Triangulation between Bernoulli Distribution and Laplacian Autoregressive Model to Predict Probability of Increase in Stock Price

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Abstract Stocks are one of the aspects that affect the economy in the world. In the stock market, the price of a stock changes every time. Investors who are able to predict the increase in stock prices will tend to get profit, but investors who are unable to predict the increase in stock prices will tend to experience losses. This study aims to find statistical models and use them to predict the probability of an increase in the price of a stock in the stock market. The method used in this study is a literature study regarding stochastic models and selected suitable stochastic models. This study finds stochastic models and uses them to predict the probability of an increase in stock prices. The novelty in this research lies in the stochastic model triangulation. The increase in stock prices is predicted using two stochastic models, namely: Bernoulli and autoregressive. The decision on stock prediction is determined by the results of the triangulation of the two methods. The proposed method in this study has the advantage that decision making is based on more than one stochastic model. The results of this study can be applied to the financial sector, especially in the stock market.

Keywords Autoregressive, Bernoulli, Laplace Noise, Stock Price

1. Introduction

Stocks are one of the things that affect the economy of people in the world [1]. Shares can be traded on the stock market, for example the Indonesia Stock Exchange. According to economic law, stock prices will increase when the number of investors who buy shares is larger than the number of investors who sell shares. Conversely, the stock price will fall when the number of investors buying shares is smaller than the number of investors who sell shares. Investors who are able to predict the increase in stock prices will tend to get profit, but investors who are unable to predict the increase in stock prices will tend to lose.

Stochastic models have been investigated and used as a
tool to predict stock prices, one of which is an autoregressive model. For example, an autoregressive model is a time series model used to forecast economic output such as the gross national product of the United States [2] and the autoregressive model is applied for forecasting on several market indices and asset return [3]. On the other hand, the Bernoulli distribution can be used to model the increase in stock prices. In various studies, researchers use one method without considering other methods in decision making. If only one method is used, the results of the research are usually less accurate.

This study aims to find stochastic models and use them to predict the probability of an increase in stock prices by means of triangulation methods. This study focuses on the Bernoulli distribution and autoregressive models. Decision making regarding the probability of an increase in stock prices is based on the results of triangulation.

The main contribution of this research lies in providing information, especially potential investors before buying and selling stocks so that potential investors do not lose money. This is reinforced by the results of research that stock market predictions are an important and valuable topic in finance [4]. The novelty in this study is the use of the triangulation method so that the accuracy of the results is better.

2. Literature Review

This literature study briefly reviews stocks, the autoregressive model, and the Bernoulli distribution.

2.1. Stocks

Stocks are proof of ownership of a company. Several researchers have conducted research on the topic of stocks, for example: [4], [5], [6], and [7]. In [4], the authors present a new data mining algorithm based on back propagation neural networks and its application to stock price predictions. To improve the efficiency of the traditional back propagation network, the authors do this by revising the perspective error, and modifying the incentive function and adaptive learning speed so as to avoid local minimums and reduce training time [4]. Whereas in [5], the authors propose a decomposition-forecast-synthesis (DFS) model in predicting stock prices. The prediction is based on the characteristic analysis of the stock price time series, combined with a stock price prediction model. These studies show that an important problem in research related to stocks is how to predict stocks in the future.

2.2. Autoregressive Model

Suppose \( x_t, \ldots, x_n \) are \( n \) time series data. These time series data have an autoregressive model if it meets the equation [8]:

\[
x_t + \sum_{i=1}^{p} \phi_i x_{t-i} = z_t
\]

In equation (1), \( p \in \{1,2,\ldots\} \) and \( \phi_1, \ldots, \phi_p \in \mathbb{R}^p \) are the model coefficients. The random variables \( z_t, \ldots, z_n \) are the noise for the model. The noise for the model has a distribution. In [9], the noise is assumed to be normally distributed. In [10], noise is assumed to have a Laplace distribution. Whereas in [11], noise is assumed to have an exponential distribution. This research adopts the Laplace distribution noise. In this paper, autoregressive model with Laplace noise is used to model stock prices.

