A Case Study of Teaching Real-world Problems Related to Exponential and Logarithmic Equations to Develop Students' Problem-solving Competency

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Abstract Today, the task of developing competencies, including the ability to solve problems for students, becomes very important. In mathematics education, teaching mathematical problems with effective content can be considered as a teaching method that brings a very positive effect on the education, training and capacity development for students. This study aimed to promote students' ability to resolve real-world problems through teaching exponential and logarithmic equations. An experimental sample consisted of 40 12th grade students at a continuing education centre in Vietnam. The primary instructional activities designed to develop these students' resolving capacities included warm-up, experience, practice, reinforcement and expansion. The data collected were attached to the student's assignments as well as worksheets and qualitatively analyzed to assess students' ability to deal with problems by components such as understanding the problem, acquiring information from practical situations; converting information from real-world situations on mathematical models; searching strategies for solving mathematical models; implementing a solution strategy to find results; and moving from results of solving mathematical models to solutions of real-world problems. Experimental results showed that students considerably increased their ability to cope with problems with logarithmic and exponential equations. Besides, concerning the spirit and learning attitude, students became active, optimistic and self-conscious in the problem-solving stages of the teaching process. Additionally, the implications of showing the relationship between mathematics and practice were drawn to emphasize the real-life applicability of the science of mathematics.

Keywords Exponential and Logarithmic Equations, Mathematics Education, Problem-solving, Real-world Problems

1. Introduction

Teaching based on competency has today become a hot topic in education. It is getting hotter and hotter as governments in countries are working to find solutions to education reform and more accurately measure student achievement. Also, the definition of competency has many different conceptions by many educators [16,22]. Most schools claim that they are teaching competency-oriented but cannot define it precisely, so redefining the concept of learning based on competency development. What is competency-based teaching, and what makes it different? An essential characteristic of capacity building education measures the abilities of students instead of studying time and grades. Students show progress by demonstrating their competencies,
which means they must demonstrate mastery of knowledge and skills (called competencies) in a particular subject, no matter how long it takes. While conventional models can still measure competence, they have to be based on time, each semester, school year, the topics are arranged by level. So, while most traditional schools have a fixed learning period (according to the school year), competency development teaching allows us to keep learning the same and allow time to change learning. Consequently, these competencies contribute to the sustainable development of each individual [4], including the research competency mentioned in the research of I. F. Yarullin & N. A. Bushmeleva [24].

It is recognized that each student has different skills, skills, interests, needs and backgrounds. That is why teaching helps to understand this fact and find approaches that are appropriate for every student. Unlike the one-size- all approach, it allows students to apply what they have learned, through the connection between lesson and life. Teaching helps to understand this and find strategies that are suitable for all students. For some students, capacity development teaching allows accelerating the completion of the curriculum, saving time and effort of learning.

Among many students' capacities, the ability to solve problems is one of the crucial competencies that students need in the learning process [8]. Additionally, the ability to handle problems has been identified as one of the standard qualifications that students need to develop through the subjects. Therefore, training for students to discover, pose and deal with problems encountered in learning, in the lives of individuals, families and communities is not only meaningful in terms of teaching methods but also must be set as an education and training goal. In problem-solving teaching, students both grasp new knowledge and understand the process of acquiring that knowledge, develop positive, creative thinking, and be prepared with an adaptive capacity with social life, timely detecting and rationally solving emerging problems. In other words, problem detection and decoding of teaching is an excellent way of training students to identify and resolve problems. Accordingly, strategies are needed to improve students' problem-solving skills [13].

Many educators around the world have studied teaching to handle problems has been identified as one of the need in the learning process [8]. Additionally, the ability problems is one of the crucial competencies that students need to improve students' problem-solving skills [13].

D. Căprioară [3] researched on a group of seniors in middle school to know the importance of students in solving math problems, their preferred type of problem, and their level of achievement when solving math problems. Moreover, the author clarified the nature of the problems students encounter when solving problems from perception to self - regulation challenges. Also, V. Geiger et al. [5] conducted a study to identify factors related to mathematics, cognition, society and the environment that allowed students to successfully model the process, i.e. form, compute, and solve a real-world problem. The research results also generated new theoretical and practical insights into the role of facilitators in the succession of mathematical tasks to lead to student development as modellers.

P. Brčka & M. Valentová [2] carried out a case study with 72 junior high school students. Content analysis was used to evaluate the data in this qualitative study. Furthermore, five creative problems developed by Smith were employed by researchers as data collection tools. The applied problems were reviewed according to Polya's problem-solving stage. The problems were assessed in steps such as understanding the problem, choosing the strategy, utilizing the chosen approach and evaluating the solution. While examining students' homework problems, it was found that they were more successful at solving problems they had met in the past, or that situations were more similar to the problems they addressed. It was reported that the majority of students had difficulty in solving unfamiliar problems in curriculum and textbooks.

G. Larina [9] analyzed 83 algebra problems that teachers gave to middle school students to clarify the relevance of the situation and mathematical model. As a result, only a few problems seemed to satisfy the typical characteristics of problems in the real world. Thus, part of the tasks that the teacher assigned to the students did not meet the standards of the real-world problem outlined in the theoretical model.

Mulyono & R. Hadiyanti [11] deployed a study to examine and analyze the quality of problem-based learning and investigate metacognitive problem-solving abilities. This study employed a mixed research method with 12th graders. The results showed that students with an implicit level of metacognitive use were able to complete tasks given, but did not understand the strategy used. Students with a degree of cognitive service were able to solve problems, could build new knowledge through problem-solving to indicators, understanding problems, and identifying strategies used. Meanwhile, students using cognition could apply a variety of appropriate techniques to resolve problems and achieve high performance and outcome indicators. The authors also suggested that teachers needed to know about students' metacognitive activity as well as its relationship to problem-solving.

