The old and new Concepts of Physics
An open-dialogue journal

www.uni.lodz.pl/concepts

THE CONCEPT OF MASS, QUARKS, AND PHASE SPACE

P. Żenczykowski
Dept. of Theoretical Physics, Institute of Nuclear Physics
Polish Academy of Sciences
Radzikowskiego 152, 31-342 Kraków, Poland
e-mail: zenczyko@iblis.ifj.edu.pl

(Received 14 November 2005; accepted 1 December 2005)

Abstract

We point out the conceptual problems related to the application of the standard notion of mass to quarks and recall the arguments that there should be a close connection between the properties of elementary particles and the arena used for the description of classical macroscopic processes.

Motivated by the above and the wish to introduce more symmetry between the coordinates of position and momentum we concentrate on the classical nonrelativistic phase space with $\mathbf{p}^2 + \mathbf{x}^2$ as an invariant. A symmetry-based argument on how to generalize the way in which mass enters into our description of Nature is presented and placed into the context of the phase-space scheme discussed.

It is conjectured that the proposed non-standard way of relating "mass" to the variables of the classical phase space is actually used in Nature, and that it manifests itself through the existence of quarks. Some properties of this proposal, including unobservability of "free quarks" and the emergence of mesons, are discussed.

1 Introduction

The present paper has its roots in a long-held suspicion that the discrete quantum properties of quarks and other elementary particles are closely linked to the properties of a continuous arena used for the description of classical macroscopic processes. This arena is usually thought to be provided by the spacetime. However, as in physics one deals with Nature only through a language chosen for its description, this arena may depend on the language chosen. Thus, as the present paper is concerned with the idea of introducing more symmetry between the position and momentum coordinates of classical nonrelativistic physics, the arena in question appears to be that of classical phase space, and the language that of a Hamiltonian formalism, in which momentum and position are treated as independent variables.

Some aspects of this idea were considered already in ref. [1], where it was pointed out that as the group of rotations in the three-dimensional space, treated as an automorphism group of an underlying algebra, singles out the algebra of quaternions, so the group of transformations in the six-dimensional phase-space under some additional assumptions points to the algebra of octonions. With the physical meaning of octonion nonassociativity fairly obscure, this route does not lead far at present, however.

In the following the idea of introducing more symmetry between position and momentum is pursued at a familiar associative level, tailored to admit a fairly clear interpretation of at least some points.

The paper is organized as follows. In the next Section various problems with the standard concept of mass as applied to quarks are presented. In this way the observational basis suggesting the need for a different approach to the issue of quark mass is laid out. Section 3 provides general arguments for a close connection between the properties of elementary particles and the classical arena on which these actors are thought to play. Then, phase-space symmetries are considered in some detail, and an argument is presented on how to generalize the way in which mass enters into our description of Nature. It is then conjectured that the proposed non-standard way of relating the concept of mass to macroscopic variables is actually used in Nature, and that it manifests itself through the existence of quarks.

In Section 4 the arguments of the preceding Section are used to generalize the Hamiltonian of a standard spin-1/2 particle to the case of a non-standard mass concept. Then, charge-conjugation is discussed and a simple-minded treatment of a quark-antiquark system is presented. Section 5 points to a 25-years-old scheme with composite leptons and quarks, which exhibits vague (even though superficial) similarity to our proposal. Our final remarks are contained therein as well.

2 The Problem of Mass

The standard concept of mass, originally introduced for the description of large classical objects moving along well-defined trajectories and responding in a well-defined way when subject to external forces, may be applied to free individual elementary particles such as leptons, photons, or hadrons when appropriate care is taken to account for their quantum properties.

Validity of this extrapolation from large and classical to small and quantum is in particular confirmed by the successes of quantum electrodynamics (QED), our best microscopic theory, when applied to electron, muon or the τ lepton.

However, while this theory permits us to predict some experimental numbers (such as e.g. lepton magnetic moments) with astonishing accuracy, thus corroborating in detail the structure of the theory, and in particular the way mass enters into it, it does not say anything about the ratios of lepton masses. In fact, the question "Who ordered that?" asked by I. Rabi when muon was discovered over half a century ago, remains completely unanswered in the nowadays widely accepted Standard Model (SM) of elementary particles, in which the appearance of several types of leptons and the problem of their masses constitute a part of a larger puzzle.

2.1 Free Parameters of the Standard Model

The Standard Model has had many spectacular successes in correlating and explaining a vast body of elementary particle data. However, being in its very essence a non-Abelian generalization of the Abelian theory of quantum electrodynamics, the Standard Model inherits the limitations of the latter, thus in particular being unable to provide any prediction for the ratios of lepton masses. In addition to lepton masses, it also contains several further parameters with values completely unexplained: e.g. quark masses, mixing angles and phases. Furthermore, in the realm of strong interactions, the accuracy of the agreement of SM predictions (or SM-inspired expectations) with experiment is in general significantly lower.

These shortcomings of the Standard Model may be viewed as indicating that the solution to the problem of why elementary particles have these or other mass values etc. may require New Physics. At present this term is used mainly to denote any kind of departure from the Standard Model, tacitly assumed to be describable within a field-theoretic approach. However, with no such departure having been identified as yet, nothing is really known about "New Physics". Thus, the latter may also involve going beyond the realm of quantum field theory.

Now, the SM quark mass parameters are introduced into the fundamental Lagrangian in a complete analogy with lepton masses. In its essence, therefore, the SM quark mass is a standard classical concept. Other SM parameters (e.g. mixing angles, phases) are more quantum-like in their nature.

The present paper deals with the conjecture that the extrapolation of the standard classical concept of mass to quarks, assumed in the SM as an input, may not be wholy warranted. As pointed out above, the concept of mass was originally introduced for the description of the behaviour of *individual* classical particles; with appropriate shift to the language of quantum physics the use of this concept was then extended to the description of the behaviour of *individual* elementary particles observed *far away* from the region of interaction. Quarks, however, are supposed to be permanently confined in hadrons and have never been observed as individual particles far away from the region of interaction. Thus, the applicability of the standard concept of mass to quarks may be questioned. I believe that the fact that free quarks have not been observed hints that quark theories should not be based on the standard concept of mass. This belief is supported by conceptual problems encountered in the Standard Model when the standard concept of mass is applied to quarks. These problems shall be presented in Section 2.3.

In order to avoid imminent charges that the advocated point of view denies the applicability of the concept of mass to quarks, it should be strongly stressed right from the beginning that it is accepted herein that a parameter of the dimension of mass and therefore *some* concept of mass may be assigned to a quark. Indeed, despite the fact that separate individual quarks have not been observed (and consequently that their mass cannot be directly measured in an experiment similar to one that can be used for leptons or hadrons), there are many indirect indications in favour of this view. What in my view remains open is the question whether the direct, straightforward application to quarks of the *standard* classical concept of mass, together with all implicit properties of such a mass, constitutes the proper extension of the latter concept into the subhadronic world. I think that the appropriate question here is: "what is the proper way of assigning a parameter of the dimension of mass to a quark?". In other words, I think that the present way of extrapolating the standard concept of mass to quarks requires modifications.

In fact, for many years there was a heated debate whether quarks should be conceived of as ordinary particles or as mathematical entities permitting a successful description of hadrons. Development over the years proved that quarks interact in a point-like manner, much like the leptons, thus shifting the concept of a "quark" more and more into the realm of "ordinary" particles. However, there still remains the question how much of this shift is really necessitated by the experimental information available, and how much appears to represent our unjustified expectation only. Interaction of quarks in a point-like manner, and the applicability to quarks of the classical concept of mass constitute two separate issues. Therefore, the latter should not be assumed on the basis of the former. Accordingly, following the general view that the issue of masses constitutes a weak point of the SM (or rather a point beyond that scheme), and in line with the arguments presented above, I believe that the SM requires some reinterpretation as far as quark masses are concerned.

It is hoped that the argument later on, and in fact the whole paper, will clarify my views in the matter, provoke some feedback, and perhaps help in the development of the ideas presented below.

2.2 Masses from Higgs Mechanism

In the Standard Model, all elementary particles acquire their masses through the Higgs mechanism. One cannot consider this mechanism as representing a solution of the problem of mass as defined above (i.e. explanation of the ratios of elementary particle masses). Indeed, the Higgs mechanism merely shifts the problem to that of the interaction of Higgs particles with matter fields. There is no apparent bonus resulting from this shift: the couplings of Higgs particles to matter fields remain as unconstrained as the masses and other parameters of the Standard Model.

Although the Higgs mechanism constitutes a crucial ingredient of the Standard Model, allowing for renormalizability with massive gauge bosons, it has its shortcomings. On the theoretical side, it generates a constant term which contributes to the energy density of the vacuum 55 orders of magnitude larger than observed. On the experimental side, Higgs particle has not been observed as yet. On the philosophical side, introduction of such a particle may be viewed as resulting from human mind's tendency to explain the natural phenomena in terms of material objects rather than in terms of abstract concepts, a tendency that has misled us several times in the past. Thus, the introduction of Higgs particle(s), while solving a problem in the description of weak interactions, is regarded by many physicists with some suspicion.

In fact, attempts to dispose of Higgs particle became more often recently. There are proposals in which no such particle is present [2]. If Higgs particle is not found soon, alternative approaches will certainly be pursued more often. Conversely, even if Higgs particle were found, we would be still very far from the solution of the problem of mass.

New hints on how to deal with the issue of mass could perhaps be unveiled through a search for regularities in mass matrices for leptons and quarks. Interesting attempts to derive lepton and/or quark mass matrices from some underlying Ansätze were made [3]. Some of these papers accept implicitly that the introduction of quark mass "á la lepton mass" is fully legitimate. However, as already hinted at above, I think that it brings about serious conceptual problems, to be exposed in more detail below.

