

“TIME OPERATOR”: THE CHALLENGE PERSISTS

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Abstract

Contrary to the conviction expressed by J. Kijowski [Phys. Rev. A **59**, 897 (1999)] and shared in some other papers, the reasons to look for the ‘time operator’ in the context of the standard quantum doctrine of orthogonal projectors and self-adjoint observables are highly questionable. Some improved solutions in terms of POV measures invite critical discussions as well.

The time of a quantum event is a random variable. The fact inspired a patient quest for the *time operators* [1, 2, 3, 4, 5, 6, 7, 8, 9, 10], originally following the orthodox idea of quantum observables [11, 12], subsequently reformulated in terms of the positive operator valued (POV) measures. In some occasions, when an operator was born with an anatomic defect, it was corrected by a ‘self-adjoint surgery’ and a *final solution* was announced (see also cf. [8]). In his essay of 1999 J. Kijowski [13] closes the cycle by announcing that the final solution was known indeed since 1974 [2]. Strangely, his comment seems like a step back toward the most traditional orthodoxy, comparing even with his 1974 paper. The *observables* are again represented by self-adjoint operators, the *probabilities* given by the spectral measures, etc. The parts of Kijowski’s solution had been adopted by other authors, though were differently composed. In this comment we try to find out whether the Kijowski solution, or some reformulated proposals can be indeed conclusive.

We start with the orthodox trend. The first attempts at constructing the time operator in the traditional form of Dirac and v. Neumann [11, 12] (circumventing the Pauli theorem [18]), pretended to check the universality of the existing formalism. The next attempts, though non-relativistic, might have an additional sense: to keep the space and time variables on equal footing, preparing the ground for the hypothetical space-time quantization [14, 3, 15].

The resulting *time operators* show some basic similarities. For the free particle, they all depart from the classical formula $t = -q/p$ (where q and p are the position and momentum, we put $m = 1$); the differences consist in methods “to make them self-adjoint”. In case of Kijowski [2, 13], the operator is constructed in two steps: (1) the kinetic energy operator $H = p^2/2$ of Schrödinger’s quantum mechanics in 1 space dimension is replaced by the *pseudoenergy*

$$\Xi = \Xi(p) = \text{sgn}(p) \frac{p^2}{2} = \frac{1}{2} p|p| \quad , \quad (1)$$

with continuous eigenvalues $\xi \in \mathfrak{R}$; and (2) the *pseudotime* Θ (with continuous eigenvalues $\theta \in \mathfrak{R}$) is defined as the operator canonically conjugate to Ξ

$$\Theta = -i \frac{\partial}{\partial \xi} \quad . \quad (2)$$

Following the orthodox rules, Kijowski applies the standard axioms originally formulated for the instantaneous measurements [11]. Thus, he uses the spectral projectors $P(\Delta)$ of Θ on the intervals $\Delta \subset \mathfrak{R}$ in a traditional way, to define the probabilities $p_\phi(\Delta) = \langle \phi | P(\Delta) | \phi \rangle$ of the particle arrival in the time intervals Δ for any normalized initial state ϕ (cf. [13], p. 898, formula (5)). Note, that the original approach of [2] was modified in [3, 4], then modernized in [5, 6, 7] and subsequent papers (cf. [10, 16, 17]) in terms of the POV probability measures. Due to its nice self-adjoint form, the Kijowski operator Θ (and its descendants) are close to winning the competition for the best “time of arrival”. However, before comparing the merits of various formal constructions solving the problem, we would like to know *what is precisely the problem which they solve?*

Like all other observables, the *time operator* should represent some measurement. Several types of physical experiments might be intuitively associated with the concept of “time”, and if these intuitions differ, so may the operators. Unfortunately, a realistic background is almost absent in most papers on the time operator, written in almost Aristotelean spirit.

A widely shared idea seems to be that the *time measurement* is performed by a *waiting detector*, programmed to register a certain definite event (e.g., the arrival of a microobject to the sensitive part of the apparatus). One of simplest such devices is a flat, macroscopic screen, producing a sharp, brilliant spark when hit by a microobject. Is this an adequate physical model behind the “time operator”? Not necessarily; some questions are still open and they can make a lot of difference!

