The Power of Test of Jennrich Statistic with Robust Methods in Testing the Equality of Correlation Matrices

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Abstract Jennrich statistic is a method that can be used to test the equality of 2 or more independent correlation matrices. However, Jennrich statistic begins to be problematic when there is presence of outliers that could lead to invalid results. When exiting outliers in data, Jennrich statistic implications will affect Type I errors and will reduce the power of test. To overcome the presence of outliers, this study suggests the use of robust methods as an alternative method and therefore, it will integrate the estimator into Jennrich statistic. Thus, it can improve the testing performance of correlation matrix hypotheses in relation to outlier problems. Therefore, this study proposes 3 statistical tests, namely *J_s*-statistic, *J_m*-statistic, and *J_{mad}*-statistic that can be used to test the equation of 2 or more correlation matrices. The performance of the proposed method is assessed using the power of test. The results show that *J_m*-statistic and *J_{mad}*-statistic can overcome outlier problems into Jennrich statistic in testing the correlation matrix hypothesis. *J_{mad}*-statistic is also superior in testing the correlation matrix hypothesis for different sample sizes, especially those involving 10% outliers.

Keywords Correlation, Jennrich Statistic, Robust Correlation Methods, Outlier

1. Introduction

In statistical analysis, testing several correlation matrices play a vital role to measure the linear relationship

between different set of groups. This estimation of parameter correlation matrices from samples serves a particularly important role in high-dimensional data analysis, including the principal component analysis [1], [2], discriminant analysis [3], clustering analysis [4], and regression analysis [5], [6]. The importance of equality of several correlation matrices has been shown in many areas such as economics, financial market, medicine, and social science [7].

Generally, testing the equality of correlation matrices have been proposed and studied by several researchers. The early development in testing the correlation matrices methods can be found in many literature, for example Box (1949) [8], Kullback (1967) [9], Aitkin et al. (1968) [10], Aitkin (1969) [11], and Jennrich (1970) [12]. Jennrich statistic is a method for testing the equality of 2 or more independent samples correlation matrices [12]–[14]. This test is constructed based on the likelihood ratio test proposed by Kullback (1967) [9] and it is applied to a sequence of independent samples of correlation matrix based on samples from multivariate normal distribution [15]. This statistic also has much better computational and distributional properties [16].

Jennrich statistic, on the other hand, relies heavily on Pearson correlations in calculating test statistical values. Although Pearson correlation is the most frequently used method to determine the strength of relationships between variables because of its superior performance, especially when data follow a normal distribution with linear relationships, these coefficients have problems when data sets contain outliers [13], [17]. Statistical results will be

poor if outliers are present. The presence of these outliers resulted in damaging effects for statistical analysis with increase of variance error and decrease of statistical test power [17]–[19]. If there are outliers in the data, the regression analysis will result in a divergence from normality, making the forecast value invalid [20]. As a result, Jennrich statistic is very sensitive to outliers, which can lead to erroneous conclusions. Jennrich statistic implications will affect type I errors and reduce the power of test. A robust method can be used as an alternative to reducing the influence of outliers. By using the robust estimator, classical estimator can be replaced to reduce the influence of outliers [21]. Studies initiated by Huber related to robust methods have been successfully applied to statistics especially in multivariate analysis [22]. More importantly, this method performs well with data drawn from non-normal distribution and could give approximately the same results as the classical method even when there is absence or presence of small proportion of outliers in data [23].

The hypothesis of 2 correlation matrices will be tested using traditional Jennrich statistics. However, this statistic is highly sensitive to the outliers. Therefore, this study will integrate robust correlation to overcome outlier problems into Jennrich statistic in testing correlation matrix hypothesis. In addition, this study also focuses on developing robust Jennrich statistic. Then, the performance of Jennrich statistic by integrating the robust correlation coefficient in the presence of outliers will be evaluated using the power of test with simulation study.