G. Özsoy et al. [14] investigated the correlation between students' reading level and math problem-solving skills by qualitative research method. The study subject consisted of six third graders with different reading levels. Data were collected through reading texts and related student work when they addressed problems. The first finding was that the problem-solving skills of the students changed with their level of reading. The authors, R. E. Simamora et al. [15] researched learning materials developed based on guided discovery learning models, including local culture integration. The results of the experiment revealed that the material-based guided
discovery learning improved students' problem-solving abilities significantly. Mathematical teachers were recommended here to try to develop suitable learning materials and to integrate local culture into math learning.

2. Theoretical Framework

2.1. Real-world Problems in Mathematics Education and Their Role

A problem with real content (also called a real-world problem or a problem associated with reality) is a problem that in assumptions or conclusions has content related to fact. According to author H. X. Thanh [17]: "A practical situation is the type of situation in which its object contains elements that bring real content, including human impact activities to real transformation. Practical situations are the type of situation in which to address it needs human-historical and socially physical activities to change nature and society".

For a "real situation" to become a "real-world problem", it is necessary to identify the requirements that need to be solved from the situation and identify the facts of the object that make the problem. For example, "A person who saves 100 million VND in a bank needs to find time for this person to earn double the money". This problem is known as a practical one. "A person deposits 100 million VND into the bank with an interest rate of 11% per year. Assuming this interest rate does not change, and the interest after the term is added to the capital. Ask after how many years they earn double the original capital?"

This content is a problem in the real world designed to handle the current situation above.

When setting up this problem, one must select and gather relevant data such as capital amount, interest rate, interest calculation method, to make assumptions for the problem (yes many other factors in the situation were ignored, not included in the problem). In fact, in teaching mathematics in high school, often practical concerns are expressed right under a real-world problem, that is, students are frequently asked to deal with real-world problems with infrequent circumstances.

From the above points of view, it can be understood that the content of "practice" here has the scope of:

(1) Mathematics practice: a new situation posed due to the development needs of mathematics is also the "practical" root of new knowledge, often appearing in textbooks.

(2) Practices in other subjects in high school such as Physics, Chemistry, Geography, Biology: a practical situation posed by the needs of that subject is the root of "practice" of a mathematical knowledge often appearing in textbooks, math textbooks as well as in other topics.

(3) Practices in science, engineering: stemming from the need to solve a problem in the field of science and technology, people need "mathematical tools", and thus leads to the need to build and use knowledge of mathematics.

(4) Finally, practical situations in life in general: derived from the demands of real-life around people, one needs tools of mathematical knowledge. To clarify the relationship between mathematics and practice in this situation and help general students understand the application of mathematical tools is not easy.

Since then, to fit into the practice of teaching and learning practical problems in high school, the problem with actual content is understood by the problems due to all real situations. The acts mentioned above bring about but are limited to the scope of "containing elements of mathematical tools" and the explanation and clarification of "practical sources or applications of mathematics" is only of relative meaning. Correspondingly, students have to comprehend it all.

The role and meaning of real-world problems

According to author L. V. Tien [18], mathematical knowledge must not be given but must be created and built starting from mathematical solving activities of students. Indeed, students develop their mathematical understanding by solving mathematical problems. Studying mathematics is learning to raise, present and resolve problems; learn to review problems in the light of theoretical tools arising from the demands to cope with problems.

In teaching, each problem used has a specific purpose and function. Some roles and meanings of real-world problems are found as follows:

(1) If practical problems with attractive, possible situations are used to start, putting the problem into a new lesson or consolidating knowledge will stimulate curiosity and desire to address students' questions. Thereby, students see the two-way relationship between mathematics and practice creating excitement, motivating math learning for students.

(2) Also, they aid students in proper awareness of the origin and practical value of mathematics. As a consequence, they know the instrumental role of mathematics in real life.

(3) Then, they contribute to the development of general and specific competencies for mathematics, especially problem-solving competency - one of the essential skills in Vietnamese education.

(4) And finally, teachers regularly collect and design practical problems that will enhance teachers' understanding of their expertise, thus contributing to innovating teaching methods and assessing students' mathematical performance.

2.2. Problem-solving Competency

The ability to solve problems is one of the necessary...
common competencies needed for each person to be able to survive in society at all times. Hence, the formation and development of this capacity for high school students are vital. As defined in the PISA assessment [23], problem-solving competency is "the ability of an individual to understand and resolve a problem situation when the solution is not clear. It involves participation in that problem - demonstrating the potential to be a positive and constructive citizen." A similar concept has been put forward by M. A. Bhat [1].

According to author N. C. Toan [10], solving the problem is "intellectual activity, considered the highest level of complexity and awareness, because it is necessary to mobilize all intellectual capacities of personal. When solving the problem, the subject needs to mobilize memory, perception, reasoning, conceptualization, language and use emotions, the motivation for belief in their abilities and the capability to control emotions".

Author T. Vui [23] argues that an individual can resolve problems in a way that uses cognitive processes to tackle and deal with real-life contexts across disciplines where the path to finding a solution is not immediately apparent. Not only mathematics, science or reading are the areas of knowledge or programming relevant. Concerning authors' perceptions of the ability to solve problems, the capability of high school pupils to address their problems is their ability to coordinate the implementation of their personal experience, knowledge, and ability to deal with situations in their studies and their positive attitude to life.

The meaning of forming and developing problem-solving capacity for students

For students, the formation and development of problem-solving competence support students in understanding and mastering the actual content of the lesson. Hence, students can increase their cultural knowledge and improve it. The creation and development of problem-solving competency help students know how to apply collective knowledge into real life. The production of problem-solving capacity also assists students in forming communication skills, organization, thinking ability, cooperative spirit, and community integration.

