Unsteady Couette Flow Past between Two Horizontal Riga Plates with Hall and Ion Slip Effects

S. Nasrin^{1,*}, R. N. Mondal¹, M. M. Alam²

¹Department of Mathematics, Jagannath University, Dhaka-1100, Bangladesh

²Mathematics Discipline, Khulna University, Khulna-9208, Bangladesh

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Abstract Riga plate is the span wise array of electrodes and permanent magnets that creates a plane surface and produced the electromagnetic hydrodynamic fluid behavior and mostly used in industrial processes with fluid flow affairs. In cases where an external application of a magnetic or electric field is required, better flow is obtained by the involvement of the Riga plate. Riga plate acts as an agent to reduce the skin friction and enhance the heat transfer phenomena. It also diminishes the turbulent effects, so that it is possible to get an efficient flow control and it increases the performance of the machine. So the numerical investigation of the unsteady Couette flow with Hall and ion-slip current effects past between two Riga plates has been studied and the numerical solutions are acquired by using explicit finite difference method and estimated results have been gained for several values of the dimensionless parameter such as pressure gradient parameter, Hall and Ion-slip parameters, modified Hartmann number, Prandtl number, and Eckert number. In this article, the importance of the modified Hartmann number on the flow profiles is immense owing to the Riga plate. The expression of skin friction and Nusselt number has been computed and the outcomes of the relevant parameters on various distributions have been sketched and presented as well as graphically.

Keywords MHD, Riga Plate, Couette Flow, Hall Current and Ion-Slip Effects, Hartmann Number, Explicit Finite Difference

1. Introduction

The highly conducting fluids are controlled by magneto-hydrodynamics flow with the influence of an external magnetic field known as classical MHD flow control. When the fluid is too weak in electrically conducting so that Lorentz force decreases exponentially then an external electric field must be applied to achieve an efficient flow control, this is called Electro-magneto hydrodynamic (EMHD) flow. Because of the importance of magnetic fields, the Riga plate holds a key place of its tremendous applications in the field of magneto-aerodynamics, civil engineering, mechanical engineering, chemical engineering, dust or fumes in a gas, in biomechanics and rectification of groundwater, and oil. The formulation of the Riga plate is the combination of electrodes and permanent magnets create a plane surface instead of polarity and magnetization that produce a wall parallel Lorentz force to control the fluid flow that was first introduced by Gailitis and Leilausis [1]. This order minimizes the friction and pressure drag of submarines. It is also separation of the boundary layer as well as diminishing the turbulence effects and so that a better flow pattern is obtained. When the flow passes, the Lorentz force is created by gathering the electrodes and permanent magnets in the flat surface. A turbulent channel event with a small Reynolds number is utilized the Lorentz force discovered by Berger et al. [2]. Pantokratoras and Magyari [3] proposed the electro-magneto hydrodynamic free convection fluid flow with a poor conductivity along a Riga plate. Wahidunnisa

et al. [4] studied the heat source of a nanofluid flow through a Riga plate with viscous dissipation. Iqbal et al. [5] explored their idea on an electrically conducted Riga plate with viscous dissipation and thermal radiation of nanofluid with melting heat and they use the Keller Box scheme to obtain the solution. Ayub et al. [6] examined the EMHD nanofluid flow along an electromagnetic actuator or Riga plate. Ahmed et al. [7] carried out the mixed convection of a nanofluid flow along a vertical Riga plate with the effect of a strong suction. The physical problems of magneto-hydrodynamic flows with Hall and Ion-slip current have practical applications as electromagnetic flow meters, electromagnetic pumps and MHD power generator, aerodynamic heating, electrostatic precipitation, geophysics, astrophysics, and many engineering and industrial processes [8]. Javeri [9] investigated the combined effect of Hall and Ion slip currents, Joule heating, and viscous dissipation on the laminar MHD channel. Eraslan [10] expressed a distribution of temperature for the MHD channels with the Hall effect. The MHD laminar flow along a porous medium has significant applications in engineering and agricultural process, groundwater flows, petroleum industry, and oil and gas purification. A lot of research work has been held on the MHD steady or unsteady flows over a vertical porous plate with Hall and ion-slip under different physical effects has been studied of their wide applications. From the point of view, the effect of Hall current and ion-slip with heat and mass convection flows of an electrically conducting fluid has been discussed by several authors such as Seddeek and Aboeldahab[11] examined an unsteady free convection fluid flow with Hall currents effect of gray gas through an infinite porous plate where a strong transverse magnetic field is imputed perpendicularly to the plate. Debnath et al. [12] have studied the effects of Hall current on unsteady hydro magnetic flow past a porous plate in a rotating fluid system. Nimr and Masoud [13] carried out an unsteady free convection flow in a porous media along a flat plate. Krishna et al. [14] have carried out an unsteady free convective magneto-hydrodynamic flow with Hall and ion slip current effects through an accelerated inclined plate with rotation which is surrounded by a porous medium with the effect of inclined angle also with the change of reference frame. They have used Laplace transform to solve these problems analytically. Angirasa and Peterson [15] presented a numerical study on heat transfer in natural transmission from an isothermal vertical surface which is a stable layered to a fluid-saturated thermally stratified porous medium. Kumar and Singh [16] analyzed the heat transfer from a vertically isothermal surface with the impact of thermal stratification in a porous medium. The influence of MHD Couette flows with Hall and Ion slip current has a great importance of experimental and theoretical applications in magnetic material processing, astrophysics, polymer technology, heating electrostatics, nuclear engineering, pumps and power generators,

