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ELECTRON LASER ACCELERATION IN VACUUM: IS THE GAUSSIAN BEAM DESCRIPTION ADEQUATE?

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Abstract

Numerical solutions, to the fully relativistic equations of motion of a single electron injected sideways into the focal point of a tightly-focused laser beam, show that accurate electron energy gain calculations require that terms of order ε^2 and higher, where ε is the diffraction angle, ought to be included in the description of the fields of a Gaussian beam focused down to a 5 μm waist radius. It is also demonstrated that, during acceleration, the phase velocity of the accelerating field stays subluminous and, hence, the electron surfs on the laser wave and gains energy from it.

1 Introduction

As a student of A. O. Barut, I learnt from him never to rush to the computer with a physics problem to solve, but to do so only as a last resort. Work leading to the results presented in this paper has evolved with that advice as the guiding light. Over the past few years, I studied a number of laser acceleration schemes by working fully analytically. In a typical configuration, the program consisted of modeling the $\boldsymbol{E}$ and $\boldsymbol{B}$ fields of the laser beam ideally as plane waves and solving the fully relativistic Newton-Lorentz equations of motion, namely

$$\frac{d\boldsymbol{p}}{dt} = -e\left[\boldsymbol{E} + \boldsymbol{\beta} \times \boldsymbol{B}\right]; \quad \frac{d\mathcal{E}}{dt} = -ec\boldsymbol{\beta} \cdot \boldsymbol{E}, \tag{1}$$

for a single electron (mass m and charge $-e$) in the presence of those fields. In Eqs. (1) the electron's relativistic momentum and total energy are given by $\boldsymbol{p} = \gamma mc\boldsymbol{\beta}$ and $\mathcal{E} = \gamma mc^2$, respectively. Furthermore, $\gamma = (1-\beta^2)^{-1/2}$ is the Lorentz factor of the electron and $\boldsymbol{\beta}$ is its velocity normalized by c, the speed of light in vacuum.

In a number of very specialized situations, exact fully analytic expressions for the electron trajectory equations, momentum and energy were obtained from (1). The analytic solutions were possible because the space-time dependence in the fields was assumed in the combination $\eta = \omega t - \boldsymbol{k}\cdot\boldsymbol{r}$, with ω and $\boldsymbol{k}$ the frequency and wavevector, respectively, of the laser light and t and $\boldsymbol{r}$ stand for the time and space coordinates of the electron. Employing electrostatic (Gaussian) units, the (linearly-polarized) plane-wave fields of amplitude E_0 may, in general, be written in the form

$$\boldsymbol{E} = \hat{\mathbf{i}} E_0 f(\eta), \tag{2}$$

$$\boldsymbol{B} = \hat{\mathbf{j}} E_0 f(\eta), \tag{3}$$

where $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are unit vectors pointing along $+x$ and $+y$ of a Cartesian coordinate system in which the direction of propagation is $+z$, and f is a function possessing up to second-order space and time-derivatives (so that the fields may satisfy the appropriate wave and Maxwell equations). In the most trivial situation f is sinusoidal. Configurations studied within the context of the plane-wave model include the Autoresonance Laser Accelerator [1, 2, 3], the Crossed Beam Laser Accelerator [4], the Beat Wave Laser Accelerator [5, 6], and a scheme involving an added DC electric field [7, 8].

It has often been remarked in the references cited above that a plane-wave model is only an idealization, of value in at least two main respects: it helps build a clear intuitive picture for the scheme, and it plays a role in bench-marking computer codes for analysis of the scheme when a realistic representation of the fields, perhaps in terms of those of a Gaussian beam, is employed. As is well known, a plane wave has infinite extension in space and time and, as such, the problem of injecting an electron into it remains ill-defined. The same thing can be said about electron extraction out of such a beam. The fields of a plane wave also exhibit high symmetry. As a result, electron interaction with any number of field cycles results in no net energy gain [9, 10].

So, the obvious next development ought to involve employing a realistic model, e.g., a Gaussian or a Bessel beam. Fields of such beams lead to equations of motion that may only be solved numerically. In the following section the fields of a tightly focused Gaussian laser beam will be reviewed in brief.