2. Methodology

This study proposes 3 new integration methods of robust correlation into Jennrich statistic to overcome outlier problems. These 3 robust measures of correlation are Spearman's rank correlation coefficient, median correlation coefficient, and median absolute deviation correlation coefficient. The aim of this research is to integrate robust correlation into Jennrich statistic that can improve the performance of hypothesis testing of correlation matrices in relations to outlier problems. Therefore, this study focuses on developing robust Jennrich statistic. The performance of the developed robust Jennrich statistic is evaluated using power of test.

2.1. Jennrich Statistic

Jennrich statistic is used to test the equality of 2 independent samples of correlation matrices. Consider the null hypothesis $H_0: P_1 = P_2$ versus alternative hypothesis $H_1: P_1 \neq P_2$ where P_1 and P_2 are the correlation matrices. To test this hypothesis, Jennrich proposes to use this following statistic [12],

$$J = \frac{1}{2} \text{tr}(Z^2) - (Z_{dg})^t W^{-1} Z_{dg} \quad (1)$$

where

$$Z = \bar{R}^{-1}(R_1 - R_2) \sqrt{\frac{n_1 n_2}{n_1 + n_2}} \quad (2)$$

R_1 and R_2 represent sample correlation matrices with n_1 observations and n_2 observations, respectively

$$\bar{R} = \frac{n_1 R_1 + n_2 R_2}{n_1 + n_2} \text{ is the pooled correlation matrix} \quad (3)$$

$W = I_p + R_p * R_p^{-1}$ is Hadamard product of 2 matrices (4)

I_p is the identity matrix of size $(p \times p)$

Jennrich (1970) shows that the statistical test, J is asymptotically χ^2 distributed with degree of freedom $k = p(p-1)/2$ where p is dimension of the correlation matrix [12]. Therefore, the null hypothesis $H_0: P_1 = P_2$ is rejected at level of significance, α if J exceeds $\chi_{\alpha, k}^2$, the $(1-\alpha)$ -th quantile of χ^2 distribution.

2.2. Correlation Method

The Pearson correlation coefficients are commonly employed in parametric tests to assess the strength of a two-factor relationship between variables. For the nonparametric test, the Spearman rank correlation coefficient and the Kendall Tau correlation coefficient were employed to assess the strength of the relationship between the 2 variables. A robust correlation methodology can be used to compute the correlation between 2 variables as an alternate way, especially involving outlier data. Under deviation from the assumed distribution model, a strong correlation method can ensure that the statistical stability of the conclusion is high [24], [25]. Zaini and Sharif (2020) [26] compared to the performance of 5 types of correlation, namely Pearson correlation, Spearman correlation, Kendall Tau correlation, median correlation, and median absolute deviation correlation. The findings demonstrated that the correlation of the median absolute deviation has a strong relationship compared to other methods when there are large outliers in data. Therefore, based on the findings by Zaini and Sharif (2020) [26], this study will use some of these correlations to integrate into Jennrich statistic.

2.3. Proposed Method

Generally, we embed 3 correlation methods in Jennrich statistic as follows:

1. Develop Spearman Jennrich Statistic by embedding the Spearman correlation in Jennrich statistic, namely J_S -statistic:

$$J_S = \frac{1}{2} \text{tr}(Zs^2) - (Zs_{dg})^t W^{-1} Zs_{dg} \quad (5)$$

where

$$Zs = R_{S_p}^{-1} (R_{S_1} - R_{S_2}) \sqrt{\frac{n_1 n_2}{n_1 + n_2}} \quad (6)$$

R_{S_1} and R_{S_2} represent sample correlation matrices with n_1 observations and n_2 observations, respectively based on

Spearman correlation.