For teachers, the formation and development of problem-solving capacity can aid teachers in evaluating quite correctly the ability of students to absorb and their level of thinking, facilitating the classification of students precisely. Also, the teachers have the right conditions to mould the misleading, inaccurate, oriented knowledge necessary for students. They quickly know how to comment, assess, and apply theory to students' social practice. From here, they can orient the method of ideological learning for students.

Assessing students' problem-solving competency

It is possible to classify and describe the types of reference assessing problem-solving capabilities such as standard reference, output standard reference, personal reference, and criteria reference.

1. Reference, according to standards, often refers to achievements, relative rankings between individuals and more statistical nature. There are two types of comparison, including comparing the relative performance of this individual with other individuals in the same group and comparing the personal account with a representative group.

2. Reference, according to output standards, is a comparison of personal achievement against the educational Program's output standards - is the knowledge, skills and level of development of specific competency components. The students can be expected to know and achieve after completing the Program. The considered individual achievement has been produced or not yet met compared with the prescribed standards.

3. The personal reference is the type of evaluation focused on individual learner values, aspirations and expectations rather than external criteria. Personal achievements will be explained from the current performance compared to past performance.

4. Reference, according to criteria, demonstrates individual achievements according to the level of behaviour performance through the completed tasks. Results are described from the relative position of personal competence on the development path and are seen as a sign of development. Evaluation according to criteria to determine the location of individuals on the road of capacity development, determining whether students can perform a specific task.

In this paper, the researchers choose to refer to criteria to evaluate students' ability to cope with real-world problems. Assessing students' problem-solving competence is the process of teachers collecting information and analyzing collected information; awareness about the problem-solving capability of students; feedback to students, schools, families evaluation results. Since then, there is a measure to foster and train problem-solving competency for students. Students are trained and developed to solve problems in problem-solving activities.

Evaluating students' real-world problem-solving skills means assessing the components of the real-world problem — resolution of skills including the ability to understand problems, acquisition of practical situations, the capability to convert data from possible cases into mathematical models, the capacity to search for strategies for the resolution of mathematical models. Based on the learning process and activities when solving real-world problems of students, the researchers propose the evaluation criteria in the following table:
A Case Study of Teaching Real-world Problems Related to Exponential and Logarithmic Equations to Develop Students’ Problem-solving Competency

Table 1. Criteria for evaluating real-world problem-solving competency

<table>
<thead>
<tr>
<th>Component capacity</th>
<th>Evaluation criteria</th>
<th>The degree of evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Capacity to detect problems</td>
<td>Find out, identify problems to solve</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ Identify the wrong problem (0 points)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ Understanding is not the right problem; there are small errors (1 point)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ Understand the problem (2 points)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gather information related to the problem (list of mathematical data and data related to the problem)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ Collect incomplete and not fully understood (0 points)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ Collect information but it is incomplete and inaccurate (0.5 points)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ Collect full and accurate (1 point)</td>
<td></td>
</tr>
<tr>
<td>2. Capacity to propose solutions</td>
<td>Convert given information about mathematical models</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ Contact related knowledge and information (1 point)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ Express the problem in the mathematical language (1 point)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Find solutions to solve math problems</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ Wrong and ineffective solution (0 points)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ Correct solution, but with small errors (0.5 points)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ Correct and detailed solution (1 point)</td>
<td></td>
</tr>
<tr>
<td>3. Capacity to solve problems</td>
<td>+ Wrong calculation (0 points)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ Calculate correctly but the solution and tool are not optimal (1 point)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ Correct calculation and optimal solution method (2 points)</td>
<td></td>
</tr>
<tr>
<td>4. Capacity to evaluate implementation results</td>
<td>Review and select results (1 point)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Answer the requirements of real-world problems (1 point)</td>
<td></td>
</tr>
</tbody>
</table>

On that basis in Table 1, the researchers propose the achieved levels of real-world problem-solving competency of students in the following table:

Table 2. Ranking evaluation of real-world problem-solving competency

<table>
<thead>
<tr>
<th>Point</th>
<th>Classification</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>From 1 point to under 5 points</td>
<td>Weak</td>
<td></td>
</tr>
<tr>
<td>From 5 points to under 6.5 points</td>
<td>Medium</td>
<td></td>
</tr>
<tr>
<td>From 6.5 points to under 8 points</td>
<td>Fairly good</td>
<td>Criteria for converting given math information about math model that does not get 1 point or approach for solving a math problem that does not get 2 points</td>
</tr>
<tr>
<td>From 8 points to 10 points</td>
<td>Good</td>
<td>The criteria for converting given information about a mathematical model gets 1 point and the requirements for solving a math problem gets 2 points.</td>
</tr>
</tbody>
</table>

To evaluate the ability of students to address real-world problems, teachers can observe problem-solving, product research, and inquiry processes between teachers and students when solving problems and teachers (or students compare) with levels developed to assess the ability level of students. As a result, teachers can evaluate and rate the problem-solving skills of students based on Table 2.

2.3. Exponential and Logarithmic Equations in the Context of Vietnam

In Vietnamese Mathematics program [10] as well as a textbook [6], the content of exponential and logarithmic equations has a lot of practical relevance. At the section of forming the problem of creating the concept of a basic exponential equation, the textbook has applied practical examples to lead students into new knowledge. These examples point to students very interested in learning and solving this warm-up problem. However, this kind of mathematics is not used in the exercises. And through the fact that high school graduation exam in recent years, real-world problems appear more and more and the topic of exponential and logarithmic equations is also the main content to build this type of mathematics.

Thereby, it is necessary to overcome the problematic teaching situation about knowledge transfer, but apply less knowledge into practical problems, shift from teaching and content evaluation to teaching and testing. Assessment is essential in line with the student's skills approach. Enhancing the applicability of mathematics into practice helps students see the critical role of mathematics in their daily lives, and at the same time stimulates their desire to explore, then contributing to renewing the
teaching method.

