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STERN, BARUT, AND GERLACH TODAY

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Abstract

We review some original and some current questions about the Stern Gerlach effect in light of Asim Barut's work.

1 Introduction

The original Stern-Gerlach experiment was performed more than eighty years ago, but the Stern-Gerlach effect continues to fascinate because it contains such a wealth of beautiful physics. Today, one can think of many reasons to find significance in the Stern-Gerlach effect. Although the motivation for performing and discussing Stern-Gerlach experiments and the interpretation of the result may have varied over the years, the questions raised have never ceased to be fundamental. Giving satisfactory answers to these questions is still challenging. As a physicist who was deeply committed to the development of a clear representation of the microscopic domain, Asim Barut could not fail to address the issues raised by Stern and Gerlach and incorporate them in his own work. In this paper in his honor, we attempt to place his contribution right at the center of the work of Stern and Gerlach, by analyzing how his publications on the subject and the theories he advocated can help us answer some of the fundamental questions raised by Stern and Gerlach.

2 Asim Barut's Challenge

We believe that the description of physical phenomena must be formulated in as rigorous and as compact mathematical way as possible, but the mathematics must have a clear physical interpretation and must abstract the nature as close as possible

This admonition, given at the beginning of his monograph "Geometry and Physics" [Baru89], seems particularly suited to the description of the Stern-Gerlach phenomenon. The large number of papers that continues to appear on the subject is an indication of the fertility of the physics and of the complexity of the mathematical description of the separation of a beam into its angular momentum components under the influence of magnetic fields. The fact that the subtler points keep being rediscovered also means that a definitive treatment, convincingly elegant, and applicable in all generality, is still lacking.

3 The Stern-Gerlach Experiment

3.1 The Original Experiment

The historical Stern-Gerlach experiment consists in work done in Frankfurt-am-Main between the Fall of 1921 and the Spring of

1922 separately, and jointly, by Otto Stern and Walther Gerlach in their attempt to measure the magnetic birefringence of silver vapor. Simplifying somewhat, one can say that Stern conceived the original experiment and that Gerlach made its successful realization possible. Excellent accounts of the historical circumstances of the experiments are available [Wein95b, Frie98, Frie03]. The work did much more than provide an answer to the original question (it turns out that silver gas is not birefringent), and it led to the publication of six journal articles [Ster21, Gerl21, Gerl22a, Gerl22b, Gerl24, Gerl25], all of which were published in the German literature in the mid nineteen twenties, before the appearance of the concepts of "intrinsic spin", "quantum mechanics", "entanglement", or "quantum measurement".

3.2 The Reinterpretation of Stern-Gerlach

This last remark is not just a historical anecdote, but a challenging issue to consider by all who use the Stern-Gerlach effect to illustrate any one of these critical aspects of our current description of microscopic physics. Not only did Stern-Gerlach occur in the absence of any awareness of the concepts mentioned above, but it conspicuously did not lead to their invention or discovery either. There is, to my knowledge, no indication whatsoever that Stern-Gerlach facilitated or directly motivated the work of de Broglie, Schrödinger, Heisenberg or Dirac. It played no role in the invention of duplicity by Pauli or in that of the spin-model by Goudsmit and Uhlenbeck. It seems that it took, in fact, two years *after* the introduction of spin before anyone established a connection between this model and Stern and Gerlach's experiment. Similarly, although Stern-Gerlach can now be viewed as a first illustration of entanglement [Rein99], this is a fairly recent realization. It did not seem to occur to Schrödinger or to the authors of the EPR paper at the time of their respective contributions. One paper did in fact anticipate the new physics hiding in the Stern-Gerlach results [Eins22], mostly however by questioning the nature of the preparation of the beam during the experiment. The prevalent view, including that of Stern and Gerlach themselves, seemed to have been that the experiment was a spectacular confirmation of Bohr and Sommerfeld's theory of space quantization, a view that is not always appreciated in modern pedagogical accounts of the "surprising discovery of half-integer spin". We know from a comment in his biographical notes on Alfred Landé [Baru05] that Asim Barut was better informed on the historical context for the intricate development of these concepts. Even the invention of half-integer quantum numbers by Landé in Frankfurt a few months earlier, does not seem to have played any role in the work of Stern and Gerlach. Also, from his work on Sommerfeld's relativistic treatment of the hydrogen atom and on the connection between the quantum numbers in Sommerfeld, Schrödinger, and Dirac theory, Barut certainly understood why Stern expected to find an *even* number of lines for the ground state of the silver alkali, even before the experimental data were obtained.