Work on schemes to accelerate charged particles using high-intensity laser beams is currently gathering momentum [11, 12, 13]. The importance of such efforts can never be underestimated in view of the fact that conventional accelerators have become too large and too expensive to construct and operate. Recent progress in laser technology [14, 15, 16, 17] lends a lot of impetus to those efforts.

After a brief review of the fields of the Gaussian laser beam conducted in Sec. 2, results from some calculations will be presented in Sec. 3 that will show the need to include high-order terms in those fields if the dynamics of the electron is to be accurately described. In Sec. 4 further light will be shed on the acceleration mechanism, while keeping the main aim of the paper in mind, by demonstrating the existence of regions along the electron trajectory over which the phase velocity is subluminous, thus allowing the electron to ride on the wave and gain energy from it. Our conclusions are summed up eventually in Sec. 5.

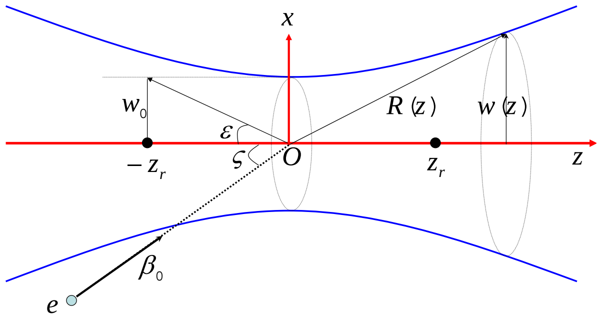

Figure 1: The Gaussian-beam geometry. Propagation is along $+z$, polarization is along x and O is a stationary focus. Shown also is the electron (labeled by e) directed at the beam focus, from a point in the xz plane, with a scaled speed β_0.

2 The fields

Figure 1 exhibits the main features of a Gaussian beam propagating along the z axis and having a stationary focus at the origin O. The beam is assumed to have circular cross sections in the xy plane (of radius w_0 at focus and $w(z) = w_0\sqrt{1 + (z/z_r)^2}$ elsewhere, where $z_r = kw_0^2/2$ is the Rayleigh length and k is the wavenumber). In the figure, the diffraction angle is defined as $\varepsilon = w_0/z_r$ and $R(z) = z + z_r^2/z$ is the radius of curvature of a wavefront crossing the beam axis at z. We borrow below the electric and magnetic field components of the beam shown in Fig. 1 from Ref. [18] (see also Refs. [19, 20, 21]). In addition to the transverse components, and as a result of focusing, the laser fields develop axial components that must be taken into account. In place of the Cartesian coordinates x and y, scaled ones defined by

$$\xi \equiv \frac{x}{w_0}; \quad \upsilon \equiv \frac{y}{w_0}, \tag{4}$$

will be employed. In terms of the new coordinates, and working within the electrostatic (Gaussian) system of units throughout, the electric field components may be written as

$$\begin{aligned}
E_x &= E\left\{S_0+\varepsilon^2\left[\xi^2 S_2-\frac{\rho^4 S_3}{4}\right]\right. \\
&\quad +\varepsilon^4\left[\frac{S_2}{8}-\frac{\rho^2 S_3}{4}-\frac{\rho^2(\rho^2-16\xi^2)S_4}{16}\right. \\
&\quad \left.\left.-\frac{\rho^4(\rho^2+2\xi^2)S_5}{8}+\frac{\rho^8 S_6}{32}\right]\right\}, \qquad (5)\\
E_y &= E\xi\upsilon\left\{\varepsilon^2\left[S_2\right]+\varepsilon^4\left[\rho^2 S_4-\frac{\rho^4 S_5}{4}\right]\right\}, \qquad (6)\\
E_z &= E\xi\left\{\varepsilon\left[C_1\right]+\varepsilon^3\left[-\frac{C_2}{2}+\rho^2 C_3-\frac{\rho^4 C_4}{4}\right]\right. \\
&\quad \left.+\varepsilon^5\left[-\frac{3C_3}{8}-\frac{3\rho^2 C_4}{8}+\frac{17\rho^4 C_5}{16}-\frac{3\rho^6 C_6}{8}+\frac{\rho^8 C_7}{32}\right]\right\}. \qquad (7)
\end{aligned}$$

