Model Reduction of Power System by Modified Balanced Truncation Method

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Abstract In this article, we are exploring and implementing the new model order reduction (MOR) method for Large-Scale Linear Dynamic System (LSLDS) to achieve reduced order. These technologies are designed to better understand and explain LSLDS based on the Modified Balanced Truncation Method (BTM). This refers to continuous/discrete LTI structures that are minimal / non-minimum. This reduced method allows MOR to preserve complete parameters with reasonable accuracy. The approach is based on the maintenance of dominant system modes and a relatively small state truncation. As the reduced-order model (ROM) is derived from the retention of dominant modes, the reduction remains stable. The main demerit of the balanced truncation method is that the ROM, stable states, does not match the original structures. By modified BTM to narrow the deviations in the ROM transfer function matrix, a gain factor is added to adjust the steady-state values of the reduction system without altering the dynamic behaviour of the system. The proposed method has been successfully applied to a real-time single area power system with ease of extension to a discrete-time case and the results obtained show the efficacy of the method. Application model and the results obtained indicate the effectiveness of the methodology. The time response of the system has been demonstrated by the proposed method, which proves to be excellent match, effectiveness and superiority compared to the response of other approaches in the literature review of the original system.

Keywords Model Order Reduction, Power System, Gain Factor, Balanced Truncation Method, Steady State

1. Introduction

Electric power technology is emerging as a result of electricity demand. As the most demanding artificial device, the entire power system consists of many processes, such as output, transmission and distribution. Systems are built in different areas, typically linked by transmission lines or linkages to tie-lines, painless areas [1][2]. This causes the system model to have an extremely high order. Both regional frequency and tie-line exchange variations in such interconnected power systems networks are often induced by unforeseen demand only for power loads, uncertain system parameters, modelling error and disturbances due to various environmental conditions[3][4]. Thus, during transient disturbances such as faults, line travel or any overload, the reliability of the power system is necessary to maintain the specified synchronism and voltage levels[5][6][7]. It is therefore capable of minimising the model of the power system so that simulation and controller design can be simplified [8].

Electrical energy technology is the biggest problem in all respects concerning the dynamic behaviour of a larger system [9][10][11]. All these complex and large systems with traditional techniques are difficult to model. The hybrid system has also been shown to be large (extensive) if it is intended for computational evaluation and practical reasons in different coordinated systems or small systems [12][13][14]. If not, the system is large and so large that conventional modelling, analysis, System design and

computing approaches cannot provide the appropriate solutions with reasonable realistic computing activities [15][16][17]. The study of such a physical device begins with a structured model that can be considered an enthusiast example. The control objective encourages the development and evaluation of a model [16][18][19]. In the first part, high-level negotiations on new technologies and the basic rules for a dense model system are considered both in view and in the industry[13][20]. Mostly in two ways, multiple MOR approaches have been given to frequency and time domains [21]. Researchers' methods of reduction have many advantages and drawbacks to them. A common flaw in the method is that even if the Higher-Order System (HOS) is stable, the reduced-order system is unstable [22][11][23]. Moreover, most of the time, the steady-state matching of the original system with its lower order representation fails. Additional drawbacks are poor accuracy in normal high-frequency ranges that may have non-minimum phase attributes [24][25][26] [27]. There are several methods available even more in the literature to the frequency and time-domain model of HOS such as a ROM algorithm using Pade Approximation [28][29][30][31][32][16][33]. MOR of state LTI systems based on the theory of balanced realization was firstly recommended by [34] in which the balanced realization term has been chosen to derive a balance between the controllability and observability of states in its system configuration[35]. The balanced truncated (BT) reduced model achieved by a balanced realized model following the truncation of less controllable and less observable states. It has been found that the model thus identified does not retain the DC gain or the steady-state of the actual (original) system[36]. Poor removal of the subsystem was introduced to maintain the balanced truncation benefit of DC gain with SPA [37][38][39][40][41][42][43][44]. The preservation of DC BT minimal system gains has been defined through a singular perturbation approach that is applicable when the system to be reduced is stable, minimal and internally balanced [45][44]. In this, the investigator suggested a modified BT approach applicable to LSLDS with outstanding DC gain matching [46]. The benefit of the approach lies not only to the matching of steady state but its applicability continuous to discrete time system as well, which has been confirmed through examples taken from literature.