In the first place, should the screen register the particles arriving from one or from both sides? A natural scenario would be the one-sided screen [19, 20] (a microparticle approaches to the screen from its sensitive side, to be registered at some moment t). This, of course, involves the assumption that the screen is impenetrable. However, the idea does not seem to prevail (except perhaps Marchewka and Schuss [20]). Anyhow, it is not stated by Kijowski, nor by Grot et al. [3]; neither we find it in majority of papers on the time of arrival. Let us therefore adopt an opposite view: we shall consider a *two sided* screen, which does not discriminate half-space, and is sensitive to particles crossing in both directions.

If so, the next question is: can the particle sneak across the screen undetected? Presumably so (should the screen preclude the tunneling, we would return to the previous option). In fact, the image of particle circulating on both sides of the screen seems implicit in almost all constructions of the “time operators” and POV measures. Enough to look at the proposed time probability distributions (see an interesting review [10]); the authors don’t care to split the packet into two space separated, non interfering components on two sides of the screen, but they care a lot to split it into the positive and negative momentum parts (*right* and *left movers*). This is of course not the same.

The next question is less trivial: is the evolution of the particle before the moment of “arrival” affected by the existence of the screen? The problem seems crucial in the approach of Kijowski and other authors trying to construct the time measurement either in terms of projectors or POV-measures. When reading [2] one might guess that the measuring apparatus is a maximally *non-intrusive* device, which waits inconspicuously, without perturbing the particle until the moment when the particle is detected. So, is it completely transparent? Does the particle obey the free evolution until detected? Such a hypothesis seems to emerge from Kijowski comment (see [13], p. 898 left, 1.7-5 from bottom), though his formulation is a bit enigmatic: “On the other hand”, he writes, “any quantum state (...) undergoes the *standard* ‘chronological evolution’ from time t_1 to t_2 , described by the Schrödinger equation”. However, what does it mean? Does the particle follow an equation which takes into account the influence of the screen? Or does it obey the *free Schrödinger equation* until detected? The last option is visibly privileged in [2], Ch. 5, p. 367, where the probability density on the time axis is constructed out of the *freely evolving* wave packet ϕ_t . The fact is considered so obvious that the formula $A(t) = F(\phi_t)$ on p. 367 is not even included into the list of formally stated axioms. The same idea, apparently, is shared by other authors who develop the POV formalism [6, 7, 10]. Yet, obvious it is not!

If the particle survived without being detected until the time t , then the algorithm for the detection probability $A(\tau)$ at any later moment $\tau \geq t$ should take into account the already achieved state ϕ_t . However, *what is ϕ_t* ? By assuming that it is the output of the free

evolution, Kijowski extrapolates quite radically the classical picture. *The particle moves along the straight line, knowing nothing about the detector, until it makes a direct hit.* The image is suggestive, but it neglects some important facts. A characteristic property of quantum objects is that they diffract: so an obstacle (detector) affects the propagation even if there was no absorption. If we forget this, there is no quantum mechanics, so why to worry about the “time operator”? Yet, this is only the beginning of the troubles.

If the particle propagates in presence of a *waiting detector* (screen, etc.), a new difficulty arises. Let's not forget that in quantum measurement theory, *no news is news*. If the detector shows a visible effect, the state of the microobject is *reduced*. However, if the detector *does not react, although it could*, the state of the microobject is as well reduced (the reduction *by the absence of an effect*; see Dicke [21]). The fact is essential for the well known locality paradoxes (see Elitzur and Vaidman [22]), for the techniques of ‘seeing in the dark’ [23], decisive also for the *time of event* of Blanchard and Jadczyk (BJ). They ask: *“Is the very presence of the counter reflected by the dynamics of the particles that pass the detector without being observed?”* ([24], p. 619, Sec. 2.3; see also [25]). The question, of course, is crucial for the waiting screens. The positive answer to BJ implies that ϕ_t varies not only due to the free motion and/or diffraction on the screen but also due to the progressive reduction process. The fact is considered as well in the quantum information and cryptography (cf. an ingenious observation of the experimental group in Geneva, on the photon state which undergoes a non unitary transformation, just because the photon *might have been absorbed* [26]).