geophysical and industrial fields. From this point of view, an unsteady Couette flow of an incompressible fluid with Ion-slip effect and with the influence of an external uniform magnetic field which is perpendicular to the plates is discussed in [17] and the same flow with uniform suction and injection past between two parallel porous plates with heat transfer is discussed in [18] by Attia. Kumar et al. [19] examined the Couette flow in three-dimensional with heat transfer through a porous medium bounded by infinite vertical porous plates. From the great interest in unsteady laminar Couette flow of its great applications in environmental, industrial, biomedical, engineering, nuclear reactors, and oil purifications, many researchers [20,21,22] have expressed their views on its. Limthanakul and Pochai [23,25] have given an idea about groundwater flows. They discussed a two dimensional model of polluted groundwater measured around a land fill. Koroleva [24] studied a flow of Stokes-Brinkman system through an ensemble of porous cylindrical particles with different viscosity by the cell method.

Drawing motivation from the above studies, the study aims to investigate the simultaneous effects of the unsteady Couette flow past between two horizontal Riga plates with Hall and ion slip current with electromagnetic field. The explicit finite difference method has been used as a main tool to solve the problem. Also, MATLAB R2015a has been used to calculate the results. The obtained results of different parameters have been shown graphically.

2. Mathematical Formulation

An incompressible laminar flow of viscous fluid between two horizontal parallel Riga plates has been considered, where one of which is moving and the other is at rest. Let the lower plate be rest at $\tilde{y} = 0$ and the upper plate is moving at a distance $\tilde{y} = d$ with a velocity $\frac{u_0}{l}$.

Let the direction of the flow be taken along the $\tilde{x}$ - axis, $\tilde{y}$ - axis is perpendicular to the flow and the plates are parallel to the $\tilde{x}\tilde{z}$ - plane. As a generalized Couette flow, the applied pressure gradient $\frac{\partial \tilde{p}}{\partial \tilde{x}} (=P)$ on the fluid is constant. Consider the plate is long enough in x - direction and the fluid motion is unsteady so that the fluid may be treated as two dimensional $\left(i.e. \frac{\partial}{\partial \tilde{z}} = 0 \right)$. Therefore the continuity equation leads to $\tilde{u} = \tilde{u}(\tilde{y})$. Plates are fixed at two constant temperatures T_1 for the lower plate and T_2 for the upper plate, where $T_2 > T_1$. The initial temperature of the fluid is assumed to be equal to the temperature of the lower plate T_1 . Due to the effect of Hall and Ion-slip current, the generalized Ohm's law may state as follows:

$$\mathbf{J} = \sigma(\mathbf{q} \wedge \mathbf{B}) - \frac{\beta_e}{B_0}(\mathbf{J} \wedge \mathbf{B}) + \frac{\omega_e \tau_e \beta_i}{B_0^2}(\mathbf{J} \wedge \mathbf{B}) \wedge \mathbf{B}$$

Where, $\omega_e \tau_e = \beta_e$ may treat as Hall parameter. The physical model is shown in Fig.1.