The two primary mathematical forms associated with exponential and logarithmic equations include \( a^x = b \) and \( \log_a x = b \). In particular, solving the above types of math requires students to understand the related concepts such as solutions, sets of equations, equivalent equations and consequences; equivalence transformations, consequences. Also, students need to distinguish two solution methods called exponentiation and logarithm. The exponentiation method increases the solution set, and the logarithm process reduces the solution set. In other words, students performing the exponential way can lead to a consequence equation.

**Objectives of the Study**

1. Develop students' problem-solving competency to use exponential and logarithmic equations as a tool to resolve relevant real-world problems.
2. Help students to realize the role and meaning of mathematics in practice.

**3. Materials and Methods**

**3.1. Participants**

The subjects were students who have learned knowledge about exponential and logarithmic equations and were students of the Center for Vocational Education - Continuing Education in Cho Lach District, Ben Tre Province, Vietnam. The experimental class was the chosen class in the centre, with good and excellent students, and was relatively more regular than the other classes.

**3.2. The Process of Research**

The process of the study can be illustrated in Figure 1 below.

Step 1: It is necessary to find out about students' competencies before organizing a teaching experiment. Below is the year-end average score table for the experimental class.

<table>
<thead>
<tr>
<th>Score Range</th>
<th>No.</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.5 ≤ score &lt;8</td>
<td>26</td>
<td>65%</td>
</tr>
<tr>
<td>8 ≤ score ≤ 10</td>
<td>13</td>
<td>32.5%</td>
</tr>
<tr>
<td>5 ≤ score &lt; 6.5</td>
<td>1</td>
<td>2.5%</td>
</tr>
</tbody>
</table>

With such scores in Table 3, the experimental class is expected of enough basic qualifications and competencies to participate in the process of solving practical problems because they are not easy for many students.

Step 2: Thanks to the results of step 1 together with the content analysis of exponential and logarithmic equations in grade 12 textbooks, a lesson plan is designed with the desire to promote real-world problem-solving, forming and developing relevant student competencies.

Step 3: Next, to see the effectiveness of the lesson plan, students need to be organized to teach according to this lesson plan.

Step 4: In this step, problem-solving competency will be analyzed and synthesized thanks to solving practical problems integrated with the lesson plan taught.

Step 5: It's time to re-assess students' problem-solving abilities to some degree, so they need to be taking part in a test. Components of problem solvability will be thoroughly analyzed to measure its progress. In this part, the competence assessment scale mentioned in the theoretical framework will be utilized to evaluate each component capacity of the problem-solving competencies accurately.

In this step, the problem below is considered as a tool to get data, from which to assess students' ability to solve mathematical problems.

**Figure 1.** Diagram illustrating the steps of the study
**Problem:** A particular breed of a rabbit was introduced to a small island eight months ago. Currently, the number of rabbits on the island is estimated at 4100 and doubled after three months. How soon does the rabbit population reach 20,000?

Students answer the following requirements:
1. Find out the problem, collect information from the given problem.
2. Convert given information about a mathematical model.
3. Look for a strategy to solve the mathematical model.
4. Present the solution to the problem in the form of a mathematical model.
5. Consider and choose the results to answer the original problem.

Pre-analysis of five requirements

- Item 1: The purpose is to evaluate the ability to detect the problem and gather information from the original problem. Here, students need to determine what the requirement of the problem is. Students are expected to understand information about the number of rabbits or the relationship between time and rabbit growth and double the number of rabbits in three months. The researchers make two suggestions in questions 2 and 3 to evaluate the proposed solution.

- Item 2: Concerning finding a way to solve the problem, students are required to fulfill this requirement. Students need to establish a mathematical model of the problem, which is to find time $t$ in the formula $n(t) = n_0 2^{t/3}$. This capacity is also known as mathematical modelling capacity.

- Item 3: When a mathematical model has been established in item 2, the initial problem is said to resolve the equation $20000 = n_0 2^{t/3}$ to find $t$. It requires students to apply knowledge and skills learned to analyze and find solutions.

- Item 4: Students present the solution to the "pure mathematics" problem. For item 5, students need to consider and select the results of solving the mathematical model. Students should pay attention to the number of months that should be a natural number that satisfies the requirements of the initial problem.

3.3. Designing the Lesson Plan

The lesson plan is designed to include four main activities such as warm-up, experience, practice and reinforcement, expanse.

(1) Activity 1: Warm-up (4 minutes).

The purpose is to create a learning attitude for students, helping them be aware of their learning tasks and interested in learning new lessons. Teachers create learning situations based on mobilizing students' knowledge and experiences related to issues to be discovered. From there, it offers assistance to students in thinking and expressing their notions about the problem they are about to learn. Consequentially, the questions or tasks in the warm-up activity are questions, or open-ended, which do not need students to have complete answers.

The teacher will show on the slide containing the pictures with the questions in Figure 2 below. These questions should not be answered and should only be considered by students. These questions aim to introduce the objective of the lesson, which can create curiosity and excitement in learning.

![Figure 2. Two pictures in the warm-up activity](image)

(2) Activity 2: Experience (16 minutes)

The teacher will show on the slide containing the content of the problem

“The probiotics B. Subtilis reproduce in a dichotomous pattern with a generation time (the time from the birth of a cell to a population doubling of cells) of 30 minutes. Assume that the probiotic count is not killed during dichotomy. With the number of probiotics in the community initially one probiotic, how many hours does it take for an initial B. Subtilis cell to divide to the number of probiotics obtained after the following turn:

- $a$. $2^{10}$ beneficial bacteria.
- $b$. $5^{20}$ beneficial bacteria.

