad \mathbf{u}_z^* = -\mathbf{u}_z'; \quad \Rightarrow \quad \mathbf{u}^* = -\mathbf{u}';$$

so, in a positive time flow, *the velocity vectors related to a pseudotachyonic frame and to its paraframe are symmetrical.*

Of course, the constancy of the speed of light is preserved in pseudotachyonic transformations – as one should expect, since it is a basic fundament of Special Relativity. However, in this case, we obtain the very important relation

$$\begin{cases} u = c \quad \Leftrightarrow \quad u^* = -c \\ u = -c \quad \Leftrightarrow \quad u^* = c. \end{cases} \tag{8}$$

Now, if the velocity $\mathbf{u}$ is parallel to the xx axis, we may write:

$$u^* = \frac{c^2 - u.v}{v - u} \quad \text{and, identically,} \quad u = \frac{c^2 - u^*.v}{v - u^*}. \tag{9}$$

We may also write both the equations (9) in a single one:

$$u.v + u^*.v - u.u^* - c^2 = 0, \tag{9.a}$$

which is (as in the bradyonic transformation case, studied in Appendix B) the equation of an equilateral hyperbole [see figure 2, for $v = 1.5c$].

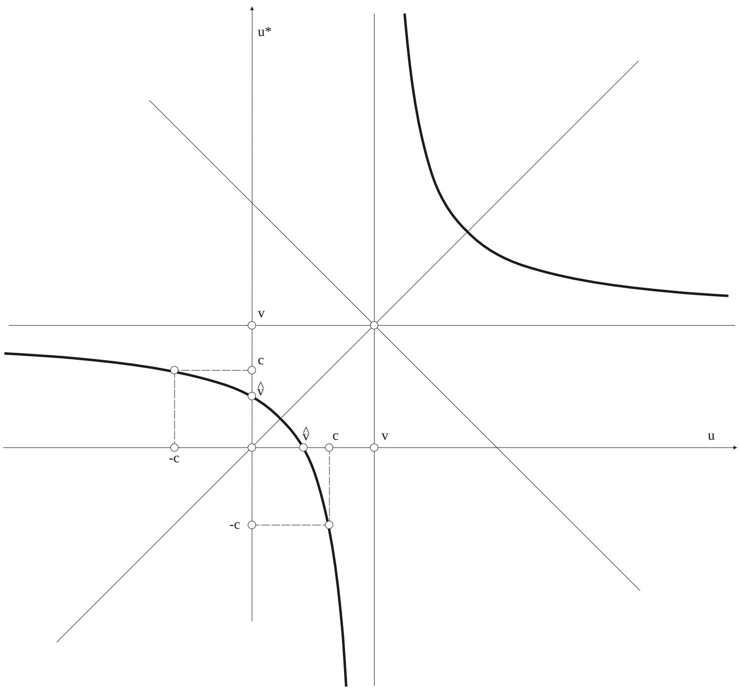

Figure 2: Velocity Pseudotachyonic Transformation

In particular, we may conclude that

$$u^* = 0 \quad \Leftrightarrow \quad u = \tfrac{c^2}{v}.$$

This means that the tachyonic frame *S"* must be detected in *S* as its associated frame *S**, which has a subluminal velocity $\hat{v}$ (read *"hat v"* or *"co-v"*) given by:

$$\hat{v} = \tfrac{c^2}{v} \quad \text{or} \quad v.\hat{v} = c^2. \tag{10}$$

Thus, if we write

$$\hat{\beta} = \hat{v}/c, \quad \text{then} \quad \hat{\beta} = 1/\beta.$$

We'll call this speed $\hat{v}$ the **associated velocity** or the **co-velocity** of v. It is also, as we have just seen, the *detection velocity* of *S"*.

Observe that, inversely, $u = 0 \Leftrightarrow u^* = \frac{c^2}{v}$; therefore, S is detected in *S** with the same velocity $\hat{v}$. This phenomenon, related to the above-mentioned invariance of the equations applied to inverse transformations, physically derive from the inversion of the time axis.

It is worthy of notice, besides, that if *S"* is tachyonic and *S** is its associated frame, then, according to the generalised transformation law, $u'' = (u - v) \,/\, \left(1 - u.v/c^2\right)$, and also to the one expressed by (9), we obtain for the same velocity u in S the relation

$$u''.u^* = -c^2;$$

so, if one of the two velocities is subluminal, the other is tachyonic. We'll say that u'' e u^* are **anti-associated velocities**. Remark that the frame S has, relatively to *S"*, a velocity $u'' = -v$. As a consequence, in relation to the *associated* *S**, the same frame S must, in fact, have a velocity $u^* = \hat{v}$ (which has just been pointed out and is the reason why the inverse transformations $S^* \rightarrow S$ and $S \rightarrow S^*$ behave in an identical manner).

One must keep in mind that the frame S is also tachyonic in relation to *S"*. It shall then be detected in *S"* as its (pseudotachyonic) associated frame, one we may symbolise by S° and which velocity will be equal to $-\hat{v}$. Therefore, the relationship $S^\circ \leftrightarrow S''$ *is not the same* than $S \leftrightarrow S^*$. This is an important issue that deserves quite a reflection and a deepen study.