Figure 1. Physical model of the system

A uniform magnetic force is generated by the Riga plate. The Lorentz force $\vec{f} = \mathbf{J} \wedge \mathbf{B} \approx \sigma(\mathbf{E} \wedge \mathbf{B})$ is defined as magnetic forces. According to the Grinberg hypothesis this magnetic forces are defined as follows:

$$\vec{f} = \mathbf{J} \wedge \mathbf{B} = \left(\frac{\pi}{8} J_0 M_0 e^{-\frac{\pi}{l} y}, 0, \frac{\pi}{8} J_0 M_0 e^{-\frac{\pi}{l} y} \right)$$

Under the above assumptions of Couette flow, and Bousniqués approximations, it is found the dimensional forms of the momentum and energy equations are:

$$\frac{\partial \tilde{u}}{\partial \tilde{t}} - v_0 \frac{\partial \tilde{u}}{\partial \tilde{y}} = -\frac{P}{\rho} + v \frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2} + \frac{\pi}{8\rho} (1 + \beta_e - \beta_e \beta_i) J_0 M_0 e^{-\frac{\pi}{l} \tilde{y}} \quad (1)$$

$$\frac{\partial \tilde{w}}{\partial \tilde{t}} - v_0 \frac{\partial \tilde{w}}{\partial \tilde{y}} = v \frac{\partial^2 \tilde{w}}{\partial \tilde{y}^2} + \frac{\pi}{8\rho} (1 - \beta_e - \beta_e \beta_i) J_0 M_0 e^{-\frac{\pi}{l} \tilde{y}} \quad (2)$$

$$\frac{\partial \tilde{T}}{\partial \tilde{t}} - v_0 \frac{\partial \tilde{T}}{\partial \tilde{y}} = \frac{k}{\rho C_p} \frac{\partial^2 \tilde{T}}{\partial \tilde{y}^2} + \frac{\mu}{\rho C_p} \left(\frac{\partial \tilde{w}}{\partial \tilde{y}} \right)^2 \quad (3)$$

The corresponding boundary conditions are

$$\begin{aligned} t \leq 0 : & \quad u = 0, \quad w = 0, \quad T = T_1 \text{ for all } y \geq 0 \\ t > 0 : & \quad \begin{cases} u = 0, & w = 0, \quad T = T_1 \text{ at } y = 0 \\ u = \frac{\pi v}{l}, & w = 0, \quad T = T_2 \text{ at } y = d \end{cases} \end{aligned} \quad (4)$$

2.1. Similarity Analysis

The following non-dimensional variables are used to make the dimensionless form of the equations (1) to (4).

$$x = \frac{\pi}{l} \tilde{x}, \quad y = \frac{\pi}{l} \tilde{y}, \quad u = \frac{l}{\pi v} \tilde{u}, \quad w = \frac{l}{\pi v} \tilde{w}, \quad t = \frac{\pi^2 v}{l^2} \tilde{t},$$

$$\theta = \frac{\tilde{T} - \tilde{T}_1}{\tilde{T}_2 - \tilde{T}_1}$$

Applying these into the equations (1)-(4), yields

$$\frac{\partial u}{\partial t} - V_0 \frac{\partial u}{\partial y} = \alpha + \frac{\partial^2 u}{\partial y^2} + H_r (1 + \beta_e - \beta_e \beta_i) e^{-y} \quad (5)$$

$$\frac{\partial w}{\partial t} - V_0 \frac{\partial w}{\partial y} = \frac{\partial^2 w}{\partial y^2} + H_r (1 - \beta_e - \beta_e \beta_i) e^{-y} \quad (6)$$

$$\frac{\partial \theta}{\partial t} - V_0 \frac{\partial \theta}{\partial y} = \frac{1}{P_r} \frac{\partial^2 \theta}{\partial y^2} - E_c \left(\frac{\partial w}{\partial y} \right)^2 \quad (7)$$