4 More Stern-Gerlach Experiments

4.1 Electron Beams

Whereas in the original Stern-Gerlach experiment, the angular momentum in question is that of the silver atom, it is now understood that this angular momentum is, in fact, that of the forty-seventh, or valence, electron. Which part of the entity one should attribute the angular momentum to, was certainly not a trivial question in the nineteen twenties, and, even after the introduction of the spin quantum number by Pauli, there was disagreement as to whether it represented an intrinsic property of the electron (Goudsmit and Uhlenbeck) or just one of its spectroscopic properties when bound to a nucleus or atomic core (Pauli). The argument continued in a modified form when it was proposed to strip the Stern-Gerlach electron, so to speak, of its silver core and perform a magnetic separation of a beam of charged electrons. This proposal led to years of discussion, which are continuing today [Garr99]. Does the interaction between the fields and the charges of the particles in an electron beam just make for a technically more difficult experiment, or do electrons just not exhibit their spin freely when moving classically? Brillouin proposed to modify the experiment into a longitudinal Stern-Gerlach configuration to bypass objections of Lorentz force blurring, Pauli argued against it, using uncertainty principle arguments, and Bohr turned the issue into a principled question as to the nonobservability of electron spin in any classical context, such as that of Stern-Gerlach trajectories. Barut did not witness the more recent developments but he was well acquainted with the measurement of the electron anomaly and the work of Dehmelt [Vand86, Dehm86] that has motivated this latest research. He also emphasizes the fact that his improved theory only applies to neutral magnetic dipole moments [Baru88b].

4.2 Molecular and Particle Beams

One of the outcomes of the Stern-Gerlach experiment was the development of the very fertile field of atomic and molecular beam spectroscopy and the direct or indirect determination of magnetic moments of atoms, molecules, and atomic constituents. In particular it led to the surprise announcement that the proton was not a simple Dirac particle. A Stern-Gerlach experiments with neutrons was eventually performed in 1954 [Sher54], allowing a direct determination of that particle's magnetic moment. At this stage, one can only dream of Stern-Gerlach experiment with neutrinos, so as to probe that particle's magnetic moment, a key ingredient in Barut's model for leptonic narrow resonances [Barut75], neutrino scattering [Barut82], and the generation problem in particle physics [Barut78].

4.3 Spinor Condensates Separation

On the other hand the recent experimental study of the spin structure of Bose-Einstein condensates (BECs), including beamless Stern-Gerlach-like separation of the spin components, is advancing rapidly [Higb05]. It shows the remarkable ability of Stern-Gerlach to reinvent itself. The optical imaging techniques should allow one to visualize the spin separation process in a way that seems to illustrate the ensemble interpretation of quantum mechanics on a single system.

4.4 The Problem of Composites

The problem of coupling the angular momenta of the components of a quantum entity to find the total angular momentum by using group theoretical methods and its generalization to other internal degrees of freedom was also at the heart of Asim Barut's program for a kinematic *and* dynamic theory of elementary particles [Basr85] and his interest in composite spin was particularly sharpened in connection with the proton spin crisis (see, e.g., [Hans03]). An extension to the many particles of BECs raises the issue as to whether a second quantized description is necessary to capture the essential dynamics [Law98]. Eliminating the need for quantizing the electromagnetic field was the essential motivation behind Barut's self-energy quantum electrodynamics [Baru88a].

5 Classical Spin, Quantum Nonlocality

Other aspects of the Stern-Gerlach effect that were central in Barut's work were the issue of the classical description of the spin degrees of freedom and the many bridges between the classical and the quantum formalism. Spin, for Barut, was also a tool to access the nonlocal correlations in quantum mechanics [Baru88b]:

[Stern-Gerlach]... is the experiment par excellence, the prototype of a quantum measurement, in which the eigenstates of an observable are separated, the reduction of the wave packet takes place, and the system is in an eigenstate after the measurement.

Let us emphasize here that a classical trajectory description of Stern-Gerlach is sufficient to show the breakdown of classical theory, but that adding a quantum condition to select the two surviving beams from the continuous distribution is unsatisfactory as a quantum explanation. Quantum treatments of Stern-Gerlach, the first of which is found in Bohm's text [Bohm51] typically treat the magnetic field approximately, whether they acknowledge it or not.

