

DECAYS OF SPACELIKE NEUTRINOS

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Abstract

In this paper we consider the hypothesis that neutrinos are fermionic tachyons with helicity $\frac{1}{2}$. We propose an effective model of interactions of these neutrinos and analyze dominant effects under the above hypothesis: the three-body neutrino decay, the radiative decay and briefly discuss β -decay with tachyonic antineutrino. We calculate the corresponding amplitudes and the mean life time for decays, as well as the differential decay width.

1 Introduction

In the last two decades one of the most interesting problems in particle physics is the issue of neutrino masses [1]. The flavor oscillations [2, 3, 4] are the indirect evidence for non-zero neutrino masses. On the other hand, the experiments devoted to direct measurement of the neutrino mass permanently yield the negative mass squared for the electron neutrino [5, 6] and the muon neutrino [7] (for a review, see also [8]). In particular, the most sensitive neutrino mass measurement involving electron antineutrino, is based on fitting the shape of beta spectrum. An as yet not understood event excess near the spectrum endpoint can be explained [9] on the ground of the hypothesis of the tachyonic neutrinos [10]. The last Mainz results do not give any hint for a positive neutrino mass squared and also no hint for any tachyonic mass. However, as long as we do not see a signal for a non-zero neutrino mass, a tachyonic neutrino mass can not be excluded by Mainz experiment data [11, 6]. Moreover, one of the aims of the KATRIN experiment (under preparation) is the search for physics beyond the Standard Model, among other things for tachyonic neutrinos [12, 13]. However, if the negative antineutrino mass squared in tritium beta decay is genuine, then it is hard to understand this phenomenon using conventional particle physics ideas. Note also that the oscillation experiments cannot decide between the positive and negative neutrino mass squared because the oscillation frequency is the same in the both cases (see Appendix E).

In this paper we continue investigation of an unconventional possibility that neutrinos are fermionic tachyons. This hypothesis was proposed firstly by Chodos et al. [10]. In the paper by Rembieliński [14] a causal classical and quantum theory of tachyons was elaborated and next applied by Ciborowski and Rembieliński to explanation of the results of tritium β -decay experiments [9, 15]. In the paper [9] physical consequences of the three body decay $\nu_\ell \rightarrow \nu_\ell + \nu_{\ell'} + \bar{\nu}_{\ell'}$ was also discussed. This paper is devoted to systematical calculations of decay rates for the two processes with participation of neutrinos: the radiative decay $\nu_\ell \rightarrow \nu_\ell + \gamma$ and the three body decay $\nu_\ell \rightarrow \nu_\ell + \nu_{\ell'} + \bar{\nu}_{\ell'}$, both conserving the lepton flavor; we briefly discuss also the β -decay. Notice, that the first two reactions are kinematically admissible for tachyonic neutrinos only. It is remarkable that, as was stressed in [9], the emission by neutrino ν_ℓ a $\nu_{\ell'}\bar{\nu}_{\ell'}$ pair can be an additional,

qualitatively similar process to flavor oscillations. Moreover, Ehrlich [16, 17, 18] showed that the hypothesis that the electron neutrino is a tachyon is in a good agreement with the observed cosmic ray spectrum. As we will see (Sec. 5), the calculated mean times of life for the aforementioned decays take reasonable values in the regions of experimentally available energies. Notice however that applicability of these results to the description of the time evolution of a neutrino flux is restricted at this level because both of the decays form cascades. Therefore they demand further investigation as nontrivial stochastic processes leading to different than Geiger–Nuttall decay law. Moreover, the mostly hopeful program is to involve both oscillations and three body decay.

2 Preliminaries

In this chapter we give a brief presentation of the causal theory of tachyons proposed in the paper [14].

It is a common opinion that existence of space-like particles is in contradiction with special relativity. This is because of the Einstein causality violation and consequently because of a number of very serious difficulties like causal paradoxes, inconsistent kinematics with impossibility of a covariant formulation of the Cauchy problem, unbounded from below energy spectrum and many others. Even more problems arise on the quantum level, like the problems with construction of covariant asymptotic spaces of states, vacuum instability, etc.

However, as was shown in the paper [14] it is possible to agree special relativity with the tachyon concept. The main idea is based on the well known fact that the definition of the time coordinate depends on the synchronization scheme [19, 20, 21, 22] which, in turn, is a convention related to the assumed one-way light velocity (only the average of the light velocity over closed paths has an operational meaning; in special relativity this average is frame independent). Taking into account this freedom, it is possible to realize Poincaré group transformations in such a way that the constant-time hyperplane is a covariant notion [14]. In that synchronization (named absolute synchronization scheme) it is possible to overcome all the difficulties which appear in the standard approach.

Let us stress main features of this nonstandard scheme. Firstly, *it is fully equivalent to the standard formulation of the special relativity*

if we restrict ourselves to the timelike and lightlike trajectories, i.e., if we exclude tachyons. If we admit space-like trajectories (tachyons are included) then one inertial frame is distinguished as a preferred frame. Thus *the relativity principle is broken, however the Poincaré symmetry is still preserved in that case*. The proper framework to this construction is the bundle of Lorentzian frames [23]. For this reason the transformation law for coordinates involves velocity of distinguished frame. The preferred frame can be locally identified with the comoving frame in the expanding universe (cosmic background radiation frame), i.e., the reference frame of the privileged observers to whom the universe appears isotropic [24]. In Ref. [25] it was shown that the preferred frame can be eventually identified by means of the Einstein–Podolsky–Rosen correlation experiment.

Now, the velocity of the Solar System deduced from the dipole anisotropy of the background radiation is about 350 km/sec [26, 27], so it is almost at rest relatively to the preferred frame. Therefore, with a good approximation, we can perform calculations in the preferred frame.

To be concrete, the interrelation with coordinates in the Einstein synchronization (x_E^μ) is given by

$$x_E^0 = x^0 + u^0 \vec{u} \cdot \vec{x}, \quad \vec{x}_E = \vec{x}. \quad (1)$$

Here u^μ is the four-velocity of the preferred frame with respect to the observer. However the corresponding interrelations between velocities \vec{v}_E and \vec{v} obtained from (1) are singular for superluminal velocities. Furthermore, the Lorentz group transformation law for x_E ($x'_E = \Lambda x_E$) implies by means of (1) the corresponding transformation law for x [23].

Now, on the quantum level, the corresponding quantum mechanics and quantum field theory can be formulated with help of the bundle of Hilbert spaces associated with the aforementioned bundle of frames [23]. The Fock construction can be done in this framework and unitary orbits can be classified [14]. For usual particles they coincide with the standard unitary representations of the Poincaré group. For tachyons they are induced from $SO(2)$ group (instead of $SO(2,1)$) and labelled by the helicity. The following two facts, true only in absolute synchronization, are extremely important for quantization of tachyons:

- invariance of the sign of the time component of the space-like four-momentum, i.e., $\epsilon(k^0) = \text{inv}$, which follows from the transformation law (see [14, 23]);
- existence of a covariant lower energy bound; in terms of the contravariant space-like four-momentum k^μ ($k^2 < 0$) this lower bound is exactly zero, i.e., $k^0 \geq 0$ as in the timelike and lightlike case.

This is the reason why an invariant Fock construction with a stable vacuum can be done in this case on the quantum level [14]. Notice that the measure

$$d\mu(k, \kappa) = d^4k \theta(k^0) \delta(k^2 + \kappa^2) \quad (2)$$

is Poincaré covariant in the absolute synchronization scheme.

3 Fermionic tachyons with helicity $\lambda = \pm \frac{1}{2}$

To construct tachyonic field theory describing field excitations with the helicity $\pm \frac{1}{2}$, we assume that our field transforms under Poincaré group like bispinor (for discussion of transformation rules for local fields in the absolute synchronization see [14, 28]); namely

$$U(\Lambda)\psi(x, u)U(\Lambda^{-1}) = S(\Lambda^{-1})\psi(x', u'), \quad (3)$$

where $S(\Lambda)$ belongs to the representation $D^{\frac{1}{2}0} \oplus D^{0\frac{1}{2}}$ of the Lorentz group. Because we are working in the absolute synchronization, it is convenient to introduce an appropriate (absolute-covariant) base in the algebra of Dirac matrices as

$$\gamma^\mu = T(u)^\mu{}_\nu \gamma_{\text{E}}^\nu, \quad (4)$$

where γ_{E}^μ are standard γ -matrices, while $T(u)$ is determined by means of the relation $x^\mu = T^\mu{}_\nu(u)x_{\text{E}}^\nu$ and the Eq. (1). Therefore

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}(u)I, \quad (5)$$

where the metric tensor is

$$g_{\mu\nu}(u) = [T^T(u)\eta T(u)]_{\mu\nu} = T(u)_\mu{}^\alpha \eta_{\alpha\beta} T(u)^\beta{}_\nu, \quad (6)$$

with $\eta = \text{diag}(+, -, -, -)$.