2. Methodology for Rom

2.1. Problem Statement

Find the following State Space matrix n represented by a SISO system.

2.1.1. Brief description

This brief discussion focuses on computing the low-order approximation estimation of linear dynamic

systems.

Consider the system of continuous-time-LTI SISO state-space as specified

$$\Sigma: \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases} \Leftrightarrow \Sigma := \left(\begin{array}{c|c} A & B \\ \hline C & D \end{array} \right) \quad (1)$$

where $x \in \mathbb{R}^n$ is n -dimensional of the system state(1), $y \in \mathbb{R}^p$ is the system output of Σ , $u(t)$ is the input vector, respectively, $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $C \in \mathbb{R}^{p \times n}$ and $D \in \mathbb{R}^{p \times m}$. $x \in \mathbb{R}^n$ are state variables.

where n is a large-value system order, which is a high-dimensional system, with n extending from several tens/hundreds to thousands as available in large-scale dynamic system control systems [47][48][34][14].

Consider the transfer function of the n -dimensional SISO Dynamical system [49]. Equation (1) is in the form of a transfer function.

$$G(s) = C[sI_n - A]^{-1} B + D \quad (2)$$

$$G(s) = \frac{N(s)}{D(s)} = \frac{n_0 + n_1s + n_2s^2 + \dots + n_{m-1}s^{m-1}}{d_0 + d_1s + d_2s^2 + \dots + d_ns^n} \quad (3)$$

$$\text{or } G(s) = \frac{\sum_{i=0}^{m-1} n_i s^i}{\sum_{i=0}^n d_i s^i} \quad (4)$$

Where m is less than n and numerator polynomial (n_i) are constants coefficient and denominator polynomial (d_i) of the original system, correspondingly it is accordingly assumed that the state of the system is such as all the roots lie in the left half of the s -map.

The gain factor of the Original system is k

$$k = \frac{n_0}{d_0} \quad (5)$$

The finding ROM $G_r(s)$, which in some way has approximated the original system ($G(s)$) and preserves essential parameters of the original system. Such a solution corresponds as closely as possible to the solution of the system for a similar form of input. The ROM is given by

$$\Sigma_r: \begin{cases} \dot{x}_r(t) = A_r x_r(t) + B_r u_r(t) \\ y_r(t) = C_r x_r(t) + D_r u_r(t) \end{cases} \Leftrightarrow \Sigma_r := \left(\begin{array}{c|c} A_r & B_r \\ \hline C_r & D_r \end{array} \right) \quad (6)$$

Where r , r is less than n , so that the transfer function of the HOS [50]. The transfer function $G_r(s)$ of the reduced model is defined by

$$G_r(s) = C_r (sI_{n_r} - A_r)^{-1} B_r + D_r \quad (7)$$

$$G_r(s) = \frac{M_r(s)}{P_r(s)} = \frac{m_0 + m_1s + m_2s^2 + \dots + m_{r-1}s^{r-1}}{p_0 + p_1s + p_2s^2 + \dots + p_{r-1}s^{r-1} + p_r s^r} \quad (8)$$

The gain factor of reduced model order is k_r

$$k_r = \frac{m_0}{p_0} \quad (9)$$

$$\text{Or } R_r(s) = \frac{\sum_{j=0}^{r-1} m_j s^j}{\sum_{j=0}^r p_j s^j} \quad (10)$$

Where m_j, n_j are the unknown parameters scalar constant of the ROM. These parameters are to be obtained while the reduction of a HOS to ROM [51].

