

EVOLUTION IN TIME OF MOVING UNSTABLE SYSTEMS

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Abstract

Relativistic quantum theory shows that the known Einstein time dilation (ED) approximately holds for the decay law of the unstable particle having definite momentum p (DP). I use a different definition of the moving particle as the state with definite velocity v (DV). It is shown that in this case the decay law is not dilated. On the contrary, it is contracted as compared with the decay law of the particle at rest. It is demonstrated that ED fails in both DP and DV cases for time evolution of the simple unstable system of the kind of oscillating neutrino. Experiments are known which show that ED holds for mesons. The used theory may explain the fact by supposing that the measured mesons are in DP state.

1 Introduction

Experimenters showed that the lifetime τ of a uniformly moving unstable particle is equal to $\tau_0\gamma$, where τ_0 is the lifetime of the particle at rest and γ is the Lorentz factor $\gamma = (1 - v^2/c^2)^{-1/2}$, e.g., see (Baily, 1977; Farley, 1992). In other words, if $F(t)$ is the decay law of the unstable particle moving in the laboratory frame and $F_0(t)$ is the decay law of the same particle at rest, then $F_0(t) = \exp(-t/\tau_0)$ and $F(t) = \exp(-t/\tau_0\gamma)$ or

$$F(t) = F_0(t/\gamma). \quad (1)$$

A usual explanation of the fact is based on the Einstein special theory of relativity and I call (1) the Einstein dilation (ED). For example, Møller (1972) sets it forth as follows:

“In view of the fact that an arbitrary physical system can be used as a clock, we see that any physical system which is moving relative to a system of inertia must have a slower course of development than the same system at rest. Consider for instance a radioactive process. The mean life τ of the radioactive substance, when moving with a velocity v , will thus be larger than the mean life τ_0 when the substance is at rest. From (2.36) we obtain immediately $\tau = (1 - v^2/c^2)^{-1/2}\tau_0$.”

This argumentation may be complemented by the following possible definition of the unit of time provided by radioactive substance: this is the time interval during which the amount of the substance decreases twice, e.g.

However, the standard clocks of the relativity theory are used when obtaining Eq. (2.36)

$$\Delta t = t_2 - t_1 = \gamma(t'_2 - t'_1) = (1 - v^2/c^2)^{-1/2}\Delta\tau$$

which Møller mentions. He begins the derivation of this equation with the phrase:

“Consider a standard clock C' which is placed at rest in S' at a point on the x' -axis with the coordinate $x' = x'_1$.”

However, such a quantum clock as an unstable particle cannot be at rest (i.e., have zero velocity or zero momentum) and simultaneously be at a definite point (due to the quantum uncertainty relation). So the standard derivation of the moving clock dilation is inapplicable for the quantum clock.

Another way of theoretical derivation may be used: to find the relativistic quantum decay law $F(t)$ of the moving particle and to compare it with the decay law $F_0(t)$ of the particle at rest. Lorentz transformation of the space-time coordinates from one inertial frame to another is not needed as well as the space coordinates themselves. The approach was employed by (Exner, 1983; Stefanovich, 1996; Khalfin, 1997; Shirokov, 2004). In these papers (below I shall refer to them as (ESKS)) the state of the moving unstable particle was described by the eigenvector Ψ_p of the momentum operator \hat{P} (Exner (1983) used a packet with almost exact momentum). One may state that the obtained decay law $F_p(t)$ is consistent with ED, Eq. (1), see Sect. 3 below. I use in Sect. 2 another definition of the moving particle: it is described by the vector Φ_v having a definite nonzero velocity \vec{v} . If the particle were stable, the vector Φ_v would coincide with Ψ_p at $\vec{p} = \vec{v}m_0\gamma$, m_0 being the particle mass. However, the unstable particle has no definite mass, it is described by a distribution over masses, see Sect. 2. Therefore, if \vec{p} is definite, then \vec{v} cannot be definite, see Eq. (12) below. The exclusion is the case $\vec{p} = 0$ when \vec{v} is also zero.

In the case of unstable particles, whose decay laws can be measured one may expect that using either Φ_v or Ψ_p should give only slightly different results. Indeed, mass distributions of such particles are concentrated in small regions near average masses m , the dimension Γ of the regions being much less than m . However, the detailed calculation of $F_v(t)$ presented in Sect. 2 provides instead the unexpected result $F_v(t) = F_0(t\gamma)$, i.e., contraction instead of dilation: particles with exact nonzero velocity decay faster than the one at rest.