Notice, however, that the Dirac conjugate bispinor $\bar{\psi} = \psi^\dagger \gamma_E^0$. Furthermore $\gamma^5 = -i\epsilon_{\mu\nu\sigma\lambda} \gamma^\mu \gamma^\nu \gamma^\sigma \gamma^\lambda / 4! = \gamma_E^5$. Let us consider the Dirac-like equation¹ [14]

$$(\gamma^5(i\gamma\partial) - \kappa) \psi = 0, \quad (7)$$

which can be derived from the Lagrangian density

$$\mathcal{L} = \bar{\psi} (\gamma^5(i\gamma\partial) - \kappa) \psi. \quad (8)$$

Dirac equation (7) implies the Klein–Gordon equation

$$(g^{\mu\nu}(u)\partial_\mu\partial_\nu - \kappa^2) \psi = 0, \quad (9)$$

related to the space-like dispersion relation $k^2 = -\kappa^2$. The equation (7) is analogous to the one of the Chodos et al. [10] Dirac-like equation for tachyonic fermion. However, contrary to the standard approach, it can be consistently quantized in the absolute synchronization scheme if it is supplemented by the covariant helicity condition [14]

$$\hat{\lambda}(u)\psi(u, k) = \lambda\psi(u, k), \quad (10)$$

with $\hat{\lambda}$ given by

$$\hat{\lambda}(u) = -\frac{\hat{W}^\mu u_\mu}{\sqrt{(Pu)^2 - P^2}}, \quad (11)$$

where

$$\hat{W}^\mu = \frac{1}{2}\epsilon^{\mu\sigma\lambda\tau} J_{\sigma\lambda} P_\tau$$

is the Pauli–Lubanski four-vector. This condition is quite analogous to the condition for the left (right) bispinor in the Weyl’s theory of the massless field. It implies that particles described by ψ have helicity $-\lambda$, while antiparticles have helicity λ . For the obvious reason in the following we will concentrate on the case $\lambda = \frac{1}{2}$.

Notice that the pair of equations (7) and (10) is not invariant under the P or C inversions separately.

¹Hereafter $u\gamma = u_\mu \gamma^\mu$, $u\partial = u^\mu \partial_\mu$, $\gamma\partial = \gamma^\mu \partial_\mu$.

Now, in the bispinor realization the helicity operator $\hat{\lambda}$ has the following explicit form [14]

$$\hat{\lambda}(u) = \frac{\gamma^5 [i\gamma\partial, u\gamma]}{4\sqrt{(iu\partial)^2 + \square}}, \quad (12)$$

where the integral operator $((iu\partial)^2 + \square)^{-\frac{1}{2}}$ in the coordinate representation is given by the well behaving distribution

$$\frac{1}{\sqrt{(-iu\partial)^2 + \square}} = \frac{1}{(2\pi)^4} \int d^4p \epsilon(u p) e^{i p x} ((u p)^2 - p^2)^{-\frac{1}{2}}. \quad (13)$$

The free field $\psi(x, u)$ has the following Fourier decomposition

$$\psi(x, u) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^4k \delta(k^2 + \kappa^2) \theta(k^0) [w(k, u) e^{i k x} b^\dagger(k) + v(k, u) e^{-i k x} a(k)]. \quad (14)$$

The creation and annihilation operators of a tachyonic fermion (a) with helicity $-\frac{1}{2}$ and an antifermion (b) with helicity $\frac{1}{2}$ satisfy almost standard canonical anticommutation relations; the nonzero ones are

$$[a(k), a^\dagger(p)]_+ = 2\omega_{\vec{k}} \delta(\vec{k} - \vec{p}), \quad (15a)$$

$$[b(k), b^\dagger(p)]_+ = 2\omega_{\vec{k}} \delta(\vec{k} - \vec{p}). \quad (15b)$$

Here $\omega_{\vec{k}} = k^0 > 0$ is the positive root of the dispersion relation $k^2 = -\kappa^2$ and \vec{k} denotes covariant components k_i , $i = 1, 2, 3$ of k . The amplitudes w and v satisfy

$$(\kappa + \gamma^5 k \gamma) w(k, u) = 0, \quad (16a)$$

$$\left(1 + \frac{\gamma^5 [k\gamma, u\gamma]}{2\sqrt{q^2 + \kappa^2}}\right) w(k, u) = 0, \quad (16b)$$

$$(\kappa - \gamma^5 k \gamma) v(k, u) = 0, \quad (16c)$$

$$\left(1 + \frac{\gamma^5 [k\gamma, u\gamma]}{2\sqrt{q^2 + \kappa^2}}\right) v(k, u) = 0. \quad (16d)$$

Furthermore, the projectors $w\bar{w}$ and $v\bar{v}$ read

$$w(k, u)\bar{w}(k, u) = (\kappa - \gamma^5 k\gamma) \frac{1}{2} \left(1 - \frac{\gamma^5 [k\gamma, u\gamma]}{2\sqrt{q^2 + \kappa^2}} \right), \quad (17a)$$

$$v(k, u)\bar{v}(k, u) = -(\kappa + \gamma^5 k\gamma) \frac{1}{2} \left(1 - \frac{\gamma^5 [k\gamma, u\gamma]}{2\sqrt{q^2 + \kappa^2}} \right). \quad (17b)$$

The above amplitudes fulfill the covariant normalization conditions

$$\bar{w}(k, u)\gamma^5 u\gamma w(k, u) = \bar{v}(k, u)\gamma^5 u\gamma v(k, u) = 2uk, \quad (18a)$$

$$\bar{w}(k^\pi, u)\gamma^5 u\gamma v(k, u) = 0, \quad (18b)$$

where the superscript π denotes the space inversion of k .

It is easy to see that in the massless limit $\kappa \rightarrow 0$ Eqs. (16) give the Weyl equations

$$k\gamma w = k\gamma v = 0, \quad \gamma^5 w = -w, \quad \gamma^5 v = -v.$$

Now, the normalization conditions (18) together with the canonical commutation relations (15) guarantee the proper work of the canonical formalism. In particular, starting from the Lagrangian density (8) we can derive the translation generators

$$P_\mu = \int \frac{d^3 \vec{k}}{2\omega_{\vec{k}}} k_\mu (a^\dagger(k)a(k) + b^\dagger(k)b(k)). \quad (19)$$

Thus we have defined a consistent Poincaré covariant free field theory for a fermionic tachyon with helicity $-\frac{1}{2}$; the Weyl's theory for a left spinor is obtained as the $\kappa \rightarrow 0$ limit. For more details of this construction see [14].

4 Dynamics

To formulate a dynamical model with tachyonic neutrino we can use, on the tree level, the phenomenological form of the weak interaction Lagrangian, namely the current-current Fermi Lagrangian. The only condition we take into account is that the massless limit $\kappa \rightarrow 0$ of our model should coincide with the standard four-fermion interaction. This leads to three natural possibilities for the part of the lepton charged current containing tachyonic neutrino:

Decays of spacelike neutrinos

$$\begin{aligned}
 \bar{\ell}(x)\gamma^\mu\frac{1}{2}(1-\gamma^5)\nu(x) + \text{h.c.} & \text{--- chiral coupling,} \\
 \bar{\ell}(x)\gamma^\mu\nu(x) + \text{h.c.} & \text{--- helicity coupling,} \\
 \bar{\ell}(x)\gamma^5\gamma^\mu\nu(x) + \text{h.c.} & \text{--- } \gamma^5 \text{ coupling,}
 \end{aligned}$$

and similarly for the neutral currents.

Recall that $\nu(x)$ satisfies the helicity condition (10) with $\lambda = \frac{1}{2}$ so in the $\kappa \rightarrow 0$ limit $\nu \rightarrow \nu_L$.

However, if we consider higher order processes (like $\nu \rightarrow \nu\gamma$) it is necessary to have a more sophisticated model like the Salam–Weinberg one. Let us notice firstly, that the Lagrangian for the massless Dirac field, under assumption of CP conservation, can be written in two different forms $\bar{\psi}i\gamma\partial\psi$ or $\bar{\psi}\gamma^5i\gamma\partial\psi$. Both forms lead to the same massless Dirac equation $i\gamma\partial\psi = 0$ because of the invertibility of γ^5 matrix. Therefore generation of a mass *via* Yukawa coupling with Goldstone fields can lead to massive fermions or fermionic tachyons, because of creation in the Lagrangian of Dirac $\bar{\psi}(i\gamma\partial - m)\psi$ or $\bar{\psi}(\gamma^5i\gamma\partial - m)\psi$ term respectively. Thus, because we have decided to treat neutrinos as tachyons, we start with the following free massless Lagrangian for a fixed lepton generation (ν, ℓ)

$$\mathcal{L}_0 = \bar{\nu}\gamma^5i\gamma\partial\nu + \bar{\ell}i\gamma\partial\ell. \quad (20)$$

After splitting ν and ℓ into left-handed and right-handed pairs $\nu_{L,R}, \ell_{L,R}$, it is easy to see that the (compact) invariance group of the Lagrangian \mathcal{L}_0 is exactly the $(SU(2) \times U(1))_L \times U(1)_R$ group! Therefore, taking into account the electric charges of ν and ℓ , we conclude that the symmetry group of \mathcal{L}_0 is $U(1)_{\text{lepton number}} \times (U(1)_Y \times SU(2)_I)$ and the left-handed fields ν_L and ℓ_L must form the weak dublet while the right-handed ν_R and ℓ_R are singlets of the weak group. *Therefore \mathcal{L}_0 leads uniquely to the standard Salam–Weinberg choice of the weak symmetry group and their realization on the lepton fields* (recall also that in the $\kappa \rightarrow 0$ limit $\nu(\kappa) \rightarrow \nu(0) = \nu_L(0)$, i.e., $\nu_R(\kappa) \rightarrow \nu_R(0) = 0$). Now, under the standard choice of the Goldstone fields (as the $U(1)_Y \times SU(2)_I$ dublet) as well as the Higgs potential and Yukawa coupling (for neutrinos like for neutral quarks) and after gauging the weak group we obtain the final Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{fermions+int.}} + (\mathcal{L}_{\text{gauge bosons}} + \mathcal{L}_{\text{Higgs+int.}} + \dots). \quad (21)$$