It thus seems, that the axioms about the *time of arrival* omit quite a number of physical aspects. It brings little comfort that they give a unique probability. On the contrary, it brings new difficulties. The screens used in laboratories, quite obviously, must differ by some sensitivity parameters. So, if the probability distribution in [2] is indeed unique, the problem arises, where is the variety of the screen parameters?¹ Trying to answer that, one faces some more questions which

¹An equivalent question was analyzed by Alcock with rather negative results [33]. It was also asked by Delgado and Muga in an opposite, optimistic spirit [4]. Later on, a pertinent remark was made by Muga, Baute, Damborenea and Egusquiza [27]: *Nevertheless, there is a clear divorce between the daily routine*

might completely frustrate the screen scenario behind the Kijowski operator (and behind the related POV measures).

Indeed, to have a unique probability distribution we must have an idealized screen, for which the physical parameters become inessential. The options are not many: **(a)** If the screen permits the particles to tunnel unperceived without any limitations, then the screen is ideal but nonexistent; **(b)** If the tunneling competes with the detection of the “arrivals” then the screen is not ideal and the probability distribution must depend on the tunneling capacity; **(c)** If the screen precludes completely the tunneling, then the screen is indeed perfect. However, in both cases **(b)** and **(c)** the particle evolution cannot be considered free before the detection [19, 24]. We thus face a mystery. What is the physical sense of Kijowski’s probabilities and the related POV measures?

In repeated discussions, several authors point out that the influence of a waiting screen (or other detectors) can be represented by a variant of an optical model with a complex potential localized in the detector volume. For the potential of an adequate structure [28, 29] the evolution of the packet on one side of the screen (or detector boundary) is unaffected by the existence of the detector, justifying the ‘free evolution hypothesis’ in [2, 13]; except that a part of the packet peacefully sinks into the screen surface. According to Muga et al [28], this absorption can be so perfect, that the influence of the complex potential can mimic every detail of the free evolution, including even the unavoidable back currents, though without producing the reflected Fourier components [28, 29]. The gradual reduction of the packet norm outside of the detector would then account for the increasing absorption probability. While the mechanism deserves further study, we doubt that it can confirm the Kijowski’s probability distribution for the two-sided screens.

The trouble is specially visible in the exceptional cases when the

of many laboratories where time-of-flight experiments are performed, and the theoretical studies on the time of arrival, which are based on the particle’s wave function without recourse to extra (apparatus) degrees of freedom. A number of “toy models” have been proposed that include simple couplings between the particle’s motion and other degrees of freedom acting as clocks or stopwatches, but they do not include any irreversibility ... (c.f. p. 1, col. II, l. 24-15 from the bottom), though they no longer return to the problem in the published article [17].

probabilities of arrival should be obvious (at least, according to the common sense). We refer to the case of *odd, normalized* packets $\phi(x, 0) \equiv -\phi(-x, 0)$ (cf. Muga and Leavens [10], Sec. 8.8, p. 404). If the screen (detector) placed at $x = 0$ is represented by an absorbing complex potential $V(x) = V(-x)$ (with a narrow support, centered at $x = 0$), the packet must remain odd for all $t > 0$, granting that ϕ_t vanishes forever at $x = 0$ due to the destructive interference, and so does the probability current [10]. According to the known statistical interpretation, it means that the absorption probability should vanish. Looking for more details, Leavens had shown [30] that the Bohmian particle circulating in such packets never arrives at $x = 0$. The same conclusion would hold in Nelson’s stochastic quantum mechanics [31] (the stochastic particle is repelled from the nodal points of ϕ_t [31, 32]). However, the conclusion does not indeed require the Bohm’s or Nelson’s theories. Through the entire history of quantum mechanics, the destructive interference was decisive to determine the probabilities on the screens. This might change if the screen (detector) precludes the free evolution (the possibility neglected in [13, 3, 4, 6, 7, 10, 16, 27, 17, 28, 29, 2]). Even so, the activated screen will cut the communication between the packet components at $x < 0$ and $x > 0$. This is not the same as to separate negative and positive momenta (which would be a nonlocal maneuver).