The corresponding boundary conditions are

$$\begin{aligned} t \leq 0 : & \quad u = 0, \quad \theta = 0, \text{ for all } y \geq 0 \\ t > 0 : & \quad \begin{cases} u = 0, & w = 0, \quad \theta = 0 \text{ at } y = 0 \\ u = 1, & w = 0, \quad \theta \rightarrow 1, \text{ at } y \rightarrow d \end{cases} \end{aligned} \quad (8)$$

where, $\alpha = -\frac{l^3 P}{\rho v^2 \pi^3}$ is the dimensionless pressure

gradient, $H_r = \frac{l^3 J_0 M_0}{8 \rho v^2 \pi^2}$ is the modified Hartmann

Number; $P_r = \frac{\rho c_p v}{k}$ is the Prandtl number ;

$E_c = \frac{v^2 \pi^2}{(\tilde{T}_2 - \tilde{T}_1) c_p l^2}$ is the Eckert Number.

3. Method of Solution

We apply the explicit finite difference method to solve the non-dimensional coupled partial differential equations (5)-(7) with the associated boundary conditions (8).

It is considered maximum length of the plate is $x_{\max} (=10)$ and distance between the plates $d=2$ i.e. $y_{\max} = 2$ as the lower plate is fixed at $y = 0$. This means x varies from 0 to 10 and y varies from 0 to 2. The finite difference schemes for the problems are as follows:

$$\frac{u_{i,j}^{k+1} - u_{i,j}^k}{\Delta t} - V_0 \frac{u_{i,j}^k - u_{i,j-1}^k}{\Delta y} = \alpha + \frac{u_{i,j+1}^k - 2u_{i,j}^k + u_{i,j-1}^k}{\Delta y^2} + H_r (1 + \beta_e - \beta_e \beta_i) e^{-y_i}$$

$$\frac{w_{i,j}^{k+1} - w_{i,j}^k}{\Delta t} - V_0 \frac{w_{i,j}^k - w_{i,j-1}^k}{l} = \frac{w_{i,j+1}^k - 2w_{i,j}^k + w_{i,j-1}^k}{l^2} + H_r (1 - \beta_e - \beta_e \beta_i) e^{-y_i}$$

$$\frac{\theta_{i,j}^{k+1} - \theta_{i,j}^k}{\Delta t} - V_0 \frac{\theta_{i,j}^k - \theta_{i,j-1}^k}{\Delta y} = \frac{1}{P_r} \left(\frac{\theta_{i,j+1}^k - 2\theta_{i,j}^k + \theta_{i,j-1}^k}{\Delta y^2} \right) - E_c \frac{w_{i,j}^k - w_{i,j-1}^k}{\Delta y}$$

with the boundary conditions

$$t \leq 0 : u = 0, \theta = 0, \text{ for all } y \geq 0$$

$$t > 0 : \begin{cases} u = 0, w = 0, \theta = 0 & \text{at } y = 0 \\ u = 1, w = 0, \theta = 1 & \text{at } y = d \end{cases}$$

Here, the subscripts i and j refer to x and y and the superscript k refers to time t .

3.1. Shear Stresses and Nusselt Number

The effects of pertinent parameters on the local and average shear stress from the velocity of the fluid have been investigated. The non-dimensional form of the local and average shear stress for the fluid is given by the relations $\tau_L = \mu \frac{\partial u}{\partial y} \Big|_{y=0}$ and $\tau_A = \frac{1}{L} \int_0^L \mu \frac{\partial u}{\partial y} \Big|_{y=0} dx$ and for the

dust particle are given by $\tau_{pL} = \mu \frac{\partial u_p}{\partial y} \Big|_{y=0}$ and

$$\tau_{pA} = \frac{1}{L} \int_0^L \mu \frac{\partial u_p}{\partial y} \Big|_{y=0} dx$$

respectively. The rate of heat transfer at the plate is defined as the Nusselt number. The local and average

Nusselt number for the fluid are given by $Nu_L = -\mu \frac{\partial \theta}{\partial y} \Big|_{y=0}$

$$\text{and } Nu_A = -\frac{1}{L} \int_0^L \mu \frac{\partial \theta}{\partial y} \Big|_{y=0} dx.$$