3 Acceleration and force

We may continue the study of particles movement by the transformation of *acceleration*. If $\mathbf{a}^*$ is, in the frame S^*, the acceleration upon a particle that moves with velocity $\mathbf{u}^*$, $\mathbf{a}^*$ being defined by

$$a^* = \frac{du^*}{dt^*} = \frac{du^*}{dt}.\frac{dt}{dt^*} = \frac{du^*}{dt^*}.\frac{\sqrt{v^2 - c^2}}{u_x - v},$$

then, we obtain from equations (3) and (7), the following transformation table:

$$\begin{cases} a^*_x = a_x.\frac{\alpha^3.c^3}{(v-u_x)^3} \\ a^*_y = \frac{\alpha^2.c^2}{(v-u_x)^2}.\left(a_y + a_x.\frac{u_y}{v-u_x}\right) \\ a^*_z = \frac{\alpha^2.c^2}{(v-u_x)^2}.\left(a_z + a_x.\frac{u_y}{v-u_x}\right). \end{cases} \tag{11}$$

This transformation is, as usual, identical to its inverse. On the other hand, if *S'* is the o paraframe of *S**, it results

$$\mathbf{a}^*_x = \mathbf{a}'_x; \quad \mathbf{a}^*_y = \mathbf{a}'_y; \quad \mathbf{a}^*_z = \mathbf{a}'_z; \quad \Rightarrow \quad \mathbf{a}^* = \mathbf{a}';$$

If, at a certain instant, the velocity $\mathbf{u}$ is null, in the frame S, the equations above simplify considerably:

$$u = 0 \quad \Rightarrow \quad \begin{cases} a^*_x = a_x.\alpha^3/\beta^3 \\ a^*_y = a_y.\alpha^2/\beta^2 \\ a^*_z = a_z.\alpha^2/\beta^2. \end{cases}$$

Now, if $\mathbf{u}$ and $\mathbf{a}$ are both parallel to the xx axis, the same applies to $\mathbf{u}^*$ and $\mathbf{a}^*$ and then

$$a^* = a.\frac{\alpha^3.c^3}{(v-u)^3} = a.\frac{\left(v^2 - c^2\right)^{3/2}}{(v-u)^3}. \tag{12}$$

In what comes to the transformation of a force $\mathbf{F}$, defined by

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = \frac{d}{dt}(m.\mathbf{u}) = m.\mathbf{a} + \frac{dm}{dt}.\mathbf{u}m.\mathbf{a} + \frac{dm}{du}.a.\mathbf{u},$$

or, since $\frac{dm}{du} = \frac{d}{du}\left(\frac{m_0.c}{\sqrt{c^2-u^2}}\right) = \frac{m_0.c.u}{(c^2-u^2)^{3/2}} = \frac{m.u}{c^2-u^2}$,

$$\mathbf{F} = m.\mathbf{a} + \frac{m.u}{c^2 - u^2}.a.\mathbf{u},$$

this definition must be equally valid to S^* and to its paraframe S’, according to the principle of relativity. Thus, for the a velocity u measured in S, we obtain

$$\mathbf{F}' = m'.\mathbf{a}' + \frac{m'.u'}{c^2-u'^2}.a'.\mathbf{u}' \quad \text{and also} \quad \mathbf{F}^* = m^*.\mathbf{a}^* + \frac{m^*.u^*}{c^2-u^{*2}}.a^*.\mathbf{u}^*.$$

Remark that

$$\begin{cases} \mathbf{u}^* = -\mathbf{u}' \\ \mathbf{a}^* = \mathbf{a}' \\ m^* = -m'. \end{cases}$$

Concerning the last equality, one may prove — but I will not do it here — that, in fact, the pseudotachyonic transformation for mass is given by

$$m = -\frac{\beta}{\alpha}.m_0^*\,, \quad \text{or} \quad m = -\frac{m_0^*}{\sqrt{1-(1/\beta)^2}}\,;$$

therefore,

$$\mathbf{F}^* = (-m)\,.\mathbf{a}' + \frac{(-m').(-u')}{c^2-u'^2}.a'.\,(-\mathbf{u}') \quad \Rightarrow \quad \mathbf{F}^* = -\mathbf{F}'.$$

As a consequence, we may derive the tachyonic transformation for a force $\mathbf{F}$ from the respective subluminal transformation, changing the signal and replacing v for c^2/v; the result is

$$\begin{cases} F_x^* = -F_x + \left(\frac{u_y}{v-u_x}\right).F_y + \left(\frac{u_z}{v-u_x}\right).F_z \\ F_y^* = -\frac{\alpha.c}{v-u_x}.F_y \\ F_z^* = -\frac{\alpha.c}{v-u_x}.F_z. \end{cases} \tag{13}$$

If, in particular, the force is parallel to the xx axis, then $\mathbf{F}^* = -\mathbf{F}' = -\mathbf{F}$ This means that, concerning Pseudotachyonic Relativity, the transformation of a force $\mathbf{F}$ parallel to the xx axis is *anti-invariant.*

4 Transformation of the electromagnetic field

It is well known that the transformation of the electromagnetic field can be obtained trough a second order anti-symmetric tensor $\mathbf{F}_{ab} = \mathbf{F}^{ab}$, called *electromagnetic tensor*, which components are[3]

$$\mathbf{F}_{ab} = \begin{pmatrix} 0 & \mathbf{H}_z & -\mathbf{H}_y & -i.\mathbf{E}_x \\ -\mathbf{H}_z & 0 & \mathbf{H}_x & -i.\mathbf{E}_y \\ \mathbf{H}_y & -\mathbf{H}_x & 0 & -i.\mathbf{E}_z \\ i.\mathbf{E}_x & i.\mathbf{E}_y & i.\mathbf{E}_z & 0 \end{pmatrix}. \tag{14}$$

We are now interested in the pseudotachyonic transformation of the electromagnetic field. Since the tensor $\mathbf{F}_{ab}$ is anti-symmetric, it counts only 6 independent components and all the $\mathbf{F}_{aa}$ elements are necessarily null. The transformation law for a generic second order tensor is

$$\mathbf{T}'^{rs} = \sum_{ab} \mathbf{T}^{ab}.\frac{\partial x^{*r}}{\partial x^a}.\frac{\partial x^{*s}}{\partial x^b} \quad \text{or} \quad \mathbf{T}'^{rs} = \sum_{a} \frac{\partial x^{*r}}{\partial x^a}.\left(\sum_{b} \mathbf{T}^{ab}.\frac{\partial x^{*s}}{\partial x^b}\right).$$