The part of \mathcal{L} in the parentheses has the standard form (we choose convention as in [29]) while the $\mathcal{L}_{\text{fermions+int.}}$ reads

$$\begin{aligned}
 \mathcal{L}_{\text{fermions+int.}} = & \bar{\nu}\gamma^5 i\gamma\partial\nu - \kappa\bar{\nu}\nu + \bar{\ell}i\gamma\partial\ell - m\bar{\ell}\ell + eA_\mu\bar{\ell}\gamma^\mu\ell \\
 & + \frac{g}{2\cos\theta_W}Z_\mu\bar{\ell}\gamma^\mu(g_V - g_A\gamma^5)\ell - \frac{g}{2\cos\theta_W}Z_\mu\bar{\nu}\gamma^\mu\frac{1-\gamma^5}{2}\nu \\
 & - \frac{g}{\sqrt{2}}W_\mu^+\bar{\nu}\gamma^\mu\frac{1-\gamma^5}{2}\ell - \frac{g}{\sqrt{2}}W_\mu^-\bar{\ell}\gamma^\mu\frac{1-\gamma^5}{2}\nu \\
 & - \frac{g}{2\sqrt{2}m_W}\bar{\nu}((m-\kappa)I + (m+\kappa)\gamma^5)\ell G^+ \\
 & - \frac{g}{2\sqrt{2}m_W}\bar{\ell}((m-\kappa)I - (m+\kappa)\gamma^5)\nu G^- \\
 & + \frac{ig}{2m_W}(\kappa\bar{\nu}\gamma^5\nu - m\bar{\ell}\gamma^5\ell)G^0 \\
 & - \frac{g}{2m_W}(\kappa\bar{\nu}\nu + m\bar{\ell}\ell)H. \quad (22)
 \end{aligned}$$

Here $g_V = \frac{1}{2} - 2\sin^2\theta_W$, $g_A = \frac{1}{2}$, g —weak coupling constant, θ_W —Weinberg angle, G^\pm and G^0 —Goldstone bosons and H —Higgs field. The corresponding Feynman rules are listed in the Appendix A.

The following remarks are in order. First of all, the derivation of the Lagrangian (21) with $\mathcal{L}_{\text{fermions+int.}}$ given by (22) is rather heuristic one; the point is that the right component of ν , ν_R goes to zero with $\kappa \rightarrow 0$. This means that the mass generation mechanism for tachyons is not well defined by Yukawa coupling if we start with the massless Lagrangian. Intuitively, it seems that the mechanism of mass generation for the tachyonic neutrinos should be caused rather by the interaction with gravitational field. This is related to our natural assumption about coincidence of the tachyon preferred frame with the comoving frame. The next problem is the helicity condition (10) which is necessary for consistency of the free tachyon theory. Of course it can be gauged by introduction of the covariant derivatives and causes additional relations between multipoint Green functions of the theory. In such a case perturbation consistency and renormalizability of this model should be proved. We shift these unanswered questions to further investigations and here we will treat the Lagrangian (21) as an effective reasonable approximation of a more realistic theory. Because in the following we do not analyze

Decays of spacelike neutrinos

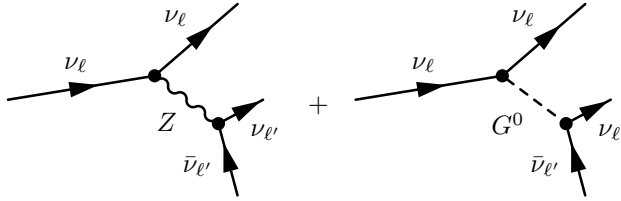


Figure 1: Neutrino three-body decay $\nu_\ell \rightarrow \nu_\ell + \nu_{\ell'} + \bar{\nu}_{\ell'}$

processes with intrinsic neutrino lines, this approximation seems to be correct.

Note that the Feynman rules derived from (21) and (22) (Appendix A), lead to the effective *chiral coupling* in the Fermi Lagrangian. Therefore, in the following all the calculations on the tree level will be done for this coupling. However, we will comment results obtained with the help of the helicity and γ^5 couplings too.

Tachyonic neutrino is in general unstable even if the leptonic flavors are conserved; namely decays of the form *tachyon* \rightarrow *tachyon* + *sth.* are kinematically admissible. If we take into account the experimental upper bounds on the neutrinos masses [8] we can select two dominant processes of this type, namely: three body decay $\nu_\ell \rightarrow \nu_\ell + \nu_{\ell'} + \bar{\nu}_{\ell'}$ and radiative decay $\nu_\ell \rightarrow \nu_\ell + \gamma$.

5 Neutrino decay $\nu_\ell \rightarrow \nu_\ell + \nu_{\ell'} + \bar{\nu}_{\ell'}$

This process can be analyzed on the tree level. The corresponding amplitude is the sum of the graphs given in Fig. 1 and leads to the effective four-fermion vertex in Fig. 2. Consequently

$$M = \sqrt{2}G_F \bar{\nu}_{\ell'}(p)\Gamma^\mu \nu_\ell(k)\bar{\nu}_{\ell'}(l)\Gamma_\mu w_{\ell'}(r), \quad (23)$$

where the momenta k , p , l and r correspond to the incoming neutrino ν_ℓ and to the outgoing neutrinos ν_ℓ , $\nu_{\ell'}$ and $\bar{\nu}_{\ell'}$ respectively. The masses of ν and $\nu_{\ell'}$ are denoted by κ and μ respectively. In our case the Feynman rules (Appendix A) lead to the chiral coupling $\Gamma^\mu = (1 + \gamma^5)\gamma^\mu/2$. Thus, by means of the polarization relations (17),

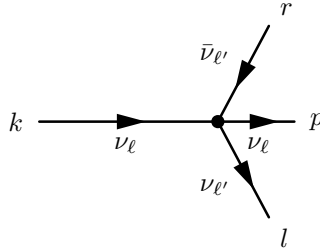


Figure 2: Effective four-fermion vertex in neutrino three-body decay $\nu_\ell \rightarrow \nu_\ell + \nu_{\ell'} + \bar{\nu}_{\ell'}$

we obtain in the preferred frame

$$|M|^2 = 2G_F^2 \frac{(|\vec{k}| + k^0) (|\vec{p}| + p^0) (|\vec{l}| + l^0) (|\vec{r}| + r^0)}{|\vec{k}||\vec{p}||\vec{l}||\vec{r}|} \times \left(|\vec{p}||\vec{l}| - \vec{p} \cdot \vec{l} \right) \left(|\vec{k}||\vec{r}| - \vec{k} \cdot \vec{r} \right), \quad (24)$$

where from dispersion relations $|\vec{k}| = \sqrt{\kappa^2 + (k^0)^2}$, $|\vec{p}| = \sqrt{\kappa^2 + (p^0)^2}$, $|\vec{l}| = \sqrt{\mu^2 + (l^0)^2}$, $|\vec{r}| = \sqrt{\mu^2 + (r^0)^2}$.

The general forms of the square of the amplitude for all couplings are given in the Appendix B. The qualitative behavior of the decay rate

$$\Gamma_{\ell\ell'} = ((2\pi)^5 2k^0)^{-1} \int d^4p d^4l d^4r |M|^2 \theta(p^0) \theta(l^0) \theta(r^0) \times \delta(p^2 + \kappa^2) \delta(l^2 + \mu^2) \delta(r^2 + \mu^2) \delta^4(k - p - l - r) \quad (25)$$

is rather similar for all three couplings. Here we present in the Fig. 3 the numerical calculations for the mean life time in the case of chirality coupling (according to our gauge model) done in the preferred frame ($u = (1, \vec{0})$). For the helicity coupling the corresponding results were given in [9]. The mean life time of the tachyonic neutrino in this process is determined by its energy $E = k^0$, as measured in the preferred frame, and masses of all three neutrino species. As we see from the Fig. 3 the life time $\tau_{e\ell}$ corresponding to the partial width $\Gamma_{e\ell}$, is