A simple unstable system is considered in Sect. 4. The oscillating neutrino may serve as an example. The usual formulae for the neutrino oscillation, e.g., see (Bilenky and Pontecorvo, 1978; Bilenky, 2004) are valid when neutrino has definite momentum \vec{p} . The corresponding oscillation is dilated as compared to the oscillation of the

neutrino with lesser momentum. However, the dilation is not Einsteinian, Eq. (1). In the case when neutrino has a definite velocity I obtain another formula for neutrino oscillation which gives the same contraction as in Sect. 2.

For summary and conclusion see Sect. 5.

2 Decay law of moving unstable state with precise velocity

Let us consider a relativistic theory which describes unstable particles, products of their decay, and the corresponding interactions. A field theory may be an example. Such a theory must contain operators of total energy and momentum \hat{H} , $\hat{\vec{P}}$ (the generators of time and space translations), total angular momentum, and generators of Lorentz boosts $\hat{\vec{K}}$. Usual Dirac's "instant form" of the theory is implied so that interaction terms are contained in \hat{H} and $\hat{\vec{K}}$:

$$\hat{H} = \hat{H}_0 + \hat{H}_{int}, \quad \hat{\vec{K}} = \hat{\vec{K}}_0 + \hat{\vec{K}}_{int}.$$

A simple example is the Lee model of the decay of particle a into two stable particles b and c : $a \rightarrow b + c$. The interaction terms are of the threelinear kind $\hat{a}\hat{b}^\dagger\hat{c}^\dagger + H.c.$ (momentum indices of destruction-creation operators are omitted). The moving particle a is usually described by the eigenvector $\hat{a}_p^\dagger\Omega_0$ of the free Hamiltonian \hat{H}_0 (Ω_0 is the "bare" vacuum and \hat{a}_p^\dagger is the creation operator of the particle a with momentum \vec{p}). When $\vec{p} = 0$ the vector $\hat{a}_0^\dagger\Omega_0$ describes an unstable particle at rest.

Alternatively a moving unstable state may be described by the eigenvector $\Phi_v = L_v\hat{a}_0^\dagger\Omega_0$ of the velocity operator $\hat{\vec{V}} = \hat{\vec{P}}/\hat{H}$. Here L_v is the Lorentz transformation from the frame where the velocity of the state $\hat{a}_0^\dagger\Omega_0$ is zero to the frame where the state has the velocity v (e.g., see Gasiorowicz, 1966). The state $\hat{a}_p^\dagger\Omega_0$ differs from Φ_v : $\hat{\vec{V}}$ does not commute with \hat{H}_0 and, therefore, \hat{H}_0 eigenvector $\hat{a}_p^\dagger\Omega_0$ has no definite velocity.

V. Stefanovich in private communication noted that the state Φ_v has an admixture of decay particles and, therefore, is not a pure one-unstable-particle state. Indeed, generators of L_v contain threelinear

(interaction) terms of the kind $\hat{a}\hat{b}^\dagger\hat{c}^\dagger$ and consequently $L_v\hat{a}_0^\dagger\Omega_0$ contains a term of the kind $\hat{b}^\dagger\hat{c}^\dagger\Omega_0$. This fact does not hinder my purpose which is to consider an example of an unstable state with a definite velocity even if it were not a pure one-particle state. Looking ahead note that Eqs. (9) and (16) below show that Φ_v is an really unstable state in the sense that the nondecay amplitude $(\Phi_v, \Phi_v(t))$, see Eq. (8), vanishes as $t \rightarrow \infty$.

To obtain decay laws, the usual solution of the Schroedinger equation (in the Schroedinger picture) is used: if at $t = 0$ the initial state is ψ , then the state $\psi(t)$ at $t > 0$ is $\psi(t) = \exp(-i\hat{H}t)\psi$.