2.3 Problems with Quark Mass

Let us return to the concept of quark mass and how it enters the present-day theories. Within the Standard Model quark interactions are described by quantum chromodynamics (QCD), the non-Abelian gauge theory of interacting quarks and gluons. The concept of quark mass enters here at the level of Lagrangian in much the same way as in quantum electrodynamics. Given the enormous success of quantum electrodynamics, the mathematical elegance of QCD - a non-Abelian generalization of QED, and the asymptotic freedom property of QCD, so appropriate for the description of characteristic features observed in the scattering at large momentum transfers in terms of hadron constituents, this way of introducing quark masses seems completely natural.

A problem appears, however, when one attempts to make a connection between quark masses introduced in the way just described, and quark masses extracted in various ways from the experimental data. Since individual quarks are not observed, any such extraction must involve a great deal of theory.

However, there is no good theoretical way to make the connection in question. Quark masses appearing in the Lagrangian (the so-called "current" quark masses, especially those of the light quarks) cannot be extracted from data at large momentum transfers, where theory is under better control, for two reasons. First, in this region these masses are negligibly small in comparison to the momenta in question. Second, we do not know how to make a theoretically sound (i.e. based on the Lagrangian only) connection between quarks and the experimentally observed hadrons. Because of the first obstacle quark masses have to be (and are) extracted from low-energy and low-momentum transfer data. However, it is widely agreed that in this region the second obstacle is particularly troublesome and that the perturbative approach to strong interactions of quarks must become invalid on account of confinement effects, with nonperturbative effects becoming dominant. Unfortunately, since the latter effects are not calculable they could affect the extraction in an unknown way. In order to proceed, one has to make here additional assumptions of phenomenological nature, thus leaving the solid ground of fundamental theory, and admitting other ingredients besides the First Principles. At the very least, therefore, the values of quark mass parameters extracted in this way are uncertain in a serious way. This prevents sound confirmation of the concepts used and opens doubts as to the meaning of the parameters extracted.

The simplest way of proceeding (and the one used very often) is to essentially forget about the confinement of quarks and to perform the calculations perturbatively using Dirac bispinors for external quarks and standard Dirac propagators for internal quarks. This is being done in the quark parton model and in all perturbative Standard Model calculations, bringing in a serious conceptual problem. Indeed, the pole present in the standard Dirac propagator, occurring at momentum square equal to quark mass squared and corresponding to a free classical particle, should be *absent* in the calculations as free quarks (in asymptotic states) are not observed in the real world. Consequently, one wonders what is the meaning of all perturbative calculations in which standard Dirac propagators and bispinors are used for quarks. Yet, it is precisely through such calculations with standard Dirac quarks and their propagators that information about quark masses is being extracted from the data. Below we briefly recall a few important calculations of this type.

2.3.1 External quarks (in initial/final hadrons)

Current quark masses

These are the masses used in the fundamental Lagrangian. For example, the ratios of the current masses of light (u, d, s) quarks were extracted from the combinations of the masses of kaons and pions [4] using standard Dirac bispinors for external quarks. The relevant procedure considers quark currents, and in particular the axial current $\bar{q}\gamma_\mu\gamma_5 q$, applying to its divergence the Dirac equation $\hat{p}_\mu\gamma^\mu q = m_{current}q$. This leads to expressions proportional to the sums of current quark masses, and the possibility of extracting them from the data under some additional assumptions. Note, however, that using the Dirac equation means that we adjust quark momenta and put quarks on their mass-shells, as appropriate for free particles. With momentum well-defined only when the associated wave is sufficiently long in ordinary space, one should expect this procedure to be invalid for quarks "confined to a small region of space within a hadron". Yet, it is such treatment of currents which, when further ingredients are added, leads from hadron masses to the nowadays widely accepted low values of current masses for the u, d, s quarks, with u, d current masses of the order of a few MeV. While there is no doubt that important information is being extracted in this way from the data, its meaning is blurred by the conceptual inconsistency indicated above, raising in particular the question of the meaning of the mass(-like) parameters thus extracted.

Constituent quark masses

In many papers the calculations were performed using the so-called "constituent" quark masses. These are not the masses present in the underlying Lagrangian, but the effective masses approximately equal to one third of the mass of an appropriate baryon, or a half of the mass of the corresponding vector meson. They are thought to take into account in some effective way the effects of the confining interactions. Thus, the effect of the confining interaction is thought to contribute some $330 - 350\ MeV$ to the effective quark masses. While these masses, being defined independently from the Lagrangian, do not have such a strong theoretical basis as the current masses, they work equally well in their respective domain of applicability.

Originally, the constituent quark masses appeared when the magnetic moments of octet baryons (i.e. proton, neutron etc.) were considered. Amazingly, the assumption of constituent quarks described by Dirac bispinors and treated as *free* particles (i.e. with $p_\mu \gamma^\mu q = m_{const} q$), when supplied with the appropriate symmetry structure of three-quark baryonic states and the principle of additivity of contributions from the individual quarks resulted in a very good description of baryon magnetic moments. One faces here a conceptual problem again: the treatment of constituent quarks as free Dirac particles leads to good results despite the fact that individual free quarks are not observed, and that constituent quarks are supposed to describe confined quarks. Many attempts were proposed in order to overcome such objections. For example, within the approach of a bag model, the size of magnetic moments is set by the size of a bag to the interior of which the quarks are confined. Despite such attempts, no improvement in the description of magnetic moments followed. In fact, the best parameter-free model for baryon magnetic moments is that of Schwinger [5], in which the size in question is set by the (experimentally measured) masses of vector mesons, with the rest of the success ensured by the spin-flavour symmetry of baryon wave functions alone.

2.3.2 Internal quarks

Perturbative treatment of the Standard Model leads to the appearance of quark propagators which may occur in loop and/or tree diagrams. The resulting formulae depend on quark masses present in these propagators.

Masses in loop propagators

An example of a calculation depending on the mass present in loop propagators only is furnished by the prediction of the charmed quark mass. This mass was predicted through the use of box diagrams with internal charmed quarks [6], and applied to uncharmed external states. When experimental data were combined with the formulae obtained in this way, a mass value of about 1.5 - 2.0 GeV was predicted. This mass was believed to be mainly the current mass, as only small ($330 - 350\ MeV$ or 20%) corrections from confining interactions were expected. Subsequent experimental discoveries of charmed particles with effective charmed quark mass around 1.5 GeV provided arguments in favour of the applicability of such calculations.

Since the current masses of the u, d, s quarks, determined as described earlier, are small when compared to the $330 - 350\ MeV$ scale, for these quarks it is the constituent quark masses that are expected to be appropriate for loop calculations. In fact, the constituent masses of "light" quarks were successfully used in a series of loop calculations of meson formfactors [7].

Masses in tree propagators

The results of loop diagram calculations suggest that quark poles are present. The unobservability of quarks suggests, however, that these poles should be absent. This contradiction can be studied in tree diagrams, where the existence of quark poles may be directly tested through the brehmstrahlung process.

The question if it is justified to use standard Dirac propagators for intermediate constituent quarks in photon brehmstrahlung was resolved as a by-product of the studies of weak radiative decays of hyperons (WRHD).

Calculations of the WRHD amplitudes in the framework of the constituent quark model (only u, d, or s quarks were involved) appropriately generalized to describe weak interactions, with poles due to intermediate constituent quarks and thus testing the meaning of the constituent-quark description of baryon magnetic moments somewhat deeper, have resulted in the firm prediction of a nearly maximal positive asymmetry in the $\Xi^0 \to \Lambda\gamma$ decay [9]. This turned out to be in complete disagreement with a nearly maximal negative asymmetry found later in experiment [10].

The origin of the incorrect prediction has been traced to the presence of the constituent quark pole. A correct description [11] of experimental data on WRHD requires abandoning the treatment of constituent quarks á la leptons, i.e. with mass-involving standard propagators, in favour of Gell-Mann's concept of $SU(3)_L \times SU(3)_R$ symmetry of currents [12], involving in its bare form no quark masses at all, in clear analogy to the Schwinger's treatment of baryon magnetic moments [5].

In effect, weak radiative hyperon decays showed that the treatment of constituent quarks as ordinary Dirac particles with standard propagators, is generally incorrect as it leads to a sharp disagreement with experiment, i.e. to artefacts. What remains of the constituent quark approach is the description of external baryons in terms of an appropriate spin-flavour wave function, with the dominant role played by its symmetry, and with the concept of constituent quark mass understood (or better: defined) just as a half of the corresponding vector meson mass, in agreement with the original ideas of Schwinger [5].

The situation presented above is deeply dissatisfying, not only because of the existence of the conceptual problems and internal inconsistencies related to the use of quark mass, but also because usually they are either unnoticed or swept away "under the carpet".

There are some hints, however. Namely, it seems that whereas the use of external quarks endowed with standard mass leads to conceptual problems and, similarly, the use of standard quark propagators in tree diagrams results in artefacts, in loop calculations the concept of poles associated with quarks seems to work, at least effectively and to some extent.

This suggests that the concept of quark mass (and/or the related pole) should undergo some reinterpretation so as to still allow for some poles (or their analogues) in the loop diagrams while clearly forbidding standard quark pole contributions in the tree diagrams. This could be achieved e.g. by admitting the concept of mass to hadrons only (consider e.g. [8], or the idea of hadronic bootstrap supplemented with some quark degrees of freedom). On the other hand, if mass parameters are to be assigned more directly to quarks, it does not seem appropriate to lay fault with the dynamical confinement of quarks alone as the latter should kill the poles everywhere, i.e. both in the tree and in the loop amplitudes. In the following we will present an idea, induced by considerations outside of the Standard Model, which forbids the appearance of standard poles by construction while still admitting for quarks the concept of a mass parameter and which, as we hope, could shed a different light on the issue of quark mass.

3 Mass and Space

3.1 Fundamental Mass / Parameters

Ultimately, the resolution of the problem of mass requires finding a theory which would assign dimensionless numbers to all mass ratios. In order to predict particle masses, this theory should be accompanied by at least one additional constant of the dimension of mass, which would set up the mass scale. (The Higgs appproach fits into this philosophy as well, one of its problems lying in the said prescription for dimensionless numbers).