5.1 Magnetic Fields

It is important to recognize that the magnetic field plays two different roles in Stern-Gerlach. The inhomogeneous field causes the splitting. Because magnetic inhomogeneity is necessarily multidimensional in the absence of magnetic charges (div $B = 0$), the splitting happens in several directions simultaneously. The homogeneous field is used classically to simplify the problem, for it removes the splitting in all but one direction, through the averaging process of precession. In quantum theory however, the simultaneous splitting in several directions could lead to the simultaneous determination of incompatible spin components. The homogeneous field prevents this from happening. Whereas the precession argument can still be made for the *expectation values* of spin, a more complete treatment is needed for the observables themselves. This begs for the question of what happens in a purely inhomogeneous field [VanH05]. It is remarkable that in a Heisenberg picture treatment of the spin evolution in a purely inhomogeneous field, Scully, Lamb and Barut [Scul87] find that the wave packet exhibits deflection in both directions perpendicular to the beam axis! Of course a purely inhomogeneous field is locally equivalent to the superposition of a homogeneous field and an inhomogeneous field. The issue of the magnitude of the homogeneous part of the field needed to select the axis along which the measurement will take place, then becomes a matter of controlling the transverse size or cross-section of the beam.

5.2 Nonlinearities

Barut's concern with an underlying mechanism for the Stern-Gerlach outcome is also apparent in an earlier publication [Baru84], where he obtains a separation of the spin components using the property of nonlinearity in two different contexts. On the one hand, he generalizes a previously used nonlinear wave equation, with special solutions that separate in space, to its two-component (Pauli) version, making sure that the separated spatial components contain different spin projections. This one-dimensional solution would need to be dynamically connected to the magnetic field to be a realistic representation of the Stern-Gerlach effect. In the second proposal, he assumes that the classical spin vector obeys an equation, formally identical to the Lorentz-Dirac equation, with its nonlinear term arising from radiation-reaction. The result is a dynamical evolution of the randomly oriented spin vectors toward one of only two spin orientations, up and down.

5.3 A Continuing Search

Barut revisited the Stern-Gerlach experiment in a subsequent publication [Baru88b], again motivated by the need to provide sensible answers to quantum nonlocality using a concrete model. In this paper Barut develops what he calls the "naive classical theory of the Stern-Gerlach experiments" whose shortcomings he then emphasizes, based on the approximate description of the field, and on the lack of joint treatment of the orbit and spin variables. He then discusses how such a description can be given, both classically and quantum mechanically in the Heisenberg language. In both cases he conjectures that the actual solution might be more subtle and unexpected than the study of averages seems to indicate. The use of Stern-Gerlach devices in the experimental verification of nonlocal correlations might therefore need to be reevaluated.

As Barut conjectured, a detailed joined treatment of spin and spatial variables is very tricky and exact analytical solutions are still eluding us. Separable equations in momentum representation lead to differential equations with a singularity structure beyond the hypergeometric and Heun classes [Sten05]. Recent treatments based on a semiclassical theory [Bate02] and on quantum mechanics in the Schrödinger picture [Pote05], approach the problem perturbatively and numerically. The number of different theoretical descriptions of the Stern-Gerlach experiment keeps growing and there is no indication that a consensus will be reached soon!

6 In Conclusion

In this paper I have tried to connect Stern-Gerlach questions, old and new, to some recurring themes in Barut's scientific oeuvre. The survey is necessarily incomplete. The references to his publications are illustrative rather than comprehensive. Barut was very prolific and an exhaustive bibliography is unfortunately currently unavailable. In addition, the task would take me beyond the intended scope of this contribution. To do Barut's work justice, one should certainly also look into the implications of describing Stern-Gerlach as a scattering process and, although atomic beams are slow moving, introduce consistent relativistic dynamics, including the classical and quantum Zitterbewegung, where applicable.

I have no personal recollection of ever discussing Stern-Gerlach with Asim Barut. What I find in his work is not a definitive treatment of Stern-Gerlach, but a message encrypted in his very distinct approach to physical problems: there are many ways to describe one experiment, creative solutions are needed and we shouldn't be complacent. The goal should be nothing short of an exact, analytic solution, but there is much to be learned from simplified models.

The fact that a really good question (Stern), combined with a powerful experimental technique (Gerlach and Stern), would challenge versatile theoretical thinking (Barut), comes as no surprise. That this thinking can be organized into new questions, questions which new experimental techniques can address, is exhilarating, for it reminds us that great achievements from the past can still provide inspiration for scientific enquiry and discovery many years later. The combined work of Gerlach, Barut and Stern, is very much alive and a profound source of inspiration today.

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