

MICHELSON–MORLEY EXPERIMENT REVISITED*

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Abstract

The idea of the Michelson–Morley experiment is theoretically reanalyzed. Elementary arguments are put forward to precisely derive the most general allowable form of the directional dependence of the one-way velocity of light.

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According to XIX-century physics light was supposed to propagate in the aether, a mysterious medium devised especially for this purpose. As light was to travel with respect to the aether with a fixed velocity, an experiment was suggested to detect the dependence of the velocity on the direction in a moving frame of the laboratory on the Earth. The experiment was proposed and the first time performed by Albert Michelson (a Nobel laureate, American physicist born in Poland) [1]. The experiment is continually being repeated, known as the Michelson–Morley (MM) experiment, with ever increasing accuracy and improved technical realization (see, e.g. [2]). The result is always the same, negative, i.e. no dependence of the velocity of light on the direction has been ever detected, at least, this is a generally accepted (but sometimes disputable [3]) conclusion. As a (standard) consequence, the velocity of light is the same in each inertial frame, in any direction, and no aether exists.

Strictly speaking, the experimental results are usually translated into the theoretical statement saying that the *average* velocity of light around any closed path is constant and equal to the universal constant c . According to [4] (see also Chapt. 1.3 in [5]) this bound yields the following directional (angular) dependence of the one-way velocity of light

$$c(\theta) = \frac{c}{1 - \lambda \cos \theta}, \quad (1)$$

where θ is the angle between the direction of light and “the direction of anisotropy”, and λ is a parameter belonging to the interval from 0 to 1.

An elementary theoretical analysis of the actual MM experiments, proposed in this paper, yields a more general solution. Among other things, we show that the negative result of the MM experiments imposes milder constraints on the directional dependence (anisotropy) of the one-way velocity of light. In particular, the statement that the average velocity of light along any closed path is constant is too strong from our point of view because it does not strictly follow from actual MM experiments.

Let us briefly recall the idea of the MM experiment. In any, traditional or modern, version of the experiment we compare the differences of the time of the travel of light in two orthogonal directions (vertical and horizontal, say) in two positions (primary and final)

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differing by 90° . To arrive at our result we assume full generality. Therefore, let us introduce the following notation (see Fig. 1):

- c_{\pm}^{\perp} — forward vertical velocity of light,
 - c_{\pm}^{\parallel} — backward vertical velocity of light,
 - $c_{\pm}^{\prime\prime}$ — forward horizontal velocity of light,
 - c_{\pm}^{\prime} — backward horizontal velocity of light,
- and
- L_1 — primarily vertical route,
 - L_2 — primarily horizontal route.

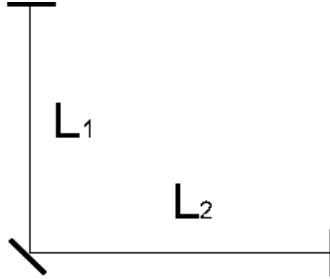


Figure 1: The Michelson–Morley interferometer.

In primary position, the vertical travel of light takes

$$t_1 = \frac{L_1}{c_{+}^{\perp}} + \frac{L_1}{c_{-}^{\perp}}, \quad (2)$$

whereas in horizontal direction

$$t_2 = \frac{L_2}{c_{+}^{\prime\prime}} + \frac{L_2}{c_{-}^{\prime\prime}}. \quad (3)$$

The difference is

$$\Delta t = t_2 - t_1 = L_2 \left(\frac{1}{c_{+}^{\prime\prime}} + \frac{1}{c_{-}^{\prime\prime}} \right) - L_1 \left(\frac{1}{c_{+}^{\perp}} + \frac{1}{c_{-}^{\perp}} \right). \quad (4)$$

After rotation, we have

$$\Delta t' = L_2 \left(\frac{1}{c_{+}^{\perp}} + \frac{1}{c_{-}^{\perp}} \right) - L_1 \left(\frac{1}{c_{+}^{\prime\prime}} + \frac{1}{c_{-}^{\prime\prime}} \right). \quad (5)$$

The change one could possibly observe is of the form

$$\Delta t^* = \Delta t' - \Delta t = (L_1 + L_2) \left[\left(\frac{1}{c_{\pm}^{\perp}} + \frac{1}{c_{\pm}^{\parallel}} \right) - \left(\frac{1}{c_{+}^{\parallel}} + \frac{1}{c_{-}^{\parallel}} \right) \right]. \quad (6)$$