Note, that the trouble is not limited to the odd $\phi(x, t)$. To illustrate this, one can choose as well the symmetric, normalized packet $\phi(x, 0) = \phi(-x, 0)$, which remains symmetric and normalizable for $t > 0$ (Fig. 1). Once again, there is no need to worry about the free evolution currents, since there are no such currents at $x = 0$. If some particles cross to the right and some other to the left there must be also particles in the superposed states of crossing in both directions. Were the evolution of the packet indeed free, then its both parts (for $x < 0$ and $x > 0$) would conserve their norms (no absorption!) However, should the absorption occur (e.g. due to the two sided analogue of the sinking mechanism described in [28]), the packet norm would be decreasing. A question arises: can this happen in such a way that the freely evolving packet is just multiplied by an attenuating c -number factor $\lambda(t)$, $|\lambda(t)| < 1$, without losing its shape, so that the renormalized packet $\lambda(t)^{-1}\phi(x, t)$ would again evolve freely? This turns impossible, since then the packet propa-

gation would be affected in a non-local way, by adding to the free Schrödinger's Hamiltonian $H = p^2/2$ an imaginary time dependent constant (independent of x) instead of the localized screen potential $V(x)$. It means that the vicinity of the screen must cause an essential change of the packet shape, visible even after renormalizing ϕ_t : so, the packet evolution cannot be classified as “free” [29]. The same argument shows that the proximity of a one sided, perfectly absorbing screen must as well deform the packets.

It thus looks that the complex models with imaginary peaks (or barriers) cannot reproduce Kijowski's nor other distributions derived axiomatically from the free evolution law.

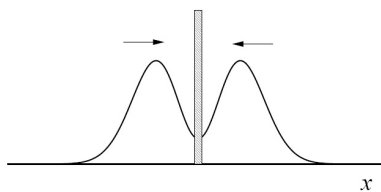


Figure 1: A symmetric wave packet on both sides of the waiting screen. Either the screen is transparent, or the shape of the packet must be affected by the existence of the screen even if there was no detection.

Strangely enough, a chance to rescue some operational aspects of [2, 13, 3, 4] might lie in an indifferent direction, i.e., close to the option **(a)**. Perhaps, the acts of *crossing the screen* by the microparticle should be understood as idealized events, developing in some virtual (Platonic) reality, without the need of physical observation (cf. also [10], Sec. 8.8, p.407). The possibility of detecting such unspoiled events would occur in an asymptotic limit, not for strong, but inversely, for very “weak detectors”, for tiny screens, with almost negligible chances of registering anything. In such situation, the *arrival* would not be a synonym of the *detection*. The screen would seldom react, leaving the free evolution practically unperturbed. Yet, should the (weak) detection attempts be patiently repeated for ample sequences of initially identical wave packets, the (conditional) proba-

bility distribution on the time axis would start to emerge. Assuming that the particle will be at all detected, it would tell which detection moments are more and which are less probable. In this *scenario* the “crossing states” [4, 5, 7, 6, 8, 16], would be a kind of asymptotic states for an extremely weak screen-particle coupling (though anchored at the finite time moments $t \in \mathbb{R}$). The results concerning the time of arrival presented in [2, 3] would be indeed the first order approximation for the action of the detector in the *interaction picture* - where the term *interaction* refers to the *weak* particle-detector coupling. (Notice that this is merely a hypothesis, though consistent with some elements which appear in Muga et al.; see [29], p. 253, formula (A.8)). The influence of the stronger, physically realistic detectors (or screens) on the evolution of the undetected particles would correspond to the next approximation steps. The original Kijowski formulae could hold approximately (in a weak coupling limit) if the packet with positive momenta arrives to the screen from the far left, or else, a packet with negative momenta comes from the far right; cf. [4] (the problem is, of course, how weak can be the particle-screen coupling before we arrive at the quantum description of the measuring devices, with all immanent paradoxes)? In spite of all doubts, the idea looks almost as a unique chance to give some operational sense to the known “time of arrival” distributions. However, it also reveals a gap in the scriptures.