At first consider the initial state $\Phi_0 = \hat{a}_0^\dagger\Omega_0$. Let us expand Φ_0 over those eigenvectors φ_μ of \hat{H} which are simultaneously \hat{P} or \hat{V} eigenvectors with zero eigenvalue (note that \hat{P} and $\hat{V} = \hat{P}/\hat{H}$ commute with \hat{H}). The corresponding \hat{H} eigenvalues may be called masses and are denoted by μ : $\hat{H}\varphi_\mu = \mu\varphi_\mu$

$$\Phi_0 = \int_\mu c(\mu)\varphi_\mu, \quad c(\mu) = (\varphi_\mu, \Phi_0). \quad (2)$$

Then we have

$$\Phi_0(t) \equiv \exp(-i\hat{H}t)\Phi_0 = \int_\mu c(\mu) \exp(-i\mu t)\varphi_\mu \quad (3)$$

and the nondecay (survival) amplitude is

$$A_0 \equiv (\Phi_0, \Phi_0(t)) = \int_\mu |c(\mu)|^2 \exp(-i\mu t). \quad (4)$$

In Sections 2 and 3 I deal with states for which survival amplitudes vanish as $t \rightarrow \infty$. This property holds only if the convolution \int_μ in Eqs. (2)-(4) is integral over continual μ values. Besides, the spectrum of \hat{H} must be bounded from below. So \int_μ may be understood as the integral $\int_0^\infty d\mu$.

The vectors φ_μ may be endowed with other indices (e.g., spin ones), upon which \hat{H} eigenvalues do not depend. I do not write out these degeneration indices.

Note that the used definition of survival probability $|A_0(t)|^2$ is a particular case of a more general definition, see, e.g., (Exner, 1983).

The latter reduces to $|A_0(t)|^2$ because the used initial states are eigenvectors of conserving operators \hat{P} or \hat{V} . In Sect. 4, I shall deal with survival amplitudes that do not vanish as $t \rightarrow \infty$ but oscillate.

Now let us consider the decay law of the initial state $\Phi_v = L_v \hat{a}_0^\dagger \Omega_0$, see above. Applying the operator L_v to both parts of Eq. (2) one obtains the expansion of Φ_v over vectors $L_v \varphi_\mu \equiv \varphi_{v\mu}$:

$$\Phi_v = \int d\mu c(\mu) \varphi_{v\mu}, \quad \hat{H} \varphi_{v\mu} = E_{v\mu} \varphi_{v\mu}, \quad \hat{V} \varphi_{v\mu} = \vec{v}_{v\mu} \varphi_{v\mu}. \quad (5)$$

Let us show that $\varphi_{v\mu}$ is \hat{H} eigenvector corresponding to the eigenvalue $E_{v\mu} = \mu\gamma$, $\gamma = (1 - v^2)^{-1/2}$. Indeed, one has

$$L_v^{-1} \hat{H} L_v = (\hat{H} + \vec{v} \hat{P}) \gamma \quad (6)$$

(\hat{H} and \hat{P} make up a 4-vector). Therefore,

$$H L_v \varphi_\mu = L_v (\hat{H} + \vec{v} \cdot \hat{P}) \gamma \varphi_\mu = \gamma \mu L_v \varphi_\mu.$$

The equations $\hat{P} \varphi_\mu = 0$ and $\hat{H} \varphi_\mu = \mu \varphi_\mu$ have been used.

Respectively, in place of Eqs. (3) and (4) one gets

$$\Phi_v(t) = \int d\mu c(\mu) \varphi_{v\mu} \exp(-i\mu\gamma t), \quad (7)$$

$$A_v(t) \equiv (\Phi_v, \Phi_v(t)) = \int d\mu |c(\mu)|^2 \exp(-i\mu\gamma t). \quad (8)$$

Note. When calculating Eq. (7) the orthonormalization equation $(\varphi_{v\mu_1}, \varphi_{v\mu_2}) = \delta(\mu_1 - \mu_2)$ is used, implying unit normalization of \hat{V} eigenvectors. This is the case if \hat{V} has a discrete spectrum analogously to the spectrum the momentum has when the system is implied to be in a large space volume and usual periodicity conditions are imposed (or the volume opposite boundaries are identified).

Comparing Eq. (8) with Eq. (4) one obtains the following relation of survival amplitudes:

$$A_v(t) = A_0(\gamma t). \quad (9)$$

The same relation holds for the probabilities $F_v(t) = |A_v(t)|^2$ and $F_0(t) = |A_0(t)|^2$:

$$F_v(t) = F_0(\gamma t). \quad (10)$$

So one gets contraction instead of dilation, Eq. (1), if a moving unstable state has a definite velocity. Remark that in order to obtain Eq. (10) one needs not know φ_μ or $c(\mu)$, see Eq. (2); relation (10) is true for any decay interaction.

In order to discuss this unexpected result I write out the corresponding survival amplitudes for the state ψ_p with exact momentum.