Decays of spacelike neutrinos

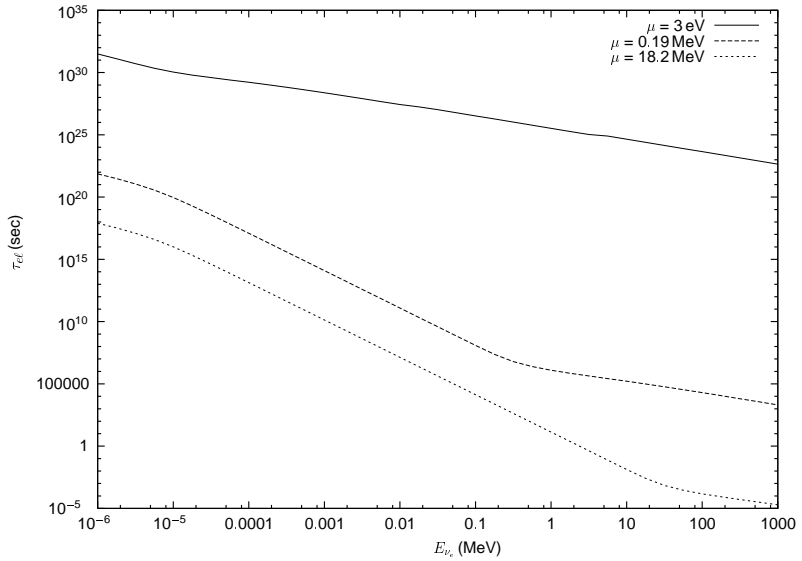


Figure 3: The mean time life $\tau_{e\ell}$ of electron neutrino in three-body decay into electron neutrino and neutrino–antineutrino pair of type ℓ for $\kappa = 3 \text{ eV}$ and masses $\mu_{\ell} = 3 \text{ eV}$, $\mu_{\ell} = 0.19 \text{ MeV}$, and $\mu_{\ell} = 18.2 \text{ MeV}$

a decreasing function of the initial energy and mass of the neutrino ν_ℓ . Note that the dashed and dotted lines in the Fig. 3 correspond to the upper bounds on the ν_μ and ν_τ masses [30]. Since the $\Delta(m^2)$ for the different neutrino flavors does not exceed a few eV, the mean life times τ_{ee} , $\tau_{e\mu}$ and $\tau_{e\tau}$ are almost equal and their dependence on neutrino energy is given by the most upper (solid) line in the Fig. 3.

It is important to stress that:

- The decay $\nu_\ell \rightarrow \nu_\ell + \nu_{\ell'} + \bar{\nu}_{\ell'}$ has the character of an emission by ν_ℓ a neutrino–antineutrino pair $\nu_{\ell'}\bar{\nu}_{\ell'}$; as a consequence the resulting neutrino flux contains neutrinos and antineutrinos of all possible flavors. Therefore this process can simulate the neutrino flavor oscillations. Notice that for massive and massless neutrinos such decays are kinematically forbidden.
- The above process is repeated and forms a cascade. Therefore, as was mentioned in the Introduction, it demands more sophisticated treatment as a nontrivial stochastic process. However after the neutrino energy degradation it is slowing down with subsequent decays; notice that, for low energies in the preferred frame, neutrinos are almost stable (Fig. 3).

Finally, let us analyse the influence of the neutrino three body decay on the neutrino flux. The effect, which we predict below, can be complementary effect on the intensity of solar neutrino flux together with neutrino oscillations and the MSW mechanism [31, 32, 33].

As it is evident from the Fig. 3, in the case of electron neutrino ($\kappa_e \sim 3 \text{ eV}$) decay (for energies 1–10 MeV and time taken as Sun–Earth distance/ $c \sim 500 \text{ sec}$) to obtain the significant effect, the mass of the resulting $\nu_{\ell'}$ should be of the order of 1 MeV. On the other hand the neutrino oscillation experiments suggest that the masses of at least of two neutrino flavors must lie very near each other. Therefore let us consider the contribution of the one $\nu_e \rightarrow \nu_e + \nu_h + \bar{\nu}_h$ decay only, where ν_h denotes a possible heavy neutrino. Our aim is

to analyze the cascade process

$$\begin{array}{r}
 \nu_e \quad \rightarrow \quad \nu_e \quad + \nu_h \quad + \bar{\nu}_h \\
 \qquad \qquad \qquad \downarrow \\
 \qquad \qquad \qquad \nu_e \quad + \nu_h \quad + \bar{\nu}_h \\
 \qquad \qquad \qquad \downarrow \\
 \qquad \qquad \qquad \nu_e \quad + \nu_h \quad + \bar{\nu}_h \\
 \qquad \qquad \qquad \downarrow \\
 \qquad \qquad \qquad \vdots
 \end{array}$$

taking into account that probabilities of the possible subsequent decays $\nu_h \rightarrow \nu_h + \nu_e + \bar{\nu}_e$ and $\bar{\nu}_h \rightarrow \bar{\nu}_h + \nu_e + \bar{\nu}_e$ are not significant in the investigated energy regime.

This greatly simplifies treatment of this stochastic process leading to the following evolution equation for the electron neutrino flux:

$$\hbar \frac{d\Phi_t(E)}{dt} = -\Gamma_E \Phi_t(E) + \int_E^\infty dE' \frac{d\Gamma_{E'}}{dE} \Phi_t(E'). \quad (26)$$

Here $\Phi_t(E)$ denotes the electron neutrino flux of energy E after time t , Γ_E is the total decay rate for the initial neutrino of energy E and $d\Gamma_{E'}/dE$ is the differential decay rate of the decaying neutrino ν_e with energy E' into outgoing electron neutrino with energy E . The equation (26) has the solution as a formal power series

$$\Phi_t = e^{tF/\hbar} \Phi_0 \quad (27)$$

with $F(E, E') = -\Gamma_E \delta(E - E') + \theta(E' - E) d\Gamma_{E'}/dE$.

6 Radiative decay of the tachyonic neutrino $\nu_\ell \rightarrow \nu_\ell + \gamma$

This process can play an important role in cosmology; in particular, as was noticed in [34, 35, 36], it can affect the cosmic background radiation in the submillimeter region, leading to its distortion.

Let us denote the four-momenta of the initial and final neutrinos by k and p respectively, their masses by κ , while photon has the four-momentum $q = k - p$. The kinematics of this process is very simple; namely $k^2 = -\kappa^2$, $p^2 = -\kappa^2$ and for the photon on the mass

shell $q^2 = (k - p)^2 = 0$ so $kp = -\kappa^2$, $qk = qp = 0$. Let us consider this kinematics in the preferred frame $u = (1, \vec{0})$. It is immediate to see that $\cos \theta = \vec{k} \cdot \vec{q} / (|\vec{k}| |\vec{q}|) = k^0 ((k^0)^2 + \kappa^2)^{-1/2}$. Therefore the outgoing photons are emitted on the cone determined by the angle 2θ . Notice that for $k^0 \gg \kappa$, $\cos \theta \approx 1$ so the process is approximately collinear in that case.

The amplitude has the form

$$M^\lambda(p, k) = \Omega^\mu(p, k) e_\mu^\lambda(q), \quad (28)$$

where e_μ^λ is the polarization vector for the photon with helicity λ . The $\Omega^\mu(p, k)$ should satisfy transversality and neutrality conditions. The first one follows from the electromagnetic current conservation and has the form

$$q_\mu \Omega^\mu(p, k) = 0, \quad (29)$$

while neutrality condition means that the neutrino charge is zero and consequently the corresponding matrix element vanishes:

$$\Omega^\mu(k, k) = 0. \quad (30)$$

By means of the equations of motion for the spinors \bar{v} and v (16c), the most general form of the $\Omega^\mu(p, k)$ reads

$$\begin{aligned} \Omega^\mu(p, k) = \bar{v}(p) [& (F_V + F_A \gamma^5) \gamma^\mu + (G_{1V} + iG_{1A} \gamma^5) k^\mu \\ & + (G_{2V} + iG_{2A} \gamma^5) p^\mu] v(k), \quad (31) \end{aligned}$$

where the form factors F and G are in general functions of q^2 . Notice that the form factors G_{iA} control CP non-invariant terms in the Eq. (31). Now, taking into account the transversality (29) and neutrality (30) we obtain the following constraints

$$F_V = 0, \quad (32a)$$

$$F_A = \kappa(G_{1V} + G_{2V}), \quad (32b)$$

$$(G_{2A} - G_{1A})q^2 = 0, \quad (32c)$$

$$(G_{2V} - G_{1V})q^2 = 0. \quad (32d)$$

Therefore, because of the analyticity of G_{iV} and G_{iA} in the point $q^2 = 0$, we obtain

$$\Omega^\mu(p, k) = -\frac{1}{2} \bar{v}(p) [\gamma^\mu, \gamma^\nu] q_\nu (G_V + iG_A \gamma^5) v(k), \quad (33)$$

where we denoted $G_V \equiv G_{1V} = G_{2V}$, $G_A \equiv G_{1A} = G_{2A}$ and the equations of motion (16c) was used again. Therefore the square of the amplitude M^λ , after summation over final polarizations of the photon (Appendix D), takes the form:

$$|M|^2 = 4\kappa^4 \left(|G_V|^2 + |G_A|^2 \right) \times \frac{(uq)^2}{\sqrt{(u(k-q))^2 + \kappa^2} \sqrt{(uk)^2 + \kappa^2}} \quad (34)$$

where we have used the form of the polarization operator (17b).

In the framework of our $SU(2) \times U(1)$ gauge model, the diagrams contributing to the process $\nu_\ell \rightarrow \nu_\ell + \gamma$, to one-loop order, are shown in the Fig. 4.