This constant then plays the rôle of a "fundamental" mass. Alternatively, the latter may be constructed from other parameters of the theory, deemed to be "more fundamental". Then, the relevant parameters do not have to have the dimension of mass themselves.

3.2 Quantum Numbers and Space

The origin of the proliferation of elementary particles and their quantum attributes constitutes a problem untouched by the Standard Model. The variety of particles and their quantum numbers, mass parameters, etc., are duly taken into account in the SM, but remain completely unexplained in terms of any simple single underlying principle.

When searching for such a principle one should note that some quantum attributes of elementary particles are clearly associated with the properties of classical macroscopic continuous space in which these particles are envisaged as "moving". One can single out here the concept of particle spin and the corresponding classical notion of macroscopic rotation. Likewise, the notion of chirality is related to the existence of left-right symmetry in the macro-world. Similar connection between properties of particles and space exists also in general relativity, where matter (i.e. particles) defines and modifies the properties of space.

The above examples suggest that other particle attributes (e.g. particle types and their quantized masses) should also be somehow connected to the properties of the macroscopic space. In fact, it is philosophically very tempting to conjecture that this should hold for all particle attributes. According to this philosophy, properties of particles should correspond to properties of space and vice versa. Many attempts were pursued along that way. Attempts to unite Poincaré and internal symmetries were shown to lead to a "no-go" situation. However, what any such no-go theorem really proves is that the one particular way (or class of approaches) considered in the theorem is not acceptable. All other possibilities of connecting space and particle properties (conceptually perhaps completely different) are not restricted in any way and remain fully open until proven otherwise.

Arguments have also been presented that the macroscopic continuous classical space and the elementary particles themselves should not only be closely connected with each other but actually constructed from a deeper level - the underlying quantum pregeometry [13, 14, 15]. While not pursuing such arguments in detail, I accept their resulting philosophy, i.e. the conclusion that space and particle attributes should be intimately related, and that to any given particle attribute there should correspond a certain attribute of classical macroscopic arena used as a background for the description of physical processes. In this context it is appropriate to quote here the words of Penrose: "I do not believe that a real understanding of the nature of elementary particles can ever be achieved without a simultaneous deeper understanding of the nature of spacetime itself" [13].

Now, it was argued above that theory should forbid the appearance of standard quark propagators in the tree diagrams, while still admitting some meaning to their presence in the loops. The simplest way to satisfy the first condition is to forbid standard quark propagators right from the beginning. If one still wants to assign some concept of mass to quarks, one has to detach the concept of quark mass from that of the standard pole. This seems to require a broader theoretical structure. Consequently, I think that this provides us with a hint that one needs to use a broader concept of underlying macroscopic space.

3.3 Mass and Phase Space

Since our intuition seems to work best at the classical macroscopic level, it is at this level, as I believe, that the generalization procedure should be started, with the world of quantum attributes to be reached only later.

As noted in the Introduction, the arena used for the description of classical processes depends on the language chosen. If one chooses here the Hamiltonian formalism, it is the phase space - with independent position and momentum coordinates - which becomes the arena. The change of background from ordinary space to phase space seems to be the simplest generalization possible.

Now, let us note that the description of Nature in terms of the concept of spacetime as the arena on which processes may be visualised, required the connection of such manifestly different notions as space and time into a single construct. In the end this was achieved in particular through the use of the velocity of light c as a dimensional constant permitting time and space to be measured in the same units so that they may be transformed into each other, as suggested by the properties of Maxwell equations.

Keeping in mind the conjectured connection between the properties of elementary particles and the classical background needed for their description, the string-like properties of hadrons suggest an introduction of a different dimensional constant κ of dimension $[GeV/cm]$, which would make it possible to treat momentum and position in a more symmetric manner, and admit their mutual transformations.

In a future quantum theory, yet to be constructed, the presence of both this constant and the Planck constant should be sufficient to set the scale for all quantized masses. For the time being, however, I will consider momentum and position as commuting variables. Over 50 years ago the idea of introducing more symmetry between the canonically conjugated position and momentum coordinates of nonrelativistic physics led M. Born [16] to his reciprocity theory of elementary particles, in which he considered the concepts of fundamental length and fundamental momentum, and introduced the "reciprocity" transformations $x \rightarrow p$, $p \rightarrow -x$.

The first decision one has to made when introducing κ is to decide whether one should begin with the four-dimensional spaces of four-momentum and spacetime coordinates, or rather with the three-dimensional spaces of momenta and positions only. Below I argue that there are reasons for choosing the second alternative, at least at the beginning. In fact, the classical Hamiltonian formalism exhibits symmetry between the three-dimensional momentum and position coordinates. Time, however, occupies a distinguished position: it is a parameter upon which the position and momentum coordinates depend. An even more pronounced difference between time and the position and momentum coordinates exists in quantum mechanics, where time is still a $c-$number parameter, while space (momentum space) coordinates become operators. The present-day relativistic field theory, which does unite special relativity and quantum physics, achieves this in a rather formal way which may be argued to be unsatisfactory at the conceptual level [17, 18, 19, 20]. Indeed, Finkestein called it a $c - q$ theory, a merger of classical and quantum concepts [21]. Although local relativistic field theory is enormously successful, predicting also the existence of experimentally confirmed quantum correlations between spatially separated events, the entailed issue of an instantaneous quantum reduction of a non-local state vector does not seem to be in accord with the original spirit of relativity [22]. The latter problem highlights the essential difference between time and space in spite of these notions being united into the concept of the Minkowskian spacetime. Furthermore, one has to keep in mind that local Lorentz-equivalent frames are not fully equivalent in Nature:

the background radiation is isotropic in one such frame only. Given this situation I think that it is appropriate to restrict our considerations to the realm of nonrelativistic physics hoping that a proper treatment of relativistic effects can be achieved at a later stage.

3.4 Symmetry Transformations

The introduction of κ permits the combination of separate invariants $\mathbf{p^2}$ and $\mathbf{x^2}$ into a single form $\mathbf{p^2} + \mathbf{x^2}$, in which position and momenta coordinates enter on a completely equal footing, and suggests subsequent consideration of all transformations that leave this form invariant. These transformations clearly include standard three-dimensional rotations and ordinary reflections. As shown in [1] the set of all transformations which leave the form $\mathbf{p^2}+\mathbf{x^2}$ invariant, under the restriction that also the Poisson brackets $\{p_i, p_k\}$, $\{x_i, x_k\}$, $\{p_i, x_k\}$ are to be form invariant, forms the group $U(1) \otimes SU(3)$. The $U(1)$ factor takes care of reflections ($\mathbf{p}' = -\mathbf{p}$, $\mathbf{x}' = -\mathbf{x}$) and the reciprocity transformations of Born ($\mathbf{p}' = \mathbf{x}$, $\mathbf{x}' = -\mathbf{p}$), the former being the squares of the latter; while the $SU(3)$ group constitutes the extension of the group of proper rotations, where the latter are understood as simultaneous rotations of $\mathbf{p}$ and $\mathbf{x}$.

In the six-dimensional space of

$$(p_1, p_2, p_3, p_4, p_5, p_6) \equiv (p_1, p_2, p_3, x_1, x_2, x_3) \equiv \mathbf{p} \oplus \mathbf{x}$$

with the fifteen $SO(6)$ generators G_{ik} $(= -G_{ki})$ given by

$$(G_{mn})_{ik} = \delta_{mi}\delta_{nk} - \delta_{mk}\delta_{ni}, \tag{1}$$

the eight $SU(3)$ generators are as follows:

$$F_1 = -H_3 = G_{15} + G_{24}, \tag{2}$$
$$F_2 = -J_3 = G_{12} + G_{45}, \tag{3}$$
$$F_3 = R_1 - R_2 = G_{41} - G_{52}, \tag{4}$$
$$F_4 = -H_2 = G_{34} + G_{16}, \tag{5}$$
$$F_5 = J_2 = G_{13} + G_{46}, \tag{6}$$
$$F_6 = H_1 = G_{62} + G_{53}, \tag{7}$$
$$F_7 = J_1 = G_{32} + G_{65}, \tag{8}$$
$$F_8 = (R_1 + R_2 - 2R_3)/\sqrt{3} = (G_{41} + G_{52} - 2G_{63})/\sqrt{3}. \tag{9}$$

These generators satisfy the commutation rules of the $su(3)$ Lie algebra

$$[F_i, F_k] = 2f_{ikj}F_j, \tag{10}$$

with totally antisymmetric structure constants f_{ikj} equal to 1 for $ikj = (123)$; 1/2 for $ikj = (147), (165), (246), (257), (345), (376)$; $\sqrt{3}/2$ for $ikj = (458), (678)$; and zero otherwise. The $U(1)$ generator is

$$R = R_1 + R_2 + R_3 = G_{41} + G_{52} + G_{63}. \tag{11}$$

The question now emerges in what way this $U(1) \otimes SU(3)$ group should be used to generalize the old notions. In contemporary particle physics, when an enlarged symmetry group is introduced, the action of new group generators produces new objects (e.g. fundamental particles) from the old ones. Standardly, this is being done at the level of a smallest-dimensionsal irreducible spinorial representation of the group. There is no necessity, however, to employ the enlarged symmetry group in exactly this way (for a remotely related remark see e.g. [23]).

3.5 Momentum and Mass

The "$\mathbf{p} \oplus \mathbf{x}$" scheme as defined above does not yet really distinguish between momentum and position coordinates. It is only when the standard concept of mass is introduced that a real difference between $\mathbf{p}$ and $\mathbf{x}$ appears. Namely, we observe that for individual objects separated from each other by large distances, be it in the macroworld or in the world of elementary particles, energy of directly observed free clasical objects (quantum particles) is defined by their mass and *momenta*, whether via a relativistic or a nonrelativistic formula. Thus, the standard concept of mass may be said to be directly associated with the concept of momentum $\mathbf{p}$, not position $\mathbf{x}$. In other words, the six-dimensional vector $(p_1, p_2, ...p_6)$ is divided into two triplets in such a way that one of the triplets ("$\mathbf{p}$") is associated with mass.