2. Develop median Jennrich Statistic by embedding median correlation method in Jennrich statistic, namely Jm -statistic:

$$Jm = \frac{1}{2} \text{tr}(Zm^2) - (Zm_{dg})^t W^{-1} Zm_{dg} \quad (7)$$

where

$$Zm = Rm_p^{-1} (Rm_1 - Rm_2) \sqrt{\frac{n_1 n_2}{n_1 + n_2}} \quad (8)$$

Rm_1 and Rm_2 represent sample correlation matrices with n_1 observations and n_2 observations, respectively based on median correlation.

3. Develop median absolute deviation Jennrich Statistic by embedding median absolute deviation correlation method in Jennrich statistic, namely $Jmad$ -statistic:

$$Jmad = \frac{1}{2} \text{tr}(Zmad^2) - (Zmad_{dg})^t W^{-1} Zmad_{dg} \quad (9)$$

where

$$Zmad = Rmad_p^{-1} (Rmad_1 - Rmad_2) \sqrt{\frac{n_1 n_2}{n_1 + n_2}} \quad (10)$$

$Rmad_1$ and $Rmad_2$ represent sample correlation matrices with n_1 observations and n_2 observations, respectively based on median absolute deviation correlation.

2.4. Simulation Data

We used simulation data to obtain p -value of the Jennrich statistic and proposed methods. We used simulation study by using R programming version 3.4.2. The simulation study is executed based on 10,000 replications to obtain an accuracy of a power approximation. For that purpose, we;

1. generate 2 n random data from p -variate normal distribution with a mean vector of $\mathbf{0}$ and covariance matrix, $\Sigma = 1$,
2. calculate the statistical test value of proposed methods,
3. replicate the simulation in step (1) and (2) for 10,000 times,
4. identify the critical value by determining the cut of value of 95% percentiles of proposed methods,
5. generate another n random data from p -variate normal

distribution with mean vector $\mathbf{0}$ and covariance matrix, $\Sigma = 0.1, 0.2, 0.3, \dots$, until 0.9,

6. calculate the statistical test value of propose method,
7. replicate the simulation in steps (5) and (6) for 10,000, and
8. calculate the power of test.

In each data set, we consider different number of variables, p with $p = 2, 5, 10, 20, 30$, and 50 as well as different size of samples with $n = 20, 30, 50$, and 100. For all pairs of variables and sample sizes, this study uses the same correlation shift, r of $r = 0, 0.1, 0.2, 0.3 \dots 0.9$.

2.5. Power of Test

The procedure is continued by comparing the power of test between the classical and the proposed methods. The concept of power is used to make a comparison between different statistical testing procedures, where the most powerful test will have a higher number of rejections of the null hypothesis [27]. Generally, the power of test is known as $1 - \beta$, where β is the Type II error probability. This power of test indicates the probability of rejecting a null hypothesis when it's false, or the probability of not making a type II error. Thus, the power of test is the ability of a test to correctly reject the null hypothesis. Yue et al. (2002) [28] estimated the power of test as:

$$\text{Power of Test} = \frac{R_n}{B}, \quad (11)$$

where

R_n is the number of experiments that fall in the rejection region, and

B is the total number of experiments

3. Results and Discussion

3.1. Performance Evaluation Based on Power of Test

Table 1 displays the power of test for comparing correlation using classical Jennrich statistic with proposed method; J_s , J_m , and $Jmad$ -statistic, different of sample sizes, ($n = 10, 20, 30, 50$, and 100), and different level of correlation, ($r = 0.1, 0.2 \dots 0.9$).