3.2. Steady-state Solution

The behavior of the various entities on the velocity and temperature profiles has been elaborated graphically. Fig.2(a), Fig.2(b), and Fig.2(c) illustrate the fluid velocities and temperature distributions u , w and θ for different time τ . The computations have been carried out for the different time such as $\tau = 1, 3, 6$ and 8 with the time step size $\Delta t = 0.0005$. It is observed from figures that there is no change after $\tau = 8$ for u , w and θ . Fig.2(d) and Fig.2(e) display the validity of the grid pairs on the velocity u and temperature θ . It has shown the velocity and temperature distributions for three grid pairs $(m, n) = (40, 40)$, $(m, n) = (45, 45)$ and $(m, n) = (50, 50)$ with time $\tau = 8$ and time step $\Delta t = 0.0005$. There is also negligible change among these grid pairs so that any one grid pair is acceptable to find the steady-state solution. It has been seen that the same situation occurs for the other distributions. The steady-state solution has been performed for the values of $\tau > 7$. In the present analysis, the following graphs have been established for the choice of time $\tau = 8$ with the grid pair $(m, n) = (50, 50)$ and time step $\Delta t = 0.0005$.

Figure 2(a). Time sensitivity on primary velocity u

Figure 2(b). Time sensitivity on secondary velocity w

Figure 2(c). Time sensitivity on temperature profile θ

Figure 2(d). Mesh sensitivity on primary velocity u

Figure 2(e). Mesh sensitivity on temperature profile θ

3.3. Effects of Various Parameters

To study the physical situation of the problem, it is mentioned that the figures (a), (b), and (c) of Fig.3 to Fig.7 presented the distribution of velocity, local shear stress and average shear stress along the x-direction respectively. Figures (a), (b), and (c) of Fig.8 to Fig.11 depicted the distribution of velocity, local shear stress and average shear stress along y-direction respectively. And figures (a), (b), and (c) of Fig.12 to Fig.17 represented the distribution of temperature, local Nusselt number, and average Nusselt number respectively.

The effects of dimensionless pressure gradient α on the velocity is shown in Figs.3(a)-(c). Since $\alpha = -l^3 P / (\rho \nu^2 \pi^3)$, it means that $\alpha > 0$ whenever a constant pressure gradient P decreasing in the direction of motion; in that case, the velocity u has an increasing effect with the increase of α over the entire width between the plates. But $\alpha < 0$ influence that P is increasing in the direction of motion and the velocity u has occurred back-flow, which been shown in Fig.3(a). The same situation occurred for the local and average shear stress on the fluid which have shown in Figs.3(b) and (c) respectively.

Figure 3(a). Effect of pressure gradient α on primary velocity Figure 3(b). Effect of pressure gradient α on local shear stress τ_{xL} Figure 3(c). Effect of pressure gradient α on average shear stress

In the figures, Figs.4(a)-(c) represented the performance of Hall parameter β_e on the primary velocity u , local shear stress τ_{xL} and on the average shear stress τ_{xA} respectively.

It is showed that the primary velocity u , local and average shear stress τ_{xL} and τ_{xA} are all increasing with the increasing values of Hall current β_e .

Figure 4(a). Effect of Hall current β_e on primary velocity u

Figure 4(b). Effect of Hall current β_e on local shear stress τ_{xL}

Figure 5(b). Effect of Ion-slip β_i on local shear stress τ_{xL}

Figure 4(c). Effect of Hall current β_e on average shear stress τ_{xA}

Figure 5(c). Effect of Ion-slip β_i on average shear stress τ_{xA}

The impact of Ion-slip parameter β_i on the primary velocity u , local shear stress τ_{xL} and the average shear τ_{xA} stress are displayed respectively in Figs.5(a)-(c). It is observed that the primary velocity u , local and average shear stress τ_{xL} and τ_{xA} all are decreased with the increase of β_i .

Figure 5(a). Effect of Ion-slip β_i on primary velocity u

Figs.6(a)-(c) depict that the effect of modified Hartmann number H_r on the primary velocity u , local shear stress τ_{xL} and the average shear stress τ_{xA} respectively. It is evident that u , τ_{xL} and τ_{xA} are all rises with the influence of H_r .

Figure 6(a). Effect of modified Hartmann number H_r on primary velocity u

Figure 6(b). Effect of modified Hartmann number H_r on local shear stress τ_{xL}

Figure 6(c). Effect of modified Hartmann number H_r on average shear stress τ_{xA}

Characteristics of the Suction velocity V_0 on the primary velocity u , local shear stress τ_{xL} and the average shear stress τ_{xA} are presented in Figs.7(a)-(c) respectively. In Fig.7(a), it is observed that the velocity u has a cross flow where the velocity has an increasing effect within the interval $0 < y < 1$ (approx.) thereafter it has a decreasing effect, whereas τ_{xL} and τ_{xA} are increased with the increasing effect of V_0 .