When the "$\mathbf{p} \oplus \mathbf{x}$" philosophy admitting transformations between $\mathbf{p}$ and $\mathbf{x}$ is accepted, one naturally asks whether this division into momenta coordinates associated with mass, and position coordinates not associated with this notion is unique, or not. In fact, the "$\mathbf{p} \oplus \mathbf{x}$"

scheme suggests that the above division may not be unique, and that the momentum and position coordinates could be treated on a more equal footing. I shall now argue how this can be done. First, however, let us note that transformations of the $U(1) \otimes SU(3)$ group were constructed to effect the $\mathbf{p} \leftrightarrow \mathbf{x}$ transformations, not to act upon mass itself. Consequently, mass (an invariant of the group of ordinary rotations) should also be an invariant of the constructed group.

Within the "$\mathbf{p} \oplus \mathbf{x}$" philosophy, the (unknown) mechanism generating standard particle masses must somehow divide the 6-dimensional object "$\mathbf{p} \oplus \mathbf{x}$" into a pair $\{\mathbf{p}, \mathbf{x}\}$ of canonically conjugated 3-dimensional variables $\mathbf{p}$ and $\mathbf{x}$, with one of these ($\mathbf{p}$) directly associated with the concept of mass. However, the division of the 6-dimensional object "$\mathbf{p} \oplus \mathbf{x}$" into two 3-dimensional canonically conjugated objects (i.e. into a pair {(generalized momenta), (generalized positions)}) may proceed in several ways, leading not only to the standard form

$$\{(p_1, p_2, p_3), (x_1, x_2, x_3)\}, \tag{12}$$

but also to

$$\{(p_1, x_2, -x_3), (x_1, -p_2, p_3)\}, \tag{13}$$

or to

$$\{(-x_1, p_2, x_3), (p_1, x_2, -p_3)\}, \tag{14}$$

or to

$$\{(x_1, -x_2, p_3), (-p_1, p_2, x_3)\}, \tag{15}$$

where the signs take into account in particular the requirement that the Poisson brackets of new momenta and positions are to be the same as before. Clearly, there are other possibilities, including the cases when all p_i and x_k in the above pairs are interchanged, such as eg. (for Eq.(13))

$$\{(x_1, -p_2, p_3), (-p_1, -x_2, x_3)\}. \tag{16}$$

There are two sets of such choices of canonical momenta and positions: one with an odd number of standard momentum components in the new canonical momentum as in Eqs (12-15), and one with an even number of standard momentum components, as in Eq.(16).

All such new pairings of $p_1, p_2, p_3, x_1, x_2, x_3$ into the canonically conjugated momenta and positions may be obtained from the standard pairing $\{\mathbf{p}, \mathbf{x}\}$ via the action of appropriate elements from the said $U(1) \otimes SU(3)$ group. For example, $SU(3)$ rotation by 90^o using the generator H_1, followed by an appropriate ordinary rotation simultaneously in planes (x_2, x_3) and (p_2, p_3), leads to the first of three pairs given above, i.e. to Eq.(13), which is also obtained by an appropriate rotation by F_3. Similarly, analogous rotation using H_2 followed by an ordinary rotation or $(F_3 + \sqrt{3}F_8)/2$ (H_3 or $(F_3 - \sqrt{3}F_8)/2$) leads to Eq.(14) (respectively: Eq.(15)).

Transformations of the standard pairing $\{\mathbf{p}, \mathbf{x}\}$ using the $SU(3)$ group lead therefore to the first set of additional pairings, i.e. to three-dimensional generalized momenta in which one component is a component of the standard momentum coordinate, while the remaining two components are the components of the standard position coordinate. The cases with $\mathbf{p}$ and $\mathbf{x}$ interchanged (two standard momenta coordinates and one standard position coordinate constituting a generalized momentum together) are generated from the former cases by the $U(1)$ generator R.

As already mentioned, transformations generated by R lead in particular to the reciprocity transformations, i.e. they exchange $\{\mathbf{p}, \mathbf{x}\}$ into $\{\mathbf{p}', \mathbf{x}'\} = \{-\mathbf{x}, \mathbf{p}\}$. Contrary to the $SU(3)$ transformations generated by H_i, the transformations induced by R are clearly acceptable by the condition of rotational invariance. Furthermore, $\mathbf{p'^2} = \mathbf{x^2}$ could also be made acceptable by translational invariance upon an interpretation of $\mathbf{x}$ as a difference of position coordinates of two objects. Despite that, in our macroscopic world we do not seem to observe objects for which energy and mass are connected with the standard position coordinates $\mathbf{x}$ in a way analogous to that in which they are connected with the standard momentum coordinates $\mathbf{p}$. In fact, momentum "$\mathbf{p}$" was defined as this subtriplet of six variables $p_1, p_2, ..., x_3$ which associates the concept of mass to energy, contrary to the other subtriplet, "$\mathbf{x}$". This suggests that consideration of transformation generated by R may lead outside the realm of objects with masses.

Thus, it is only through the $SU(3)$ factor, the minimal simple-group extension of the group of standard rotations, that one arrives at the proposal for how to generalize the link between the concepts of energy, mass and ordinary momentum. While I have no idea how the mass-generating mechanism actually operates, the $SU(3)$ symmetry present in the "$\mathbf{p} \oplus \mathbf{x}$" scheme suggests a unique way of applying the standard concept of mass to 1+3 types of divisions of standard momenta and position coordinates into pairs of canonically conjugated triplets. Accordingly, there are three additional choices for generalized momenta, besides the ordinary momentum $\mathbf{p}$, which may be linked to the concept of mass. Each of these three choices clearly violates ordinary rotational invariance (translational invariance is satisfied if $\mathbf{x}$ is understood as denoting the difference of position coordinates). Thus, the objects (if any) for which mass is linked with generalized momenta of the type $(p_1, x_2, -x_3)$ certainly cannot belong as individual objects to our macroworld since the latter is rotationally invariant. These objects could, however, belong to the macroworld as unseparable components of composite objects, provided the latter are constructed in such a way that they satisfy all invariance conditions (rotational etc.) requested.

It is now very tempting to conjecture here that the three additional types of objects, linking mass to three choices for generalized momenta and related to each other by rotations, correspond to quarks. The unobservability of individual quarks would then be directly related to this lack of rotational (and possibly translational) invariance. An argument against the above proposal claiming that strong interactions are known to be invariant under the transformations from both the homogenous and inhomogenous Poincaré group, including 3-dimensional translations and rotations, is not a valid one. Indeed, we do know that any hadron, when probed by objects exhibiting all required transformation properties (e.g. a photon, a W-boson, etc.) does exhibit transformation properties compensating those of a probe, so that the whole interaction is rotationally and translationally invariant. However, this *always* concerns the interaction with hadron *as a whole*, and *never* with an *individual* quark (in the SM it is the interaction with a *colour-singlet superposition* of quark currents, which - as far as colour is concerned - possesses the properties of a hadron, and not those of an individual quark). If the individual quarks *conspire* in ensuring the requested transformation properties of hadrons, the whole scheme could be a viable one. Obviously, the conspiration mechanism should ensure that somehow the quarks of a hadron might be described by rotationally covariant entities - i.e. ordinary spinors. The difference with the standard approach would then be that the objects of definite mass are not the objects with standard spinorial transformation properties. This type of property is well known in the Standard Model where quark states in which weak currents are diagonal and quark states in which quark mass matrix is diagonal are not the same, but related by a unitary transformation. It may be that the only essential difference of the present proposal with respect to the SM is the relaxation of the standard way of treating quark masses. However, further changes might also be needed.

Symmetry and simplicity of the scheme make me find it hard to believe that Nature has not utilized the above possibility. Rather, the problem seems to me to be how to put the above ideas into an appropriate mathematical form, and how to develop the latter. In the next sections a simple construction will be proposed, which satisfies certain of the requirements discussed above.

This construction is of a $c-q$ type in the classification of Finkelstein, and thus is most probably a great oversimplification, in particular when the complexity of hadron physics is recalled. Yet, it exhibits a series of interesting qualitative properties and admits a simple interpretation. Although it does have some shortcomings and is presumably a toy model only, I think it is worth presenting.

4 Generalizing Dirac Hamiltonian

The content of the previous section suggests that the basic inputs of present theories, which are based on the standard link between the concept of mass and momentum, should be appropriately generalized. In particular this concerns the Dirac equation. Indeed, no $SU(3)$ "$\mathbf{p} \oplus \mathbf{x}$" symmetry is present in the Dirac equation: when the latter is written down in momentum representation, it is completely oblivious to space and time. It exhibits connection with space(-time) only through the quantum/wave route with the help of the Planck constant of dimension $GeV \cdot cm$. When restricted to momentum-space representation the Dirac Hamiltonian does not exhibit any quantum features, its properties being of a purely geometric classical nature.

Below we shall start with the Dirac Hamiltonian in momentum space and then proceed to act on it with the $SU(3)$ transformations discussed above. Now, one may argue that there is an inconsistency here: the Dirac equation is relativistic, while the proposed phase-space approach is nonrelativistic. For our purposes, however, what is important in the Dirac Hamiltonian is 1) its algebra, and 2) the fact that it leads to antiparticles. Now, as far as the algebra is concerned, the Dirac's trick of doubling the size of matrices (from 2×2 to 4×4), although originally needed to linearize the form $\mathbf{p}^2 + m^2$, is also needed to represent reflections (a nonrelativistic concept, needed in the $\mathbf{p}^2 + \mathbf{x}^2$ scheme as well), since these cannot be described by Pauli matrices alone. Thus, the matrices

$$\alpha_k = \begin{bmatrix} 0 & \sigma_k \\ \sigma_k & 0 \end{bmatrix} = \sigma_k \otimes \sigma_1, \tag{17}$$

$$\beta = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \sigma_0 \otimes \sigma_3, \tag{18}$$

with σ_k being Pauli matrices, and β needed to represent the reflection through $\alpha_k \to \beta\alpha_k\beta = -\alpha_k$, should be appropriate in our nonrelativistic case as well.