Similarly, the magnetic field components are given by

$$\begin{aligned}
B_x &= 0, \qquad (8)\\
B_y &= E\left\{S_0+\varepsilon^2\left[\frac{\rho^2 S_2}{2}-\frac{\rho^4 S_3}{4}\right]\right. \\
&\quad \left.+\varepsilon^4\left[-\frac{S_2}{8}+\frac{\rho^2 S_3}{4}+\frac{5\rho^4 S_4}{16}-\frac{\rho^6 S_5}{4}+\frac{\rho^8 S_6}{32}\right]\right\}, \qquad (9)\\
B_z &= E\upsilon\left\{\varepsilon\left[C_1\right]+\varepsilon^3\left[\frac{C_2}{2}+\frac{\rho^2 C_3}{2}-\frac{\rho^4 C_4}{4}\right]\right. \\
&\quad \left.+\varepsilon^5\left[\frac{3C_3}{8}+\frac{3\rho^2 C_4}{8}+\frac{3\rho^4 C_5}{16}-\frac{\rho^6 C_6}{4}+\frac{\rho^8 C_7}{32}\right]\right\}. \qquad (10)
\end{aligned}$$

In Eqs. (5)-(10), the Gaussian amplitude and sinusoidal terms are given, respectively, by

$$E = E_0 \frac{w_0}{w} \exp\left[-\frac{r^2}{w^2}\right], \tag{11}$$

$$S_n = \left(\frac{w_0}{w}\right)^n \sin(\psi + n\psi_G); \quad n = 0, 1, 2, \ldots, \tag{12}$$

$$C_n = \left(\frac{w_0}{w}\right)^n \cos(\psi + n\psi_G). \tag{13}$$

Furthermore, $k = \omega/c$, E_0 is a constant amplitude, $r^2 = x^2 + y^2$, and $\rho = r/w_0$. Some of the details leading to Eqs. (5)-(10) may be found in the Appendix of Ref. [18]. These equations were derived from a vector potential polarized along x and having a time dependence of the form $e^{i\omega t}$, with ω the frequency. The remaining symbols in Eqs. (5)-(10) have the following definitions

$$\psi = \psi_0 + \psi_P - \psi_R + \psi_G, \tag{14}$$

$$\psi_P = \omega t - kz, \tag{15}$$

$$\psi_G = \tan^{-1}\left(\frac{z}{z_r}\right), \tag{16}$$

$$\psi_R = \frac{kr^2}{2R}, \tag{17}$$

$$R(z) = z + \frac{z_r^2}{z}. \tag{18}$$

Note that ψ_0 is an intial phase, $\psi_P = \eta$ is the plane wave phase, ψ_G is the Guoy phase associated with the fact that a Gaussian beam undergoes a total phase change of π as z changes from $-\infty$ to $+\infty$, and that ψ_R is the phase associated with the curvature of the wave fronts. The fields given above satisfy Maxwell's equations $\boldsymbol{\nabla} \cdot \boldsymbol{E} = 0 = \boldsymbol{\nabla} \cdot \boldsymbol{B}$, plus terms of order ε^6 [19]. Before the fields to order ε^5 are employed in any calculations, let us take a look at the low-order fields. These fields are first constructed from Eqs. (5)-(18) and then the resulting expressions are compared with those employed by other authors. To order ε^0 the fields read

$$\boldsymbol{E} = \hat{\mathbf{i}} E_0 \frac{w_0}{w} e^{-r^2/w^2} \sin\psi, \tag{19}$$

$$\boldsymbol{B} = \hat{\mathbf{j}} E_0 \frac{w_0}{w} e^{-r^2/w^2} \sin\psi. \tag{20}$$

The important thing to note is that, to this order, the fields are purely transverse; they have no longitudinal components. Thus Eqs. (19) and (20) can not be relied upon in describing the fields of a tightly-focused laser beam. However, when one goes to order ε^1, forward (longitudinal) components of both fields begin to establish presence. To that order, the fields are

$$\boldsymbol{E} = E_0 \frac{w_0}{w} e^{-r^2/w^2} \left[\hat{\mathbf{i}} \sin\psi + \hat{\mathbf{k}} \frac{x}{z_r} \cos(\psi + \psi_G) \right], \tag{21}$$