The negative result of the MM experiment formally means $\Delta t^* = 0$. (Our analysis is purely theoretical, and we are not going to engage into the debate whether the equality $\Delta t^* = 0$ is experimentally well-established or not, see, e.g. [3].) A very simplified argumentation would say that the equality $\Delta t^* = 0$ implies $c_{\pm}^{\perp} = c_{\pm}^{\parallel} = c_{+}^{\parallel} = c_{-}^{\parallel} = c$. A bit less simplified argumentation presumes a strictly defined form of c_{\pm}^{\perp} , c_{\pm}^{\parallel} following from geometrical analysis (the Pythagorean theorem and geometrical addition of velocities) of the movement of inertial systems with respect to the aether, i.e.

$$c_{\pm}^{\perp} = \sqrt{c^2 - v^2} \quad (7)$$

and

$$c_{\pm}^{\parallel} = c \mp v, \quad (8)$$

where c — velocity of light with respect to the aether, v — velocity of the inertial system with respect to the aether. The possibility (7)-(8) is also excluded by virtue of the experimental fact $\Delta t^* = 0$, where Δt^* is defined by Eq. (6). But there are infinitely many other possibilities consistent with Δt^* defined by Eq. (6) equal to zero (a particular subset of the possibilities is given by Eq. (1)). The aim of the paper is to quantify this fact.

We can rewrite the equation $\Delta t^* = 0$ with Δt^* defined by Eq. (6) in the following form

$$z_{+}^{\perp} + z_{-}^{\perp} = z_{+}^{\parallel} + z_{-}^{\parallel}, \quad (9)$$

where for simplicity we use inverses of the corresponding velocities, $z_{+,-}^{\perp,\parallel} \equiv 1/c_{+,-}^{\perp,\parallel}$. Eq. (9) is a functional equation with continuum of solutions.

Let us consider the two-dimensional (unphysical) case, first. The simplest way to solve the problem in two dimensions is to analyze a picture (see, Fig. 2). The length of the vector \vec{z} in Fig. 2 corresponds to the inverse of the one-way velocity of light in the direction of \vec{z} . Thus, e.g. the circle corresponds to a constant (independent of the

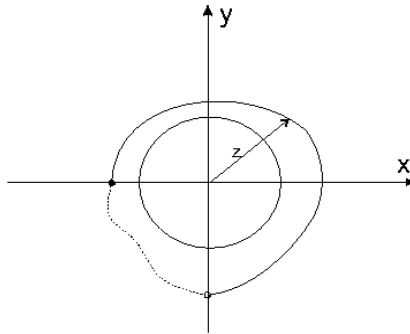


Figure 2: A general solution of the two-dimensional problem, i.e., a solution of the functional equation (9).

direction) velocity of light. Since Eq. (9) is a single equation with four unknowns, the three unknowns are arbitrary and determine the fourth one. For example, the three coordinates: of the black dot, of the intersections of the solid curve with positive y and positive x axes respectively are arbitrary, and they determine the coordinate of the white dot satisfying Eq. (9). Analogously, the whole solid curve in Fig. 2 is practically arbitrary (it should only be unique as a continuous function of the angle) and determines the (dotted) segment with negative coordinates x and y . As a side remark, we observe that changing the angle between the arms of the MM interferometer (usually it is 90°) changes the dotted segment. Namely, the angle between the vectors \vec{z} pointing at the black dot and the white dot, respectively, is equal to the greater angle between the arms of the interferometer. In principle, the whole curve could be discontinuous in two dotted points. But we can easily avoid this possibility by appropriate deformation of the primary solid curve. We could also require the mirror symmetry of the curve. For example, the axis of the symmetry could be interpreted as the direction of the movement of the laboratory frame, in the spirit of the aether philosophy. A general solution would be determined by an arbitrary solid curve (see Fig. 3) starting at the black dot (as in Fig. 2) and terminating at the inter-

section with the line $y = x$. Mirror reflection with respect to the line $y = x$ reproduces the rest of the curve except the last quarter, which should be constructed according to (9) as described earlier in this paragraph. One can easily check that, thanks to the mirror symmetry, this time, the whole curve is automatically continuous provided the primary segment is continuous.