The term $\alpha_k p_k$ is rotation- and reflection- invariant and is clearly appropriate for the linearization of both the relativistic and nonrelativistic expressions for Hamiltonians, both containing $\mathbf{p^2}$. It is only when β, multiplied by the mass parameter m, is added to $\alpha_k p_k$ and the whole expression is identified with a Hamiltonian, i.e.

$$H_D = \alpha_k p_k + \beta m, \tag{19}$$

that one recovers a relativistically covariant expression. We will use the above Hamiltonian not only because of its simplicity when compared to the analogous Hamiltonian obtained when one linearizes the Schrödinger equation [24], but mainly because it leads *in a standard way* to antiparticles and the well-known procedure of charge conjugation which we shall need. What is important here is the form of the relativistic relation to be linearized, in which energy, mass and momentum enter in squares, unlike in the nonrelativistic case for which both energy and mass enter in a linear fashion:

$$2mE = \mathbf{p}^2. \tag{20}$$

Should one insist on a manifestly nonrelativistic treatment, application of $SU(3)$ transformations to the linearized Schrödinger equation proceeds in a way fairly analogous to the Dirac case discussed below. It is only when charge conjugation (hence antiparticles) is considered that a difference in the treatment of mass is needed (see Section 4.2). Alternatively, we may argue that we do not really need the full relativistic-invariance of the Dirac Hamiltonian: we may restrict ourselves to very large ratios of mass to momentum, when the Dirac Hamiltonian may be considered as approximating the strictly nonrelativistic case with the important addition that it involves negative energy solutions to be interpreted later as antiparticles.

In order to proceed with $SU(3)$ transformations on the Dirac Hamiltonian we have to introduce $\mathbf{x}$ in addition to $\mathbf{p}$ (and m), and linearize the $\mathbf{p}^2+\mathbf{x}^2$ $(+m^2)$ form, still treating $\mathbf{p}$ and $\mathbf{x}$ as commuting variables. In order to do this we have to enlarge Dirac matrices by doubling their size and introducing

$$A_k = \alpha_k \otimes \sigma_0 = \sigma_k \otimes \sigma_1 \otimes \sigma_0, \tag{21}$$
$$B = \beta \otimes \sigma_0 = \sigma_0 \otimes \sigma_3 \otimes \sigma_0, \tag{22}$$
$$B_k = \quad = \sigma_0 \otimes \sigma_2 \otimes \sigma_k, \tag{23}$$

with matrices B_k associated with x_k. The above matrices satisfy the conditions:

$$A_k A_l + A_l A_K = 2\delta_{kl}, \tag{24}$$
$$A_k B_l + B_l A_K = 0, \tag{25}$$
$$B_k B_l + B_l B_K = 2\delta_{kl}, \tag{26}$$
$$A_k B + B A_k = 0, \tag{27}$$
$$B_k B + B B_k = 0, \tag{28}$$
$$BB = 1. \tag{29}$$

Note that because of the requirement that all the $p_i x_k$ mixed terms are to vanish, and that $\mathbf{x}$ (and hence B_k) changes sign under reflection, B_k has to contain σ_2 as a second factor in the tensor product in Eq.(23).

Applying now the $SU(3)$ transformation induced by H_1 (followed by a rotation in (2,3)-planes), i.e.

$$(p_1, p_2, p_3; x_1, x_2, x_3) \rightarrow (p_1, x_2, -x_3; x_1, -p_2, p_3), \tag{30}$$
$$(A_1, A_2, A_3; B_1, B_2, B_3) \rightarrow (A_1, B_2, -B_3; B_1, -A_2, A_3), \tag{31}$$

we arrive at the following counterpart of the Dirac Hamiltonian:

$$H_R = A_1 p_1 + B_2 x_2 + B_3 x_3 + Bm. \tag{32}$$

This Hamiltonian is not invariant under standard rotations, and - if x_2, x_3 are not position differences - it is not translationally invariant either. Restoration of rotational (and/or translational) symmetry would require addition of appropriate terms. We shall discuss this in due time. Now, in line with the three possible ways of choosing generalized momenta (cf. Eqs.(13,14,15)) two further Hamiltonians may be constructed apart from the Hamiltonian of Eq.(32), namely:

$$H_Y = B_1 x_1 + A_2 p_2 + B_3 x_3 + Bm \tag{33}$$

and

$$H_B = B_1 x_1 + B_2 x_2 + A_3 p_3 + Bm. \tag{34}$$

The subscripts distinguish betwen these Hamiltonians, and were thought of as abbreviations for "Red", "Yellow", and "Blue", proposed in anticipation that the above Hamiltonians will perhaps turn out appropriate for the description of quarks.

Note that under the above proposal the $SU(3)$ group in question, which offers a generalization of the rotation group, does provide a set of transformations leading from the old objects to the new ones. However, this occurs at the level of a pair of vectors **p** and **x** (or axial vectors, because $SU(3)$ is not concerned with reflections), not at the level of the spinorial representation of the rotation group [23].

4.1 Restoring Rotational Invariance

Consider now an ordinary rotation of the reference frame around the third axis by an arbitrary angle ϕ. Matrices **A** and **B** transform under this rotation in the same way as vectors **p** and **x**. Below we denote the transformed matrices and vectors with a prime sign. Hamiltonian H_B remains then form-invariant:

$$H_B \equiv H_B(x_1, x_2, p_3) = H'_B(x'_1, x'_2, p'_3), \tag{35}$$

where the prime sign in H'_B refers to the transformed matrices A'_k, B'_k, whereas Hamiltonians H_R and H_Y acquire the following look:

$$\begin{aligned} H_R(p_1,x_2,x_3) &= c^2 H'_R(p'_1,x'_2,x'_3)+s^2 H'_Y(x'_1,p'_2,x'_3)+ \\ &\quad +sc(B'_2x'_1+B'_1x'_2-A'_2p'_1-A'_1p'_2), \end{aligned} \tag{36}$$

$$\begin{aligned} H_Y(x_1,p_2,x_3) &= c^2 H'_Y(x'_1,p'_2,x'_3)+s^2 H'_R(p'_1,x'_2,x'_3)- \\ &\quad -sc(B'_2x'_1+B'_1x'_2-A'_2p'_1-A'_1p'_2), \end{aligned} \tag{37}$$

with $s=\sin\phi$, $c=\cos\phi$. The sum of the Hamiltonians H_R+H_Y is clearly form-invariant:

$$H_R(p_1,x_2,x_3)+H_Y(x_1,p_2,x_3)=H'_R(p'_1,x'_2,x'_3)+H'_Y(x'_1,p'_2,x'_3), \tag{38}$$

which is true if p_1,p_2 (x_1,x_2) constitute two components of a *single* vector $\mathbf{p}$ (respectively $\mathbf{x}$), as is implicit on the r.h.s. of Eqs. (36,37). It is now obvious that the Hamiltonian:

$$H=H_R(p_1,x_2,x_3)+H_Y(x_1,p_2,x_3)+H_B(x_1,x_2,p_3) \tag{39}$$

is form-invariant under arbitrary three-dimensional rotations. Consequently, with the Hamiltonian as above, it does not matter whether we perform the division of the six-dimensional object $\{\mathbf{p},\mathbf{x}\}$ into the three pairs of triplets of Eqs(13,14,15) of a given frame of reference

$$(R) \qquad \{(p_1,x_2,-x_3),(x_1,-p_2,p_3)\}, \tag{40}$$

$$(Y) \qquad \{(-x_1,p_2,x_3),(p_1,x_2,-p_3)\}, \tag{41}$$

$$(B) \qquad \{(x_1,-x_2,p_3),(-p_1,p_2,x_3)\}, \tag{42}$$

or do this in the rotated frame for $\{\mathbf{p}',\mathbf{x}'\}$ according to:

$$(R') \qquad \{(p'_1,x'_2,-x'_3),(x'_1,-p'_2,p'_3)\}, \tag{43}$$

$$(Y') \qquad \{(-x'_1,p'_2,x'_3),(p'_1,x'_2,-p'_3)\}, \tag{44}$$

$$(B') \qquad \{(x'_1,-x'_2,p'_3),(-p'_1,p'_2,x'_3)\}. \tag{45}$$

In the above considerations x_1 (x_2, x_3) occurs in two Hamiltonians: H_Y and H_B (H_R and H_B, H_R and H_Y). In principle one could introduce here two different position-type vectors $\mathbf{x}$ and $\mathbf{y}$ so that each of the three Hamiltonians H_R,H_Y,H_B would depend on three independent components of generalized momenta:

$$H_R(p_1,x_2,y_3), \quad H_Y(y_1,p_2,x_3), \quad H_B(x_1,y_2,p_3).$$

The analog of Eq.(39) would be still rotationally invariant. Although this extension might be relevant, it is not needed for the presentation of our main idea. Consequently, below we shall have one position-type vector only ($\mathbf{x}$). If the requirement of translational invariance is imposed, $\mathbf{x}$ has to be understood as a difference of position coordinates. We shall come back to this issue in Section 4.3.

4.2 Charge Conjugation

As mentioned earlier, the Dirac Hamiltonian illustrates our ideas probably in the simplest way, in particular permitting also the standard introduction of the operation of charge conjugation. In order to discuss the latter along the standard lines we need to replace the up-to-now classical variables of $\mathbf{p}$ and $\mathbf{x}$ with operators.

First, we recall how charge conjugation operation is effected when electromagnetic interaction is added to the Dirac Hamiltonian:

$$H_D = A_k(p_k - e\mathcal{A}_k) + Bm + e\mathcal{A}_0, \tag{46}$$

where $\mathcal{A}_\mu$ denotes the electromagnetic field.