2.3. Bernoulli Distribution

Suppose \( y \) is the random variable. The random variable \( y \) is said to have a Bernoulli distribution if the random variable has the following probability function [12]:

\[
f(y|\theta) = \theta^y(1-\theta)^{1-y}
\]

for \( y \in \{0,1\} \) and is zero for the other \( x \). In this distribution, \( \theta > 0 \) is a parameter. The Bernoulli distribution is generally considered an analytical tool for making decisions in many disciplines that have particular theoretical and practical importance [13]. In this study, the increase in stock prices will be modeled by the Bernoulli distribution.

2.4. Forecasting

Forecasting is an activity to predict an event in the future based on current and previous data. Forecasting is very necessary to improve the effectiveness and efficiency of planning in various fields, including economics and finance. There are two quantitative forecasting approaches, namely: explanatory and time series [14]. In contrast to explanatory forecasting which assumes a cause and effect relationship between one variable and another, time series forecasting does not assume a relationship between variables. Time series forecasting methods and time series models can be found in [8]. This article uses time series forecasting.

3. Materials and Methods

This research is included in the type of pure research. As an example of the application, this study uses 3 stocks listed in LQ 45 for the period August 2020 - January 2021. LQ45 was chosen because the stocks that are members of the LQ45 have good criteria in market capitalization, transaction value, listed on the Indonesia Stock Exchange, financial condition, growth prospects, and free weights. LQ45 stocks are evaluated for their performance every 6 months.

The data collection method uses observation guidelines on the stock market on the Indonesia Stock Exchange.
Data are collected through the Spot application provided by Sucor Sekuritas. The collected data were analyzed using Bayesian inference statistics. A reversible jump MCMC algorithm [15] was used to find the Bayes estimator which cannot be determined analytically.

The research procedure is as follows: (a) literature study related to stochastic models and selection of suitable stochastic models, (b) description of the theory of estimation on the parameters of stochastic models, making algorithms, (d) collecting stock price data on the stock market, (d) and data analysis to predict the probability of an increase in stock prices.

### 4. Results and Discussion

#### 4.1. Results

This study produces an estimation procedure using the Bernoulli distribution, the Autoregressive model, and its application for prediction of stock prices, particularly stocks in LQ45.

##### 4.1.1. Bernoulli

Suppose $x_1, \ldots, x_n$ represents the stock price on the Indonesia Stock Exchange for $n$ days. Then, suppose $y_1, \ldots, y_n$ are the results of recording the corresponding increase in stock prices. If the stock rises from the previous day, the increase in stock price is 1. Whereas if the stock falls or remains from the previous day, the increase in stock price is 0. The increase in stock price $y$ is modeled as Bernoulli distribution with parameter $0 < \theta < 1$. Here, $\theta$ is the probability of an increase in the stock price. The likelihood function of the increase in stock prices $y_1, \ldots, y_n$ is

$$f(y_1, \ldots, y_n | \theta) = \prod_{i=1}^{n} f(y_i | \theta)$$

$$= \theta^{\sum_{i=1}^{n} y_i} (1 - \theta)^{n - \sum_{i=1}^{n} y_i}$$

The prior distribution of the probability of an increase in stock price $\theta$ is the Beta distribution with parameters $a$ and $b$. The Beta distribution was selected because it is a conjugate prior. The distribution of the probability of an increase in the share price $\theta$ can be written as follows:

$$\pi(\theta) = \frac{\Gamma(a + b)}{\Gamma(a) \Gamma(b)} \theta^{a-1} (1 - \theta)^{b-1}$$

Using the Bayes theorem, the posterior distribution of the probability of an increase in stock price $\theta$ can be expressed as follows:

$$\pi(\theta | y_1, \ldots, y_n) = \frac{\Gamma(u + v)}{\Gamma(u) \Gamma(v)} \theta^{u-1} (1 - \theta)^{v-1}$$

In this posterior distribution, $u = a + \sum_{i=1}^{n} y_i$ and $b = n + \beta$. Let $\hat{\theta}$ be a Bayes estimator of the probability of an increase in stock price $\theta$ is

$$\hat{\theta} = \frac{u}{u + v}$$

#### 4.1.2. Laplacian Autoregressive Model

The recording results of stock prices $x = (x_1, \ldots, x_n)$ are modeled with an autoregressive model with order $p$, so this stock price meets the equation (1). Here, the noise $z_{t}$ ($t = 1, \ldots, n$) is assumed to have a Laplace distribution with the parameter $\beta$ [10]. Therefore, the likelihood function of the data $x$ is