3. Balanced Truncation Method

The BR method Realization with the controllability grammian (CG) and observability grammian (OG) equal to a diagonal matrix Σ are essential mechanisms for the balancing and adaptation of the composite system [16]. There exists a transformation (T) such that transformed CG and OG is equal to a diagonal matrix. Such as a realization is called a balanced realization or inside balanced realization. Internal balancing of a given realization is the first step into a class of methods for MOR, called balanced truncation method (BTM) [32].

3.1. The Reduction Algorithm Steps Using the BT Approach is Given Below

Balanced truncation (BT) Algorithm [34][52]

Input: (A, B, C, D)

Output: Proposed ROM (A_r, B_r, C_r, D_r)

1. Calculate $X = RR^T$ and $Y = LL^T$
2. Calculate the SVD

$$L^T R = [U_1, U_2] \begin{bmatrix} \Sigma_1 & \\ & \Sigma_2 \end{bmatrix} [V_1, V_2]^T,$$

With $\Sigma_1 = \text{diag}(\sigma_1, \dots, \sigma_l)$, $\Sigma_2 = \text{diag}(\sigma_{r+1}, \dots, \sigma_n)$.

3. Calculate the ROM

$$(A_r, B_r, C_r, D_r) = (T^T A T, T^T B, C T, D)$$

With $W = LU_1 \Sigma_1^{-1/2} \in \mathbb{R}^{n \times l}$, $T = RV_1 \Sigma_1^{-1/2} \in \mathbb{R}^{n \times l}$.

Properties

(A_r, B_r, C_r, D_r) is asymptotically stable

Error bound: $\|G_r - G\|_{H_\infty} \leq 2(\xi_{r+1} + \dots + \xi_n)$

Must solve the large -scale Lyapunov equations

Now the system is balanced which is partitioned as [53][14][48][54].

$$G_{Bal}(s) = \left[\begin{array}{c|c} TAT^{-1} & T^T B \\ \hline CT & D \end{array} \right] \quad (11)$$

$$= \left[\begin{array}{c|c} A_{Bal} & B_{Bal} \\ \hline C_{Bal} & D_{Bal} \end{array} \right] \quad (12)$$

Balanced which is partitioned as strong subsystem and weak subsystem

$$= \underbrace{\begin{bmatrix} A_{11} & B_1 \\ C_1 & D_r \end{bmatrix}}_{\text{Strong}} + \underbrace{\begin{bmatrix} A_{22} & B_2 \\ C_2 & 0 \end{bmatrix}}_{\text{Weak}} \Leftrightarrow \Sigma: \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \quad (13)$$

Where

A_{11} and Σ_1 are reduced matrix ($r < n$)

It is called 'direct-reduction' (DR) reduction of the balanced system. Several well-known outcomes that are related to approximation are offered [38].

4. A Modified Balanced Truncation Method

The ROM (6) may be closer (approximately) to the HOS (1) well in the high-frequency band, but generally, it may not do well in the steady-state output deviation between the ROM and the HOS [46]. To bring the ROM into line with the original system, we need to change the conventional BTM in that section [23].

Here considers only the single-input single-output (SISO) linear system, in this step only taken as the transfer function of both systems. Let $G(s)$ and $G_r(s) = R_r(s)$ be the transfer function of the system (1) and reduced model (6). To reconcile the steady-state value of the ROM, the gain factor is introduced to the ROM [46]. Gain factor K as follow as

$$K = \frac{G(s)}{R_r(s)} \Big|_{s=0} \quad (14)$$

$$K = \frac{k}{k_r} = \frac{d_0}{\frac{m_0}{p_0}} = \frac{n_0}{d_0} \times \frac{p_0}{m_0} \quad (15)$$

At this point, n_0 and m_0 are positive, d_0 and p_0 are nonzero, because the system is controllable, observable, and stable [44]. Then the transfer function of the reduced model becomes the following.

$$\hat{G}_r(s) = KG_r(s) = KR_r(s) \quad (16)$$

Steady-state differences may be reduced by utilizing the gain factor, but the dynamic behaviour of the reduced model cannot be consistent with that of the original system [55].