$$\boldsymbol{B} = E_0 \frac{w_0}{w} e^{-r^2/w^2} \left[\hat{\mathbf{j}} \sin\psi + \hat{\mathbf{k}} \frac{y}{z_r} \cos(\psi + \psi_G) \right], \tag{22}$$

where $\hat{\mathbf{k}}$ is a unit vector in the direction of $+z$. In the system of units employed in this paper, $E_x = B_y$. Furthermore, E_z and B_z are identical, apart from the dependence on x in the former and on y in the latter. To the best of our knowledge, other authors [22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32] have only been using fields accurate to order ε^1 in their analysis of various schemes of laser acceleration of electrons in vacuum. For example, apart from the important (but, otherwise, arbitrary) initial phase, ψ_0, the electric field components E_x and E_z, employed in their publications, are the same as those given in Eq. (21). One of our aims in this paper is to demonstrate that when a beam of present-day power is focused to a waist radius of 5 μm or less, an accurate description of its fields should include terms of order ε^2 and higher. As can be clearly seen from Eqs. (5)-(10), the electric field develops a second transverse component, an E_y, when one goes to order ε^2 and beyond.

The full fields, containing terms of order up to ε^5, have recently been used in investigation of a number of laser acceleration schemes [18, 33, 34, 35]. However, the question of whether one needs to go to that order in ε has not been fully addressed. This will be done in the next section.

3 Appropriate order in the diffraction angle

In this section, the energy gained by an electron, injected sideways, as shown in Fig. 1, into the focal region of a tightly-focused laser beam, will be calculated. Inclusion of terms higher in order than ε will be shown to be essential if the gain is to be accurately calculated. Typically, the electron will be injected from a point in the xz plane with coordinates (x_0, z_0) at the speed β_0 (in units of c, the speed of light in vacuum) and directed at an angle ζ with respect to the propagation direction. Subsequent motion in the laser fields will be governed by Eqs. (1) which may now be coupled into the following single equation for the scaled velocity $\boldsymbol{\beta}$

$$\frac{d\boldsymbol{\beta}}{dt} = \frac{e}{\gamma mc}\left[\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{E}) - (\boldsymbol{E} + \boldsymbol{\beta}\times\boldsymbol{B})\right]. \quad (23)$$

This equation is equivalent to three coupled differential equations for the velocity components, which qualify for numerical solution using a Runge-Kutta routine. The energy gain is defined as

$$G(t) = mc^2(\gamma - \gamma_0), \quad (24)$$

where γ_0 is the Lorentz factor at $t = 0$. In a typical gain calculation, Eq. (23) will be integrated from time $t = 0$, corresponding to the instant the electron leaves the point in the xz plane with coordinates (x_0, z_0), to a time $t = 10^5\ T$, where T is the laser period. The value of γ at $t = 10^5\ T$ then goes into Eq. (24) for the gain.

Variation of the gain with the initial phase ψ_0 is shown in Fig. 2. Note in Fig. 2(a) that the line corresponding to the fields of order up to ε stands out and underestimates the gain. When the order ε^2 to ε^5 terms are added, four lines show up which are not easily resolved from one another but are markedly different from the line corresponding to the order ε fields. When focusing to a waist radius $w_0 = 7\ \mu$m a similar result (in terms of the general shape and relationship between the lines) obtains in Fig. 2(b). In Fig. 2(c) the beam is focused down to $w_0 = 6\ \mu$m. Here the four lines begin to show obvious discrepancies. Finally, as one focuses down to $w_0 = 5\ \mu$m the lines completely lose resemblance to one another, as can be seen in Fig. 2(d).

As a second attempt at demonstrating the inadequacy of the fields to order ε we show in Fig. 3 electron energy gain and trajectory results pertaining to the parameters of Fig. 2, albeit with $w_0 = 6\ \mu$m and $\psi_0 = 90°$. It is clear that neglecting the terms in the fields of order ε^2 and beyond results in a completely different environment for acceleration. This shows in both the trajectory and gain results. Adding the terms of order ε^3 and higher does not seem to alter the trajectory and gain by much, from the corresponding state of affairs when terms of order up to ε^2 only are retained in describing the fields.