3 Decay law of unstable state with precise momentum

I define Ψ_p in analogy with Eq. (5) by expansion over common eigenvectors $\psi_{p\mu}$ of the operators \hat{H} and \hat{P} :

$$\Psi_p = \int d\mu c(\mu) \psi_{p\mu}, \quad \hat{H} \psi_{p\mu} = E_{p\mu} \psi_{p\mu}, \quad \hat{P} \psi_{p\mu} = \vec{p} \psi_{p\mu}. \quad (11)$$

Here the coefficients $c(\mu)$ do not depend on p just as in Eq. (5). One may assume that $\psi_{p\mu} = L_{p\mu} \varphi_\mu$, where $L_{p\mu}$ is the operator of the Lorentz transformation of the zero velocity state φ_μ into the frame where the velocity of the state is equal to $\vec{p}/\sqrt{p^2 + \mu^2}$, i.e., corresponds to the momentum \vec{p} . One may verify that $L_{p\mu} \varphi_\mu$ is \hat{P} eigenvector with eigenvalue \vec{p} . In the same way as before, see Eq. (6), one may demonstrate that $\psi_{p\mu}$ is \hat{H} eigenvector with eigenvalue $E_{p\mu} = \sqrt{p^2 + \mu^2}$:

$$\begin{aligned} \hat{H} L_{p\mu} \varphi_\mu &= L_{p\mu} L_{p\mu}^{-1} \hat{H} L_{p\mu} \varphi_\mu = L_{p\mu} (\hat{H} + \vec{v}_{p\mu} \hat{P}) \gamma_{p\mu} \varphi_\mu = \\ &= \mu \gamma_{p\mu} L_{p\mu} \varphi_\mu, \end{aligned}$$

where $\mu \gamma_{p\mu} = \mu [1 - p^2/(p^2 + \mu^2)]^{-1/2} = \sqrt{p^2 + \mu^2}$.

The value $\sqrt{p^2 + \mu^2}$ for $E_{p\mu}$ was obtained in a different way in (Shirokov, 2004).

Let us stress that φ_μ is a stable state (with a definite mass) so $\psi_{p\mu}$ is also the state with a definite velocity $\vec{v}_{p\mu}$ corresponding to

momentum \vec{p} . Meanwhile Ψ_p is not eigenstate of \hat{V} . Indeed,

$$\hat{V}\Psi_p = \int d\mu c(\mu) \hat{P}/\hat{H}\psi_{p\mu} = \vec{p} \int d\mu c(\mu) (p^2 + \mu^2)^{-1/2} \psi_{p\mu}. \quad (12)$$

So the r.h.s. of (12) is not proportional to $\Psi_p = \int d\mu c(\mu) \psi_{p\mu}$.

Using Eq. (11) one obtains for survival amplitudes

$$A_p(t) \equiv \langle \Psi_p, e^{-i\hat{H}t}\Psi_p \rangle = \int d\mu |c(\mu)|^2 \exp(-it\sqrt{p^2 + \mu^2}), \quad (13)$$

$$A_0(t) \equiv \langle \Psi_0, e^{-i\hat{H}t}\Psi_0 \rangle = \int d\mu |c(\mu)|^2 \exp(-i\mu t). \quad (14)$$

Note that when $\vec{p} = 0$ the state $\Psi_p(t) = \exp(-i\hat{H}t)\Psi_p$ coincides with $\Phi_p(t)$, see Eq. (3).

Now the survival law $A_p(t)$ is not connected with $A_0(t)$ by such a simple relation as $A_v(t)$ does, see Eq. (9). To compare $A_p(t)$ with $A_0(t)$, one has to calculate $A_p(t)$ and $A_0(t)$ separately. For this purpose one needs to know $|c(\mu)|^2$. The Breit-Wigner distribution

$$|c(\mu)|^2 = \frac{\Gamma}{2\pi} [(\mu - m)^2 + \Gamma^2/4]^{-1} \quad (15)$$

was used in (ESKS). Let us write out approximate expressions for $A_0(t)$ and $A_p(t)$ which are valid for time not too short and not too long when the decay laws are exponential (Shirokov, 2004)

$$A_0(t) \cong \exp(-imt - \frac{1}{2}\Gamma t), \quad (16)$$

$$A_p(t) \cong \exp(-im\gamma_m t - \frac{1}{2}\Gamma t/\gamma_m), \quad \gamma_m \equiv \sqrt{p^2 + m^2}/m. \quad (17)$$