$$f(x | p, \phi^{(p)}, \beta) = \left( \frac{1}{2\beta} \right)^{n-p} \exp \left[ -\frac{1}{\beta} \sum_{t=p+1}^{n} \sum_{i=1}^{p} \phi_i x_{t-i} + x_t \right]$$

where $\phi^{(p)} = (\phi_1, \ldots, \phi_p)$. Let $r^p = (r_1, r_2, \ldots, r_p)$ represent the partial autocorrelation functions corresponding to the autoregressive model. Let $F$ denote a transformation from $\phi^{(p)} \in S_p$ to $r^p \in (-1,1)^p$ where $S_p$ is the stationarity area of the autoregressive model [16]. Using reparameterization, the likelihood function of data can be written as

$$f(x | p, r^p, \lambda, \beta, v) = \left( \frac{1}{2\beta} \right)^{n-p} \exp \left[ -\lambda^{\max-p} \frac{1}{2\beta} \frac{v^{u}}{\Gamma(u)} \beta^{-(u+1)} \exp - \frac{v}{\beta} \right]$$

Therefore, the posterior distribution for $p, r^p, \lambda, \beta, v$ is

$$\pi(p, r^p, \lambda, \beta, v | x) = \frac{C^p_{\lambda}(1 - \lambda)^{p-1} \frac{1}{2\beta} \frac{v^{u}}{\Gamma(u)} \beta^{-(u+1)} \exp - \frac{v}{\beta}}{f(x | p, r^p, \lambda, \beta, v)}$$

Posterior distribution simulation is carried out in 2 stages, namely conditional distribution simulation of $(\lambda, \beta, v)$ if given $(p, r^p)$ and conditional distribution simulation of $(p, r^p)$ if given $(\lambda, \beta, v)$. Since the conditional distribution of $(\lambda, \beta, v)$ if given $(p, r^p)$ can be recognized easily, the simulation of the conditional
distribution of \((\lambda, \beta, v)\) if given \((p, r^{(p)})\) can be done as follows:

- \(\beta \sim \text{Inverse Gamma}(n - p, v + \sum_{i=1}^{n} F^{-1}(x_{i-1}) + x_i)\)

- \(\lambda \sim \text{Binomial}(p + 1, p_{max} - p + 1)\) \(v \sim G\left(\frac{1}{\beta}\right)\).

- \(v \sim \text{Gamma}\left(\frac{1}{\beta}\right)\).

However, the conditional distribution of \((p, r^{(p)})\) if given \((\lambda, \beta, v)\) has a complex form, then the conditional distribution simulations of \((p, r^{(p)})\) if given \((\lambda, \beta, v)\) were estimated using the reversible jump MCMC algorithm. The algorithm uses 3 kinds of transformation, namely: coefficient change, order birth, and order death. More complete and detailed calculations can be found in [10]. The resulting Markov chain is then used to estimate parameters \((p, r^{(p)}, \lambda, \beta, v)\).

4.1.3. Prediction of the Probability of an Increase in Stock Prices

Price data of stocks A, C, and D are presented in Figure 1, Figure 2, and Figure 3. Stocks A is included in the LQ45 index and these stocks are listed on the Indonesia Stock Exchange. The stock price taken is the stock price at closing. The Indonesian stock exchange is open Monday to Friday so that it is open for 5 days every week minus national holidays. Recording is carried out from January 4, 2021 to April 30, 2021.

The stock price data in Figure 1, Figure 2, and Figure 3 are analyzed respectively using two ways. The first way, the increase in stock prices is modeled by the Bernoulli distribution with the probability parameter of an increase in stock prices \(\theta\). The prior distribution of \(\theta\) is a Beta distribution with the parameters \(a = 1\) and \(b = 1\). This study shows that the Bayes estimators of the probability of an increase in stock prices are \(\hat{\theta} = 0.42\), \(\hat{\theta} = 0.38\), and \(\hat{\theta} = 0.39\), respectively. The second way, the stock price is modeled with an autoregressive model [10]. Autoregressive model parameters include model order \(p\), model coefficient \(\phi^{(p)}\) and noise variance \(\sigma^2\). Bayes estimator of the order model, order coefficient and noise variance are calculated using the reversible jump MCMC algorithm. The reversible jump MCMC algorithm is run for 100 000 iterations with a burn-in period of 20 000 iterations. This study provides the results that the estimators of the model order are \(\hat{p} = 2\), \(\hat{p} = 1\), and \(\hat{p} = 2\). Each of these order estimators is obtained by taking the order that has the most frequency in the order histogram (Figure 4, Figure 5, and Figure 6).
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The forecast value of stock A, stock C, and stock D is presented in Table 2.