Now, the transversality and neutrality conditions (29) and (30) can be treated as renormalization conditions. By means of the Feynman rules listed in the Appendix A we can easily calculate the form factors G_A and G_V :

$$G_A = 0, \quad (35)$$

as we expected because of the CP invariance of this model, while

$$G_V = \frac{eg^2\kappa}{64\pi^2} [I_a + I_b + I_c + I_d + I_e + I_f], \quad (36)$$

where (see Appendix C), under the experimental conditions for lepton masses $m \ll m_W$ and for corresponding neutrino masses $\kappa \ll m_W$,

$$\begin{aligned} I_a &= \frac{4}{3m_W^2}, \\ I_b &= -\frac{7}{6m_W^2}, \\ I_c &= \frac{2m^2}{m_W^4} \ln \left(\frac{m_W}{m} \right)^2, \\ I_d &= \frac{5m^2 + \kappa^2}{6m_W^4}, \\ I_e + I_f &= \frac{1}{2m_W^2}. \end{aligned}$$

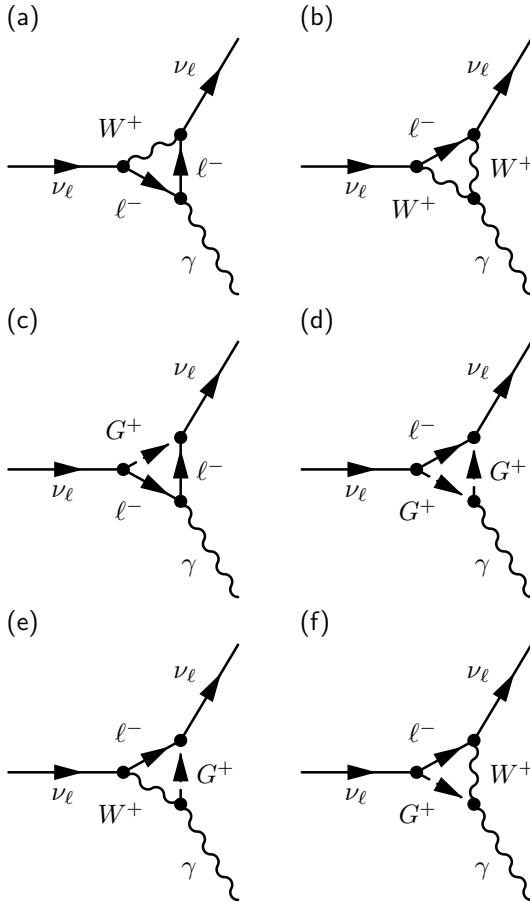


Figure 4: Contributions to the radiative decay $\nu_\ell \rightarrow \nu_\ell + \gamma$ to one-loop order

Thus in this approximation $I_c + I_d$ can be omitted and the final form of G_V reads

$$G_V = \frac{eg^2\kappa}{96\pi^2 m_W^2} = \frac{\kappa G_F}{6\pi} \sqrt{\frac{\alpha}{2\pi}}. \quad (37)$$

Consequently

$$\Omega^\mu = -\frac{\kappa G_F}{12\pi} \sqrt{\frac{\alpha}{2\pi}} \bar{v}(p) [\gamma^\mu, \gamma^\nu] q_\nu v(k) \quad (38)$$

and

$$|M|^2 = \frac{\alpha\kappa^6 G_F^2}{18\pi^3} \frac{(uq)^2}{\sqrt{(uk)^2 + \kappa^2} \sqrt{(u(k-q))^2 + \kappa^2}}. \quad (39)$$

Notice, that (38) implies that the magnetic moment μ_ν of the neutrino ν is given by

$$\mu_\nu = \frac{\kappa G_F}{3\pi} \sqrt{\frac{\alpha}{2\pi}} = \frac{G_F m_e \kappa}{3\sqrt{2}\pi^2} \mu_B, \quad (40)$$

where μ_B is the Bohr magneton and m_e the electron mass. (N.B. the value of μ_ν is of the same form as for the massive neutrino; the difference lies in the numerical factor only [37]). To calculate the decay rate we integrate $|M|^2$ over the phase space:

$$\Gamma = (8\pi^2 k^0)^{-1} \int d^4q d^4p |M|^2 \theta(q^0) \theta(p^0) \times \delta(q^2) \delta(p^2 + \kappa^2) \delta(k - p - q). \quad (41)$$

We shall do it in the preferred frame, that is we put $u = (1, \vec{0})$. Inserting $|M|^2$ given by (39), we obtain the energy spectrum of the emitted photons as:

$$\frac{d\Gamma}{dq^0} = \frac{\alpha G_F^2 \kappa^6 (q^0)^2 \theta(k^0 - q^0) \theta(q^0)}{288\pi^4 k^0 ((k^0)^2 + \kappa^2) \sqrt{(k^0 - q^0)^2 + \kappa^2}}. \quad (42)$$

Finally, the total decay rate reads:

$$\Gamma = \frac{\alpha\kappa^6 G_F^2}{576\pi^4 k^0 ((k^0)^2 + \kappa^2)} \left(4\kappa k^0 - 3k^0 \sqrt{(k^0)^2 + \kappa^2} + (2(k^0)^2 - \kappa^2) \ln \frac{\sqrt{(k^0)^2 + \kappa^2} + k^0}{\kappa} \right). \quad (43)$$

Table 1: Upper bounds on the neutrino magnetic moment

	from (40)	from [8]
μ_e	$< 4.27 \times 10^{-19} \mu_B$	$< 1.0 \times 10^{-10} \mu_B$
μ_μ	$< 2.70 \times 10^{-14} \mu_B$	$< 6.8 \times 10^{-10} \mu_B$
μ_τ	$< 2.59 \times 10^{-12} \mu_B$	$< 3.9 \times 10^{-7} \mu_B$

Note that the experimental upper bounds on the neutrino masses applied to the magnetic moment formula (40) give results which are in agreement with the experimental data [8] (cf. Table 1).

Now, for illustration we present the mean life time $\tau = \hbar/\Gamma$ for this process as a function of the energy k^0 in Fig. 5. From (43) it is easy to see that for $k^0 = \xi\kappa$, with $\xi \approx 2.6899$ the mean time of life has a minimum. The minimal value τ_{\min} , as the function of the mass κ , reads

$$\tau_{\min}(\kappa) \simeq \frac{902\pi^4 \hbar}{\alpha G_F^2} \kappa^{-5} \quad (44)$$

and is presented in Fig. 6. Finally, in Fig. 7 differential rate $d\Gamma/dq^0$ as the function of the photon energy q^0 and the initial energy k^0 is given.

7 β -decay with tachyonic antineutrino

The tritium β -decay with tachyonic antineutrino was discussed in the papers by Ciborowski and Rembieliński [9, 15]. Here for a completeness we present only a brief view on results obtained in [15]. As in the standard case, the two graphs shown in the Fig. 8 contribute to this process on the tree level, where dominant is the first one. Effectively, for energies much less than m_W , it reduces to the four-fermion interaction with the amplitude square given by chiral coupling

$$|M_{\text{ch}}|^2 = 2G_F^2 \text{Tr} \left[u_e \bar{u}_e \gamma^\mu \frac{1 - \gamma^5}{2} w \bar{w} \gamma^\nu \frac{1 - \gamma^5}{2} \right] \\ \times \text{Tr} \left[u_p \bar{u}_p \gamma_\mu (1 - g_A \gamma^5) u_n \bar{u}_n \gamma_\nu (1 - g_A \gamma^5) \right], \quad (45)$$

Decays of spacelike neutrinos

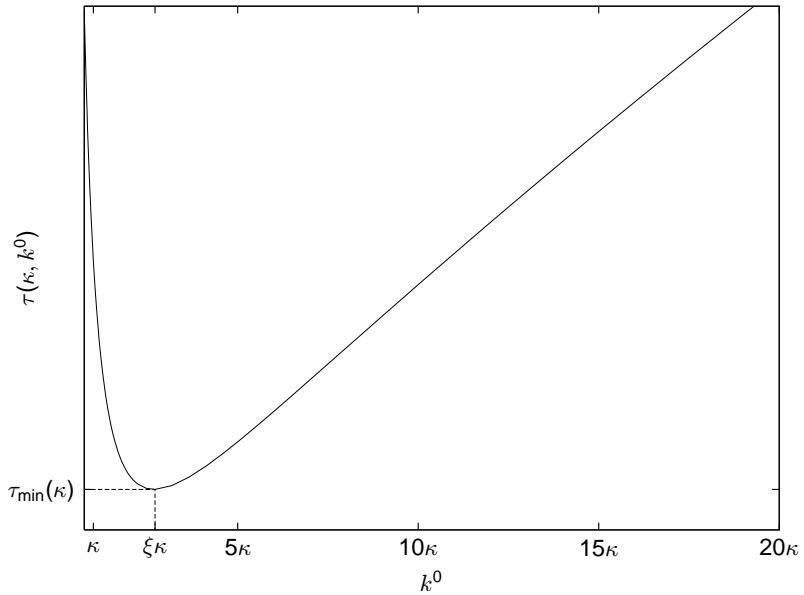


Figure 5: Mean life time τ in the radiative decay $\nu \rightarrow \nu\gamma$ as the function of the neutrino mass κ and the initial energy k^0 . Here $\xi \approx 2.6899$