Depending on the final value theorem, the final output of the reduced model can be calculated as the final output of the specified step input.

$$\lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow \infty} \left(s \cdot G_r(s) \cdot \frac{1}{s} \right) = \frac{m_0}{p_0} \quad (17)$$

It is well known from equation (17) that only the constant terms of the numerator polynomial and denominator polynomial are associated with the steady-state value of y . To ensure the stability of the ROM,

the constant term p_0 of the denominator polynomial must not be changed. For this reason, we propose that the gain factor K should be included in front of the constant term m_0 of the numerator polynomial. In this way, the transfer function of the reduced model is modified as follows

$$\hat{G}_r(s) = \frac{\hat{M}_r(s)}{P_r(s)} = \frac{K(m_0 + m_1s + m_2s^2 + \dots + m_{r-1}s^{r-1})}{p_0 + p_1s + p_2s^2 + \dots + p_{r-1}s^{r-1} + p_rs^r} \quad (18)$$

From (18), we can calculate the final output of the reduced model $\hat{G}_r(s)$ as follows

$$\lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow \infty} \left(s \cdot \hat{G}_r(s) \cdot \frac{1}{s} \right) = \frac{Km_0}{p_0} = \frac{n_0}{d_0} \quad (19)$$

The steady-state output of the reduced model is equal to that of the original systems.

This Modified Balanced Truncation Method can also be extended to a continuous/discrete-time MIMO system to a reduced model for a large-scale dynamic system [46].

4.1. Discrete-time Stable System by Extended Modified Balanced Truncation Method

Many Approaches in the continuous and discrete-time system to run in parallel [56] suggested Modified Balanced Truncation Method[46] is also extended to the reduced discrete system by converting the discrete-time system into an equivalent continuous-time system using bilinear mapping and by then reducing it. The reduced system is retransforming to the discrete system using reversing bilinear transformation without losing behaviour. In the last section, we have shown that Modified balance Truncation of a balanced system is fully compatible with Moore's [57] direct truncation method in the continuous-time case. We must now examine whether this is still true for the discrete-time system to do so, we must first determine some of the bilinear mapping characteristics between the complex s-plane and the Z-plane.

We begin with a minimal and stable system with a transfer function, continuous-time, linear, time-invariant (LTI)

$$G(s) = C[sI_n - A]^{-1}B + D \quad (20)$$

Assumes that discrete-time, LTI, minimal and stable system

$$G(z) = H(ZI_n - \Phi)^{-1}G + J \quad (21)$$

Via the following bilinear transformation, it is obtained from the continuous-time system [14]. The bilinear transformation is characterised by

$$s = \frac{z-1}{z+1} \Leftrightarrow \left(\text{inverse: } z = \frac{1+s}{1-s} \right) \quad (22)$$

Therefore, the matrices

$$\Sigma_c = \begin{bmatrix} A & B \\ C & D \end{bmatrix}, \Sigma_d = \begin{bmatrix} F & G \\ H & J \end{bmatrix} \quad (23)$$

Which maps the left -half complex s-plane, $\text{Re}(s) \leq 0$, onto the unit circle $|z| \leq 1$ in the z-plane, we then have the relations

$$\left. \begin{aligned} A &= (F + I_n)^{-1}(F - I_n) \\ B &= \sqrt{2}(F + I_n)^{-1}G \\ C &= \sqrt{2}H(F + I_n)^{-1} \\ D &= J - H(F + I_n)^{-1} \end{aligned} \right\} \quad (24)$$

Or, in another form for bilinear transformation.

$$G(s) = C[sI_n - A]^{-1}B + D \xleftrightarrow{z = \frac{1+s}{1-s}} G(z) = H(ZI_n - \Phi)^{-1}G + J \quad (25)$$