Table 1. Power of Test for Different Methods and Different Correlation Shifts

Method	n	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Jennrich Statistic	10	0.0076	0.0134	0.0298	0.0565	0.1034	0.176	0.286	0.5198	0.7883
	20	0.0111	0.025	0.071	0.1618	0.3154	0.5165	0.7721	0.9517	0.9987
	30	0.0115	0.0492	0.114	0.282	0.5252	0.7983	0.9536	0.9969	1
	50	0.0188	0.0844	0.2586	0.555	0.8394	0.9761	0.9986	1	1
	100	0.0392	0.2242	0.609	0.9155	0.9962	1	1	1	1
Spearman Jennrich Statistic, J_s	10	0.0113	0.0172	0.028	0.0523	0.0891	0.158	0.2689	0.4038	0.6496
	20	0.0129	0.0311	0.0726	0.1448	0.2603	0.4579	0.6997	0.8841	0.9861
	30	0.0171	0.0489	0.1206	0.2631	0.4626	0.7166	0.9046	0.9839	0.9998
	50	0.0244	0.088	0.2374	0.5048	0.7635	0.9456	0.9926	0.9999	1
	100	0.0386	0.2045	0.5479	0.8546	0.9862	0.9993	1	1	1
Median Jennrich Statistic, J_m	10	0.0356	0.0388	0.0452	0.0524	0.0697	0.0961	0.1291	0.2006	0.3333
	20	0.0342	0.0453	0.0637	0.0953	0.1294	0.1816	0.2898	0.4303	0.6857
	30	0.0352	0.0524	0.0742	0.1219	0.1949	0.3104	0.4662	0.6703	0.89
	50	0.0365	0.0709	0.1194	0.2117	0.3523	0.5288	0.7218	0.8934	0.9901
	100	0.0509	0.11	0.239	0.4425	0.6419	0.8478	0.9648	0.9982	1
Median Absolute Deviation Jennrich Statistic, J_{mad}	10	0.0389	0.0353	0.0456	0.0507	0.0713	0.0833	0.1295	0.17	0.2797
	20	0.0357	0.0483	0.0564	0.0838	0.1274	0.1966	0.2843	0.3985	0.6457
	30	0.041	0.054	0.0789	0.1286	0.1925	0.3004	0.4513	0.6464	0.8933
	50	0.0434	0.0689	0.1219	0.204	0.3397	0.527	0.7117	0.8943	0.9898
	100	0.0518	0.1021	0.2321	0.4048	0.6526	0.8474	0.9605	0.998	1

Based on Table 1, we found that,

- for each method, the power value will increase if the sample size increases and the correlation shift also increases.
- the power value of classical Jennrich statistic is always greater than other statistics when the correlation is 0.4 and above for different sample size.
- the power value of J_{mad} -statistics is better compared to others when the correlation is low ($r = 0.1$) for different sample size.
- for small sample size, $n = 10$, classical Jennrich statistics becomes powerful when the correlation is 0.8. Meanwhile J_s -statistic become powerful when $r = 0.9$. For J_m -statistic and J_{mad} -statistic, both statistics are not powerful for all correlation shifts.
- for small sample size $n = 20$, classical Jennrich statistics become powerful when the correlation is 0.6. Meanwhile J_s -statistic become powerful when $r = 0.7$, J_m -statistic and J_{mad} -statistic become power when $r = 0.9$.
- for sample size $n = 30$, classical Jennrich statistics become powerful when the correlation is 0.5, J_s -statistic become powerful when $r = 0.6$, J_m -statistic become power when $r = 0.8$ and J_{mad} -statistic become powerful when $r = 0.8$.
- for large sample size $n = 50$, classical Jennrich statistics and J_s -statistic become powerful when the correlation is 0.4. Meanwhile J_m -statistic and J_{mad} -statistic become powerful when $r = 0.6$.

- for large sample size $n = 100$, classical Jennrich statistics and J_s -statistic become powerful when the correlation is 0.3, meanwhile J_m -statistic and J_{mad} -statistic become powerful when $r = 0.5$.

3.2. Comparison Power of Test for Jennrich Statistics Using Different Percentage of Outlier

Previously, we compared classical Jennrich statistic with the proposed method; J_s , J_m , and J_{mad} -statistic without the presence of outliers. Next, 2 data sets are formed where the first data set does not contain an outlier, while the second data set is formed by entering different percentages stating 10%, 20%, 30%, 40%, and 50% of sample sizes. Then, we calculated the power of test of the hypothesis test to test the equality of 2 independent correlation matrices. The outcome of the power of test for Jennrich statistic, J_s , J_m , and J_{mad} -statistic with different percentage of outlier with 10%, 20%, 30%, 40%, and 50% are presented in Table 2 until Table 6.