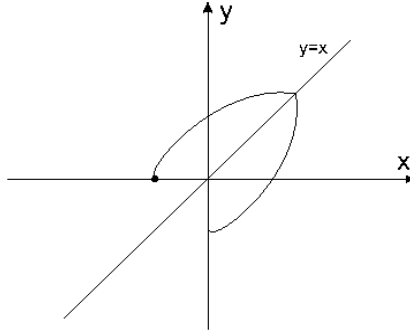


Figure 3: A general axially symmetric two-dimensional solution.

Up to now we have been considering a (n unphysical) two-dimensional construction. It appears that in three dimensions constraints are a bit stronger, and the three-dimensional case is qualitatively quite different. Therefore, we present now an explicit three-dimensional analysis. We will determine directional dependence of the one-way velocity of light consistent with MM-type experiments. Thus, roughly, we are interested in (generally) non-constant (continuous) functions on a sphere \mathcal{S}^2 , $z(x) \neq const$, defining (the inverse of) the velocity of light in the direction associated to $x \in \mathcal{S}^2$, i.e., if we place the interferometer in the center of the interior of \mathcal{S}^2 , the values of the all four z 's (z_{\pm}^{\perp} , z_{\pm}^{\parallel}) are given by the values of the function $z(x)$ for x belonging to the points of \mathcal{S}^2 corresponding to appropriate axes of the interferometer. The only constraint for the values of $z(x)$ is given by Eq. (9). For any configuration of the interferometer two z 's correspond to the upper hemisphere of \mathcal{S}^2 and the other two to the lower one. The two upper points uniquely determine the positions of the other two. Therefore, we can confine ourselves to consideration

of the two points on the upper hemisphere. We can project the upper hemisphere onto a two-dimensional disc \mathcal{D}^2 . Let us now define an auxiliary function on \mathcal{D}^2 , $\bar{z}(y)$, $y \in \mathcal{D}^2$, where informally $\bar{z} = z_+ + z_-$, i.e. the value of \bar{z} is the sum of the opposite z 's. It is easy to show that \bar{z} has to be a constant function on \mathcal{D}^2 . It indirectly follows from Eq. (9) by virtue of transitivity. Namely, we can connect arbitrary two points B and C on \mathcal{D}^2 , and compare the corresponding values of \bar{z} using an additional auxiliary point A, and next apply Eq. (9) to the both pairs (see “kitty” Fig. 4). The pair B, C does not, in general, corresponds to a position of the interferometer, but the both pairs B, A and C, A, by construction, do. Since $\bar{z}(B) = \bar{z}(A)$ and $\bar{z}(C) = \bar{z}(A)$, then $\bar{z}(B) = \bar{z}(C)$, and $\bar{z} = \text{const}$, i.e. $\bar{z} = 2c^{-1}$.

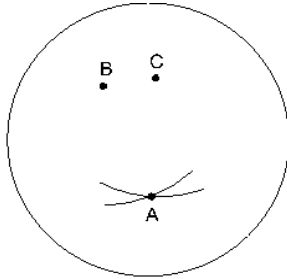


Figure 4: The “angular distance” between B and A, as well as between C and A equals 90° because the both pairs correspond to possible positions of the interferometer.

The constancy of \bar{z} is a strong restriction, not having a counterpart in two dimensions. To decipher this restriction let us now consider an auxiliary function z_+ on \mathcal{D}^2 corresponding to one of z 's entering the sum defining \bar{z} , say z_+ , then $z_- = \bar{z} - z_+$. The (continuous) function z_+ is almost arbitrary and the only restriction, of topological nature, is coming from the fact that the sum of the values of z_+ on opposite sides of the boundary of \mathcal{D}^2 should be equal $2c^{-1}$.