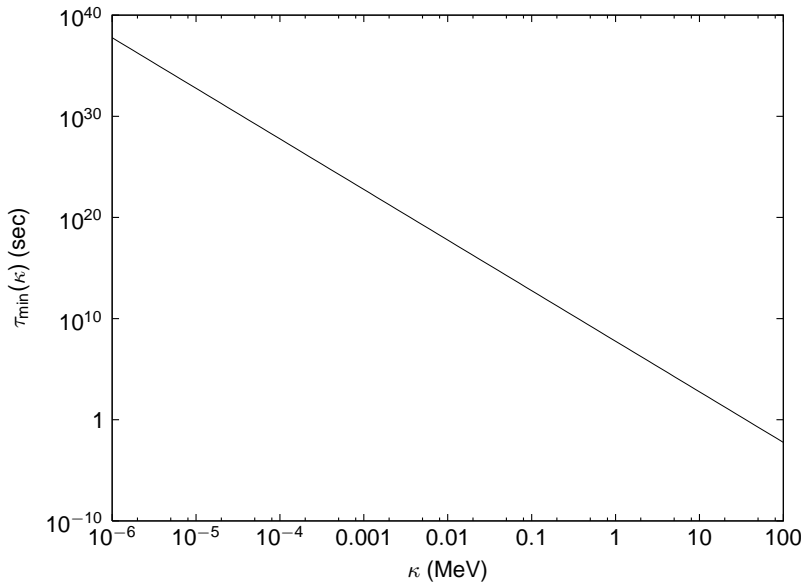


Figure 6: Minimal mean time of life τ_{\min} for the radiative decay $\nu \rightarrow \nu\gamma$ as the function of the neutrino mass κ

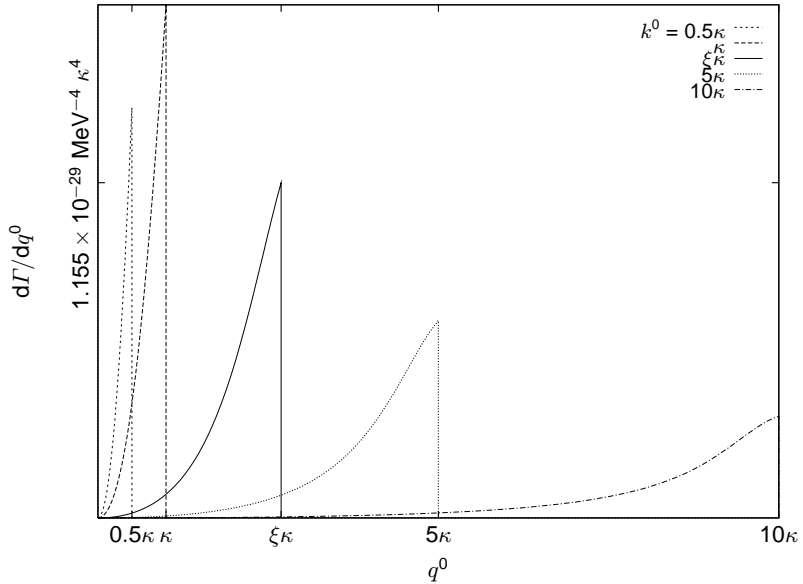


Figure 7: Differential rate $d\Gamma/dq^0$ for the radiative decay $\nu \rightarrow \nu\gamma$ as the function of the photon energy q^0 close to the minimum of the mean time of life

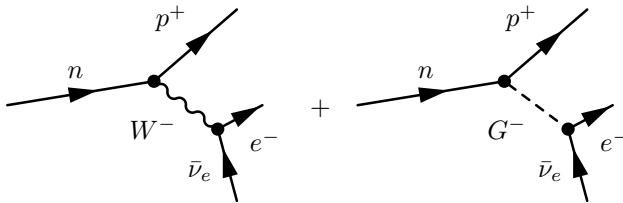


Figure 8: Graphs contributing to the β -decay

where the axial coupling $g_A \approx 1.25$; in the case of helicity coupling, factors $(1 - \gamma^5)/2$ are replaced by 1. The explicit calculation of the electron spectrum for chiral and helicity couplings as well as comparison with the experimental data are given in [15]. Let us stress only that the mysterious bump near the end point is reproduced on the theoretical plot; also the second anomaly problem in the tritium β -decay can be solved with help of the tachyonic hypothesis [15].

8 Conclusions

In this paper we have analyzed the introductory results related to tachyonic neutrino hypothesis. We do not pretend to explain all the problems with neutrino physics by means of this hypothesis. However, from the behaviour of the calculated decay rates for three dominant processes: emission of a $\nu\bar{\nu}$ pair by a neutrino, radiative decay $\nu \rightarrow \nu\gamma$ and β -decay, it follows that this possibility cannot be ruled out of considerations. Moreover such a hypothesis provides us with an additional mechanism simulating flavor oscillations.

Acknowledgement

One of us (J.R.) thanks to Jacek Ciborowski for interesting discussions.

A Vertices in our gauge model

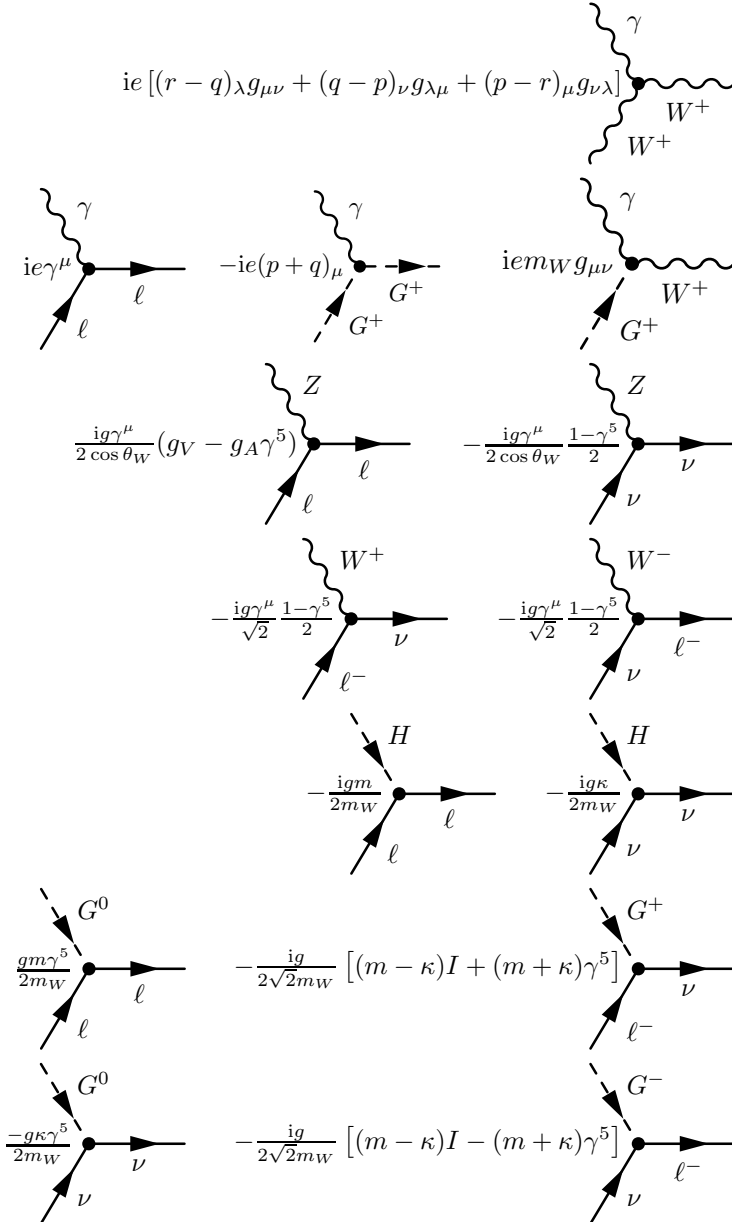
The vertices necessary for calculation of amplitudes for three processes considered here in our gauge model are shown in the Table 2. The rest of the Feynman rules in the convention adopted in this paper is given, e.g., in the Ref. [29].

B Amplitudes for neutrino decay $\nu_\ell \rightarrow \nu_\ell\nu_{\ell'}\bar{\nu}_{\ell'}$ for all couplings

The square of the amplitude for the chiral coupling is

$$|M|^2 = 2G_F^2 m^{\mu\nu} n_{\mu\nu}, \quad (46)$$

Table 2: Vertices in our gauge model used in this work



where

$$\begin{aligned}
 m^{\mu\nu} = & \frac{1}{2} \left(1 + \frac{(uk)}{\sqrt{(uk)^2 + \kappa^2}} \right) \left(1 + \frac{(up)}{\sqrt{(up)^2 + \kappa^2}} \right) \\
 & \times (k^\mu p^\nu + k^\nu p^\mu - (kp)g^{\mu\nu} - i\varepsilon^{\mu\nu\alpha\beta} k_\alpha p_\beta) \\
 & + \frac{\kappa^2}{2\sqrt{(up)^2 + \kappa^2}} \left(1 + \frac{(uk)}{\sqrt{(uk)^2 + \kappa^2}} \right) (k^\mu u^\nu \\
 & + k^\nu u^\mu - (uk)g^{\mu\nu} - i\varepsilon^{\mu\nu\alpha\beta} k_\alpha u_\beta) \\
 & + \frac{\kappa^2}{2\sqrt{(uk)^2 + \kappa^2}} \left(1 + \frac{(up)}{\sqrt{(up)^2 + \kappa^2}} \right) (p^\mu u^\nu \\
 & + p^\nu u^\mu - (up)g^{\mu\nu} + i\varepsilon^{\mu\nu\alpha\beta} p_\alpha u_\beta) \\
 & + \frac{\kappa^4}{2\sqrt{(uk)^2 + \kappa^2} \sqrt{(up)^2 + \kappa^2}} (2u^\mu u^\nu - g^{\mu\nu})
 \end{aligned}$$