Based on Table 2, for a small sample size ($n = 10$), when outlier data are in the dataset, the power of test for Jennrich statistic increased from 0.7396 when there is only 10% outlier to 0.9736 when there are 50% outliers in the data. This means that hypothesis testing using Jennrich statistic is very sensitive to the presence of outliers. However, J_m -statistic and J_{mad} -statistic do not show high values of power of test for detecting a false null hypothesis when

there is presence of outlier. *Jm*-statistic only show high values of power of test when there are 50% data outliers. *Jmad*-statistic, on the other hand, do not show high power of test values even with the presence of data outliers. These results show that *Jmad*-statistic are not affected by up to 50% increase in sample size and still can detect a false null hypothesis when performing hypothesis tests between 2 correlations, while *Jm*-statistic are not affected by up to 40% outliers when performing hypothesis tests between 2 correlations.

Next, we continued to look at the power of test when we increase the number of sample size in the dataset. For sample size $n = 20$, the power test value for Jennrich statistics is still high although with the presence of only 10% outliers which is 0.9996. Meanwhile *Js*-statistic showed a high value of power of test (0.5286) at the presence of 30% outliers. *Jm*-statistics and *Jmad*-statistics also showed high power of test values when there is presence of 30% outliers of 0.684 and 0.5286, respectively. This indicates that when the sample size is increased, all test statistics in detecting a false null hypothesis begin to demonstrate effects with the presence of 20% outliers.

Table 2. Power of test for different statistics with outliers for $n = 10$

	$n = 10$				
outlier	10%	20%	30%	40%	50%
Jennrich statistic	0.7396	0.8961	0.9428	0.9692	0.9736
<i>Js</i> -statistic			0.663	0.7536	0.8529
<i>Jm</i> -statistic	0.0004	0.0212	0.1491	0.3772	0.5913
<i>Jmad</i> -statistic	0	0.0053	0.0854	0.2993	0.4172

Table 3. Power of test for different statistics with outliers for $n = 20$

	$n = 20$				
outlier	10%	20%	30%	40%	50%
Jennrich statistic	0.9996	0.9999	1	1	1
<i>Js</i> -statistic	0.4644	0.0486	0.5286	0.8272	0.8908
<i>Jm</i> -statistic	0.0684	0.308	0.684	0.8779	0.9567
<i>Jmad</i> -statistic	0.0022	0.0486	0.5286	0.8272	0.8908

Table 4. Power of test for different statistics with outliers for $n = 30$

	$n = 30$				
outlier	10%	20%	30%	40%	50%
Jennrich statistic	1	1	1	1	1
<i>Js</i> -statistic	0.923	0.997	0.9999	1	1
<i>Jm</i> -statistic	0.1406	0.5388	0.9093	0.9862	0.9973
<i>Jmad</i> -statistic	0.0121	0.2015	0.8699	0.9839	0.9925

Next, we continued increasing the sample size to $n = 30$. As in Table 4, the power of test value for Jennrich statistic still gives high values with the presence of only 10% outliers. *Js*-statistic also showed high value of power of test at the presence of 10% outliers. *Jm*-statistics showed high

value of power of test when there is a presence of 20% outliers and *Jmad*-statistics maintained a high power of test values when there is presence of 30% outliers. This indicates that *Jmad*-statistics are not affected by the increase in sample size and still can detect a false null hypothesis when there is presence of 20% outliers. *Jm*-statistics can also detect a false null hypothesis when there is presence of only 10% outliers.