If two packets, composed of the “right” and “left movers” arrive simultaneously at the screen and are not coherent, the probabilities can be simply added, justifying the well known POV measures [10]. If however they are two *coherent parts* of a single packet on both sides of the detector, then it is not at all obvious how the weak coupling with the screen can destroy the coherence (see again Leavens [30], p. 844, col. II, above (31)²). It is even less obvious why a “soft screen” should destroy the interference precisely between the *right* and *left movers* (i.e, between the Fourier components). Both assumptions are arbitrary, moreover, they are not identical. The tacit conviction

²Commenting on Grot et al. [3], Leavens writes: “*For the theory based on Bohmian mechanics, interference between the two time-evolved components of the pure state (19) has a dramatic effect on the distribution of arrival times*”(…) *I find it a cause for concern that this important ingredient of the theory apparently emerges as a consequence of the regulation.* Indeed, it is difficult to disagree, even though the Bohm’s theory is lateral to the problem.

about their equivalence, exaggerates the role of the ‘Fourier thinking’ in the problem involving the space localization.

Quite evidently, the question about the “time of arrival” for a coherent, finite norm packet extending on both side of the screen was never solved. As it seems, the very concept of a *perfect screen* bears an intrinsic *antinomy*. Either the screen is impenetrable (“strong”); then it affects the packets even before the detection. Or the screen (detector) is extremely delicate (“soft”); then all operators and POV measures based on the “interference ban” are misleading. They were a useful intent, but the story is not yet written.

* * *

While the future of the subject is unknown, it becomes clear, that all intents to obtain the *time observable* in the orthodox form of a self-adjoint operator (in spite of the best stratagems to avoid the Pauli theorem [18]) lead to a blind alley. The resulting operators are typically plagued by some little but persistent difficulties which might look accidental; besides they all suffer some basic defect which seems common for the whole family. As to the Kijowski proposal of 1999 [13], in the first place, it is handicapped by an artificial form of Θ , the continuous spectrum operator, representing the *time* no better than $\Xi(x) = \text{sgn}(x)x^2/2$ imitates the harmonic oscillator! Observe, that already the classical limit of Θ :

$$\Theta = -\frac{z}{|p_z|} \tag{3}$$

gives a false time value for 50% of classical trajectories.

To see what happens if one takes “to the letter” its quantum version (2) (see [13]), we have simulated the evolution of an odd wave packet, vanishing all the time at $x = 0$. The corresponding Kijowski probabilities do not seem to reflect the destructive interference at 0. Obeying rather the superposition ban between the “right” and “left” movers, they produce two sharp peaks on the pseudo time axis (Fig. 2). As also becomes obvious, the probability formula (5) in [13] evolves covariantly with respect to $\exp(-it\Xi)$ but not with respect to $\exp(-itH)$ (once again, $\Xi(p)$ represents H no better than $\Xi(x)$ approximates the harmonic oscillator!).

“Time operator”: the challenge persists

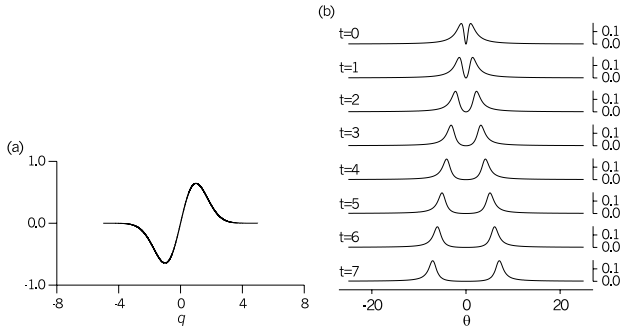


Figure 2: The evolution of an odd packet seen on the θ axis. a) The initial packet in the q representation. b) The Kijowski probability on the θ axis forms two sharp peaks, ignoring the destructive interference. The peaks subsequently separate, ignoring the covariance. The probability distribution was corrected by POVM of Delgado and Muga, but the destructive interference was not recovered.