We obtain from the equations (3) the partial derivatives:

$$\frac{\partial x^{*1}}{\partial x^1} = \frac{\beta}{\alpha}; \quad \frac{\partial x^{*4}}{\partial x^4} = -\frac{\beta}{\alpha}; \quad \frac{\partial x^{*1}}{\partial x^4} = \frac{\partial x^{*4}}{\partial x^1} = \frac{i}{\alpha}; \quad \frac{\partial x^{*2}}{\partial x^2} = \frac{\partial x^{*3}}{\partial x^3} = 1,$$

all the others being nulls; as a consequence, if $\mathbf{T}^{ab}$ is anti-symmetric, keeping in mind that $\mathbf{T}^{ba} = -\mathbf{T}^{ab}$ (which implies that $\mathbf{T}^{aa} = 0$) the final result is

$$\begin{array}{ll} \mathbf{T}^{*12} = \frac{\beta.\mathbf{T}^{12} - i.\mathbf{T}^{24}}{\alpha} & \mathbf{T}^{*14} = -\mathbf{T}^{14} = \\ \mathbf{T}^{*13} = \frac{\beta.\mathbf{T}^{13} - i.\mathbf{T}^{34}}{\alpha} & \mathbf{T}^{*24} = -\frac{\beta.\mathbf{T}^{24} + i.\mathbf{T}^{12}}{\alpha} \\ \mathbf{T}^{*23} = \mathbf{T}^{23} & \mathbf{T}^{*34} = -\frac{\beta.\mathbf{T}^{34} + i.\mathbf{T}^{13}}{\alpha}. \end{array} \tag{15}$$

Remark that, in relation to the S^* paraframe, *S'*, we'll have

$$\left\{ \begin{array}{l} \mathbf{T}^{*12} = \mathbf{T}'^{12} \\ \mathbf{T}^{*13} = \mathbf{T}'^{13} \\ \mathbf{T}^{*23} = \mathbf{T}'^{23} \end{array} \right. \quad \text{and} \quad \left\{ \begin{array}{l} \mathbf{T}^{*14} = -\mathbf{T}'^{14} \\ \mathbf{T}^{*24} = -\mathbf{T}'^{24} \\ \mathbf{T}^{*34} = -\mathbf{T}'^{34}. \end{array} \right.$$

Finally, if we apply the formulae (15) to the electromagnetic tensor (14), we obtain for the transformation of the components **E** and **H** the expressions:

$$\begin{cases} \mathbf{E}^*_x = -\mathbf{E}_x \\ \mathbf{E}^*_y = -\frac{\beta.\mathbf{E}_y - \mathbf{H}_z}{\alpha} \\ \mathbf{E}^*_z = -\frac{\beta.\mathbf{E}_z + \mathbf{H}_y}{\alpha} \end{cases} \quad \text{and} \quad \begin{cases} \mathbf{H}^*_x = \mathbf{H}_x \\ \mathbf{H}^*_y = \frac{\beta.\mathbf{H}_y + \mathbf{E}_z}{\alpha} \\ \mathbf{H}^*_z = \frac{\beta.\mathbf{H}_z - \mathbf{E}_y}{\alpha}. \end{cases} \tag{16}$$

Once again, note that, if *S'* is the paraframe of S^*, we'll have the following equalities:

$$\begin{cases} \mathbf{E}'_x = -\mathbf{E}^*_x \\ \mathbf{E}'_y = -\mathbf{E}^*_y \\ \mathbf{E}'_z = -\mathbf{E}^*_z \end{cases} \quad \text{and} \quad \begin{cases} \mathbf{H}'_x = \mathbf{H}^*_x \\ \mathbf{H}'_y = \mathbf{H}^*_y \\ \mathbf{H}'_z = \mathbf{H}^*_z. \end{cases}$$

This is valid, of course, for the frame S itself, which is the paraframe of S^* with velocity zero. We may see, then, that there is an interesting connexion between the components **E** and **H** for correspondent electromagnetic *radiation* and presumably *anti-radiation*: equal to **H**, they are symmetrical to **E**.

As one can confirm, and as usual, the inverse pseudotachyonic transformation is formally identical to (16):

$$\begin{cases} \mathbf{E}_x = -\mathbf{E}^*_x \\ \mathbf{E}_y = -\frac{\beta.\mathbf{E}^*_y - \mathbf{H}^*_z}{\alpha} \\ \mathbf{E}_z = -\frac{\beta.\mathbf{E}^*_z + \mathbf{H}^*_y}{\alpha} \end{cases} \quad \text{and} \quad \begin{cases} \mathbf{H}_x = \mathbf{H}_x \\ \mathbf{H}_y = \frac{\beta.\mathbf{H}^*_y + \mathbf{E}^*_z}{\alpha} \\ \mathbf{H}_z = \frac{\beta.\mathbf{H}^*_z - \mathbf{E}^*_y}{\alpha}. \end{cases} \tag{16.a}$$

5 Tachyonic information?

At last, we'll briefly study the crucial problem of information and communication. The classical question is: *can information be transmitted faster-than-light?* And the answer is: ***certainly not!***