In order to transform H_D to a form in which its antiparticle content is explicit (i.e. in which the positive energy solution corresponds to antiparticles) we have to change the relative sign between p_k and $e\mathcal{A}_k$. This is standardly obtained via complex conjugation applied to both c-numbers and operators with the properties: $i \to -i$, $\mathbf{x} \to \mathbf{x}$, $t \to t$, $H \to -H$, $\mathbf{p} \to -\mathbf{p}$, $A_k \to A_k^*$ ($A_{1,3}^* = A_{1,3}$, $A_2^* = -A_2$), $B \to B^* = B$, $\mathcal{A}_\mu \to \mathcal{A}_\mu^* = \mathcal{A}_\mu$, and $m \to m$, $e \to e$ [in order to describe antiparticles when linearizing the Schrödinger equation one would have to use $m \to -m$ (instead of $m \to m$) when $E \to -E$ (c.f. Eq.(20)]. Application of these rules to Eq.(46) leads to

$$H' = A_k^*(p_k + e\mathcal{A}_k) - Bm - e\mathcal{A}_0. \tag{47}$$

When the following matrix is introduced ($\tau^2 = 1$):

$$C = -i\sigma_2 \otimes \sigma_2 \otimes \tau = -C^{-1}, \tag{48}$$

which satisfies

$$CBC^{-1} = -B, \tag{49}$$

$$CA_k^*C^{-1} = A_k, \tag{50}$$

the above Hamiltonian may be transformed to the standard form with an opposite sign of charge:

$$H' \rightarrow CH'C^{-1} = A_k(p_k + e\mathcal{A}_k) + Bm - e\mathcal{A}_0, \tag{51}$$

as appropriate for the explicit description of antiparticles.

Now, when complex conjugation (defined as above) is applied to H_R (with $B_k \rightarrow B_k^*$ and $B_{1,3}^* = -B_{1,3}$, $B_2^* = B_2$), one obtains the analogue of Eq. (47) (we skipped the electromagnetic field)

$$H'_R = A_1 p_1 - B_2 x_2 + B_3 x_3 - Bm. \tag{52}$$

For H_Y and H_B one similarly obtains:

$$H'_Y = B_1 x_1 + A_2 p_2 + B_3 x_3 - Bm, \tag{53}$$

$$H'_B = B_1 x_1 - B_2 x_2 + A_3 p_3 - Bm. \tag{54}$$

Choosing now

$$\tau = \sigma_2, \tag{55}$$

which leads to:

$$CB_k^* C^{-1} = B_k, \tag{56}$$

or

$$CB_{1,3}C^{-1} = -B_{1,3}, \tag{57}$$

$$CB_2 C^{-1} = B_2, \tag{58}$$

we find that H'_R, H'_Y, and H'_B are transformed to

$$H_{\bar{R}} = A_1 p_1 - B_2 x_2 - B_3 x_3 + Bm, \tag{59}$$

$$H_{\bar{Y}} = -B_1 x_1 + A_2 p_2 - B_3 x_3 + Bm, \tag{60}$$

$$H_{\bar{B}} = -B_1 x_1 - B_2 x_2 + A_3 p_3 + Bm, \tag{61}$$

(Choices $\tau = \sigma_0, \sigma_1, \sigma_3$ do not lead to a rotationally invariant form simultaneously for all three Bx terms in $H_{\bar{R}}, H_{\bar{Y}}, H_{\bar{B}}$.) Note that neither an ordinary rotation nor reflection ($A_k \rightarrow BA_kB = -A_k$; $B_k \rightarrow BB_kB = -B_k$) can bring $H_{\bar{R}}$ into H_R (i.e. $A_1 p_1 \rightarrow A_1 p_1$ and $-B_3 x_3 \rightarrow +B_3 x_3$) etc. Thus, even without considering the charge explicitly, $H_{\bar{R}}$ represents an object different from that described by H_R. In summary, when we are given a Hamiltonian for a particle, we form a Hamiltonian for its antiparticle by replacing e and $\mathbf{x}$ with $-e$ and $-\mathbf{x}$ while keeping the rest of the particle Hamiltonian unchanged.

4.3 Quark-Antiquark Systems

We are now in a position to consider a Hamiltonian for a quark-antiquark system. In order to construct this Hamiltonian one has to add the Hamiltonians of system components. Here one encounters a problem whether the matrices A_i, B_k for quarks and those for antiquarks are distinct or identical. The simplest possibility is that they are identical. Here, one may point out a parallel to the current-field identity of hadronic physics, according to which the electromagnetic vector current $j_\mu = \bar{q}\gamma_\mu q$, involving matrices $\gamma_i \leftrightarrow BA_i$ is identical to the vector meson field, built out of $q\bar{q}$ pair. Consequently, we propose to add the Hamiltonians of red quarks and antiquarks as follows:

$$H_R + H_{\bar{R}} = A_1(p_1 + \bar{p}_1) + B_2(x_2 - \bar{x}_2) + B_3(x_3 - \bar{x}_3) + 2\ B\ m, \quad (62)$$

where bars over momenta or positions identify these as *physical* antiquark variables. Given the complexity of hadronic physics even in the meson spectrum only, this proposal is most probably an oversimplification, but it exhibits several interesting features. For x_k ($\bar{x}_k$) we may now admit the position coordinates themselves and not their differences only. (If an overall position coordinate X_1, canonically conjugate to $P_1 = p_1 + \bar{p}_1$ from $H_R + H_{\bar{R}}$, were included in the definitions of x_1 and $\bar{x}_1$ in H_Y, $H_{\bar{Y}}$, H_B, and $H_{\bar{B}}$ through $x_1 \to x_1 - X_1$, $\bar{x}_1 \to \bar{x}_1 - X_1$, the dependence on X_1 would cancel in $H_Y + H_{\bar{Y}}$ and $H_B + H_{\bar{B}}$ anyway.) The total rotationally and translationally invariant Hamiltonian $H_{q\bar{q}}$ of a quark-antiquark system is therefore:

$$H_{q\bar{q}} = H_R + H_{\bar{R}} + H_Y + H_{\bar{Y}} + H_B + H_{\bar{B}} = \mathbf{A}\cdot\mathbf{P} + 2\,\mathbf{B}\cdot\Delta\mathbf{x} + 6\,B\,m, \quad (63)$$

where $\mathbf{P}$ denotes the total momentum of a system, and $\Delta\mathbf{x}$ - quark-antiquark "position difference". Thus, one has to add contributions from all three colours and from both quarks and antiquarks to obtain a rotationally and translationally invariant Hamiltonian. Hamiltonian (63) provides a particular mathematical realization of the idea of conspiration mechanism which, as I believe, ensures that the individual rotationally and translationally non-invariant quarks combine to form fully acceptable states.

If we divide the total Hamiltonian into quark and antiquark contributions H_q and $H_{\bar{q}}$, we observe that while either of them lacks translational invariance, it is rotationally invariant, e.g.

$$H_q = \mathbf{A} \cdot \mathbf{p} + \mathbf{B} \cdot 2\mathbf{x} + 3\ B\ m. \tag{64}$$

With the first and third factors in the direct product of 2×2 matrices in the explicit expressions for $\mathbf{A}, \mathbf{B}, B$ transforming under rotations in the same way, this Hamiltonian should be appropriate for a description of a system of total spin 0 or 1. However, its coupling to a photon goes solely through the first factor, i.e. is that of a spin 1/2 object, as needed for a quark.

Let us also point out that with

$$\gamma_5 \equiv -iA_1A_2A_3 = \sigma_0 \otimes \sigma_1 \otimes \sigma_0, \tag{65}$$

anticommuting with both B and $\mathbf{B}$, both the Bm and $\mathbf{B} \cdot \mathbf{x}$ terms are not chirally-invariant, a property exhibited by mass terms in standard formulations. It is perhaps also worth noting here that the analogues of definition (65), appropriate for individual "coloured" quarks, and defined in their respective subspaces through:

$$\gamma_{R5} = \quad -iA_1B_2B_3 = \quad \sigma_1 \otimes \sigma_1 \otimes \sigma_1, \tag{66}$$

$$\gamma_{Y5} = \quad -iA_2B_3B_1 = \quad \sigma_2 \otimes \sigma_1 \otimes \sigma_2, \tag{67}$$

$$\gamma_{B5} = \quad -iA_3B_1B_2 = \quad \sigma_3 \otimes \sigma_1 \otimes \sigma_3, \tag{68}$$

anticommute with the quark mass terms Bm as well, parallelling a similar property of γ_5 (however, the full meaning of these properties is not clear).

Translational invariance requires that we add the quark and antiquark contributions $H_{q\bar{q}} = H_q + H_{\bar{q}}$ as proposed above. This procedure maintains 0 and 1 as possible values for the total spin of the composite system, and satisfies the condition of the additivity of quark and antiquark charges.

Then, taking the square of $H_{q\bar{q}}$ á la Dirac, we obtain

$$E^2 = \mathbf{p}^2 + 4\ \Delta\mathbf{x}^2 + (6m)^2, \tag{69}$$

with the $4\ \Delta\mathbf{x}^2 + (6m)^2$ playing the rôle of meson mass squared.

There are several interesting qualitative features appearing in a rudimentary form in the above construction, with some of the important ones being:

- Additivity of quark charges. As usual, the charges of individual quarks have to be adjusted so as to lead to correct meson charges.
- Additivity of quark masses. First quark models often had such a property built in by hand. Later it was viewed as an approximation.
- Appearance of a "string" between points describing the "locations" of a quark and an antiquark. This was an argument in favour of introducing κ of dimension GeV/cm. An analogous constant (when Planck constant is added) appears in the original dual string model of mesons [25, 26], which introduces a constant of the dimension of cm^2 in its definition of string action. This dual string model exhibits various features in qualitative agreement with the observed properties of hadronic amplitudes.
- Objects exhibiting well-defined properties of one type do not have well-defined properties of another type.