Figure 7(a). Effect of Suction velocity V_0 on primary velocity u

Figure 7(b). Effect of Suction velocity V_0 on local shear stress τ_{xL}

Figure 7(c). Effect of Suction velocity V_0 on average shear stress τ_{xA}

Figure 8(a). Effect of Hall current β_e on secondary velocity w

Figure 8(b). Effect of Hall current β_e on local shear stress τ_{xL}

Figure 8(c). Effect of Hall current β_e on average shear stress τ_{zA}

Figs.8(a)-(c) explain the influence of Hall current parameter β_e on the secondary velocity w , local shear stress τ_{zL} and the average shear stress τ_{zA} respectively. It has shown that the increasing values of β_e the secondary velocity w , local shear stress τ_{zL} and average shear stress τ_{zA} become decreases.

The effects of Ion-slip parameter β_i on the secondary velocity profile w , local shear stress τ_{zL} and the average shear stress τ_{zA} are exposed in Figs.9(a)-(c) respectively. It has shown for the increasing values of β_i , the secondary velocity profile w , local and average shear stress τ_{zL} and τ_{zA} are decreased.

Figure 9(a). Effect of Ion-slip β_i on secondary velocity profile w

Figure 9(b). Effect of Ion-slip β_i on local shear stress τ_{zL}

Figure 9(c). Effect of Ion-slip β_i on average shear stress τ_{zA}

The variations of modified Hartmann number H_r on the secondary velocity w , local shear stress τ_{zL} and on the average shear stress τ_{zA} are depicted in Figs.10(a)-(c). With the increasing effect of H_r , it has found that the secondary velocity w , local and average shear stress τ_{zL} and τ_{zA} are decreased.

Figure 10(a). Effect of modified Hartmann number H_r on secondary velocity w

Figure 10(b). Effect of modified Hartmann number H_r on local shear stress τ_{zL}

Figure 10(c). Effect of modified Hartmann number H_r on average shear stress τ_{zA}

Figs.11(a)-(c) are plotted to explain the behavior of suction velocity V_0 on the secondary velocity w , local shear stress τ_{zL} and the average shear stress τ_{zA} respectively. It has shown that the back flow of w with the increasing values of suction velocity V_0 , whereas τ_{zL} and τ_{zA} are displayed decreasing effect.

Figure 11(a). Effect of suction velocity V_0 on secondary velocity w

Figure 11(b). Effect of suction velocity V_0 on local shear stress τ_{zL}

Figure 11(c). Effect of suction velocity V_0 on average shear stress τ_{zA}

To find the variations of Hall current parameter β_e on the temperature θ , local Nusselt number Nu_L and the average Nusselt number Nu_A have been demonstrated in Figs.12(a)-(c) respectively. It is found from the figures that for the increasing effect of β_e , temperature profile θ is increased whereas Nu_L and Nu_A showed the inverse effects.

Figure 12(a). Effect of Hall current β_e on temperature profile θ

Figure 12(b). Effect of Hall current β_e on local Nusselt number Nu_L

Figure 12(c). Effect of Hall current β_e on average Nusselt number Nu_A

The effects of Ion-slip current parameter β_i on the temperature θ , local Nusselt number Nu_L and the average Nusselt number Nu_A are plotted in Figs.13(a)-(c) respectively. It has shown that for the increasing effect of β_i , temperature θ is increased whereas Nu_L and Nu_A showed the decreasing effects.

Figure 13(a). Effect of Ion-slip β_i on temperature profile θ

Figure 13(b). Effect of Ion-slip β_i on local Nusselt number Nu_L

Figure 13(c). Effect of Ion-slip β_i on average Nusselt number Nu_A

It is visualized from the figures Figs.14(a)-(c) that for increasing values of modified Hartmann number H_r on the temperature θ depicted rising effect and the local Nusselt number Nu_L and the average Nusselt number Nu_A are demonstrated decreasing effects.