Table 2. The value of the stock forecast for the next day of stock A, stock C, and stock D

<table>
<thead>
<tr>
<th>Stocks</th>
<th>$\bar{x}_{83}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6460</td>
</tr>
<tr>
<td>C</td>
<td>4380</td>
</tr>
<tr>
<td>D</td>
<td>3168</td>
</tr>
</tbody>
</table>

Based on Table 2, the prediction value shows that the three stock prices will tend to decline. If the probability prediction using the Bernoulli distribution is compared with the stock price prediction using the autoregressive model, the prediction results are mutually reinforcing. Therefore, these results indicate that the stock prices of the three stocks will not increase on the 83rd day.

4.2. Discussion

For stocks A, C, and D, the probability of the share increase is less than 0.5. This means that the stock price is less likely to increase. These results are supported by predictions using an autoregressive model with Noise Laplace, which indicates the prediction of stocks will decline. Based on the results of the two data analysis methods, the price of A, C, and D stocks will have a chance not to increase. These results are in line with the method in [9]. In [9] the researcher uses an autoregressive model with normal noise. This means that investors do not sell stocks the next day. Conversely, investors should buy stocks the next day because the price is cheaper due to the decline in stock prices. The downward trend in some of this is due to foreign investors selling more shares than buying shares on the 81st and 82nd days.

Based on the results of the 3 stocks above, order is low (less than 3 days). This means that the 83rd day's stock value is only determined by the stock price of the previous day or two. Potential investors only need to look at the data one day or two in advance to make a decision whether to sell or buy shares. For the three stock samples, the proposed method can predict stock prices. This method is generally applicable to stocks in LQ45. Therefore, to predict stock prices in LQ45, this method can be used as a consideration. To get a more accurate prediction result, other available methods can be considered so that the decisions taken are the ones recommended by the majority of the methods.

This method uses original data so that the original data can be accessed when the research results are published. For confidential data, this method needs to be developed by transforming data. The data analyzed are data that are transformed so that the confidentiality of the original data is maintained.

![Order histogram of stock A](image1)

![Order histogram of stock C](image2)

Estimates of the model coefficient and corresponding noise variance of stock A, stock C, and stock D are presented in Table 1.

Table 1. Estimates of model coefficients and variances noise of stock A, stock C, and stock D

<table>
<thead>
<tr>
<th>Stocks</th>
<th>$\phi^p$</th>
<th>$\sigma^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(0.14, 0.85)</td>
<td>$2.42 \times 10^5$</td>
</tr>
<tr>
<td>C</td>
<td>(0.95)</td>
<td>$4.18 \times 10^5$</td>
</tr>
<tr>
<td>D</td>
<td>(0.24, 0.75)</td>
<td>$7.41 \times 10^4$</td>
</tr>
</tbody>
</table>

The estimated value of the autoregressive model parameters in Table 1 is used to predict the stock price one day in the future, for example for stock A is

$$\bar{x}_{83} = 0.14x_{82} + 0.85x_{81}$$
$$= 0.14(6525) + 0.85(6525)$$
$$= 6460$$
5. Conclusions

This research studies stock price stochastic modeling and uses stochastic models to predict the probability of an increase in stock prices. This study focuses on the Bernoulli distribution with the beta prior distribution and the autoregressive model with Laplace noise. Further research can be carried out by considering other prior distributions so that the best prior distribution can be found. In addition, further research can also be carried out by considering other noise so that the best noise model can also be found. Selection of the best prior distribution and noise model will have a positive impact on the accuracy of the prediction of the probability of an increase in stock prices. This research provides benefits in finance and other fields related to forecasting. The advantage of this research lies in the use of the triangulation method to determine the probability of an increase in stock prices. The results of the research can be applied as a consideration for investors before buying a stock so that the shares purchased bring profit. To minimize losses, investors are recommended to know in advance the probability of an increase in the price of the shares to be purchased so that investors can choose stocks that have a high probability of an increase.

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