For the description of baryons one needs an extension of the algebra of A_k, B_k matrices (still with SU(3) symmetry properties). Indeed, in the construction discussed before there are only two 2×2 matrix factors transforming under rotation in the standard way, while three such factors are needed to admit the description of the spin of three baryon quarks. The corresponding matrices should be multiplied by the available translationally-invariant three-vectors. Thus, in addition to the triplet of momentum coordinates one needs two triplets of position coordinate differences. However, from six canonical position coordinates ($\mathbf{x}$, $\mathbf{y}$) in general present in the three Hamiltonians H_R, H_Y, H_B one can form only one triplet of position differences $\mathbf{x} - \mathbf{y}$. Consequently, it is not clear to me how the baryon-describing Hamiltonian should be constructed. Perhaps one should use the same triplet of position differences in both spinorial subspaces. This would suggest that in baryons one space degree of freedom is actually frozen. Such a possibility was discussed at length in baryon phenomenology [27, 28], and is a viable option in light of the experimental information. Or maybe the overall position coordinate $\mathbf{X}$ canonically conjugated to the total momentum $\mathbf{P}$ should be somehow introduced (a kind of analogue of the "string junction" in the string model of baryons [29]). A deeper insight on how to proceed with baryons would be very helpful.

5 Final Remarks

The approach of the preceeding Section proposes a particular mathematical realization of the arguments of Section 3. It is formulated at the $c-q$ level and for this reason it is presumably still a toy model. However, it exhibits several very interesting qualitative properties, which should follow in a natural way from any more involved approach including the possibly underlying quantum pregeometry.

The considered scheme exhibits some (though somewhat superficial) similarities with the rishon model of leptons and quarks [30], in which leptons of a given generation (e.g. e^+, ν_e) are constructed from rishons T (of charge $+1/3$) and V (of charge 0) as TTT and VVV, while coloured quarks of the same generation, i.e. u_r, u_y, u_b, and $\bar{d}_r, \bar{d}_y, \bar{d}_b$ are the (ordered) composites VTT, TVT, TTV and TVV, VTV, VVT respectively. The rishon model suggests that our scheme should be doubled to incorporate weak isospin. In line with the general idea that the attributes of elementary particles should be somehow linked with the properties of the classical arena used for the description of Nature, one should try to propose a correspondence betwen this doubling and phase-space properties. Most probably it should be related to a relative relation between the momentum and position coordinates, as our scheme, contrary to the standard approach, admits considering their independent transformations. However, at this stage we prefer not to speculate on this subject any further. Still, it should be mentioned that the rishon model was incorporated in a much more elaborated scheme involving the standard $SU(3)_C \times U(1)_{EM}$ gauge invariance [31].

I hope that the proposed approach provides an interesting suggestion concerning the unobservability of quarks and the concept of quark mass. Obviously, the ideas put forward herein may constitute a mirage only. Should these ideas turn out to be relevant, one could look further, guided by the conjecture that the problem of mass is intimately connected or even identical to the problem of space (phase-space) quantization.

References

[1] P. Żenczykowski, Int. J. Theor. Phys. 29, 835 (1990).

[2] R. Delbourgo, G. Thompson, Phys. Rev. Lett. 57, 2610 (1986); B. Kniehl, A. Sirlin, Eur. Phys. J. C16, 635 (2000); M. E. Shaposhnikov, P. Tinyakov, Phys. Lett. B515, 442 (2001).

[3] see e.g. H. Fritsch, "The problem of mass and mass generation" hep-ph/9605300; H. Fritzsch "The symmetry and the problem of mass generation" hep-ph/9703242; H. Fritzsch, Zhi-zhong Xing, Prog. Part. Nucl. Phys. 45, 1 (2000); Y. Koide, hep-ph/0506247; W. Królikowski, hep-ph/050943.

[4] J. Gasser, H. Leutweyler, Phys. Rep. 87, 77 (1982).

[5] J. Schwinger, Phys. Rev. Lett. 18, 923 (1967); P. Żenczykowski, Z. Phys. C28, 317 (1985).

[6] M. K. Gaillard, B. W. Lee, Phys. Rev. D10, 897 (1974).

[7] S. B. Gerasimov, Yad. Fiz. 29, 513 (1979) [Sov. J. Nucl. Phys. 29, 259 (1979)]; R. Tarrach, Z. Phys. C2, 221 (1979); L. Ametller, A. Bramon, Ann. Phys. 154, 308 (1984); Ll. Ametller, C. Ayala, A. Bramon, Phys. Rev. D29, 916 (1984); however, see also [8].

[8] J. Steinberger, Phys. Rev. 76, 1180 (1949).

[9] J. Lach, P. Żenczykowski, Int. J. Mod. Phys. A10, 3817 (1995); P. Żenczykowski, Phys. Rev. D40, 2290 (1989).

[10] A. Lai et al., Phys. Lett. B584, 251 (2004).

[11] P. Żenczykowski, Nucl. Phys. Proc. Suppl. 115, 24 (2003); Acta Phys. Pol. B34, 2683 (2003).

[12] M. Gell-Mann, Physics 1, 63 (1964).

[13] R. Penrose, "Structure of spacetime", p.121 in Batelle Rencontres, C. M. DeWitt and J. A. Wheeler, eds., New York, 1968.

[14] R. Penrose, "Angular momentum: an approach to combinatorial spacetime", p. 151 in *Quantum Theory and Beyond*, T. Bastin, ed., Cambridge University Press, Cambridge, 1971.

[15] J. A. Wheeler, "From relativity to mutability", pp. 227, 235 in *The Physicist's Conception of Nature*, J. Mehra, ed., D. Reidel, Dordrecht, Holland, 1973.

[16] M. Born, Review of Modern Physics 21, 463 (1949).

[17] E. P. Wigner, Review of Modern Physics 29, 255 (1957).

[18] P. A. M. Dirac, "The development of the physicist's conception of Nature", p.11 in *The Physicict's Conception of Nature*, J. Mehra, ed. Reidel, Dordrecht, Holland, 1973.

[19] G. F. Chew, "Bootstrap idea and the foundations of quantum theory" p.141 in *Quantum Theory and Beyond*, T. Bastin, ed., Cambridge University Press, Cambridge, 1971.

[20] D. Finkelstein, G. McCollum, "Unified quantum theory", pp.19,45 in *Quantum Theory and the Structures of Time and Space*, L. Castell, M.Drieaschner, C. F. von Weizsäcker, eds., C. Hanser, Munich, Germany, 1975.

[21] D. Finkelstein, Phys. Rev. D5, 320 (1971).

[22] R. Penrose, "The Emperor's New Mind", Oxford University Press, 1989.

[23] C. N. Yang, in his paper "Some concepts in current elementary particle physics" in *The Physicict's Conception of Nature*, J. Mehra, ed. Reidel, Dordrecht, Holland, 1973, when talking about $SU(3)_{flavour}$ of the sixties, writes: "My personal prejudice is that if we finally should have an understanding of SU(3), it will turn out to be an understanding on quite a different footing from that of SU(2)"

[24] J. M. Lévy-Leblond, "Galilei group and Galilean invariance" in *Group Theory and its Applications*, vol. II, E.M.Loebl. Ed., Academic Press, New York, 1971, pp.221-299.

[25] G. Veneziano, Phys. Rept. 9, 199 (1974).

[26] C. Rebbi, Phys. Rept. 12, 1 (1974).

[27] P. Żenczykowski, Phys. Lett. B80, 298 (1979); Z. Phys. C5, 319 (1980).

[28] R. Koniuk, N. Isgur, Phys. Rev. Lett. 44, 845 (1980); Phys. Rev. D21, 1868 (1980).

[29] G.C. Rossi, G. Veneziano, Nucl. Phys. B123, 507 (1977).

[30] H. Harari, Phys. Lett. B86, 83 (1979).

[31] H. Harari, N. Seiberg, Nucl. Phys. B204, 141 (1982).

The old and new Concepts of Physics
An open-dialogue journal

www.uni.lodz.pl/concepts

Comment on THE CONCEPT OF MASS, QUARKS AND PHASE SPACE

Andrzej Horzela
H.Niewodniczański Institute of Nuclear Physics
Polish Academy of Sciences
ul. Eliasza-Radzikowskiego 152, PL 31342 Krakow, Poland
e-mail: Andrzej.Horzela@ifj.edu.pl

The notion of inertial mass (called simply 'mass' in what follows) is defined in classical mechanics through the Newton's second law of dynamics. According to this rule, the mass of a material object is a proportionality coefficient which relates the acceleration of the object and the force which acts on it, causing changes in the velocity which the object moves with. An equivalent definition may be given if one introduces one half of the mass as a coefficient linking the kinetic energy of the object with its velocity squared. Both these definitions mean that mass is an individual property of the object, in general unpredictable in any theoretical way, and operationally determined for each object that can be separated from other objects. In this context, separation means that the object can be distinguished from its environment and can be observed as an individual, its velocity can be measured and all forces acting on it can be identified and determined. Such a scheme, completed with the principle of equivalence of inertial and gravitational masses, works perfectly well in macroscopic physics and has enabled physicists not only to determine unambiguously the masses of observable physical objects, but also to forecast the existence of unobservable massive objects. The mathematical formalism of celestial mechanics allowed us to determine their masses from careful observations of the motion of visible astronomical objects perturbed by interaction with the invisible ones, thus providing us with the most spectacular examples: the prediction of the existence of Pluto and Neptune, and the discoveries of black holes and planets outside the Solar System. The inclusion of relativity does not lead to difficulties - in fact, it is the rest mass m_0 that plays the role of a (constant) mass parameter characterizing the object, while the so-called relativistic mass is not a mass, but a velocity-dependent function calculable from the formula $m = \frac{m_0}{\sqrt{1-v^2/c^2}}$. Similarly, in nonrelativistic quantum mechanics one does not find any serious problems if one wants to define mass. Although the Heisenberg uncertainty principle excludes the possibility of a simultaneous measurement of particle position (needed for determining force) and velocity, one can use the correspondence principle and define mass as a parameter entering quantum Hamiltonians exactly in the same way as in the case of classical Hamiltonians. This method works perfectly well and gives unique results for objects of quantum origin yet observable through classically measurable effects - we determine masses of elementary particles by studying their traces left in various detectors, we extract masses or mass-like parameters in condensed matter physics while investigating thermodynamical properties, we find masses of atomic nuclei through studying the properties of their emitted radiation. However, the problem gets more complicated when we analyse quantum systems while basing our considerations on the principles of quantum field theory. In quantum field theory, masses characterize field quanta - the latter being objects which have no analogy within classical field theory, in which we deal with waves only, without taking the matter-wave duality into account. Mathematically, masses of the quanta of elementary fields can be introduced as parameters labelling representations of the Poincare group, subsequently identified with experimentally measured masses of elementary particles. This constitutes a unique and trustworthy procedure as long as we can identify the elementary particle which passes through the detector -