4.2. Error Bound Analysis of ROM

If the reduced system has been obtained of the original system, then the modelling error transfer function is well-defined via

$$E_r(s) = [G_o(s) - G_r(s)] \quad (26)$$

The actual infinity norm (H_∞) error bound in r^{th} a ROM may be calculated by taking H_∞ of $E_r(s)$. The actual and theoretical infinity error bounds are given by

$\|E_r(s)\|_\infty$ and $\leq 2 \sum_{i=r+1}^{n=\text{order of system}} \sigma_i$ respectively, Furthermore, the actual amount of error bound is continuously less than or equal to the theoretical amount of error bound. Bound of the system[47][14][2]. Thus

$$\|E_r(s)\|_\infty = \|[G_o(s) - G_r(s)]\|_\infty \leq 2 \sum_{i=r+1}^n \sigma_i \quad (27)$$

it is also known as a Priori Error bound.

5. Numerical Experiments

Example 1: Consider the third-order transfer function of a real-time single-area power system model taken from previous work as [58][59] [60][1][61] with a non-reheated turbine as

In the form of Frequency domain

$$\frac{250.00}{s^3 + 15.880s^2 + 42.460s + 106.20} \quad (28)$$

Expression of Finding the State Space Form

$$\frac{Y(s)}{U(s)} = \frac{250.00}{s^3 + 15.880s^2 + 42.460s + 106.20} \quad (29)$$

$$[s^3 + 15.88s^2 + 42.46s + 106.2]Y(s) = U(s) \quad (30)$$

Taking Inverse Laplace Transform, we Obtain

$$\ddot{y}(t) + 15.880\dot{y}(t) + 42.460\dot{y}(t) + 106.20y(t) = u(t) \quad (31)$$

Let the chosen state variable be

$$x_1(t) = y(t) \quad (32)$$

$$x_2(t) = \dot{y}(t) = \dot{x}_1(t) \quad \text{and} \quad (33)$$

$$x_3(t) = \ddot{y}(t) = \ddot{x}_1(t) \quad (34)$$

Substituting equation (33) in equation (31), we obtain

$$x_3(t) = -106.20x_1(t) - 42.460x_2(t) - 15.880x_3(t) + 250u(t) \quad (35)$$

Representing equation (33) and (35) in matrix form, we obtain the state equation as

In the form of time domain

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -106.20 & -42.460 & -15.880 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t) \quad (36)$$

$$y(t) = \begin{bmatrix} 250 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \quad (37)$$

Comparing with the standard state-space equation, we obtain

$$\left. \begin{aligned} A &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -106.20 & -42.460 & -15.880 \end{bmatrix}, B^T = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \\ C &= \begin{bmatrix} 250 & 0 & 0 \end{bmatrix}, D = 0 \end{aligned} \right\} \quad (38)$$

It can be demonstrated that the system can be controllable, observable and state. We get Hankel's unique matrix by calculation.

$$\sigma = \begin{bmatrix} 2.0694 & 0.9222 & 0.0299 \end{bmatrix} \quad (39)$$

From the matrix σ , it can be proved that $\sigma_2 \gg \sigma_3$. The first-second singular values are important here, and then the third singular values have rapidly decayed to become insignificant. Therefore, the order of reduction has been chosen as 2nd order.

The Steps of reduction algorithm using the BT approach are given below

Step1: The controllability grammians (G_c) and the observability grammians (G_o) of the system are calculated as for a stable, the controllability G grammians ramming of (A, B) is defined the observability grammians of (C, A). These grammians can be calculated by solving the Lyapunov equations for controllability grammians (G_c) [44],