This is one of the major problems theorists encounter when facing the "hypothetical tachyons". In fact, it is quite easy to conclude, *using the ordinary Relativity transformations*, that a tachyon travels back in time. Therefore, one supposes that tachyons, if they interact with ordinary particles (*bradyons*), may have the capacity of sending messages to the past — this possibility bringing an unobservable collapse of Nature! Langevin was perhaps the first to argue in this sense: if an event (**A**) is anterior to another one (**B**) in frame S $[\Delta t = t_B - t_A > 0]$, then, in certain circumstances — the distance between **A** and **B** being superior to the one covered by light in the laps of time Δt — there is the real possibility of finding a (subluminal) frame S' in which the event **B** is posterior to **A**: $\Delta t' = t'_B - t'_A < 0$. So, if there is a physical action that propagates faster-than-light, then the event **B**, supposed caused by **A** through this action, would be (in frame S') anterior to its cause, which is obviously absurd. To avoid this, Langevin postulated the speed of the light c as a limit to any speed, not only for frames or particles but also for the transmission velocity of any kind of physical action — that is to say, he postulated the non-existence not only of *tachyons* but of any *tachyonic action.*

Remark that, in his cogitations, Langevin seemingly never considered the possibility of S' to be a tachyonic frame. The analysis would be different and the result, perhaps, somewhat uncomfortable. The usual theory of tachyons also doesn't consider but subluminal frames and faster-than-light particles with imaginary energy or imaginary rest mass. Anyway, the question here is not exactly the existence or not of tachyons or tachyonic actions but the *possibility of influence* of such particles or actions.

We have concluded in this paper that any tachyonic frame, particle, wave or action cannot be *detected* as tachyonic ones but only as its associated ones, which have a lower-than-light velocity, $\hat{v} = c^2/v$. **We'll never be able to detect a tachyonic action but only its correspondent 'image'. Therefore, it's just these "anti-actions" (like antiparticles, the 'images' of tachyons) that can interact in our world.** Tachyonic entities cannot have any direct effect upon bradyonic ones (and vice-versa) because lengths, time lapses, energies or masses expressed by imaginary numbers are not measurable quantities, and so they have no physical signification. This is the profound meaning of the concept of *"detection"* often mentioned in this paper. Finally, in this way (and not by a postulate), the speed of the light, c, really constitutes a speed limit and the principle of special relativity remains a fundamental law of nature.

6 Conclusion

Instead of studying tachyons in traditional subluminal reference frames, we assumed the existence of tachyonic frames. By a reinterpretation of Lorentz transformations, this led us to the conclusion that a tachyonic frame (and particle) can only be detected as a subluminal one we may consider its 'image'. We studied the pseudotachyonic description of motion and of electromagnetic field and, finally, made a reflection on the problem of information/communication. We are now ready to go further and to apply these conclusions to the study of antiparticles, as bradyonic 'images' of tachyons.

A Appendix: the signal option

In section 1.2, I have justified why it is legitimate to assume that Lorentz transformations remain valid for $|\beta| > 1$. I'll discuss here the choice of signals in equations (2). The reasoning of Einstein himself[1] in deducing the formulae of those transformations, particularly the argument of symmetry, is also valid, from a mathematical point of view, if $|\beta| > 1$. As a consequence, we obtain the equations (1) for a tachyonic frame (S").

However, in this case, the interpretation I have proposed (the interchange of space-time coordinates) would leave us to accept, on principle, each of the two alternatives

$$\begin{cases} x^* = i.c.t'' \\ i.c.t^* = x'', \end{cases} \tag{A.1}$$

or

$$\begin{cases} x^* = -i.c.t'' \\ i.c.t^* = -x'', \end{cases} \tag{A.2}$$

since the equation of the invariant ds is quadratic. The non-symmetric alternatives

$$\begin{cases} x^* = i.c.t'' \\ i.c.t^* = -x'', \end{cases} \quad \text{or} \quad \begin{cases} x^* = -i.c.t'' \\ i.c.t^* = x'', \end{cases}$$

must be excluded because, if we consider one of them, this would introduce an arbitrary *difference in nature* between the two reference frames, S and S". Else, why choose the first transformation and not the second? The inconsistency of such choice and of the alternatives themselves is the reason why they are unacceptable. In brief, each alternative implicates a symmetry break which would violate the principle of relativity.

Remark that considerations, like those presented by Einstein, which are valid for bradyonic transformations, do not necessarily apply to pseudotachyonic transformations. For instance, all the *bradyonic* transformations must conduce to identities if we consider $v = 0$ (that is, $S' \equiv S$). This situation cannot obviously occur if S" is tachyonic. But then we can resort to a corresponding situation:

$$v \to \infty \qquad \Leftrightarrow \qquad \hat{v} \to 0,$$

in which S itself is the paraframe of the associated frame S^* (S^* is immobile in S). In this case, we must obtain the transformation for any variable simply by making $v \to \infty$ in the respective equation. The mentioned correspondence does not mean, as it is clear throughout this paper, that S^* is physically identical to S. In fact, mathematically, we'll not have necessarily identity-transformations; for instance, if we'll obtain $x_i^* = x_i$ (for $i \leq 3$), in the case of time it will be $t^* = -t$:

$$\begin{cases} x^* = \lim\limits_{v\to\infty} \frac{x.\beta - c.t}{\alpha} = \lim\limits_{v\to\infty} \frac{x - c/\beta.t}{\sqrt{1-1/\beta^2}} = x \\ y^* = p_y \\ z^* = p_z \\ t^* = \lim\limits_{v\to\infty} \frac{x/c - \beta.t}{\alpha} \lim\limits_{v\to\infty} \frac{x/(\beta.c) - t}{\sqrt{1-1/\beta^2}} = -t. \end{cases}$$

So, any transformation results either identical or anti-identical, according to what generally happens in the connexion between a pseudotachyonic frame and its paraframe. Therefore, the fundamental criterion to use in the choice of the signal option, (A.1) or (A.2), is the one that follows: **any transformation must be coherent in the limit in which S itself is the paraframe of S^*.** This specially implicates that, $\hat{v} = 0$ being the velocity of the frame S^*, the choice of the xx axis is totally arbitrary. So, the transformation of the component of any variable upon this space axis must be identical to the transformation upon the other two.