i.e., for all these elementary particles which do exist as asymptotic states. However, what can we say about particles which we are not able to observe as asymptotic ones? Is it justified to assign masses to them exactly in the same way as for particles which can be separated from other particles and observed as free moving individuals? Is it justified to label the mass some dimensionful parameter extracted from experimental data using complicated mathematical machinery, which from the very beginning uses undefined or ill-defined notions? This is exactly the case we face in the most sophisticated realization of the relativistic field theory, namely in the Standard Model of fundamental interactions. The matter fields of this model, leptons and quarks, appear in the formalism in exactly the same way - both are assumed to be Dirac spin 1/2 fields but their physical properties are required to be different. The electro-weakly interacting leptons exist as "ordinary" particles, observed as asymptotic free ones, while strongly and electro-weakly interacting quarks manifest themselves in bound systems only; they have fractional charges, obey the mysterious phenomenon of confinement and are required to be unobservable as free objects.

In his very interesting paper, Piotr Żenczykowski investigates this problem, asking one of the most important questions in contemporary particle physics, namely how to solve the puzzle of quark masses *i.e.,* how to define the mass of an object called quark, which - as a quantum of an elementary field - must not exist in an asymptotic state? The problem does exist because in fact theoretical particle physics uses two inequivalent definitions of quark masses - the so-called *constituent mass* calculable from hadron properties and in a sense constituting an effective mass that includes the effects of the interaction between quarks forming a hadron, and the so-called *current mass*, entering theoretical models in complete analogy to lepton mass, *i.e.,* as a parameter in Dirac equations describing quark fields. However, when solving suitable Dirac equations we adjust quark momenta and put quarks on their mass-shells, thus excluding their primary localization in a bounded region of spacetime. Therefore, the current quark masses - entering the standard field theoretical calculations as poles of suitable propagators and appearing in final formulas - should be interpreted as masses of quarks existing as free particles. They may be extracted from the experimental data and it appears that for light quarks, the current masses are almost negligible, being much smaller than the constituent masses, while for heavy quarks both masses are practically equal. However, in loop calculations in the perturbative treatment of the Standard Model one cannot use small values of the light quark masses - the correct choice, in agreement with the experimental data, is to take the constituent masses. Żenczykowski claims that such a situation is "deeply dissatisfying, not only because of the existence of the conceptual problems and internal inconsistencies related to the use of quark mass, but also because usually they are either unnoticed or swept away under the carpet". I am convinced that when making such a statement, he is obviously right. Moreover, and this is much more important, he proposes a completely different approach to the subject, whose epistemological background comes from Roger Penrose's words: "I do not believe that a real understanding of the nature of elementary particles can ever be achieved without simultaneous deeper understanding of the nature of spacetime itself". Żenczykowski goes even further and supposes that a scheme which will provide real understanding of elementary particles, in particular the understanding of their masses, should reflect the structure of phase-space, the arena on which quantum physics is realized and in which symmetries constitute the most important source of information that we can obtain when investigating the properties of physical systems. Żenczykowski's second assumption is to study the problem in nonrelativistic approximation. I consider both these assumptions fully justified. When investigating the meaning of spacetime in quantum physics, we unavoidably face the problem of the nature of time. We have to answer fundamental questions: Is time an external classical parameter or a quantity having a quantum meaning and related to some quantum time operator? Consistency of relativistic quantum physics requires the latter possibility to be satisfied but up to now nobody has found its satisfactory realization. Thus, it seems to be much more reasonable to avoid this difficulty, at least at the level of preliminary considerations, and agree that the nonrelativistic description is enough. Concerning the phase space approach, it is well known that a consistent formalism of quantum physics in phase space exists, [1], [2], and, although sometimes complicated, has been used effectively in quantum optics, where it has enabled physicists to understand numerous interesting and meaningful phenomena. I would also like to emphasize that nonrelativistic descriptions, if carefully formulated, are rich enough to contain the properties of physical systems commonly believed to be ascribed solely to relativistic theories. Nonrelativistic analogues of the Dirac equation, based on Galilean-invariant formulations of field theories, lead not only to the proper description of the electron spin and gyromagnetic ratio [3] but also to duality interpreted as an indication of the existence of antiparticles [4]. Because of that, I am not surprised that within a nonrelativistic Dirac-like formalism proposed by Żenczykowski, antiparticles appear in a natural way. This fact is very important because all known hadrons are built from quarks and antiquarks, so any model aiming at describing hadrons must contain both kinds of fundamental objects: particles and antiparticles.

The crucial idea in the framework of Żenczykowski's approach is to consider a six-dimensional quadratic form $\vec{p}^{\,2} + \vec{x}^{\,2}$. Such a choice of the quadratic form under investigation cannot be treated as arbitrary - it is the simplest quadratic form which joins two sectors of phase-space variables, introduces a universal dimensional constant in a natural way and may be interpreted as energy of the simplest solvable interacting physical system, for which interaction strength grows with distance, namely the harmonic oscillator. For this form, Żenczykowski knows the symmetry transformations that preserve the canonical Poisson brackets of momenta p_i and positions x_j - he recalls that such canonical transformations were shown to form a group $U(1) \times SU(3)$ [5]. The author's next step is to apply the universal methods of quantum field theory. According to the general rules of quantum physics, the representations of a symmetry group may be used to describe quantum systems obeying these symmetries. In the relativistic quantum field theory, fundamental representations of the Poincare group are particularly important; they are subsequently identified with basic fields because basic field equations and their solutions are required to be covariant with respect to the symmetry transformations. Thus, having defined the symmetry, according to our knowledge the most important property of any quantum system, Żenczykowski suggests one should search for a fundamental representation of the symmetry group, following the well-established way, i.e. the Dirac procedure of the linearization of form $\vec{p}^{\,2} + \vec{x}^{\,2}$. This leads to Dirac-like equations, for the case under consideration constituting 8-dimensional sets of linear equations; this is easy to understand because new Dirac matrices can be taken as an 8-dimensional fundamental representation of the 6-dimensional Clifford algebra. But the most important part of Żenczykowski's construction is based on the observation that "the standard concept of mass may be said to be directly associated with the concept of momentum $\vec{p}$, not position $\vec{x}$" which, because of the assumed symmetry between p_i and x_j, means that we can choose new generalized momenta. Such a choice may be made in three possible ways, leading to three Dirac-like equations, separately lacking rotational invariance but restoring it when summed up. What are the consequences of this fact? It appears that the Dirac-like Hamiltonian constructed for the form $\vec{p}^{\,2} + \vec{x}^{\,2}$ is a sum of three terms, each of which may be related to a fundamental field. Moreover, each of these fields has its "charge conjugated" partner, *i.e.,* when introducing electromagnetic interaction through the minimal coupling rule, we have to distinguish between charge conjugated fields. Thus, the model contains six fundamental fields and Żenczykowski proposes to identify these fields with quarks and antiquarks. Furthermore, he observes that in order to restore total rotational and translational invariance for a quark-antiquark system, one has to build a Hamiltonian, in which all fundamental contributions are present. This Hamiltonian has interesting properties - quark charges and masses appear to be additive, the properties of an object considered as a whole may be qualitatively different from the properties characterizing its components, elementary components of the composed object are linked in some nondynamical sense that Żenczykowski compares with "string properties" characterizing models frequently used in hadron physics. Moreover, total energy of the quark-antiquark system, calculable as the Hamiltonian squared, contains a sum of two terms, one of which can be identified as generated by the current quark mass being a purely mathematical parameter, while the source of the second term is the interaction. Obviously, the sum just mentioned may be identified with the mass of the meson, and we can see that the *constituent* quark mass explicitly includes effects originating from the interaction. The conclusions seem obvious - the author has constructed an interesting model, qualitatively consistent with the models of mesons, believed to be quark-antiquark systems...

Is it possible to do this in such a relatively simple way? Is it not too beautiful to be true? I think it is not, even if the model is in fact only a toy model, oversimplified and very difficult to be generalized in a way allowing a description of a three-quark system, *i.e.,* baryons. I would like to stress in favour of the model that it explores only well-defined notions and fundamental principles. It is based on the symmetry and analysis of its properties. This is, in my deep convinction, the correct way in which we will be able to understand these properties of the physical microworld that nowadays seem to us very strange and extremely difficult to be explained using standard approaches. Moreover, I believe that the difficulties, or even failures, of the standard approaches to quantum physics are frequently the consequences of orthodoxy and prejudices coming from the macroscopic, classical world, and that solutions to these problems will be found as a result of a deep analysis of classical concepts rather than as a result of constructing exotic theories often lacking realistic physical background.

References

[1] W. P. Schleich, *Quantum Optics in Phase Space* Wiley, Berlin (2001).

[2] L. E. Ballentine, *Quantum Mechanics: a Modern Development* World scientific, Singapore (1998).

[3] J.-M. Levy-Leblond, in *Group Theory and its Applications*, vol.2, E.M. Loebl, Ed., Academic Press, NY (1971).

[4] A. Horzela and E. Kapuścik, *Electromagnetic Phenomena* **3**, 63 (2003).

[5] P. Żenczykowski, *Int. J. Theor. Phys.* **29**, 835 (1990).