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SURPRISES OF QUANTUM WAVE MOTION THROUGH POTENTIALS

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Abstract

We have found that transparent potentials $V(x)$ divided at arbitrary point x' in right-left parts $V_r(x < x') = 0$, $V_l(x > x') = 0$ have remarkable unexpected symmetry. There are continuous sets of equalities for penetrabilities of quite unequal separated $V_r \neq V_l$. Generalization of these equalities to wide classes of transformed V_r, V_l potentials and even arbitrary initial potentials $V(x)$ are given. Such equalities serve us as instructive illustrations enriching our quantum intuition useful for quantum engineering, etc.

We have revealed still unmentioned facts about transparency of potentials for massive particles and photons. Although unexpected they could be simply made evident. Let us cut at arbitrary point x=x' any potential $V(x)$ which is 100% transparent at discrete resonance energy points $E = E_{res}; |T(E_{res})| = 1, R(E_{res}) = 0$. Let us consider the separate left and right hand side parts $V_l(x)$ and $V_r(x)$ of $V(x)$. Then let us designate the transparency of one isolated part $V_l(x)$ as T_l and of the other $V_r(x)$ as T_r. At resonance energies E_{res} we will have found the equalities $|T_l(E_{res})| = |T_r(E_{res})|$ which are true **independent on particular choice of** x'. So, there are continuous(!) sets of pairs (for arbitrary x') of *quite different* potentials ($V_l(x) \neq V_r(x)$ with the *same* transparency. And this appeared to have strikingly simple and universal explanation. For reflectionless initial potentials $V(x)$ **absolutely transparent at any energy value of the continuous spectrum**(soliton-like ones) the same is true at arbitrary E. In this case we have additional continual widening of equation set. The class of potentials $V(l, r)$ can also be infinitely enlarged by some special transformations. We found that the analogous equalities are true also for potentials $V_l(x)$ and $V_r(x)$ with addition of different symmetrical potentials $V_{add}(x' + x) = V_{add}(x' - x)$, e.g., rectangular steps at the point of cut.

Fortunately the exact equality of penetrabilities

$$|T_l(E_{res})| = |T_r(E_{res})|$$

can be shown both analytically and in pictures.

Consider the potential $V(x)$ giving no reflection at resonance energy $E = E_{res}$. Its 100 % transparency $|T(E_{res})| = 1)$ follows [1, 2, 3] due to mutual destructive interference of the reflected wave with the wave "decaying backward" from $V(x)$, where it was 'accumulated' in quasi-bound state.

So, we have cut the potential $V(x)$ at the point x' into two parts $V_l(x) \neq V_r(x)$. The wave function at the point of infinitely narrow potential cut (with the 'bottom' at energy $E = 0$) can be represented as two 'free' waves moving in opposite directions with different amplitudes:

$$Aexp(ikx) + Bexp(-ikx). \tag{1}$$

Removing the left part $V_l(x)$ of $V(x)$ we can consider (1) as waves: incident from left hand side and reflected from the right part $V_r(x)$.

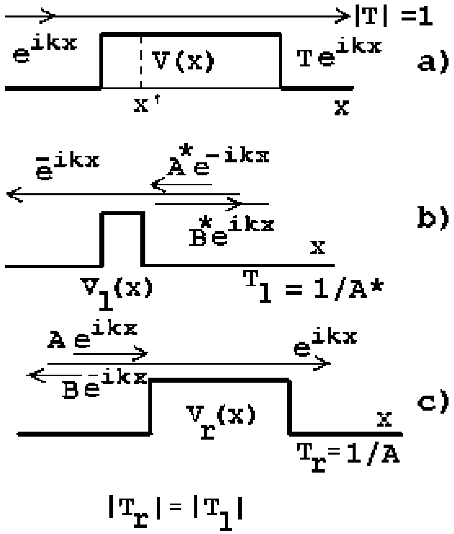

Figure 1: a) The initial rectangular potential barrier $V(x)$ is cut at the arbitrary point x'. b),c) illustrate wave penetration through separate left and right parts $V_l(x)$, $V_r(x)$, generally unequal rectangular barriers. Waves inside the cut are infinitely continued to the right (left) hand sides in b), c). Comparison of c) and d) makes evident the coincidence of transparency of $V_l(x)$ and $V_r(x)$.

Analogously if we remove the right part $V_r(x)$ and consider the complex conjugated wave

$$A^* exp(-ikx) + B^* exp(ikx) \tag{2}$$

the expression (2) conjugated to (1) can be considered as waves: incident from the right hand side and reflected from the left part $V_l(x)$. Normalizing the incident waves in both cases we get the equal modulo transition $|T_r|(E_{res}) = |1/A| = |1/A^*| = |T_l|(E_{res})$ and reflection $|R_r|(E_{res}) = |B/A| = |B^*/A^*| = |R_l|(E_{res})$ coefficients.

The following arguments can be given to ground this. If the cut would be of finite width, the real and imaginary parts of the wave function would have finite bending inside the cut. In the limit of zero width cut with finite depth this bending disappears. In this case the cut does not perturbs the solutions.