All this seems to support Allcock [33] and Oppenheim et al. [8] rather than the self-adjoint schools of Dirac & v. Neumann [11, 12]. We conclude that, in spite of its great inspiration, Kijowski’s solution is too biased by the orthodox doctrine. The attempts of Grot et al. [3] do not look more convincing. As it seems, the entire trend pays a price for excessive idealizations: (1) for the lack of a physical scenario, (2) for overestimating the role of the Fourier transforms in problems involving space and time localizations, and (3) for introducing the probabilities which are valid only when the experiment is never performed.

Even if putting these questions aside, one faces an independent difficulty, generic for all selfadjoint formalisms [11, 12]. The trouble consists in the existence of “absolute certainty states”. The fact, quite normal for the traditional (instantaneous) observables, here leads to unphysical conclusions. We shall discuss them taking *bona fide* the free evolution background of Kijowski model.

Indeed, suppose, there exists a self-adjoint “time operator” \hat{t} obeying the orthodox interpretations [11, 12]; so that the probability of the particle arrival in any time interval $\Delta = [t_0, t_1]$ is given by the

spectral projector $P(\Delta)$ [13], whose eigenstates imply the certainty. If so, the orthogonal projectors $P(\Delta)$ could be used to *assure the arrival* or to *grant the absence*. We claim that this is a completely unrealistic conclusion. In fact, let $\phi_0 \in \mathcal{H}$, $P(\Delta)\phi_0 = 0$, be an initial state at $t = 0$, granting that the microobject *will certainly not arrive* in an immediate future, i.e., in $\Delta = [0, t_1]$. Yet, if the particle obeys the free evolution equation for $t \in \Delta$ (as suggested in [2]) then, with rare exceptions [30, 34], ϕ_t will immediately develop a non vanishing current penetrating to the detector at $x = 0$. We find it entirely impossible to believe that this will traduce itself *exclusively* into the tunneling effect, while the probability of the particle detection will remain *exactly zero* in the entire interval Δ (unless, of course, the detector is completely blind!). We henceforth consider the absolute certainty states unphysical. The difficulty had been foreseen by Aharonov et al. [35], in form of an atypical time-energy uncertainty relation $\Delta t > 1/E$ (where E is the initial kinetic energy), implying that the “certainty” offered by the spectral projectors is illusory.

To clarify these doubts, we have carried out some numerical tests. For simplicity, we consider the particle in 1-space dimension. Our detector is a “sensitive eye” placed at $x = 0$. The detector can be switched on and off at will (eye open/closed) in any desired time interval. Since the correct theory must resist unfriendly tests, we have chosen our initial wave function ϕ_0 to be just a step function in θ -representation, vanishing outside of a narrow interval $[\theta_1 = 2, \theta_2 = 2.00001]$. According to [13] it should give an absolute certainty that the particle *will not be registered* for $t \leq 2$, and that it *must arrive* in $2 < t < 2.00001$. The sequence of graphics on Fig. 3 represents the free evolution of our wave function in the position representation as t approaches $t_1 = 2$. As can be seen, around half of the packet is far away from the detector for t close to 2, contradicting the “absolute certainty” of the particle arrival between $t_1 = 2$ and $t_2 = 2.00001$ (cf. also [8]). The other problem is even deeper. While approaching the time moment $t_1 = 2$ from below, ϕ_t develops an increasing tail around $x = 0$. Should the window of awareness be e.g. $[1.99, 2.00]$, it is hard to believe in the absolute impossibility of the particle detection in this window, i.e. *before* the time allowed by Kijowski projector.

Note, that the situation would not change for any different self

adjoint, positively defined Hamiltonians [36, 37], since our graphics illustrate simply the general *no go theorem* of Hegerfeldt [38, 34, 39, 40, 41].