Explaining some pedagogical options:

The problem poses two questions $a$ and $b$ for them to apply the mathematical model they built in combination with mastering the form and solving basic exponential equations $a^{ax} = b$, $(a,b > 0, a \neq 1)$. With the mathematical model manipulated in step 2 combined with finding a strategy to deal with the problem using knowledge of basic exponential equations to address, students will solve step 4 in the process of solving the problem—practical math-oriented problem-solving capacity development. Nonetheless, in two items, $a$ and $b$ force students to adapt to their solving approach to find consistent results.
The teacher prepares the following forms of work. Worksheets 1, 2 which students use to do the homework will be returned to the teacher. Study report that students use to record the teacher's combined results, then they will keep this form throughout the lesson. The teacher will give worksheet 1 (A4 paper) to each student. They will answer some questions on the worksheet in 8 minutes. The teacher should note: Students should present according to their thoughts and arguments, they should not discuss. These are survey questions with no points, so students are confident to do the test independently. Teacher collects worksheet 1 and summarizes by slide. Students record the results the teacher summarizes in the study report. There are some questions for students as below:

(1) How do you find out the problem, collect information from the given problem?

(2) Can you convert given information about the mathematical model?

(3) Why not do you look for a strategy to solve the mathematical model?

(4) Can you present the solution in the form of a mathematical model?

(5) Can you consider, choose the results to answer for the initial problem?

With these suggestions, students are hoped to form the habit of converting information from actual problems to mathematical problems. Hints are included to help students approach the capacity for modelling. Thereby, they will get a better understanding of how to save data from real-world problems to mathematical models and can apply them when encountering similar real-world problems.

(1) Activity 3: Practice (20 minutes)

The objective of this activity is to aid students in consolidating and perfecting their acquired knowledge and skills. The teacher distributes worksheet 2 to each group. In particular, the teacher will divide the experimental class into four groups. Worksheet 2 includes four problems with the assignment. Group 1 solves problem 1; group 2 solves problem 2, group 3 handles problem 3 and group 4 copes with problem 4. Each group has 13 minutes (10 minutes to discuss and fill in questionnaire number 2, and after 3 minutes, representatives of the group will briefly present their ideas and solutions). After 10 minutes, the teacher collects worksheet 2 and slides the necessary information to fill in the worksheet 2. Students observe and record in their study report and ask questions (if any) (7 minutes).

**Problem 1:** "From June 19, 2012, Agribank announced to apply a new interest rate for a 12-month term in VND for individual customers. Assume that the interest rate remains unchanged and the interest after maturity is added to the principal. Uncle Tu sends VND 10 million according to the above term, after two years, uncle Tu will receive VND 12.321.000. If Uncle Tu has 500 million VND, how many years later will Uncle Tu get double the initial capital?"

**Problem 2:** "The population growth is calculated by the formula \( P_n = Pe^{nt} \), in which \( P \) - the population of the year is taken as a milestone, \( P_n \) is the people after \( n \) years, and \( r \) is the annual population growth rate. Know that in 2016, the world population was about 7.29 billion, and the population growth rate was 1,07% (according to the Bureau of Statistics). If the population growth rate does not change, when will the world population reach 10 billion people?"

This situation is the problem of population growth; this growth also obeys the exponential law that the problem has given the model. \( n \approx 29,5 \) will be identified thanks to the formula \( P_n = Pe^{nt} \). And finally, the answer to the problem will be that by 2046, the world's population will reach 10 billion people.

**Problem 3:** "Radioactive carbon dating method - is an archaeological method used to determine the age of ancient objects. Carbon dioxide gas in the atmosphere always contains a fixed fraction of radioactive carbon (\( C-14 \)), with a half-life of about 5730 years. The plants absorb carbon dioxide from the atmosphere, which then travels to the animals through the food chain. Accordingly, all living organisms \( C-14 \) and \( C-12 \) are proportional and constant. After a plant dies, it stops assimilating \( C-14 \), and its contents begin to decay exponentially, but the amount \( C-14 \) remains constant.

This situation is the compound interest problem, the formula of compound interest \( nPP r \) = \( 11% \) \( n \approx 6,64 \) and conclude: After seven years, Uncle Tu will get double the initial capital.

**Problem 4:** "The average family income in a year - half of all households have the same or more revenue, and half have lower income. The average family income can be approximately equal to a function \( f(x) = 22751 + 82171 \ln(x+1) \), where \( x=0 \) corresponds to 1990. (Data from US Census Bureau). When will the average family income reach $50,000?"
This situation is a problem with the social field for which the students know the mathematical model. Based on the formula $f(x) = 22751 + 8217 \ln(x+1)$, the students will be comfortable to find out and conclude: Around 2017, the median family income of the United States reached $50,000.

Explaining some pedagogical options:

The researchers want to check what students have learned after this lesson. Are they able to apply the process of solving real-world problems in addressing problems related?

The problems with such actual content are exciting and show students, in particular, the role of mathematics in life. With this approach, the researchers want to form a habit of solving a real-world problem in students. The requirements outlined in the worksheet 2 are directed to guide students to approach five steps in the process of solving real-world problems according to the development orientation of problem-solving competency of students. This approach will allow students to understand this process more effectively and apply it in the face of similar practical problems. Nevertheless, for these four problems, the researchers choose four different fields that have the application of exponential and logarithmic equations, and these four problems are other from the real-world problem of knowing the mathematical model. The aim is to check that when a problem has known the mathematical model, and a problem does not know the mathematical model, which problem students will encounter obstacles and then orient to overcome that obstacle.

(4) Activity 4: Reinforcement and expansion (5 minutes)

(i) **Reinforcement:** Please repeat the steps to solve a real-world problem?

The expected answer from students: There are five steps.

   Step 1: Find out the problem, collect information from the given problem.
   
   Step 2: Convert given information about the mathematical model.
   
   Step 3: Search for a strategy to solve the mathematical model.
   
   Step 4: Present the solution to the problem in the form of a mathematical model.
   
   Step 5: Consider and select the results to answer for the initial problem.