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -106.2 & -42.46 & -15.88 \end{bmatrix} \begin{bmatrix} G_{C11} & G_{C12} & G_{C13} \\ G_{C21} & G_{C22} & G_{C23} \\ G_{C31} & G_{C32} & G_{C33} \end{bmatrix} + \begin{bmatrix} G_{C11} & G_{C12} & G_{C13} \\ G_{C21} & G_{C22} & G_{C23} \\ G_{C31} & G_{C32} & G_{C33} \end{bmatrix} \begin{bmatrix} 0 & 0 & -106.2 \\ 1 & 0 & -42.46 \\ 0 & 1 & -15.88 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} = 0$$

$$A = PP^T = \begin{bmatrix} P_{11} & 0 & 0 \\ P_{21} & P_{22} & 0 \\ P_{31} & P_{32} & P_{33} \end{bmatrix} \begin{bmatrix} P_{11} & P_{21} & P_{31} \\ 0 & P_{22} & P_{32} \\ 0 & 0 & P_{33} \end{bmatrix}$$

For observability grammians (G_o)

$$\begin{bmatrix} 0 & 0 & -106.2 \\ 1 & 0 & -42.46 \\ 0 & 1 & -15.88 \end{bmatrix} \begin{bmatrix} G_{O11} & G_{O12} & G_{O13} \\ G_{O21} & G_{O22} & G_{O23} \\ G_{O31} & G_{O32} & G_{O33} \end{bmatrix} + \begin{bmatrix} G_{O11} & G_{O12} & G_{O13} \\ G_{O21} & G_{O22} & G_{O23} \\ G_{O31} & G_{O32} & G_{O33} \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -106.2 & -42.46 & -15.88 \end{bmatrix} + \begin{bmatrix} 250 & 0 & 0 \end{bmatrix} \begin{bmatrix} 250 \\ 0 \\ 0 \end{bmatrix} = 0 \quad (14)$$

$$G_o = 1.0e+04 * \begin{bmatrix} 2.6367 & 0.5546 & 0.0294 \\ 0.5546 & 0.2129 & 0.0131 \\ 0.0294 & 0.0131 & 0.0008 \end{bmatrix}$$

Step 2. The find Cholesky factor of G_c and G_o

$$A = PP^T = \begin{bmatrix} P_{11} & 0 & 0 \\ P_{21} & P_{22} & 0 \\ P_{31} & P_{32} & P_{33} \end{bmatrix} \begin{bmatrix} P_{11} & P_{21} & P_{31} \\ 0 & P_{22} & P_{32} \\ 0 & 0 & P_{33} \end{bmatrix}$$

$$PP^T = \begin{bmatrix} P_{11}^2 & 0 & 0 \\ P_{21}P_{11} & P_{21}^2 + P_{22}^2 & 0 \\ P_{31}P_{11} & P_{31}P_{21} + P_{32}P_{22} & P_{31}^2 + P_{32}^2 + P_{33}^2 \end{bmatrix}$$

We obtain the following

$$P = \begin{bmatrix} \sqrt{A_{11}} & 0 & 0 \\ \frac{A_{21}}{P_{11}} & \sqrt{A_{22} - P_{21}^2} & 0 \\ \frac{A_{31}}{P_{11}} & \frac{(A_{32} - P_{31}P_{21})}{P_{22}} & \sqrt{A_{33} - P_{31}^2 - P_{32}^2} \end{bmatrix}$$

and therefore, the following formulas for the entries of L

$$P_{jj} = \sqrt{A_{jj} - \sum_{k=1}^{j-1} P_{j,k}^2}$$

$$P_{ij} = \frac{1}{P_{j,j}} \left[A_{ij} - \sum_{k=1}^{j-1} P_{i,k} P_{j,k} \right] \quad \text{for } i > j$$

So, we obtained

$$P_c = \begin{bmatrix} 0.0115 & 0 & 0 \\ 0.0000 & 0.0297 & 0.0297 \\ -0.0767 & 0.0000 & 0.1774 \end{bmatrix} \quad \text{and} \quad P_o = \begin{bmatrix} 162.3778 & 0 & 0 \\ 34.1572 & 31.0264 & 0 \\ 1.8122 & 2.2151 & 0.1873 \end{bmatrix}$$

Step 3. The singular value decomposition (SVD) $P_o^T P_c$ is obtained using equation (18).