In addition, terms to first order in ε underestimate the fields and, hence, the force seen by the electron along its trajectory. This may be checked by looking at the forward component, F_z, of that force derived from the first of Eqs. (1). Before that is done, though, let us have a look at the y- component of the force experienced by such an electron (injected as in Fig. 1)

$$F_y = -e(E_y + \beta_z B_x - \beta_x B_z). \tag{25}$$

However, while $B_x = 0$ everywhere, E_y and B_z vanish identically at all points on the xz- plane, where $y = 0$. Thus, $F_y = 0$ at all points on the the xz- plane. This leads to the inevitable conclusion that the electron in question will always follow a two-dimensional trajectory confined to the xz- plane. On the other hand, the z- component of the force is ($B_x = 0$)

$$F_z = -e(E_z + \beta_x B_y). \tag{26}$$

Thus, motion of the electron in the xz- plane will always be governed by E_z and B_y alone. In Fig. 3(c) the forward force component (the accelerating force [36]) is shown as a function of the forward distance of travel, z, for the electron whose gain and trajectory are shown in Fig 3(a) and (b), respectively. Clearly, the force acts over a fraction of a millimeter, delivering a huge impulse (or more, in other situations) to the electron. Fig. 3(c) also demonstrates that, when only terms of order ε are used, the force exhibits a large accelerating (positive) part followed by a small decelerating (negative) portion. When terms of order ε^2 and higher are added, the force seen by the electron appears clearly to be accelerating (positive) over the small interaction distance.

4 slippage

Another reason why some models (or some parameter sets used within the context of a particular model) may fail to account for a net energy gain and acceleration is the question regarding the size of the particle velocity compared to the phase velocity of the field. If the phase velocity of the field is to exceed the speed of light, the electron, whose speed can never reach that of light, will slip behind the field, interact with the decelerating part of the wave and consequently end up gaining no energy. However, it will be shown in the present section that the electron may encounter regions with subluminous phase velocities, surf on the field wave and get accelerated. To investigate this issue we go back to the way the field equations were derived. The vector potential from which the fields have been derived may be written as [18, 19]

$$\begin{aligned} \mathbf{A} &\approx \hat{\mathbf{i}} A_0 e^{i(\psi_0+\omega t-kz)} f e^{-f\rho^2} \left\{1+\varepsilon^2\left[\frac{f}{2}-\frac{f^3\rho^4}{4}\right]\right. \\ &\quad \left. +\varepsilon^4\left[\frac{3f^2}{8}-\frac{3f^4\rho^4}{16}-\frac{f^5\rho^6}{8}+\frac{f^6\rho^8}{32}\right]\right\}, \\ &= \hat{\mathbf{i}} A_0 \frac{w_0}{w} e^{-r^2/w^2} e^{i\varphi}; \quad A_0 = \text{constant}, \end{aligned} \tag{27}$$

and where

$$f = \frac{i}{i+z/z_r} = \alpha e^{i\psi_G}; \quad \alpha = [1+(\frac{z}{z_r})^2]^{-1/2}. \tag{28}$$

The phase of the field in Eq. (27) may now be written down as

$$\varphi(x,y,z;t) = \psi_0+\omega t-kz+\psi_G-\frac{kr^2}{2R}+\tan^{-1}\left(\frac{\Theta_i}{\Theta_r}\right), \tag{29}$$

where

$$\begin{aligned} \Theta_i &= \varepsilon^2\left[\frac{\alpha}{2}\sin\psi_G-\frac{\alpha^3\rho^4}{4}\sin(3\psi_G)\right] \\ &\quad +\varepsilon^4\left[\frac{3\alpha^2}{8}\sin(2\psi_G)-\frac{3\alpha^4\rho^4}{16}\sin(4\psi_G)\right. \\ &\quad \left.-\frac{\alpha^5\rho^6}{8}\sin(5\psi_G)+\frac{\alpha^6\rho^8}{32}\sin(6\psi_G)\right], \end{aligned} \tag{30}$$

$$\Theta_r = 1+\Lambda_r, \tag{31}$$

where Λ_r may be obtained from Θ_i simply by replacing the sine terms with the corresponding cosines. For surfaces of constant phase, one has

$$0 = d\varphi/dt = \partial\varphi/\partial t + \boldsymbol{\nabla}\varphi \cdot \mathbf{v}_{ph}, \tag{32}$$

where $\boldsymbol{\nabla}\varphi$ is understood to be evaluated at points on the particle trajectory. One may now adopt the following definition for the phase velocity along the trajectory