This simple informal discussion will be summarized and refined now in more mathematical terms. First of all, “in the first approximation”, we can observe that the constant function \bar{z} on \mathcal{D}^2 is actually a

constant function on a two-dimensional real projective surface $\mathbb{R}P^2$. It should seem obvious, because identification of opposite points of \mathcal{S}^2 provides $\mathbb{R}P^2$ by definition, or in other words, we are interested in functions on a set of rays rather than on a set of directions. For further convenience, we shift the function (by $2c^{-1}$) down to zero, which is a kind of additive normalization. What is less obvious, we claim that we now deal with a twisted real linear bundle \mathcal{B} over the base manifold $\mathbb{R}P^2$. The “shifted” \bar{z} denoted as \tilde{z} is a zero cross-section of \mathcal{B} , whereas the (shifted) function z_+ denoted as \tilde{z}_+ becomes an arbitrary continuous cross-section of \mathcal{B} . The form of (the twist) of the bundle \mathcal{B} follows from the observation that the values of \tilde{z}_+ on opposite sides of the boundary of \mathcal{D}^2 (at the identified points) should be opposite, i.e., the non-trivial element of the discrete group \mathbb{Z}_2 , coming from the corresponding principal bundle, acts in the fiber \mathbb{R} (Fig. 5). To exclude negative velocities of light, and consequently negative times of the travel of light, we can limit the real values of \tilde{z}_+ 's to the interval $|\tilde{z}_+| \leq c^{-1}$, and we can speak on the interval bundle $\bar{\mathcal{B}}$ instead of the linear bundle \mathcal{B} .

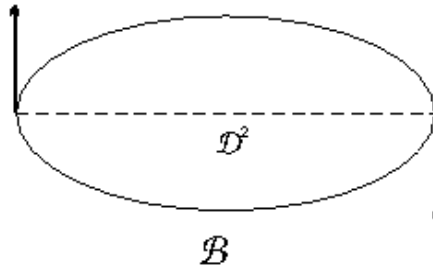


Figure 5: The non-trivial element of \mathbb{Z}_2 acts at the identified points on opposite sides of the boundary of \mathcal{D}^2 .

Recapitulating, we could state that all solutions of the problem (angular-dependent one-way velocities of light consistent with the null-effect of the MM experiment) are parameterized by cross-sections

of the non-trivial bundle \bar{B} . Obviously, non-constant solutions (non-zero cross-sections) do exist. We could also choose an axially symmetric solution on demand. Still the simplest possibility corresponding to Eq. (7) and Eq. (8) is excluded, but some mild deformations, e.g. expressed in the framework of the Mansouri–Sexl test theory with the parameters $a^2 = b^2 (1 - v^2)^2$, $d^2 = b^2 (1 - v^2)$ [6], are allowable. The two-dimensional case is even less restrictive than three-dimensional one (richer in a sense) but as unphysical is less interesting.

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Comment by

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Boguslaw Broda and Marcin Ostrowski discuss the interesting point that even a precise Null result of the Michelson-Morley experiment leaves open many possibilities for anisotropies of the speed of light, including the two-way speed of light, although the authors are mainly concerned with the one-way speed. Broda and Ostrowski also discuss the geometry of the trajectories taken by the interfering radiation as well as the geometry of the rotation and motion of the apparatus in space. Let us consider these issues separately in this commentary:

Two-way speed: That the Michelson-Morley experiments do not necessarily constrain all degrees of freedom of the two-way speed is directly acknowledged in a large part of the modern literature. Moreover, it has been tried to fill that potential gap of the verification of relativity. Modern test theories of relativity, like the standard model extension [1], have come a long way towards this aim. Therein, an anisotropy of the speed of light is a consequence of nine independent test parameters. These are related to anisotropies having different directions and symmetries, in analogy to the nice and clarifying discussion in the paper. Of these parameters, experiments without a turntable [2] measure seven. Since two parameters remain, there are still unrestricted degrees of freedom. However, at least one additional parameter can be measured by using a rotating setup [3]. Nevertheless, this point is usually buried in the rather lengthy formalisms of the SME, while the matter is interesting in its own right and can be discussed without the help (or distraction) that such an elaborate apparatus provides.

One-way speed: To measure the one-way speed of light c_{one} , one needs a clock at the light source determining t_{send} as well as at the detector, determining t_{detect} . The one-way speed would then be $c_{one}=(t_{detect} - t_{send})/L$, where L is the distance travelled. A way

to synchronize the clocks would be to emit a light pulse in the middle and set both clocks to zero when the light is detected at its location. Within relativity, this would mean that $c_{one} = c_{two}$ where c_{two} is the two-way speed of light. However, suppose the synchronization pulse is emitted at one of the clocks. Then, c_{one} would be infinite in the direction of the synchronization pulse and $c_{two} / 2$ in the other. Which synchronization convention is adopted in the paper? Note that these considerations assume no violation of relativity or any other accepted physical law. Thus, facts about nature (rather than about conventions) are best expressed in terms of c_{two} . In the remainder of my discussion, "c" and "speed of light" always mean "two way speed of light".