In spite of all objections, an interesting expression of these facts are the “states of arrival” $|\pm, t\rangle$ (also *crossing states*), studied in [4, 42, 6, 7, 10, 17, 5, 43, 8, 44, 39, 40, 41], forming an overcomplete, nonorthogonal basis. Note that the mappings $t \rightarrow |\pm, t\rangle$ establish indeed a *fuzzy structure* on the real time axis. In the recent research, such structures are often defined by mapping the points of classical differential manifolds into the families of non-orthogonal *coherent states* [45, 46] which form the overlapping, *fuzzy images* of the originally distinguishable points (for a different approach, see [47, 48, 49, 50]). The sense of the “crossing states” seems analogous. Each falling particle watches the time axis; it must chose a moment t to hit the screen surface. However, instead of the continuum of sharply defined time points, the particle ‘sees’ a family of fuzzy events in form of the *crossing states* $|+, t\rangle$ and/or $|- , t\rangle$ with nontrivial overlaps. As a result of this *fuzzy vision*, the particles cannot be instructed to hit the screen in sharply defined time intervals: i.e., the ensemble preparation does not admit the *absolute certainty states*. The same phenomenon seems generic for the time localizations of other quantum events.

Quite obviously, the waiting detectors form a new class of measurements, which have rather little to do with the traditional Dirac-v.Neumann observables. Though all this concerns a non-relativistic theory, it might mean a warning for the ‘Euclidean’ space-time quantizations [15, 51] where the space and time localizations receive an equal status (see also the discussions by León [42] and Giannitrapani [43]).

Note, that these negative conclusions might also imply some good news. Indeed, imagine a simple experiment which consists in registering the time moment in which an unstable particle decays. The result is a real number (the *decay time*); yet, an attempt to describe it in frames of the orthodox scheme (i.e., as an eigenvalue of a self-adjoint *time operator*) would lead to a wrong conclusion about the existence of initial states for which the moment of the decay can be predicted with certainty! Should this be true, the consequences could be quite dramatic. They would include a suspense story about a suitcase full of

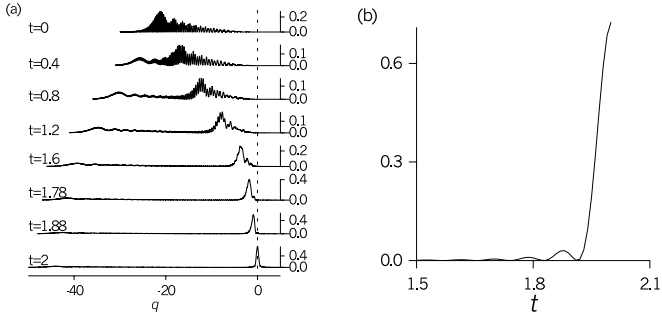


Figure 3: (a) The position representations for the wave packet initially forming a sharp step on the θ axis in the narrow interval $[2, 2.00001]$. For $t \rightarrow 2$ more than $1/2$ of the packet is far from 0. Moreover the packet develops a visible tail at $x = 0$. b) The tail absolute values at $x = 0.003$ (behind the detector) for $t < 2$, $t \rightarrow 2$. We refuse to believe that the phenomenon will translate itself only into the packet tunneling without absorption.

radioactive atoms smuggled safely through the custom control, with all atoms programmed to decay tomorrow! So, perhaps, we should not regret that the *time of events* does not obey the orthodox axioms of Dirac and v. Neumann?

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Comment on “TIME OPERATOR”: THE CHALLENGE PERSISTS

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Abstract

The criticism formulated by Mielnik and Torres-Vega in their paper, does not apply to my derivation of the *time of arrival* in Quantum Mechanics but to our present understanding of the quantum measurement theory.

In paper [1] I have expressed the following opinion:

Although, there is no room to modify the mathematically unique definition of the probability $Q_{I \times D}(\psi)$ within the standard mathematical framework of Quantum Mechanics (spectral measures, self-adjoint operators etc.), the problem is still open from the physical point of view. Indeed, the probabilistic interpretation of the quantum mechanics is based on “short” measurements of the type “What is a probability that the particle will be found within the three-dimensional volume V precisely at a given time instant t ?” How to relate them with “long” measurements, corresponding to time intervals rather than to time instants t is by no means obvious.

Bogdan Mielnik and Gabino Torres-Vega propose a deep analysis of conceptual problems which arise when one starts to inter-

pret “long” measurements in terms of the conventional framework of Quantum Mechanics. To strengthen the dramatic effects, they decided to introduce a “scapegoat” as the main *dramatis persona*. I am deeply touched by the fact, that this role has been assigned to my humble self. The prize I am proposed to get for this job is indeed high: I am allowed to consider myself, together with P. A. M. Dirac and J. von Neumann, a member of the “gang of three” – those who petrified the present “hard orthodoxy” in Quantum Mechanics, oppressing free spirits, like, e. g. , Bogdan Mielnik.