(ii) **Expansion:**

The aim is to help students never stop with what they have learned and understand that in addition to the knowledge gained in school, many things can and should continue to be determined, contributing to lifelong learning. The amount numbered in the lesson is enormous, so the calculator can not express its value completely. And in fact, students can only find out how many digits it has when written in the decimal system. Here's how to do it:

   (1) By taking the approximate value of $\log_2 \approx 0.3010$.

   Then:

   - The number $2^{336}$ has $[336 \times \log_2] + 1 = [336 \times 0.3010] + 1 = 102$ digits.
   
   (2) By taking the approximate value of $\log_5 \approx 0.6989$.

   Then:

   - The number $5^{210}$ has $[210 \times \log_5] + 1 = [210 \times 0.6989] + 1 = 147$ digits.

   The teacher will summarize the significant results and show them on the slide.

### 3.4. Data Analysis

The data collected included students' work in practical instruction as well as student tests during the reevaluation of students' problem-solving abilities. These products were carefully analyzed qualitatively to clarify how well students achieve the component competencies associated with problem-solving capacities. Additionally, the level of achievement was attributed to the score to have a basis for assessing the relevant skills according to the mentioned theoretical framework. In general, data were analyzed qualitatively to clarify students' answers and arguments. To explain how useful the experimental lesson plan is, some teachers also recorded and interpreted their observations.

Typically, researchers can follow the following procedure: designing a pre-test to evaluate the capacity of the research sample, then teaching the impact of the new method, and finally evaluating the effectiveness of this teaching method by a post-test. For evaluation, the researcher can use quantitative analysis to compare the initial results and the outputs, using mathematical statistics to quantify the two results. However, in this study, the analyzed outcomes were tied to the teacher's experimental teaching process, and the students' assignments were explained clearly and compared with the researcher's original intentions, as well as the ability to solve problems of students was assessed according to the scale given in the theoretical framework.

### 4. Results and Discussion

#### 4.1. Findings Related to Teaching the Lesson Plan

4.1.1. Activity of experience

(1) Question 1: How do you find out the problem, collect information from the given problem?

Students only had to summarize the information the problem provided to answer this question. Thirty-three students were giving correct answers. The problem was re-recorded to four students to answer question 1. Three students had false or unclear answers because they did not understand the significance of the information.

Most students understood how to summarize the problem, ensuring essential facts to build mathematical models. This step was a simple job that in basic
mathematics, teachers often practised a similar activity, which was to find assumptions, make conclusions before going into solving problems.

(2) Question 2: Can you convert given information about the mathematical model?

In question 2, there were four hints to guide students to answer question 2:

(i) Hint 1: 40 students correctly answered the hint 1. However, only 20 students presented the expected number of cells $2^1, 2^2, 2^3, 2^4, 2^5, 2^6$. The remaining 20 students presented $2, 4, 8, 16, 32, 64$.

(ii) Hint 2: How many cell division after 180 minutes? What is the number of cells?

The results of 6 divisions were correctly replied to by 40 students. Nevertheless, only 15 students saw a relationship between the number of cell division and the number of cells obtained $2^n$.

(iii) Hint 3: Which word shows that after any time, how many cell divisions? Many cells obtained?

The hint 2 was a guide to concluding the hint 3. Entirely as expected, students who saw the relationship between the number of division times and the number of cells obtained would give suggestions 3. Twenty-two students answered correct words: "The number of cells derived is $2^{n+t}$". In which, there was a student tabulated to show the relationship between time, the number of cell division, the number of cells obtained. Fourteen students did not answer hint 3.

(iv) Hint 4: To put the initial problem on the math problem, which factor should you set the variable to be?

When teaching about the variable setting, the teacher often suggested to students how to set the variable. If the variable is $x$, the value of the variable is $f(x)$. Here variable $t$ is time, so it is usually set then its value will be $f(t)$. Nonetheless, this is the number often denoted $n$ so that most students choose $nt$ to be the value of the variable $t$ to set.

After completing four hints, students rewrote the obtained mathematical models and switched from the language of the original problem to the math problem. Nearly 55% of students read the problem and completed four suggestions carefully and presented the model as required. Also, incorrect answers or not answering question 2 were found in 45% of students. These data showed that before addressing the problem, they were not focused on reading and analyzing mathematical problems.

Through the above question, it was realized that they discovered the relationship between the math problem and the practical problem and also converted the actual problem into the math problem. That was what the researchers wanted when surveying step 2 of the process. However, the results were only average. In the teaching process, therefore, teachers were believed to strengthen problems in the real world so that they could apply mathematical knowledge. Thereby, the teachers not only assisted the students in inculcating knowledge but also trained them to be capable of solving real-world problems.

(3) Question 3: Why do not you look for a strategy to solve the mathematical model?

To limit the strategy of using the testing calculator of the equation, the researchers gave relatively large numbers to determine the students to use the approach of using the testing calculator of the equation. Only 25 students issued a solution strategy. In which 22 students gave the mathematical model in sentence 2, so they proposed solutions for the mathematical model that were solutions to exponential equations. Three students answered to question 2 but still explained answer 3 using the method of solving exponential equations. The researchers temporarily examined the problem and found that the number of beneficial bacteria was increasing exponentially. Nevertheless, their mathematical knowledge was limited, so they did not create an entire mathematical model. The exponential solution method could instead be used.

(4) Question 4: Can you present the solution in the form of a mathematical model?

Furthermore, students had to use exponential equations to resolve this problem to avoid using calculators. Twenty-two students presented the right solutions. Among 22 students who answered correctly, the researchers did not find any answer that mentioned the condition or unit of the variable $t$. This activity affected the problem much later. When answering question 5, they found it challenging to consider and select the results to respond to the first problem.

(5) Question 5: Can you consider, choose the results to answer for the initial problem?