$$\beta_{ph} \equiv \frac{k}{|\boldsymbol{\nabla}\varphi|}. \tag{33}$$

This equation holds strictly for a trajectory parallel to the wave propagation direction, and only approximately for other trajectories. Consider now Fig. 3(d) in which β_z and β_{ph} are shown for the particular situation whose trajectory is shown in Fig. 3(b). Obviously, there exists a sizable portion of the trajectory over which the phase velocity is subluminous ($\beta_{ph} < 1$). Most of the energy gain and acceleration take place over that portion. The obvious conclusion from this figure is that the electron travels approximately in phase with the wave and loses little, if at all, of the energy gained as a result of the impulses alluded to above.

This section will now be concluded by presenting Fig. 4. This figure shows results similar to those of Fig. 3, albeit for a laser system of output power 10 PW (also $w_0 = 6\ \mu$m, $\zeta = 15°$, $\psi_0 = 70°$, and plots are shown for terms of order up to ε and ε^5 in the field description). To the best of our knowledge, such power has not yet been achieved in any laboratory. The figure shows that energy gain from a system like that may reach the GeV range. Our conclusion, regarding the need to include higher-order terms in the field description, is only enhanced by the results displayed in Fig. 4. A look at Fig. 4(d) lends similar support to the conclusions arrived at from Fig. 3(d).

5 Concluding remarks

The need to model the fields associated with a continuous tightly-focused laser source of present-day output power, using those of a Gaussian beam of order up to ε^5, has been demonstrated, if such a model is to successfully account for electron acceleration in vacuum.

For a beam of power 1 PW focused to a waist radius $w_0 > 5\ \mu$m, the need to include terms in the fields of order ε^2 and higher has been shown to be essential; the order ε terms alone may not be relied upon. Demonstration has been done by monitoring the energy gained by an electron, that has been injected sideways into the beam focus from a point in the polarization plane of the beam, as the initial phase of the beam is varied. This has been further supported by calculating the gain along the electron trajectory, by looking at the electron trajectory in the polarization plane and by investigating the force seen by such an electron as it samples the field points. It has been shown also that the mechanism of electron acceleration relies on momentum transferred from the field in the form of one or more violent impulses acting over a short distance, typically a fraction of one millimeter. Finally, it has been shown that a region around the beam focus develops over which the phase velocity of the wave becomes subluminous. That is where the electron catches up with the wave and acquires most of its energy gain. All of our investigations have demonstrated that, beyond that region, it simply cruises at about the same acquired speed and with almost unchanged energy over a long distance.

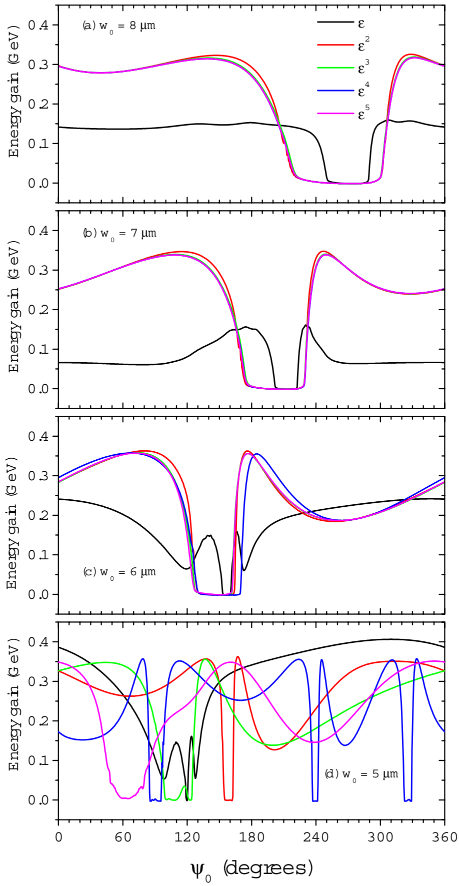

Figure 2: The electron energy gain is shown here as a function of the initial phase parameter ψ_0 for beam-waist radii decreasing from $w_0 = 8\ \mu m$ to $w_0 = 5\ \mu m$. In each figure five plots are shown corresponding to cases where terms in the fields of up to order ε^n, with $n = 1, 2, \cdots, 5$ are included. The following parameter values are common to all plots: $\gamma_0 = 7$, $\zeta = 10^\circ$, $\lambda = 1\ \mu m$, $z_0 = -5\ mm$, and the laser peak power is $P = 1\ PW$.