Consider $\beta > 1$. The option made in the alternative (A.2) satisfy this fundamental criterion. We saw it above for space and time coordinates; we can also see it, for instance, in the transformation of velocity:

$$\lim_{v\to\infty} \mathbf{u}_x^* = -\mathbf{u}_x; \quad \lim_{v\to\infty} \mathbf{u}_y^* = -\mathbf{u}_y; \quad \lim_{v\to\infty} \mathbf{u}_z^* = -\mathbf{u}_z; \quad \Rightarrow$$
$$\lim_{v\to\infty} \mathbf{u}^* = -\mathbf{u}.$$

Should we take the option (A.1), we would obtain symmetrical transformations for x* e de t*, in regard to (3):

$$\begin{cases} x^* = \frac{c.t-x.\beta}{\alpha} \\ t^* = \frac{\beta.t-x/c}{\alpha}. \end{cases}$$

but, in the limit in which S is the paraframe of S*,

$$x^* = -x, \quad y^* = y, \quad z^* = z, \quad t^* = t,$$

and also, for instance,

$$\mathbf{u}_x^* = -\mathbf{u}_x; \quad \mathbf{u}_y^* = \mathbf{u}_y; \quad \mathbf{u}_z^* = \mathbf{u}_z.$$

This would be incoherent because the transformation of the xx component of the variables is not identical to the transformation upon the other two axis. So, if instead we have considered the yy axis as the xx axis (that means, the axis to which the motion of S, with infinite speed, is parallel to) and vice-versa, the result would not be compatible with the one we obtained first. This is, of course, unacceptable.

Similar conclusions may be obtained concerning any pseudotachyonic transformation, even those not mentioned in this article. Anyway, these examples clearly mean that, for $\beta > 1$, we must keep as valid only the alternative (A.2).

It's not difficult to verify that, for $\beta < -1$, the situation is exactly the inverse. In this case it is the option (A.2) that leads to incoherencies. The correct alternative proves to be (A.1); it gives us the transformation laws

$$\begin{cases} x^* = \frac{c.t-x.\beta}{\alpha} \\ t^* = \frac{\beta.t-x/c}{\alpha}. \end{cases} \tag{A.3}$$

So, in the limit $\hat{v} = 0$, the result is

$$\begin{cases} \lim_{v\to-\infty} x^* \lim_{v\to-\infty} \frac{\beta.\left(\frac{c.t}{\beta} - x\right)}{|\beta|.\sqrt{1-(1/\beta)^2}} = x \\ \lim_{v\to-\infty} t^* \lim_{v\to-\infty} \frac{\beta.\left(t - \frac{x}{c.\beta}\right)}{|\beta|.\sqrt{1-(1/\beta)^2}} = -t. \end{cases}$$

We can easily obtain the equations for length contraction and time expansion:

$$\Delta x = -\tfrac{\alpha}{\beta}.\Delta x^*, \quad \text{or} \quad \Delta x = \sqrt{1-(1/\beta)^2}.\Delta x^*; \tag{A.4}$$

$$\Delta t = \tfrac{\beta}{\alpha}.\Delta t^*, \quad \text{or} \quad \Delta t = -\tfrac{1}{\sqrt{1-(1/\beta)^2}}.\Delta t^*; \tag{A.5}$$

the last ones being the very expressions (5) and (6) of the text.

According to (A.3), the law for the composition of velocities, in case of $\beta < -1$, is

$$\begin{cases} u_x^* = \frac{c-u_x.\beta}{v-u_x}.c \\ u_y^* = \frac{u_y.\alpha}{v-u_x}.c \\ u_z^* = \frac{u_z.\alpha}{v-u_x}.c. \end{cases} \tag{A.6}$$

The expressions $u^* = \frac{c^2-u.v}{v-u}$ and $u = \frac{c^2-u^*.v}{v-u^*}$ are still valid. From them we deduce that the detection velocity of S^* is, once again, $\hat{v} = c^2/v$ (negative, as v itself).

Finally, we must remark that the apparent incongruity in the choice of opposite signals [options (A.1) or (A.2)] relates to the following observations (mentioned in the text):

- Contrary to what happens in the connexion between bradyonic frames, the velocity of S^* in relation to S is also the velocity of S in relation to S^*: the associated velocity $\hat{v} = c^2/v$ (this fact being in the origin of the general invariance of expressions for inverse pseudotachyonic transformations);

- By the other hand, the frame S is also tachyonic in relation to S”, and so it must be detected at the frame S” as his associated S°, which velocity is equal to $-\hat{v}$. Therefore, the connexion $S^\circ \to S$ is not the same as $S \to S^*$; and, because of this, we cannot obtain the identity transformation, like we do in the bradyonic case, through a double transformation

$$S \xrightarrow{tr(v)} S' \xrightarrow{tr(-v)} S,$$

but (being $\hat{v} = c^2/v$) through each one of these transformations:

$$S \xrightarrow{tr(\hat{v})} S^* \xrightarrow{tr(\hat{v})} S, \qquad \text{or} \qquad S'' \xrightarrow{tr(\hat{v})} S^\circ \xrightarrow{tr(-\hat{v})} S''.$$

One must be aware that these conclusions are valid whatever is the option we consider, (A.1) or (A.2).

As a final conclusion, the incongruity in the choice of opposite signals is not only apparent but instead, once again, reveals a marvellous reality: if two alternatives are possible, Nature uses them both!