Table 5. Power of test for different statistics with outliers for $n = 50$

	$n = 50$				
outlier	10%	20%	30%	40%	50%
Jennrich statistic	1	1	1	1	1
<i>Js</i> -statistic	0.9947	1	1	1	1
<i>Jm</i> -statistic	0.2562	0.8132	0.992	1	1
<i>Jmad</i> -statistic	0.0309	0.4428	0.9899	0.9997	1

For large sample size $n = 50$ and $n = 100$, *Jm*-statistic and *Jmad*-statistic are not affected to detect a false null hypothesis with the presence of 10% outliers. However, only *Jmad*-statistic is not affected to detect a false null hypothesis with the presence of 20% outliers when $n = 50$. This analysis shows that *Jm*-statistic and *Jmad*-statistic can overcome outlier problems into Jennrich statistic in testing of correlation matrix hypothesis. *Jmad*-statistic is superior in testing correlation hypothesis for different sample sizes, especially those involving 10% outliers. Therefore, a robust method can be used as an alternative in reducing the influence of outliers.

Table 6. Power of test for different statistics with outliers for $n = 100$

	$N = 100$				
Outlier	10%	20%	30%	40%	50%
Jennrich statistic	1	1	1	1	1
<i>Js</i> -statistic	1	1	1	1	1
<i>Jm</i> -statistic	0.4843	0.9805	1	1	1
<i>Jmad</i> -statistic	0.0691	0.8025	1	1	1

3.3. Real Data for Testing the Equality of Two Matrices

The largest economic disturbance is happened after Great Recession in 2007 until 2009. It is about eighteen months recession which is the longest time period [29]. This recession gives a bad impact to job market, unemployment rate, social systems, and stocks market. Specifically, the stock market crash occurred in September 2008. The rise and collapse of correlation are important subject in inspecting the stock behaviour. It may help the investors utilize which stocks that impact correlation in order to improve asset allocation decisions [30](Mc Bride, 2011). To illustrate the application of *Jmad*-statistic, we used 81 stocks that have been traded at Bursa Malaysia from year 2009. All of the stocks having more than RM500 million market capital. The time series data of 81 stocks

for two different time period is retrieved from Bloomberg database. The first time period is start from Jan until June 2009 (6 months before recession end), and the second time period is start from July 2009 – December 2009 (6 months after recession end).

Then we conduct a hypothesis testing to determine the equality of two correlation matrices, $H_0 : P_1 = P_2$ and the alternative hypothesis is $H_1 : P_1 \neq P_2$. The statistical test value from *Jmad*-statistical test is 11433.08. Based on Chi-square distribution with $\frac{p(p-1)}{2} = \frac{81(81-1)}{2} = 3240$ degree of freedom, the critical values is 3108.74. Therefore, there is enough evidence to reject the null hypothesis and conclude that the Bursa Malaysia also having risen and collapse during the great recession.

4. Conclusions

The Jennrich statistic is appropriate way to test the correlation between 2 or more independent correlation matrices. However, this statistic is extremely sensitive to the presence of outliers that can reduce the test's power. To address the presence of outliers, robust correlations, such as median correlation and median absolute deviation correlation, as well as Spearman correlation, can be used to integrate into Jennrich statistic. These proposed methods are alternative methods in reducing the influence of outliers, especially in comparing correlation hypothesis testing. *Js*-statistic, *Jm*-statistic and *Jmad*-statistic are proposed and have been compared to classical Jennrich statistic using the power of test values. For perfect data, which have no outliers in the data set, all 4 statistical tests for correlation analysis show that the probability value of the test of significance will detect a low deviation from the null hypothesis when the correlation difference varies up to 0.2. In addition, *Jm*-statistic and *Jmad*-statistic have low probability values to detect a deviation from the null hypothesis when the correlation difference varies up to 0.4. When outliers exist in the dataset, *Jmad*-statistic still gives a strong performance compared to other methods. This proves that, although outliers exist in the data, the *Jmad*-statistic does not affect the hypothesis testing to compare correlation matrices. The classical Jennrich statistic also suffer greatly from the presence of outliers even though only 10% of outliers are present in the dataset.

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