and

$$\begin{aligned}
 n_{\mu\nu} = & \frac{1}{2} \left(1 + \frac{(ul)}{\sqrt{(ul)^2 + \mu^2}} \right) \left(1 + \frac{(ur)}{\sqrt{(ur)^2 + \mu^2}} \right) \\
 & \times (l_\mu r_\nu + l_\nu r_\mu - (lr)g_{\mu\nu} + i\varepsilon_{\mu\nu\alpha\beta} l^\alpha r^\beta) \\
 & + \frac{\mu^2}{2\sqrt{(ur)^2 + \mu^2}} \left(1 + \frac{(ul)}{\sqrt{(ul)^2 + \mu^2}} \right) (l_\mu u_\nu \\
 & + l_\nu u_\mu - (ul)g_{\mu\nu} + i\varepsilon_{\mu\nu\alpha\beta} l^\alpha u^\beta) \\
 & + \frac{\mu^2}{2\sqrt{(ul)^2 + \mu^2}} \left(1 + \frac{(ur)}{\sqrt{(ur)^2 + \mu^2}} \right) (r_\mu u_\nu \\
 & + r_\nu u_\mu - (ur)g_{\mu\nu} - i\varepsilon_{\mu\nu\alpha\beta} r^\alpha u^\beta) \\
 & + \frac{\mu^4}{2\sqrt{(ul)^2 + \mu^2} \sqrt{(ur)^2 + \mu^2}} (2u_\mu u_\nu - g_{\mu\nu}) .
 \end{aligned}$$

Squares of the amplitudes for helicity ($s = 0$) and γ^5 ($s = 1$) couplings read

$$|M|^2 = 2G_F^2 m_s^{\mu\nu} n_{s\mu\nu}, \quad (47)$$

where

$$\begin{aligned}
 m_s^{\mu\nu} = & g^{\mu\nu}(u) [((-1)^s \kappa^2 - kp) \\
 & + \frac{1}{\sqrt{(uk)^2 + \kappa^2} \sqrt{(up)^2 + \kappa^2}} ((-1)^s \kappa^2 ((kp) \\
 & - (uk)(up)) - \kappa^4 - \kappa^2 ((up)^2 + (uk)^2) \\
 & - (uk)(up)(kp))] \\
 & + u^\mu u^\nu \frac{2\kappa^2 (\kappa^2 - (-1)^s (kp))}{\sqrt{(uk)^2 + \kappa^2} \sqrt{(up)^2 + \kappa^2}} \\
 & + (k^\mu p^\nu + k^\nu p^\mu) \left[\frac{(uk)(up) - (-1)^s \kappa^2}{\sqrt{(uk)^2 + \kappa^2} \sqrt{(up)^2 + \kappa^2}} + 1 \right] \\
 & + (u^\mu k^\nu + u^\nu k^\mu) \frac{\kappa^2 ((-1)^s (up) + (uk))}{\sqrt{(uk)^2 + \kappa^2} \sqrt{(up)^2 + \kappa^2}} \\
 & + (u^\mu p^\nu + u^\nu p^\mu) \frac{\kappa^2 ((-1)^s (uk) + (up))}{\sqrt{(uk)^2 + \kappa^2} \sqrt{(up)^2 + \kappa^2}} \\
 & + i\varepsilon^{\mu\nu}{}_{\alpha\beta} \left[\frac{\kappa^2 ((-1)^s k^\alpha + p^\alpha) u^\beta - (uk)k^\alpha p^\beta}{\sqrt{(uk)^2 + \kappa^2}} \right. \\
 & \quad \left. - \frac{\kappa^2 ((-1)^s p^\alpha + k^\alpha) u^\beta + (up)k^\alpha p^\beta}{\sqrt{(up)^2 + \kappa^2}} \right]
 \end{aligned}$$

and

$$\begin{aligned}
 -n_{s\mu\nu} = & g_{\mu\nu}(u) [((-1)^s \mu^2 + lr) \\
 & + \frac{1}{\sqrt{(ul)^2 + \mu^2} \sqrt{(ur)^2 + \mu^2}} ((-1)^s \mu^2 ((lr) \\
 & - (ul)(ur)) + \mu^4 + \mu^2 ((ul)^2 + (ur)^2) \\
 & + (ul)(ur)(lr))] \\
 & + u_\mu u_\nu \frac{2\mu^2 (-\mu^2 - (-1)^s (lr))}{\sqrt{(ul)^2 + \mu^2} \sqrt{(ur)^2 + \mu^2}} \\
 & + (l_\mu r_\nu + l_\nu r_\mu) \left[-\frac{(ul)(ur) - (-1)^s \mu^2}{\sqrt{(ul)^2 + \mu^2} \sqrt{(ur)^2 + \mu^2}} - 1 \right]
 \end{aligned}$$

$$\begin{aligned}
 &+ (u_\mu l_\nu + u_\nu l_\mu) \frac{\mu^2 ((-1)^s(ur) - (ul))}{\sqrt{(ul)^2 + \mu^2} \sqrt{(ur)^2 + \mu^2}} \\
 &+ (u_\mu r_\nu + u_\nu r_\mu) \frac{\mu^2 ((-1)^s(ul) - (ur))}{\sqrt{(ul)^2 + \mu^2} \sqrt{(ur)^2 + \mu^2}} \\
 &+ i\varepsilon_{\mu\nu\alpha\beta} \left[\frac{\mu^2 ((-1)^s r^\alpha - l^\alpha) u^\beta + (ur)r^\alpha l^\beta}{\sqrt{(ur)^2 + \mu^2}} \right. \\
 &\quad \left. - \frac{\mu^2 ((-1)^s l^\alpha - r^\alpha) u^\beta - (ul)r^\alpha l^\beta}{\sqrt{(ul)^2 + \mu^2}} \right].
 \end{aligned}$$

C Amplitudes of processes contributing to the radiative decay

Amplitudes of processes contributing to the radiative decay described by diagrams in the Fig. 4 (a)–(f) read

$$\Omega_i^\mu = \bar{v}(p)M_i^\mu v(k), \quad (48)$$

where $i = a, b, c, d, e, f$ and:

$$\begin{aligned}
 M_a^\mu &= \frac{g^2 e}{2} \int \frac{d^4 r}{(2\pi)^4} \frac{1 + \gamma^5}{2} \gamma^\nu \\
 &\quad \times \frac{[(p-r)\gamma + m]\gamma^\alpha [(k-r)\gamma + m]}{[r^2 - m_W^2][(p-r)^2 - m^2][(k-r)^2 - m^2]} \\
 &\quad \times \gamma_\nu \frac{1 - \gamma^5}{2}, \quad (49a)
 \end{aligned}$$

$$\begin{aligned}
 M_b^\mu &= -\frac{g^2 e}{2} \int \frac{d^4 r}{(2\pi)^4} \frac{1 + \gamma^5}{2} \gamma^\beta \\
 &\quad \times \frac{[r\gamma + m]}{[r^2 - m^2][(p-r)^2 - m_W^2][(k-r)^2 - m_W^2]} \\
 &\quad \times [(2p-k-r)_\lambda g^\mu{}_\beta + (2r-k-p)^\mu g_{\lambda\beta} \\
 &\quad + (2k-r-p)_\beta g^\mu{}_\lambda] \gamma^\lambda \frac{1 - \gamma^5}{2}, \quad (49b)
 \end{aligned}$$

Decays of spacelike neutrinos

$$\begin{aligned}
 M_c^\mu &= -\frac{g^2 e}{8m_W^2} \int \frac{d^4 r}{(2\pi)^4} [(m - \kappa)I + (m + \kappa)\gamma^5] \\
 &\quad \times \frac{[(p - r)\gamma + m]\gamma^\mu[(k - r)\gamma + m]}{[r^2 - m_W^2][(p - r)^2 - m^2][(k - r)^2 - m^2]} \\
 &\quad \times [(m - \kappa)I - (m + \kappa)\gamma^5], \quad (49c)
 \end{aligned}$$

$$\begin{aligned}
 M_d^\mu &= \frac{g^2 e}{8m_W^2} \int \frac{d^4 r}{(2\pi)^4} [(m - \kappa)I + (m + \kappa)\gamma^5] \\
 &\quad \times \frac{[(p - r)\gamma + m](k + p - 2r)^\mu[(k - r)\gamma + m]}{[r^2 - m^2][(p - r)^2 - m_W^2][(k - r)^2 - m_W^2]} \\
 &\quad \times [(m - \kappa)I - (m + \kappa)\gamma^5], \quad (49d)
 \end{aligned}$$

$$\begin{aligned}
 M_e^\mu &= \frac{g^2 e}{8} \int \frac{d^4 r}{(2\pi)^4} \gamma^\mu (1 - \gamma^5) \\
 &\quad \times \frac{[r\gamma + m]}{[r^2 - m^2][(p - r)^2 - m_W^2][(k - r)^2 - m_W^2]} \\
 &\quad \times [(m - \kappa)I - (m + \kappa)\gamma^5], \quad (49e)
 \end{aligned}$$

$$\begin{aligned}
 M_f^\mu &= \frac{g^2 e}{8} \int \frac{d^4 r}{(2\pi)^4} [(m - \kappa)I + (m + \kappa)\gamma^5] \\
 &\quad \times \frac{[r\gamma + m]}{[r^2 - m^2][(p - r)^2 - m_W^2][(k - r)^2 - m_W^2]} \\
 &\quad \times (1 + \gamma^5)\gamma^\mu. \quad (49f)
 \end{aligned}$$