Out of 22 correct answers to question 4, there were only 12 answers with five correct answers. So the wrongest items were to immediately conclude the value of $t$ just found in sentence 4 was in minutes unit without changing into hours.

4.1.2. Activity of practice

In this activity, the class was divided into four groups (each group of 10 students). Each group solved a problem according to the group’s number by the process of solving the practical problem through worksheet 2. When analyzing the results obtained from the groups, The researchers recorded the results as follows. All four groups answered completely and quite accurately according to the requirements, including the work of Group 3 in Figure 3 below.
The researchers did not let the groups present themselves but let them do the work in the form of filling in the blanks in a particular order. Here, the researchers did not tie them to a specific pattern but only aimed to guide them through the steps of the process of solving real-world problems. All four problems with mathematical models were available. In comparison with the experience of the unknown mathematics model, the researchers intended to examine this. Students, who solved the obstacles immediately step 2 were to convert information from the initial problem about the mathematical model, in this practice activity, for a problem that already had a mathematical model, see if solving the actual problem had any other obstacles.

When analyzing the results of the four groups, the researchers found that all had reasonable solutions, especially in collecting information from the initial problem, finding a strategy to address or solving math problems, students did very well. From there, it was drawn that students still had difficulty in converting information from real-world problems about math problems (or modeling capacity) of students still limited. Nevertheless, the exercises of the four groups still contained some errors. Similar errors were made in the research of M. D. Haryanti et al. [7]. In group 3, the math problem was clear and precise.

Nevertheless, in answer to the results for the real-world problem, when they got the conclusion that the mummy was 2602 years ago, they did not know how to determine the date of the mummy. Group 4 had confusion in the last step when solving exponential equations $e^{\text{something}} + 1$, leading to erroneous results, deviation one year. To demonstrate this, the solution of Group 4 was illustrated in Figure 4 below.

The teacher posed questions to reinforce knowledge about the process of solving real-world problems.

+ Teacher: In my opinion, what should we do to solve a real-world problem?
+ Student: To resolve a real-world problem, first of all, we need to carefully read the problem and then determine the variable to be set.
+ Teacher: When setting variables, do we need to pay attention to anything?
+ Student: Yes, when setting the variable, we need to select the variable's condition.
+ Teacher: After setting the variable and the variable's condition, what steps do we do next?
+ Student: Yes, based on the date of the problem to denote the unknown quantities through the set variable. Then, they defined a function or a variable equation and solved the problem. Finally, they answered the practical question.

(2) Expansion Activity

The teacher posed questions to create students' curiosity and desire to learn about the knowledge they wanted to know:

Teacher: In the lesson, there appear numbers with great value such as $2^{336}$ and $5^{210}$. Do you know what the importance of these two numbers is?
Student: No.
Teacher: Up until now, when written in the decimal system, only those numbers can be found how many digits does it have. Would you like to know this determination?
Student: Yes.
Teacher: By taking the approximate value $\log 2 \approx 0.3010$. Then, $2^{336}$ has $[336 \times \log 2] + 1 = [336 \times 0.3010] + 1 = 102$ digits.
Teacher: Similar to the above, please determine how many digits there are in the number?
Student: By taking the approximate value of $\log 5 \approx 0.6989$. Then, $5^{210}$ has $[210 \times \log 5] + 1 = [210 \times 0.6989] + 1 = 147$ digits.

Through two experimental sessions and after collecting and processing relevant data, the following results were found. Concerning the problem-solving capacity, the component competencies included the ability to understand the problem, to obtain information from real situations, the ability to convert data from practical concerns about mathematical models, the ability to seek strategy for mathematical models, the ability to execute results-solving strategy and the ability to switch from mathematics resolution. The students' competencies increased significantly compared to before the experiment when they addressed mathematical concerns in experimental activities.

In terms of learning spirit as well as attitude, according to comments of teachers participating in practical teaching hours, most students were proactive, active, self-reliant and creative in problem-solving activities. A teacher wrote the following: "I am allowed to attend a teaching period with a
new form, so I enjoy and want other lessons and other subjects to apply to synchronize innovation in education, thereby improving the efficiency of teaching and learning activities”.

4.2. Results Associated with the Post-test to Evaluate Students’ Ability to Solve Real-world Problems

Based on the criteria in the theoretical framework, the students’ tests were scored on a scale of 10. From here, the researchers evaluated the learners’ ability to resolve problems when solving real-world problems.

Table 4. Statistics results of assessing students’ competency to solve problems

<table>
<thead>
<tr>
<th>Classification</th>
<th>Good</th>
<th>Fairly Good</th>
<th>Medium</th>
<th>Weak</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount</td>
<td>24</td>
<td>5</td>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>Percentage</td>
<td>60%</td>
<td>12.5%</td>
<td>5%</td>
<td>22.5%</td>
</tr>
</tbody>
</table>

Based on the data from Table 4, it showed that up to 60% of students handled problems in good practice. They all identified information and problems to be addressed. They knew how to convert from the data of the original problem about the mathematical model, the ability to propose the right solutions and present relatively good solutions. At the fair and average level, the number of students was small. Nonetheless, there were still nine students (22.5%) whose capacity to cope with problems was weak because they were not able to convert information from given problems about mathematical models. Some specific capabilities of students had to be analyzed in detail.

The students’ ability to learn problems and collect information from real-world problems was quite good. They knew the basics and information they needed. Most of them found that the problem had to be resolved; some children had written down the original problem. Since then, it was revealed that it was necessary to practice more skills to summarize the requirements of the problem neatly and concisely.

Capacity proposing solutions included two criteria for the requirements of converting problem information about the relatively satisfactory mathematical model. Eight students could not turn the problem of the mathematical model at all, thereby not solving the problem requirement. This skill was an essential and decisive criterion. They were able to transfer information about the actual problem to the mathematical model for them to seek a strategy to solve the problem and thereby address the problem.