Here m is the average (or the most probable) mass in the distribution (15). It follows from Eqs. (16) and (17) that

$$|A_p(t)|^2 \cong |A_0(t/\gamma_m)|^2, \quad (18)$$

i.e. ED holds for survival probability of an unstable particle with precise momentum. The dilation (18) is to be juxtaposed to the contraction (10). As was argued in Introduction, one may expect that the amplitudes $A_v(t)$ and $A_p(t)$ should not differ appreciably. Let us

show that this expectation is realized in a sense. Note beforehand that Eqs. (16) and (9) result in the explicit approximate expression for $A_v(t)$ when $\vec{v} = \vec{p}/\sqrt{p^2 + m^2}$

$$A_v(t) \cong \exp(-im\gamma t - \frac{1}{2}\Gamma t/\gamma). \quad (19)$$

Let us compare the exponents E_p and E_v of the corresponding exponentials in Eqs. (17) and (19)

$$E_p = -imt\gamma_m - \frac{1}{2}\Gamma t/\gamma_m, \quad E_v = -imt\gamma - \frac{1}{2}\Gamma t\gamma, \quad (20)$$

assuming that $\gamma_m = \gamma$. As $\Gamma \ll m$ the exponents coincide in the zero approximation when the terms $\sim \Gamma$ are neglected in E_p and E_v . So in this approximation the corresponding amplitudes $A_p(t)$ and $A_v(t)$ coincide and both satisfy the contraction property

$$A_p(t) \cong A_v(t) \cong A_0(t\gamma). \quad (21)$$

However, the main terms of E_p and E_v are purely imaginary and do not contribute to modules of $A_p(t)$ and $A_v(t)$. It is real parts $\frac{1}{2}\Gamma t/\gamma_m$ and $\frac{1}{2}\Gamma t\gamma$ that do contribute and determine the different dependences of $|A_p|^2$ and $|A_v|^2$ upon t , see Eqs. (18) and (10).

In the next section I will consider a simple unstable system whose time evolution is determined by the interference of the main terms defined above (the terms $\sim \Gamma$ being absent). For this system one may expect the breakdown of ED (in view of Eq. (21)) even if the system has a precise momentum.

4 Time evolution of moving two-mass state

Let us consider an unstable system at rest whose state vector ϕ is a superposition of two \hat{H} eigenvectors φ_1 and φ_2 :

$$\phi = c_1\varphi_1 + c_2\varphi_2, \quad |c_1|^2 + |c_2|^2 = 1, \quad (22)$$

$$\hat{H}\varphi_1 = m_1\varphi_1, \quad \hat{H}\varphi_2 = m_2\varphi_2, \quad m_1 \neq m_2. \quad (23)$$

The system time evolution is described by the survival amplitude

$$A_0(t) \equiv (\phi, \phi(t)) = |c_1|^2 e^{-im_1 t} + |c_2|^2 e^{-im_2 t}. \quad (24)$$

The survival amplitudes of the system with nonzero exact velocity \vec{v} and exact momentum \vec{p} are, respectively,

$$A_v(t) = |c_1|^2 \exp(-im_1 t\gamma) + |c_2|^2 \exp(-im_2 t\gamma), \quad (25)$$

$$A_p(t) = |c_1|^2 \exp(-it\sqrt{p^2 + m_1^2}) + |c_2|^2 \exp(-it\sqrt{p^2 + m_2^2}), \quad (26)$$

cf. Eqs. (8) and (13).

As examples of such a system one may take electron neutrino, e.g., see (Bilenky, Pontecorvo, 1978; Bilenky, 2004) and K_0 -meson

$$|K_0\rangle = (|K_s\rangle + |K_l\rangle) / \sqrt{2},$$

provided that $\Gamma_s = \Gamma_l = 0$, e.g., see (Perkins, 1987). Note that I deal here with the evolution of the unstable states in time, provided the source and the detector of the states are located in the same space volume. In the literature different approaches to neutrino oscillations are considered: the neutrino source and detector are separated by a distance R and one deals with the oscillatory dependence on R , see, e.g., (Dolgov, Okun, Rotaev, Schepkin, 2004) and references therein.