B Appendix: 'strange transformations' of velocities

The usual relativistic transformation rule for velocities,

$$u = \frac{u' + v}{1 + u'.v/c^2}, \tag{A}$$

gives us real values to any value of v and u', even for $|v| > c$, infinite ones for $u' = -c^2/v = -\hat{v}$, its discontinuity point. As pointed out in subsection 1.2, if $|v| > c$, this happens because the imaginary unit of dx and dt is cancelled in their quotient. However, as it has been already noticed, although the combined velocity is always real, even for bradyonic transformations ($|v| < c$), strange results appear, apparently opposite to intuition. For instance, Prof. António Brotas in his text *"An apparent inversion of time"*[6], notes:

> *"In the case of $u' > c$ and $v < c$, intuition makes us expect to find $u > u'$, in the same way it happens in the classical case and in the relativistic case for velocities u and v inferior to c. Yet, the calculus with formula (A) gives us $u < u'$. In spite of velocity u' is 'added up' to the transport velocity (v), the velocity in S is smaller!"*

And he adds:

> *"Obviously, these results, apparently opposite to intuition, cannot be really extraordinary, since they result from the ordinary transformation formulae to the coordinates of two frames which relative velocity is inferior to c. They look however quite strange and incomprehensible."*

These remarks fit in the following general statements (for $v > c$), which can be easily understood, from a straight mathematical point of view, by the study of the graphic in figure 3:

$$\begin{array}{lll} u' > c & \Rightarrow & u < u' \\ u' = c & \Rightarrow & u = u' \\ -c < u' < c & \Rightarrow & u > u' \text{ the usual gap} \\ u' = -c & \Rightarrow & u = u' \\ \hat{v} < u' < -c & \Rightarrow & u < u' \\ u' < -\hat{v} & \Rightarrow & u > u' \text{ and } u' > \hat{v}. \end{array}$$

The graphic of the usual composition rule $u = f(v, u')$ – for $|v| < c$ but allowing for u bradyonic or tachyonic velocities – is an equilateral hyperbole. It clearly displays the remarkable fact that the speed of light, $\pm c$, is not at all a discontinuity point, but a point of absolute continuity in the main branch of the hyperbole. We will study now some of the above-mentioned 'strange results'. Suppose a point **P** that is moving in the frame *S'* with **tachyonic velocity** u'. Of course, its transformed velocity, in *S*, will also be tachyonic.

1) If $u' > c$:

Really no wonder that it results $u < u'$; as a matter of fact, the *detection velocity* of **P** in *S'* is $\widehat{u'} = c^2/u'$ and so

$$u = \frac{c^2/\widehat{u'} + v}{1 - v/\widehat{u'}} = \frac{c^2 + v.\widehat{u'}}{\widehat{u'} + v} = c^2.\left(\frac{\widehat{u'} + v}{1 + v.\widehat{u'}/c^2}\right)^{-1}$$

or

$$u = c^2/\hat{u},$$

which corresponds to the following statement:

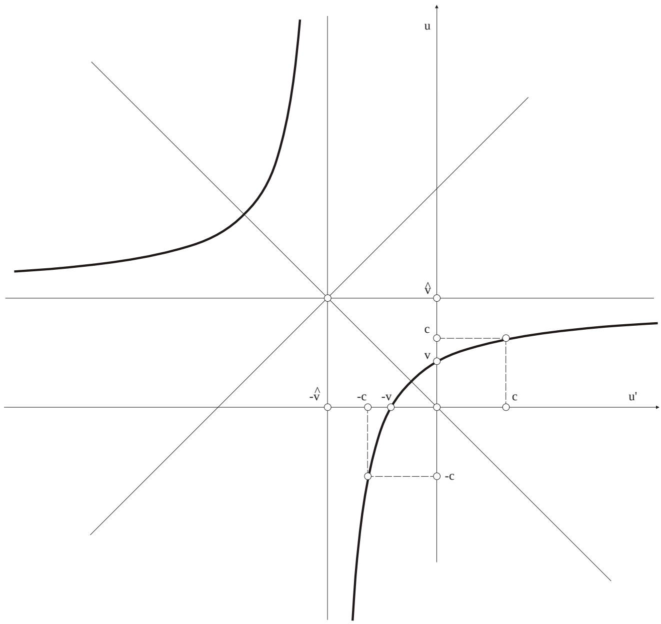

Figure 3: Velocity Bradyonic Transformation

the transformation of the co-velocity of u' is the co-velocity of the transformation of u'.

This proposition is also valid for pseudotachyonic transformations.

Now, the expression of $\hat{u}$ as a function of $\widehat{u'}$ is a banal transformation of bradyonic velocities; as a consequence, it must be (now, agreeing with intuition)

$$\hat{u} > \widehat{u'} \qquad \text{and, therefore,} \qquad u < u'.$$

An identical result may be obtained for the interval $-\hat{v} < u' < -c$.

2) The genuine discontinuity point of the function $u = f(v, u')$ is $u' = -\hat{v}$ (and similarly, in the inverse transformation, in $u = \hat{v}$). It is a tachyonic velocity corresponding to the associate velocity

$$\widehat{u'} = \tfrac{c^2}{u'} = \tfrac{c^2}{-\hat{v}} = -v \qquad \text{or, in } S, \qquad \hat{u} = 0;$$

in this case, then, the point **P** shall be *detected* in the frame S with a velocity null (which co-velocity is infinite – the asymptotic limit of $u = f(v, u')$).

3) To the values under $u' = -\hat{v}$, the velocity of P decreases and above $u' = c$ it increases, in both cases with the same asymptotic limit:

$$\lim_{u' \to 0} u = c^2/v = \hat{v}.$$

The limit to the detection velocity of **P** is, then, the associated velocity of tachyonic $\hat{v}$ – that is to say, the velocity v of the frame *S'* itself! This means that, in the frame *S'*, **P** will be detected as an immobile point; the reason to this lies in the mathematical fact that, not only in the frame S but also in *S'* (according to the principle of relativity), each velocity u and its associated one, $\hat{u}$, are inversely proportional.