The idea to become a member of an exclusive society, together with Dirac and von Neumann is, of course, delightful. But the entire deal is unfair and I do not like it at all. During the last 30 years, since I have published my result for the first time in 1974, we have had many discussions with Bogdan Mielnik. He knows very well, that I never considered my result as a physically satisfactory solution of the problem and the *passus* from my paper which opens this letter proves it undoubtedly. My construction of “arrival time” is indeed *mathematically unique and final* within the conceptual framework of the standard interpretation of Quantum Mechanics. But I always considered it as an argument for further analysis of the conceptual framework of the theory. The physical arguments given by Mielnik and Torres-Vega against this construction are just arguments against Quantum Mechanics in its present shape. Exactly *the same* arguments may be formulated against, e. g., Newton-Wigner position operator in relativistic quantum mechanics, which is again unique in the same mathematical framework (as proved by J. Kijowski and G. Rudolph, see [2]). Here, the list of physical problems is equally long. For example: if a quantum state is localized within a tiny space region in one reference frame, it is generically spread over the entire space in another reference frame. What seems to be a friendly “instantaneous measurement” in one reference frame, becomes a nasty “waiting detector” in another one.

Insisting on the self-adjointness of operators is not a political repression used by those nasty (orthodox!) mathematicians in order to restrict somebody’s freedom of expression. A self-adjoint operator implements the very notion of a simplest *quantum observable*: an increasing family of projectors $E(\lambda)$ labelled by possible values λ of the measurement. Unfortunately, at the moment there is no measure-

ment theory, which could replace this (naive and very unsatisfactory!) picture. I wish Bogdan Mielnik to find one.

If we want to improve physical predictions of quantum mechanics (i. e. that the atom in 50-th Rydberg state will remain *for ever* in this state) by taking into account radiation phenomena, we must replace quantum mechanics by the quantum field theory. But here, the situation is even worse. What is good, is that quantum observables are local and they fulfill local field equations. But the very nature of a quantum state is *highly non-local* not only with respect to space (this problem is common with quantum mechanics), but also with respect to time: in general, no observable may be assigned to a *sharp* instant of time, but must be smeared over a time interval. Thus, we are unable to give any rigorous meaning even to “instantaneous measurements” (cf. [3] and [4]).

The analysis of the problem, done by Mielnik and Torres-Vega is, in my opinion, very interesting. On the other hand, I do not accept the role of a scapegoat. Some time ago V. Delgado, J. G. Muga (see [5]) attacked me for being not sufficiently orthodox:

Our result turns out to be similar to those previously obtained by Kijowski. However, the approach by Kijowski was based on the definition of a non-conventional wave function . . . whose relation to the conventional wave function is unclear.

A similar critique was expressed by N. Grot, C. Rovelli and R. S. Tate (see [6]).

Now, Mielnik and Torres-Vega criticize my “hard orthodoxy”. I consider the above ideological attacks from contradictory directions as fully unjustified: in paper [7] I have just proved a mathematical theorem and nobody has questioned this result.

At this point one should notice that replacing spectral measures $E(\lambda)$ by positive operator valued measures (POV measures, see [8]) does not change essential features of the “orthodox” measurement theory but allows for a slightly more sophisticated experimental setting, where even the physical interaction which we use for the measurement depends upon the range of λ we are investigating. Within this framework R. Brunetti and K. Fredenhagen were able to prove the time-energy uncertainty principle (and even more!) without any specific realization of the “arrival time” observable. But again, this is a nice mathematical result, which does not answer any of physical

questions discussed by Mielnik and Torres-Vega.

Finally, let me point out just one misunderstanding: the observable Θ , so much ridiculed by the authors (see their formula (3) and the text below), gives the *correct time value* on 100% classical observables, provided we measure it for the “right-movers” and the “left-movers” separately, which we always do in the experiment.

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