In what follows, I let $c_1 = c_2 = 1/\sqrt{2}$. Then

$$|A_0(t)|^2 = \cos^2 \left[\frac{1}{2}(m_1 - m_2)t \right], \quad (27)$$

$$|A_v(t)|^2 = \cos^2 \left[\frac{1}{2}(m_1 - m_2)t\gamma \right] = |A_0(t\gamma)|^2, \quad (28)$$

$$|A_p(t)|^2 = \cos^2 \left[\frac{1}{2} \left(\sqrt{p^2 + m_1^2} - \sqrt{p^2 + m_2^2} \right) t \right] = |A_0(t/\tilde{\gamma})|^2, \quad (29)$$

$$\tilde{\gamma} = (\sqrt{p^2 + m_1^2} + \sqrt{p^2 + m_2^2}) / (m_1 + m_2).$$

The oscillatory behavior of these probabilities allows us to use the two-mass system as the quantum clock. Its unit of time may be defined as the period of oscillation (the oscillation frequency being equal to $m_1 - m_2$ in the case of $|A_0(t)|^2$), cf. with the definition of the unit of time provided by radioactive substance, see Introduction.

It follows from (27) and (29) that the time evolution $|A_p(t)|^2$ in the case of exact momentum is dilated as compared to $|A_0(t)|^2$, but the dilation is not Einsteinian if $m_1 \neq m_2$: $\tilde{\gamma}$ turns into the Lorentz factor only if $m_1 \cong m_2$. In the case of exact velocity we have the same contraction as for unstable particles, cf. Eqs. (27) and (28).

5 Conclusion

Relativistic quantum-mechanical derivation of the time evolution of moving unstable particles was considered in the papers (ESKS). There the state of the moving particle was defined as the eigenvector Ψ_p of the momentum operator \hat{P} with eigenvalue \vec{p} . It was shown that then the nondecay law satisfied approximately ED, Eq. (1). Here in Sect. 2 the moving particle is described by eigenvector Φ_v of the velocity operator $\hat{V} = \hat{P}/\hat{H}$. If the particle were stable, then Φ_v would coincide with Ψ_p if $\vec{p} = \vec{v}m_0\gamma$. The vectors do not coincide in the case of unstable particle, but one may expect that they should give only slightly different results. It was shown in Sect. 3 that Φ_v and Ψ_p give indeed the same nondecay amplitude in the zero approximation. However, the approximation does not contribute to the corresponding probability, i.e., the nondecay law $F(t)$. As a result, the laws $F_v(t)$ and $F_p(t)$ turn out to be strongly different: $F_v(t)$ is contracted as compared to the nondecay law $F_0(t)$ of the particle at rest, see Eq. (10), meanwhile $F_p(t)$ is dilated, see Eq. (18).

Section 4 deals with unstable systems which are simpler than unstable particles. Oscillating neutrino may be the example. If it has exact momentum, then a dilation follows, but it is not ED, Eq. (1). In the case of exact velocity I obtain the formula which leads to the contraction, see Eq. (28).

I conclude that relativistic quantum theory of the time evolution of moving unstable systems does not ensure ED. The theory allows the possibility of moving unstable systems whose time evolution breaks ED.

Experiments are known which show that moving mesons have longer lifetimes than the immovable ones so that ED holds (Bailey, 1977; Farley, 1992). The theory used here may explain this fact supposing that the experiments deal with mesons which are in states close to Ψ_p . In this case, the theory approximately gives ED.

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Comment on EVOLUTION IN TIME OF MOVING UNSTABLE SYSTEMS

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M.I. Shirokov points out a curious fact about relativistic systems which are uniformly moving with respect to a system at rest. Let Φ_0 denote the state of a system with zero momentum, i.e. $\hat{P}\Phi_0 = 0$ (assuming for the moment such a state exists), and let $\Phi_{\vec{v}} \equiv L_{\vec{v}}\Phi_0$ be the corresponding system with velocity \vec{v} , where $L_{\vec{v}}$ is the appropriate Lorentz transformation. Since the total Hamiltonian, \hat{H} , is a generator of the Poincaré group one has

$$L_{\vec{v}}^* \hat{H} L_{\vec{v}} = (\hat{H} + \vec{v} \cdot \hat{P}) / \sqrt{1 - v^2} \quad (1)$$

and thus, since \hat{H} commutes with \hat{P} and since $\vec{v} \cdot \hat{P}\Phi_0 = 0$, one has

$$\langle \Phi_{\vec{v}}, e^{-i\hat{H}t} \Phi_{\vec{v}} \rangle = \langle \Phi_0, e^{-i\hat{H}t/\sqrt{1-v^2}} \Phi_0 \rangle. \quad (2)$$

Hence the overlap on the l.h.s. of Eq. (2) at time t for the moving system equals the overlap for the system at rest at the later(!) time $t/\sqrt{1 - v^2}$.