The absolutely transparent potentials have attracted a wide attention of physical community (see, e.g., [4, 5]), but nobody has suspected the possibility of the wonderful special symmetry hidden in reflectionless potentials and resonance tunneling. The discovered phenomena could be verified experimentally in real samples using the already good developed technique of constructing heterogeneous structures of thin layers (by dust covering, etc.) of different materials (see, e.g.,[5]).

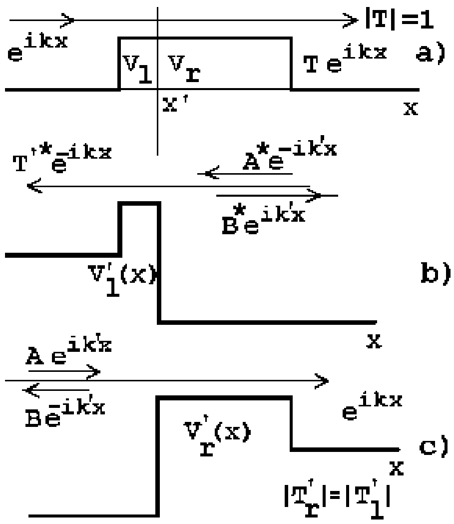

Figure 2: a) More deep cut of the initial potential $V(x)$ at the arbitrary point x' and the pictures of wave penetration through separate left and right parts ($V_l'(x)$; $V_r'(x)$ b),c) with **additional potential steps**. The waves inside the cut are continued to infinity at the right (left) hand side in Fig. b), c).

It is possible to cut potentials with "rectangular zero width cuts" and with bottoms also above or below the energy $E = 0$. Then it can be proven that the equality of transparencies is true also for potentials with additional steps, as is shown in Fig.2. It is possible even to join (from the right and left hand sides) to $V(l,r)$ potentials $V(x' \pm x)$, decreasing to zero with $x \to \pm\infty$ without violation of equality of transparencies of the transformed potentials $V(l,r) + V(x' \pm x)$, see Fig.3. It is also possible to insert into the cut of resonance transparent or reflectionless potential the Bloch waves what corresponds the addition to $V(l,r)$ of periodical tails $V_{per}(x' \pm x)$.

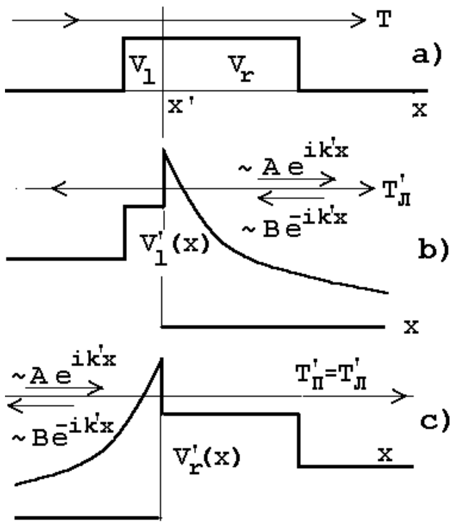

Figure 3: Left and right parts of the initial potential a) can be deformed by addition of potentials $V(x' \pm x)$, decreasing to zero with $x \to \mp\infty$ b),c) without violation of equality of transparency of the transformed potentials $V(l,r) + V(x' \pm x)$, $V(x)$ at the arbitrary point x'.

It may be useful to remind that in the case of wave motion over the half axis there will be addition of phase shifts $\alpha_l(x)+\beta_r(x) = \gamma(x)$ produced by $V(l,r)$ and where γ is the full phase shift produced by the whole initial potential.

Generalization of these results to arbitrary perturbations of periodic potentials is straightforward. So, they can be spread also to perturbations having the 100 % resonance transparency for the Bloch waves. In this case we can "insert" into the potential cut either the Bloch waves which are not perturbed or solutions of free motion, or the ones for potentials decreasing far from the cut. We must only take care that the waves do not meet obstacles as infinite sections of zones of forbidden wave motion.

This is also applicable for optics with special attention to the *energy dependence* of effective potentials (connected with refractive indexes).

We have also found the generalization of the theorem to the cases of arbitrary initial potentials $V(x)$ with $T \neq 1$ and even for complex valued $V(x)$, see [6]. Generalization of these results for arbitrary 1D-potentials give rather simple connections between T_l, R_l, T_r, R_r, T, R which do not depend on the choice of the cut position x':

$$R_l = \frac{RT_rT^* - TT_r^*R_r}{T_rT^* - R^*T_r^*TR_p}, \tag{3}$$

$$T_l = \frac{1 - |R_r|^2 R_rTT^*}{T_rT^* - R_rR^*T_r^*}. \tag{4}$$

The theory can be applied to finite-difference Schroedinger equation [7]

$$-\frac{\Psi(n+1) - 2\Psi(n) + \Psi(n-1)}{\Delta^2} + V(n)\Psi(n) = E\Psi(n). \tag{5}$$