Regarding criteria of searching for solutions to solving mathematical problems, 27 students gave more than one answer, of which 18 students provided the solution was to press the calculator. The ability to deal with the problem was shown relatively well; most of them who could identify the mathematical model presented reasonable solutions. Specifically, 25 students chose how to perform the answers for the strategy of solving exponential equations, and six lessons for selecting the presentation method were calculator presses (See a solution in Figure 5).

![Figure 5. The work of a student with the right solution due to pressing a calculator](image)

About the ability to evaluate good performance results, 30 students all chose the answer "After nearly 15 months, the number of rabbits on the island reached the number of 20,000", ten students had no answers because questions 2, 3, 4 had no solutions. The results have been selected by one student that did not meet the problems. Thus, the majority of students were able to detect the problem that needed to be solved and were able to apply knowledge of solving exponential equations to deal with real-world problems associated with the number of rabbit populations.

Evaluation of each step in the process of solving the problem, it was realized that students were still weak in step 2 - converting given information to the mathematical model. Students wanted to do this step well; it was required that they understood the problem clearly and made fair use of relevant knowledge. This stage was a critical step in the problem-solving process. If the mistaken math model were found, the next steps in the process would be affected. Particularly in step 4, namely, solving math problems, the students were highly appreciated their ability to resolve math problems related to some principal types: \( a^{\log_b x} = x \), \( \log_a x = \frac{\log_b x}{\log_b a} \), and \( \log_a (xy) = \log_a x + \log_a y \).

In particular, they were not able to check the solution in step 5, and the conclusions were inaccurate. In general, comparing between solving practical problems in experiential activities and practical problems, their ability to address real-world problems was more advanced. Accurately, in teaching activities, only 55% of students handled the problem while in the temporary problem, 75% of students solved correctly to the final result.

The problems in the lesson and the post-test were linked to certain areas, such as biology, economics and archaeology. From there, the students were able to see that knowledge of exponential and logarithmic equations was not only applied to solving pure mathematical problems but could be regarded as tools and means to solve real-world problems in the above subjects. For this reason, students would see the importance of solving math problems in everyday life.

5. Conclusions and Suggestions

In consequence, the students used the method to solve problems from the real world. However, due to frequent
contact with mathematical problems in textbooks, when approaching these types of math, they ignored the relationship between the mathematical solution and the practical solution as well as forgot about the demand to check mathematical answers with real situations to verify whether the results were reasonable or not. This vital activity was mentioned in the research of L. V. Tien et al. [19]. The lesson plan partly helped students access the process more, especially in step 1, step 2, step 3, step 5, because they got only accustomed to doing step 4 (solving math problems).

Some students still had many difficulties when performing step 2, which was converting given information about the mathematical model. Also, it was documented that because of problems with known mathematical models, they identified and resolved quickly and accurately. For problems that did not give a model, the majority of students could not provide the mathematical model, and from there, they could not cope with the problem. The researchers also organized for students to work in groups to create conditions for them to interact with each other for them to recognize their shortcomings; in studying, they would progressively perfect themselves.

The lessons not only helped them have the proper motivation to learn when studying about exponential and logarithmic equations but also supported them in seeing the role of mathematics in practice. In the teaching process, teachers should introduce problems in different fields so that they could grasp a lot of knowledge and also created conditions for them to see the role and meaning of mathematics in practice. This way was the trend of integration which the educators tried to achieve. Moreover, teachers also had to encourage learners to explore and develop a variety of ways to solve a problem to boost their thinking. Teachers organized and taught real-world problems to establish student problem-solving skills. In other words, the teachers act as enablers to help students approach real-world problems [5]. This approach was shown through the students’ proactive attitude to learning and positive comments from teachers observing the teaching period.

The results from teaching experiments and tests to assess students’ competencies allow drawing some implications. Firstly, teaching practical problems not only helps students review basic knowledge but also support students in connecting between mathematics and real life, see the meaning and importance of learning mathematics. The second way is to assist students in getting more interested in and active in math-learning by using real-world problems in the teaching process. This phase stimulates and motivates them to learn and solve problems in the world around them, thereby contributing to the development of problem-solving competency for students. Then, all of the above has a great significance in the teaching process is that it aids both teachers and learners in determining the meaning of mathematics in life and realize what basic mathematical knowledge will be used to address real-world problems. Besides, teachers can use guided discovery teaching to provide students with access to math problem-solving [15].

Some recommendations are posed for teachers and students to approach teaching that develops students' competencies in mathematics. First of all, teachers must take time to read, carefully study the overall Program and subject program, in which the requirement to master mathematics objectives and the criteria to be met on the quality and competence of students. Also, they need to achieve a firm grasp of the mathematical goal for the whole grade level before setting a lesson goal. This activity helps teachers recognize student thinking skills, identify shortcomings in part of the content to take suitable measures. The teachers then determine a particular objective for each lesson based on the content of the textbook. The teacher develops teaching activities from a textbook's objective and scope. All teaching activities must define the objectives and build the students' capacity. Furthermore, for each teaching activity, teachers must create and choose to use teaching methods and forms to suit, in each of these methods, teachers use techniques and ways of assessment to develop students' competencies.

Moreover, the intimate connection between mathematics and practice must be recognized for students. It is therefore difficult to deal with problems of practical origin, but the mathematics is applicable. Mathematical problem-solving is an activity that requires students' thinking, so they need to mobilize some thinking actions such as analyzing, comparing, synthesizing and generalizing to be able to achieve the process of successfully solving the mathematical problem [12,21]. Additionally, children need to be aware that they should not be delighted with the knowledge gained from the curriculum or textbook but have situations arising from real life as a real-world problem to handle. More specifically, the problems not only exist in mathematics but also occur in several other subjects such as economics, biology and archaeology.

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