Geometry: For the discussion of the three-dimensional case, it is interesting to note that Michelson-Morley-experiments are subject to rotations due to (i) the turntable, (ii) Earth, and (iii) Earth's orbit. These rotation axes are in general not parallel. The experiments are also subject to the Lorentz boosts associated with Earth's orbit (30km/s) and its modulation by about 300m/s (depending on geographical latitude) due to Earth's rotation. The experiments thus exhibit a complicated three-dimensional motion which make them more restrictive than in the simplified picture of rotation within a plane. These issues have been analyzed in detail and are taken into account for all modern experiments [2-4].

A different, but very interesting, way in which geometry comes into play is via the geometry of the crystals that the cavities are made of. Since the electronic structure and the electromagnetic binding forces predominantly determine the macroscopic properties of solids, a violation of Lorentz invariance in the equations of motion of the electron or in the Maxwell equations would distort the crystals, leading to additional signal components. This can be used to determine additional limits on Lorentz violation in the electron's equation of motion [5].

Broda and Ostrowski mention the possibility of experiments using light traveling along arbitrary, non-contracted paths. An experiment using a circular path in so-called whispering gallery resonators has indeed been performed, and is one of the highest precision experiments so far [4]. It could be interesting to apply Broda and

Ostrowski's considerations to these results.

Apart from the one way and two way speed issues, I find the manuscript correct and clearly written. I hope you won't find that an argument against publication, as your Journal emphasizes controversy so much.

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Reply to referee's comments

Most of modern versions of the Michelson–Morley (MM) experiment is concerned with a particular type of the two-way velocity of light. Namely, light is supposed to travel forth and back along the same way (the trajectory is a contracted loop rather than an arbitrary one), as for example, in each arm of the MM interferometer. Our paper is mainly concerned with the one-way velocity of light: constancy of the particular type of the two-way velocity of light is assumed, and a directional (angular) dependence of the one-way velocity of light is derived. The “average” velocity of light discussed in the beginning of our work is of course a two-way velocity of light. But we wanted to stress that, in contrast to “standard” MM experiments, all kinds of loops are admissible.

There seem to be some difficulties with the notion of the one-way velocity of light because no definite synchronization convention has been adopted in the paper. Since the only measurements utilized in the paper concern the two-way velocity of light a synchronization convention is superfluous. Of course, to measure the one-way velocity of light a synchronization convention is unavoidable. Moreover, the one-way velocity of light (naturally) defines a synchronization

convention, and vice versa. Therefore, the analyzed freedom of the one-way velocity of light could have no direct experimental meaning, as for example, gauge in gauge theories. Nevertheless, there are some non-trivial consequences of that freedom, and we mention two of them: a theoretical and an experimental one.

Theoretical. Since one can (naturally) relate the one-way velocity of light to a synchronization procedure, there are some restrictions placed upon general clock synchronization convention following from the constraints on directional dependence of the one-way velocity of light [4]. Moreover, one could even speculate about a kind of a possible generalization of the Lorentz transformations to the new synchronization scheme.

Experimental. We have assumed constancy only of the particular type of the two-way velocity of light (see, the first paragraph of the Reply), and an arbitrary average (two-way) velocity of light could, in principle, vary. Therefore, non-standard MM experiments, i.e. those utilizing light travelling along arbitrary, non-contracted loops, would be of potential interest. Negative results of the MM experiments for all kinds of loops would reduce our results to results of [4]. Negative results for “standard” (contracted) loops, and positive ones for non-contracted loops would reinforce our general results.

In the three-dimensional case, we compare, according to Fig. 4, arbitrary positions of the MM interferometer. Therefore, our analysis seems to be fully three-dimensional, although rather idealized.

Reference [3] of our paper has been singled out as an example of the non-orthodox conclusion concerning the MM experiment. In the paper by R.T. Cahill, *The Roland De Witte 1991 Detection of Absolute Motion and Gravitational Waves*, Prog. in Phys. **3** (2006) 60–65; Arxiv:physics/0608205, one can find bibliography to several other works of this type. Non-negative results of the (“standard”) MM experiment would partially invalidate our analysis giving more freedom to the directional dependence of the one-way velocity of light.

B. Broda and M. Ostrowski