References

[1] Hendrik A. Lorentz, Albert Einstein e Hermann Minkowski, *The Principle of Relativity* - Portuguese translation, page 55 to 62, Calouste Gulbenkian Foundation, Lisbon, 1983, 3th edition.

[2] Alonso & Finn, *Fundamental University Physics*, 1967 Addison-Wesley Publishing Company, Brasilian edition, 1972 Editora Edgar Blucher Lda.

[3] António da Silveira,*Teoria da Electricidade (Theory of Electricity) 2nd part*, National Institute for Scientific Research, 1985, re-edition, Portugal.

[4] Mark Davidson, *Motions of Classical Charged Tachyons*, Ph. Essays, Vol. 14, N. 1, March 2001.

[5] Alain Haché, *Dépasser la vitesse de la lumière*, Les Dossiers de La Recherche, n 18: L'héritage Einstein, 1905-2005, un siècle de physique, February-April 2005.

[6] António Brotas, *Uma aparente inversão do tempo (An apparent inversion of time)* and *An extension of the Lorentz Group*, unpublished papers, I think, the author kindly sent me in July 1993.

Comment by

Edward Kapuścik
Department of Physics, University of Łódź
Henryk Niewodniczański Institute of Nuclear Physics PAN
Kraków, Poland

The paper "Tachyons and Pseudotayonic Relativity" at first sight looks very strange. The Author's considerations look however less strange after observing that his "Lorentz transformations" may be obtained from the standard Einstein-Bondi radiolocation method[1],[2]. In fact, following this method in each inertial reference frame, the space and time coordinates are defined from two instants of time measured by the observer at the origin of the coordinate system. Denoting by T_1 the emission time of the light signal and by T_2 the detection time of the reflected signal, the distance from the observer to the event where the signal was reflected is given by

$$x = \frac{c}{2}\left(T_2 - T_1\right), \tag{17}$$

where c is the universal velocity of light and to the moment of reflection the instant of time

$$t = \frac{T_2 + T_1}{2} \tag{18}$$

is ascribed. The emission and reflection times for two different observers are connected by the relations

$$T_1' = kT_1, \qquad T_2' = k^{-1}T_2, \tag{19}$$

where $k > 0$ is usually related to the relative velocity of the two observers.

From (17), (18) and (19) we get the Lorentz transformations in the form

$$x' = \frac{\left(k^2+1\right)x - \left(k^2-1\right)ct}{2k} \tag{20}$$

and

$$t' = \frac{\left(k^2+1\right)t - \left(k^2-1\right)\frac{x}{c}}{2k}. \tag{21}$$

In the standard approach to special relativity the relative velocity of two observers is defined by the motion of the primed observer (for whom $x' = 0$) described in the reference system of the unprimed observer. From (20) this motion is described by the trajectory

$$x_0\left(t\right) = c\frac{k^2-1}{k^2+1}t \equiv ut, \tag{22}$$

where u is the relative velocity of two observers. It is clear that from such a definition we have

$$-c < u < +c \tag{23}$$

and expressing k in terms of the velocity u we get the standard form of the Lorentz transformations

$$x' = \frac{x - ut}{\sqrt{1-\frac{u^2}{c^2}}} \tag{24}$$

and

$$t' = \frac{t - \frac{u}{c^2}x}{\sqrt{1 - \frac{u^2}{c^2}}}. \tag{25}$$

We may however proceed also in a nonstandard way defining a parameter β in the following way

$$\beta = \frac{k^2 + 1}{k^2 - 1}. \tag{26}$$

Clearly, for $k^2 < 1$ we have

$$-\infty < \beta < -1, \tag{27}$$

while for $k^2 > 1$ we have

$$1 < \beta < +\infty. \tag{28}$$

Expressing now the parameter k in terms of β we have

$$k = \pm\sqrt{\frac{\beta + 1}{\beta - 1}}. \tag{29}$$

Substituting this expression for k into eqs. (20) and (21) we get the two forms of the Author's transformations

$$x' = \frac{\beta x - ct}{\sqrt{\beta^2 - 1}}, \tag{30}$$

$$t' = \frac{\frac{x}{c} - \beta t}{\sqrt{\beta^2 - 1}} \tag{31}$$

and

$$x' = \frac{ct - \beta x}{\sqrt{\beta^2 - 1}}, \tag{32}$$

$$t' = \frac{\beta t - \frac{x}{c}}{\sqrt{\beta^2 - 1}}, \tag{33}$$

provided we extend the validity of the relations (19) also for $k < 0$.

Therefore, the Author could start his considerations directly from these formulas without any excursion to complex numbers. Also the group theoretical properties of standard Lorentz transformations and of the Author's ones follow from relations (19) because after twice application of such relations we get

$$k\,(1,2) = k\,(1)\,k\,(2)\,, \tag{34}$$

where $k\,(1)$ and $k\,(2)$ are the corresponding k-parameters for the two subsequent relations (19) and $k\,(1,2)$ is the k-parameter for the resulting composed relation.