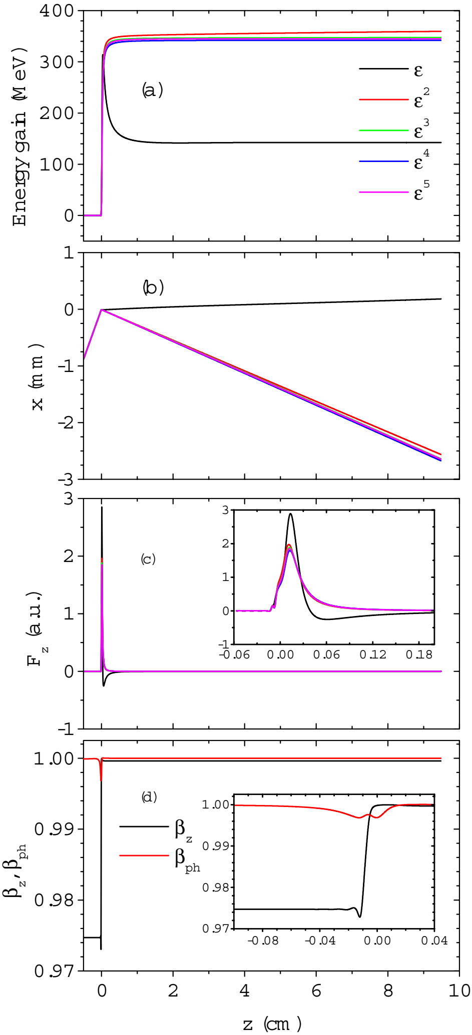

Figure 3: (a) Electron energy gain as a function of the forward distance of travel. (b) Electron trajectory in the polarization plane. (c) Forward component of the force seen by the electron. (d) The scaled electron and phase velocities. The waist radius at focus is $w_0 = 6\ \mu$m, $\psi_0 = 90^\circ$, and all initial conditions and remaining laser parameters are the same as in Fig. 2 above. The legends in (a) apply to (b) and (c) as well.

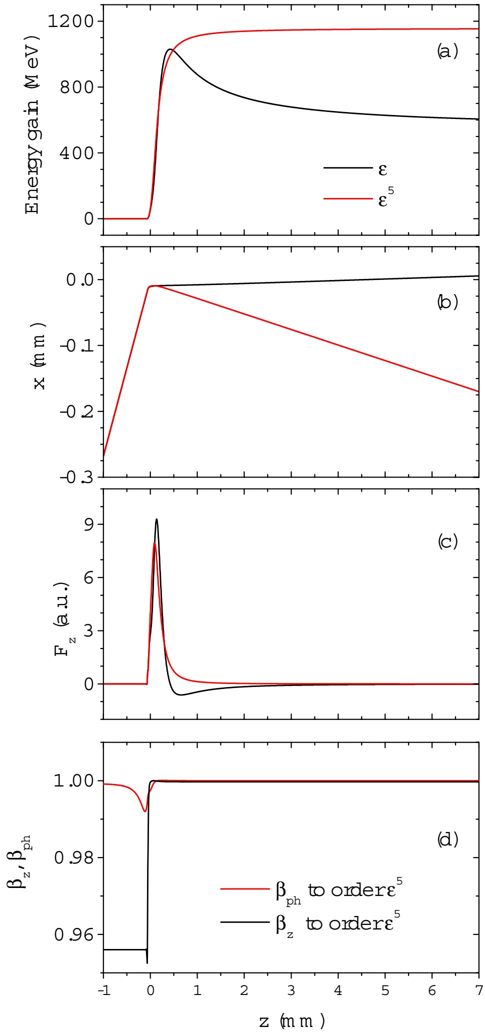

Figure 4: Same as Fig. 3 but for a power output of 10 PW, injection angle $\zeta = 15°$, and an initial phase $\psi_0 = 70°$. Plots are given for fields described by terms of orders up to ε and ε^5 only. The legends in (a) apply to (b) and (c) as well.

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