Figure 14(a). Effect of modified Hartmann number H_r on temperature profile θ

Figure 14(b). Effect of modified Hartman number H_r on local Nusselt number Nu_L

Figure 14(c). Effect of modified Hartman number H_r on average Nusselt number Nu_A

Temperature profile θ is increased due to the increasing effect of the Prandtl number P_r that has been seen in Figs.15(a) whereas Figs.15 (b) and Figs.15 (b) exhibit the reverse effect of θ .

Figure 15(a). Effect of Prandtl number P_r on temperature profile θ

Figure 15(b). Effect of Prandtl number P_r on local Nusselt number Nu_L

Figure 15(c). Effect of Prandtl number P_r on average Nusselt number Nu_A

It is observed from Figs.16(a)-(c) the effects of Eckert number E_c on the temperature profile θ , local Nusselt number Nu_L and the average Nusselt number Nu_L respectively. It has shown that θ depicts the increasing effect but Nu_L and Nu_L show the decreasing effect with the effects of Eckert number.

Figure 16(a). Effect of Eckert number E_c on temperature profile θ

Figure 16(b). Effect of Eckert number E_c on local Nusselt number Nu_L

Figure 16(c). Effect of Eckert number E_c on average Nusselt number Nu_A

The effects of the Suction velocity V_0 on the temperature θ , local Nusselt number Nu_L and on the average Nusselt number Nu_A are plotted in Figs.17(a)-(c) respectively. It has shown that with the increasing effect of V_0 , the temperature θ , local and average Nusselt number Nu_L and Nu_A showed decreasing effects.

Figure 17(a). Effect of Suction velocity V_0 on temperature profile θ

Figure 17(b). Effect of Suction velocity V_0 on local Nusselt number Nu_L

Figure 17(c). Effect of Suction velocity V_0 on average Nusselt number Nu_A

4. Conclusions

The characteristics of two horizontal parallel Riga plates in the presence of Hall and ion slip currents effects have been investigated numerically. The main findings of the study are as follows:

- i). Primary velocity profile u increases for the increasing values of pressure gradient parameter α , Hall parameter β_e and modified Hartmann number H_r , while it has a decreasing effects with the increase of Ion-slip parameter β_i . The suction velocity V_0 has a cross effect where the skin friction is increased as the suction velocity V_0 is increased.
- ii). Secondary velocity profile w decreases for the increasing values of Hall parameter β_e , Ion-slip parameter β_i and modified Hartmann number H_r . The suction velocity V_0 shows a cross effect where the skin friction coefficient is increased as the suction velocity V_0 is decreased.
- iii). The temperature profile increases with the rising values of Hall parameter β_e , Ion-slip parameter β_i and modified Hartmann number H_r , where as the local and average Nusselt numbers shows the reverse effects.

Nomenclature

- $\tilde{u}$: Velocity components along $\tilde{x}$ -axis
 $\tilde{v}$: Velocity components along $\tilde{y}$ -axis
 $\tilde{w}$: Velocity components along $\tilde{z}$ -axis
 $\tilde{\tau}$: Dimensional time
 ω_e : Cyclotron frequency

τ_e : Electron Collision time
 β_e : Hall parameter
 β_i : Ion-slip parameter
 α : Dimensionless pressure gradient
 ρ : Density of the fluid
 σ : Conductivity of the fluid
 τ : Maximum time
 E_c : Eckert number
 ν : Kinematic viscosity of the fluid
 μ : Coefficient of viscosity of the fluid
 u : Dimensionless velocity component in x-axis
 v : Dimensionless velocity component in y-axis
 B : Magnetic field vector
 B_0 : Constant magnetic field vector
 V_0 : Suction Velocity
 k : Thermal conductivity of the fluid
 c_p : Specific heat capacity at constant pressure
 J_0 : Current density
 $\tilde{T}$: Temperature of the fluid
 $\tilde{T}_w$: Constant temperature near the plate
 $\tilde{T}_\infty$: Temperature outside of the boundary layer
 Δt : Time increment
 t : Dimensionless time
 θ : Dimensionless temperature
 H_r : Hartmann number
 P_r : Prandtl number
 τ_{*L} : Local shear stress
 Nu_L : Local Nusselt number
 τ_{*A} : Average shear stress
 Nu_A : Average Nusselt number

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