The Einstein-Bondi radiolocation method can be generalized by replacing relations (19) by more general ones of the form

$$T_1' = k_1 T_1, \qquad T_2' = k_2 T_2, \tag{35}$$

where k_1 and k_2 are two a priori independent parameters. Clearly, the Einstein case corresponds to the particular relation $k_2 = k_1^{-1}$. The corresponding generalized Lorentz transformations have then the form

$$x' = \frac{x\,(k_1 + k_2) + ct\,(k_2 - k_1)}{2}, \tag{36}$$

$$t' = \frac{t\,(k_1 + k_2) + c^{-1}x\,(k_2 - k_1)}{}. \tag{37}$$

The relative velocity u of the two observers is now given by

$$u = c\frac{k_1 - k_2}{k_1 + k_2}. \tag{38}$$

This relation allows to represent the generalized Lorentz transformations (36) and (37) in the following way. First, let us express one of the k-parameter (let say k_1) in terms of the other one and the velocity u getting the relation

$$k_1 = k_2\frac{c + u}{c - u}. \tag{39}$$

Second, let us assume that the second k-parameter (in our case k_2) is a function of the relative velocity u. Then the group property (34) valid for each k-parameter separately leads to the standard relativistic composition law for the relative velocities of inertial observers and to functional equation for the function $k_2\,(u)$ of the form

$$k_2\left(\frac{u_1 + u_2}{1 + \frac{u_1 u_2}{c^2}}\right) = k_2\,(u_1)\,k_2\,(u_2)\,. \tag{40}$$

The general solution of this functional equation is of the form

$$k_2(u) = \left(\frac{1+\frac{u}{c}}{1-\frac{u}{c}}\right)^{\lambda}, \tag{41}$$

where λ is arbitrary and we end up with more general Lorentz transformations of the type

$$x' = \frac{\left(1+\frac{u}{c}\right)^{\lambda-1}}{\left(1-\frac{u}{c}\right)^{\lambda}}(x-ut), \tag{42}$$

$$t' = \frac{\left(1+\frac{u}{c}\right)^{\lambda-1}}{\left(1-\frac{u}{c}\right)^{\lambda}}\left(t-\frac{u}{c^2}x\right). \tag{43}$$

The standard Lorentz transformations (24) and (25) are obtained for $\lambda = \frac{1}{2}$ and from (39) and (41) we see that in this case indeed $k_2 = k_1^{-1}$.

A similar considerations may be performed for the generalized parameter

$$\beta = \frac{k_1+k_2}{k_1-k_2} \tag{44}$$

instead of the parameter u introduced by (38). We may however go further and consider the Einstein-Bondi method in the following more general framework. It is well known that the Einstein-Bondi synchronization condition (18) was criticized by H. Reichenbach[3] who advocated Special Relativity with unequal forward and backward velocities of light. It is easy see that the Einstein-Bondi method with two k-parameters is well adapted for such version of Special Relativity[4]. It is also possible to develop Special Relativity without introducing the concepts of light velocity *a priori* [5]. In fact, let us assume that light beams travel is described by some monotonic function of time $f(t)$ which satisfy the obvious condition

$$f(0) = 0. \tag{45}$$

Then, assuming that the light is behaving in the same way forward and backward we get the synchronization condition in the form

$$f(t-T_1) = f(T_2-t), \tag{46}$$

where the meanings of T_1, T_2 and t are the same as before we described for the Einstein-Bondi method. However, a monotonic function takes the same values only for equal arguments and from this fact the particular Einstein synchronization of clocks (18) immediately follows. The distance to the reflecting mirror is however now given by the formula

$$x = f\left(\frac{T_2 - T_1}{2}\right). \tag{47}$$

Introducing now the inverse function $g(x)$ we get the relation

$$T_2 - T_1 = 2g(x), \tag{48}$$

which together with (18) leads to

$$T_1 = t - g(x), \tag{49}$$

and

$$T_2 = t + g(x). \tag{50}$$

Any other observer who applies the same procedure will get formulas

$$x' = f\left(\frac{T'_2 - T'_1}{2}\right), \tag{51}$$

$$t' = \frac{T'_1 + T'_2}{2} \tag{52}$$

and

$$T'_1 = t' - g(x'), \qquad T'_2 = t' + g(x'). \tag{53}$$

Assuming now the same as previous relations (35) we get the following general form of the transformation rules for spacetime coordinates

$$x' = f\left(\frac{(k_1 + k_2)\, g(x) - (k_1 - k_2)\, t}{2}\right), \tag{54}$$

$$t' = \frac{(k_1 + k_2)\, t - (k_1 - k_2)\, g(x)}{2}. \tag{55}$$

Since the primed observer in his reference frame has the coordinate $x' = 0$ its trajectory in the unprimed frame will be given by the solution of the equation

$$f\left(\frac{(k_1+k_2)\,g\,(x_0\,(t))-(k_1-k_2)\,t}{2}\right)=0 \tag{56}$$

and from the condition (45) we get

$$(k_1+k_2)\,g\,(x_0\,(t))-(k_1-k_2)\,t=0. \tag{57}$$

Thus, the light beams travel according to the general law

$$g\,(x_0\,(t))=\frac{k_1-k_2}{k_1+k_2}t, \tag{58}$$

or

$$x_0\,(t)=f\left(\frac{k_1-k_2}{k_1+k_2}t\right). \tag{59}$$

Clearly, this result shows that we can develop Special Relativity without any assumption on the velocity of light. In particular, the discussion of the tachyonic physics may be quite different.

References

[1] A. Einstein; *Ann. Phys.* **17** (1905) 891

[2] H. Bondi; in *Brandeis Lectures,* vol.1, Englewood Cliffs, p. 386, 1964

[3] H. Reichenbach; *The Philosophy of Space and Time,* Dover, New York, 1957. For more recent discussion see: R. Anderson et al; Phys. Rep **295** (1998) 93-180, H. C. Ohanian; *Am. J. Phys.* **72** (2004) 141-148, A. Macdonald; *Am. J. Phys.* **73** (2005) 454-455, A. A. Martinez; *Am. J. Phys.* **73** (2005) 452-454

[4] D. Wcisło; *PhD Dissertation,* Krakow, 2006

[5] A. Horzela, E. Kapuścik and J. Kempczyński; *Phys. Essays* **5** (1992) 478