After calculations and renormalization, the integrals contributing to the form factor G_V in the Eq. (36) read

$$\begin{aligned}
 I_a &= -2 \int_0^1 dx \frac{x(x^2 - 3x + 2)}{x^2 \kappa^2 + x(m_W^2 - \kappa^2 - m^2) - m_W^2} \\
 &\simeq \frac{4}{3m_W^2}, \\
 I_b &= \int_0^1 dx \frac{x^2(2x + 1)}{x^2 \kappa^2 + x(m^2 - \kappa^2 - m_W^2) - m^2}
 \end{aligned}$$

$$\begin{aligned}
 &\simeq -\frac{7}{6m_W^2}, \\
 I_c &= \frac{1}{m_W^2} \int_0^1 dx \frac{x^2[x(\kappa^2 - m^2) - (m^2 + \kappa^2)]}{x^2\kappa^2 + x(m_W^2 - \kappa^2 - m^2) - m_W^2} \\
 &\simeq \frac{2m^2}{m_W^4} \ln\left(\frac{m_W}{m}\right)^2, \\
 I_d &= \frac{1}{m_W^2} \int_0^1 dx \frac{x(x-1)[x(\kappa^2 - m^2) + 2m^2]}{x^2\kappa^2 + x(m^2 - \kappa^2 - m_W^2) - m^2} \\
 &\simeq \frac{5m^2 + \kappa^2}{6m_W^4}, \\
 I_e + I_f &= -\int_0^1 dx \frac{x^2}{x^2\kappa^2 + x(m^2 - \kappa^2 - m_W^2) - m^2} \\
 &\simeq \frac{1}{2m_W^2}.
 \end{aligned}$$

D Sum over photon polarization vectors

Sum over photon polarization vectors (on the mass shell $q^2 = 0$) in the absolute synchronization reads:

$$\begin{aligned}
 \Pi^{\mu\nu}(q) &= \sum_{\substack{\lambda=-1 \\ \lambda \neq 0}}^1 e^\mu_\lambda e^\nu_\lambda = -\frac{1}{(uq)^2} [(uq)^2 g^{\mu\nu}(u) + q^\mu q^\nu \\
 &\quad - (uq)(u^\mu q^\nu + u^\nu q^\mu)]. \quad (50)
 \end{aligned}$$

Notice, that $q_\mu \Pi^{\mu\nu} = u_\mu \Pi^{\mu\nu} = 0$, so e^μ_λ , q^μ and u^μ form an pseudo-orthogonal basis.

E Oscillations of tachyonics vs. massive neutrinos

Here we compare the oscillation effects for the massive and tachyonic neutrinos. Calculations will be performed in the preferred frame, in which the metric tensor has the standard Minkowskian form.

Decays of spacelike neutrinos

Let us consider three flavors of neutrinos. We shall denote the neutrino flavor states by

$$|\nu_e\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} |\vec{p}\rangle, \quad |\nu_\mu\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} |\vec{p}\rangle, \quad |\nu_\tau\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} |\vec{p}\rangle. \quad (51)$$

The $|\vec{p}\rangle$ denotes the vector from the representation space of the Poincaré group, corresponding to the momentum \vec{p} .

In the basis (51) the free Hamiltonian describing neutrinos can be written as

$$H = U^\dagger H_0 U, \quad (52)$$

where U is a 3×3 unitary mixing matrix, acting in each subspace of the Hilbert space determined by momentum \vec{p} , and the Hamiltonian H_0 in the mass eigenstates basis is of the form

$$H_0 = \begin{pmatrix} \sqrt{\vec{p}^2 \pm m_1^2} & 0 & 0 \\ 0 & \sqrt{\vec{p}^2 \pm m_2^2} & 0 \\ 0 & 0 & \sqrt{\vec{p}^2 \pm m_3^2} \end{pmatrix}. \quad (53)$$

Here m_j^2 ($j = 1, 2, 3$) are the absolute values of the squares of neutrino four-momenta and the choice of the signs corresponds to the massive (+) and tachyonic (−) case; moreover $|\vec{p}| > \max_j m_j$ in the tachyonic case.

Following the standard procedure, we consider the time evolution of the neutrino state. Assume that the initial state $|\nu_i\rangle$ ($i = e, \mu, \tau$) is an eigenstate of the fractional lepton number operator. Thus, after time t the probability that we get the neutrino ν_k is given by

$$P_{\nu_i \rightarrow \nu_k}(t) = |\langle \nu_k | U^\dagger e^{-itH_0} U | \nu_i \rangle|^2. \quad (54)$$

Now, taking (54) in the limit $|\vec{p}| \gg m_j$, we can expand the matrix elements of the Hamiltonian as follows

$$\sqrt{\vec{p}^2 \pm m_i^2} \simeq p \pm \frac{m_i^2}{2p}. \quad (55)$$

Using (55) we obtain from (54)

$$P_{\nu_i \rightarrow \nu_k}(t) = \left| \sum_{j=1}^3 u_{jk}^* u_{ji} e^{\mp \frac{it}{2p} m_j^2} \right|^2. \quad (56)$$

On introducing the polar representation such that

$$u_{2k}^* u_{2i} u_{3k} u_{3i}^* = r_1(i, k) e^{i\theta_1(i, k)}, \quad (57a)$$

$$u_{3k}^* u_{3i} u_{1k} u_{1i}^* = r_2(i, k) e^{i\theta_2(i, k)}, \quad (57b)$$

$$u_{1k}^* u_{1i} u_{2k} u_{2i}^* = r_3(i, k) e^{i\theta_3(i, k)}, \quad (57c)$$

and

$$\omega_1 = \frac{m_2^2 - m_3^2}{2p}, \quad \omega_2 = \frac{m_3^2 - m_1^2}{2p}, \quad \omega_3 = \frac{m_1^2 - m_2^2}{2p}, \quad (58)$$

we can write (56) in the following form:

$$P_{\nu_i \rightarrow \nu_k}(t) = \sum_{j=1}^3 |u_{jk}|^2 |u_{ji}|^2 + 2 \sum_{j=1}^3 r_j(i, k) \cos[\omega_j t \pm \theta_j(i, k)], \quad (59)$$

where, as before, the signs + and - correspond to massive and tachyonic neutrinos, respectively. In particular, if $k = i$ then $\theta_j(i, i) = 0$ ($j = 1, 2, 3$) and

$$P_{\nu_i \rightarrow \nu_i}(t) = \sum_{j=1}^3 |u_{ji}|^4 + 2 \sum_{j=1}^3 r_j(i, i) \cos(\omega_j t), \quad (60)$$

so this probability is the same for massive and tachyonic neutrinos.

We have shown that the only difference between the neutrino oscillations in the massive and tachyonic case lies in the initial phase of oscillations $\theta_j(i, k)$. Moreover, the initial phases for tachyonic case are obtained by taking the complex conjugations of the elements of mixing matrix for the massive case (see (57a)–(57c)). In the oscillation experiments we cannot decide whether we should take the mixing matrix U or its complex conjugation U^* without taking into account the nature of the neutrinos. More precisely, we can explain the oscillations for massive or tachyonic neutrinos simply by the different choice of the mixing matrix (U or U^*). Therefore, the experimental evidence of neutrino oscillations does not distinguish between massive and tachyonic neutrinos.

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Comment on DECAYS OF SPACELIKE NEUTRINOS

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In the paper "Decays of spacelike neutrinos" the authors invoke a hypothesis according to which neutrinos are tachyons. They present calculations of decay parameters of such neutrinos. This hypothesis is interesting because it offers an explanation of the anomaly (excess of counts) near the end of the electron spectrum in the beta decay of tritium. Numerous attempts seeking conventional explanations seem unsuccessful. If neutrinos were tachyons, they would possess interesting properties in distinction to classical non zero mass neutrinos. A tachyonic neutrino might decay into two basic channels: (i) $\nu_i \rightarrow \nu_i \nu_j \bar{\nu}_j$ and (ii) in radiation channel, $\nu \rightarrow \nu \gamma$. In the paper the authors follow calculations of the widths (lifetimes) corresponding to these decay channels and present the results in the form of plots for various values of the (tachyonic) masses and energies of tachyonic neutrinos. These calculations have been done within the Standard Model, using the formalism proposed by one of the authors (J.R.) based on the Chang-Tangherlini synchronisation scheme which is the correct one in the case of tachyons. Impact of the results presented in the paper is large for neutrino physics. Firstly, if neutrinos were tachyons, the appearance of decays (i) in an accelerator neutrino

Comment

beam would lead to an effect similar to neutrino oscillations and thus this effect should be taken into account when analysing the experimental data in numerous accelerator experiments. Secondly, if neutrinos were tachyons, then their decays might have had an influence on the description of the early Universe at the stage when the energies of neutrinos were large (a subject never explored). The content of the paper is fully exhaustive and the body is written clearly. Numerous details in the appendices supplement the subject. In summary, I highly evaluate the paper and I fully support publishing it.