Shirokov now envisages Φ_0 to represent an unstable decaying system and the overlap in Eq. (2) to be the decay amplitude. He therefore obtains the seemingly paradoxical result that an unstable particle at rest decays more slowly than a moving unstable particle. To reconcile this with the contradictory experimental evidence he suggests that a moving unstable system is not described by $\Phi_{\vec{v}}$ but by an eigenstate $\Psi_{\vec{p}}$ of total momentum $\hat{\vec{P}}$.

Technically, Shirokov supposed in his paper that the momentum $\hat{\vec{P}}$ has a discrete spectrum because his system is implied to be in a large but finite volume and usual periodicity conditions are imposed (or the boundaries of the volume are identified) so that $\langle \Psi_p, \Psi_p \rangle = 1$. He supposed also that the velocity $\hat{\vec{V}}$ has a discrete spectrum so that $\langle \Phi_v, \Phi_v \rangle = 1$. See his note after his Eq.(8). However, then for quantization in a finite box Lorentz invariance is broken and his Eq. (6) does not hold as it stands. So one would have to eventually let the box go to infinity, and then it seems to be simpler to consider the infinite case right away, as I did above in deriving Eq. (2).

I would like to point out some difficulties I have with Shirokov's interpretation. First of all, imagine an observer moving with velocity $-\vec{v}$ and observing the unstable particle which is at rest in the lab frame and described by Φ_0 . Then, by Lorentz covariance, the moving observer clearly sees a state of the form $L_{\vec{v}}\Phi_0$ from above. But the suggestion of Shirokov seems to imply that a moving unstable system which is prepared in the lab frame is not of this form, and it seems to me that this would make different frames inequivalent.

Another difficulty is, in my opinion, connected with the infinite volume limit. In fact, a normalized zero-momentum state for the case of finite volume does not converge in norm when the volume tends to infinity and the overlap amplitude for the case of finite volume will converge to zero in the limit. Of course, in the infinite volume case such a zero-momentum state exists mathematically as an improper eigenvector (which can not be normalized to 1), but it is not physically realizable since in general physical states are given by normalizable vectors. Improper eigenvectors are very useful, but care has to be taken with the probabilistic interpretation when one uses non-normalizable states. Any *normalized* state in the infinite volume case, however, has also components for nonzero momentum and then

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Eq. (2) does *not* hold since there will be contributions from the $\vec{v} \cdot \hat{\vec{P}}$ term in Eq. (1).

Shirokov's observation has given much food for thought and will hopefully stimulate further research.

Reply to referee's comments.

G. Hegerfeldt points out a simple derivation of my main relation Eq. (10). In particular, his derivation does not use the expansion given by Eq. (2) of my paper.

I think that the puzzle with contraction, Eq. (10), should be removed by physical reasons rather than mathematical ones. For example, consider a simple case of neutrino oscillation. Let Φ_ν be a superposition of two mass states with energies $E_1 = m_1\gamma$ and $E_2 = m_2\gamma$, see Eqs. (23) and (5). Let $m_2 \gg m_1$, then $E_2 \gg E_1$. Suppose that Φ_ν is prepared by using the reaction $\pi \rightarrow \mu + \nu$, π being at rest. Suppose that π and μ have exact masses m_π and m_μ . It follows from energy–momentum conservation that $E_1 \cong E_2$ when $m_1, m_2 \ll m_\pi$, see G. Giunti, C. Kim, *Found. Phys. Lett.*, 2001, v. **14**, pp. 215-216. Therefore, Okun, Giunti *et al.* (see the cited paper, Sect. 5D and reference [49] therein) state that one cannot prepare the state Φ_ν in the case $m_2 \gg m_1$ (when $E_2 \gg E_1$) using π -meson. Note that the state Ψ_p may be prepared in this way. Of course, this example does not exclude the possibility of Φ_ν preparing by some other means, but π is the simplest neutrino source.

Of course, removing controversies with experiments we are left with the theoretical difficulty: Eq. (10) shows that the Einstein dilation does not follow from relativistic quantum theory.

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