How to cut the potential determined only in discrete points? We can consider the whole potential composed of two parts : the left one till point "n" and further continued by $V(n) = 0$ instead the right hand side, and the right one - from the point "n+1" and continued to the left hand side by $V(n) = 0$. So the values of solutions with $V_r(m)$, $V_l(m)$, $V(m)$ at two space points $m = n$ and $m = n+1$ can be chosen the same and they uniquely determine the same coefficients A and B

$$Aexp(Ikm) + Bexp(-Ikm) \tag{6}$$

corresponding to $V_r(m)$ for $m < n+2$; to $V_l(m)$ for $m > n-1$; and to $V(m)$ for $m = n; m = n+1$. That means the possibility to use these A, B as in the case of the continuous coordinate. The difference is that here the free solution (6) is determined on the finite space sector $[n, n+1]$ and not in an infinitely narrow cut interval. That is because the finite-difference eq. (5) has a nonlocal operator of the second derivative, so that the values of a solution at two neighbor points $m-1, m$ determine the solution in the third point $m+1$ independently of the value of the potential $V(m+1)$. Take also into account that there is only one allowed zone of finite width with continuous spectrum of scattering states [7].

Multichannel generalization of our results is possible for matrix interaction $V_{\alpha\beta}(x)$ of coupled open channel systems [8] (closed channels may violate equalities which we seek):

$$-\frac{d^2}{dx^2}\Psi_\alpha(x) + \sum_\beta V_{\alpha\beta}(x)\Psi_\beta(x) = (E - \varepsilon_\alpha)\Psi_\alpha(x), \ \ \alpha = 1, ..., M. \quad (7)$$

In this system each equation is "channel" and ε_α are the energy of channel thresholds. If $E \geq \varepsilon_\alpha$ then the αth threshold is open. The system (7) is the matrix generalization of the ordinary 1D Schroedinger equation what allows describing, e.g., the motion of complex particles with excitations of their internal degrees of freedom. The peculiarity here is that resonance transparency is possible only for special boundary conditions (rate of amplitudes of incident waves in different channels, 'eigenchannel' states). Another peculiarity is asymmetry of transparency of interaction matrices in general case in opposite directions. So, only the penetrabilities "out of the cut point at x'" toward left-right hand sides of the initial potential matrix must be equal to provide its full transparency (the same is true for the penetrabilities from outside toward the cut). Here may be useful the exact models of reflectionless matrix interactions [8]. The proof of the equal transparency of the interaction matrix left-right parts for resonant tunneling can be made quite evident by using schematic graphic representation, see Fig.4. In separated left part of the complex conjugated solution all waves have the opposite direction of motion. So the waves incident from the right hand side in both channels have the amplitudes $C^*_{1,2}$. And in the separated right part the incident/reflected waves hand side have the same absolute values of amplitudes $|C_{1,2}| = |C^*_{1,2}|$, $|D_{1,2}| = |D^*_{1,2}|$. Due to the law of total wave flux conservation we get the same penetrability of waves

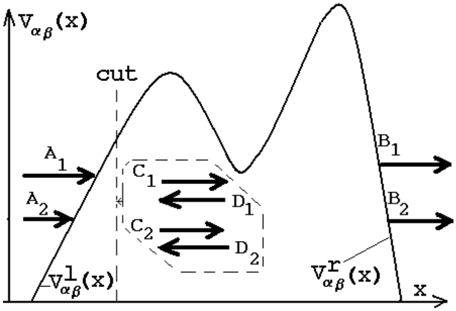

Figure 4: Schematic representation of waves in two-channels penetrating the interaction region which is cut in two parts (left and right ones).

through the left and right parts of the interaction matrix.

$$T_l = \frac{|A_1^*/k_1|^2 + |A_2^*/k_2|^2}{|C_1^*/k_1|^2 + |C_2^*/k_2|^2} = \frac{|B_1/k_1|^2 + |B_2/k_2|^2}{|D_1/k_1|^2 + |D_2/k_2|^2} = T_r. \tag{8}$$

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Comment

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Comment on SURPRISES OF QUANTUM WAVE MOTION THROUGH POTENTIALS

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The paper by V. M. Chabanov and B. N. Zakhariev describes some unknown phenomenon which occurs for all quantum mechanical systems with completely transparent potentials. The Authors start with simple examples of the rectangular potential barriers and step by step extend the results to quite general class of nontrivial potentials. The presentation given by the Authors enlarges our intuition on the nature of quantum systems.

I think that the manipulations with cutting potentials will be much more realistic if they will be accompanied by the discussion of the role of boundary conditions usually imposed at the points of discontinuities of the potentials. Removing parts of the potentials means a real intervention into the structures of the systems which, in turn, open the door for new boundary conditions and this may lead to drastic changes of wave functions describing the systems. All that makes the problem much more complicated then the Authors suggest.

I am sure that the reviewed paper, as well as other papers cited herein, is a significant contribution to the future quantum engineering.