We have

$$\begin{aligned}
 &= \underbrace{\begin{bmatrix} -0.9733 & 0.2292 & 0.0073 \\ -0.2293 & -0.9731 & -0.0205 \\ 0.0024 & -0.0216 & 0.9998 \end{bmatrix}}_U \underbrace{\begin{bmatrix} 2.0694 & 0 & 0 \\ 0 & 0.9222 & 0 \\ 0 & 0 & 0.0299 \end{bmatrix}}_\Sigma \underbrace{\begin{bmatrix} -0.7920 & 0.6082 & 0.0541 \\ -0.5787 & -0.7194 & -0.3842 \\ -0.1948 & -0.3356 & 0.9217 \end{bmatrix}}_V \\
 &= SVD \left(\underbrace{\begin{bmatrix} 0.0115 & 0 & 0 \\ 0.0000 & 0.0297 & 0.0297 \\ -0.0767 & 0.0000 & 0.1774 \end{bmatrix}}_{P_o^T} \underbrace{\begin{bmatrix} 162.3778 & 34.1572 & 1.8122 \\ 0 & 31.0264 & 2.2151 \\ 0 & 0 & 0.1873 \end{bmatrix}}_{P_c} \right) \\
 &= SVD \left(\underbrace{\begin{bmatrix} 1.7238 & 1.0134 & 0.3216 \\ -0.1699 & 0.9205 & 0.3931 \\ -0.0144 & -0.0000 & 0.0332 \end{bmatrix}}_{P_o^T P_c} \right)
 \end{aligned}$$

Where, U and V are right and left vectors, known as orthogonal columns matrix. By using a non-singular matrix W (transformation) the model can be transformed into a balancing model with help of transformation matrix.

Step 4. Compute $\sum^{-\frac{1}{2}} = \text{diag}\left(\frac{1}{\sqrt{\sigma_1}}, \frac{1}{\sqrt{\sigma_2}}, \dots, \frac{1}{\sqrt{\sigma_n}}\right)$

where $\Sigma = \text{diag}(\sigma_1, \sigma_1, \dots, \sigma_n)$

Note that $\sigma_i, i=1, 2, \dots, n$ are positive numbers

$$\sum^{-\frac{1}{2}} = \text{diag}(0.6951 \quad 1.0413 \quad 5.7868)$$

From equation (39). It is evident that $\sigma_2 \gg \sigma_3$ to choose the optimal reduced model after truncated σ_3 singular value.

$$V = \begin{bmatrix} -0.7920 & 0.6082 & 0.0541 \\ -0.5787 & -0.7194 & -0.3842 \\ -0.1948 & -0.3356 & 0.9217 \end{bmatrix}$$

Step 5. By using a non-singular matrix T, a model can be converted into a balanced model, which can be achieved as follows: the transformation matrix T is

$$\begin{aligned}
 T &= P_c V \sum^{-\frac{1}{2}} \\
 &= \underbrace{\begin{bmatrix} 0.0115 & 0 & 0 \\ 0.0000 & 0.0297 & 0.0297 \\ -0.0767 & 0.0000 & 0.1774 \end{bmatrix}}_{P_c} \underbrace{\begin{bmatrix} -0.7920 & 0.6082 & 0.0541 \\ -0.5787 & -0.7194 & -0.3842 \\ -0.1948 & -0.3356 & 0.9217 \end{bmatrix}}_V \\
 &= \underbrace{\begin{bmatrix} 0.6951 & 0 & 0 \\ 0 & 1.0413 & 0 \\ 0 & 0 & 5.7868 \end{bmatrix}}_{\sum^{-\frac{1}{2}}}
 \end{aligned}$$

$$T = \begin{bmatrix} -0.0063 & 0.0073 & 0.0036 \\ -0.0119 & -0.0222 & -0.0660 \\ 0.0182 & -0.1106 & 0.9224 \end{bmatrix}$$

Step 6. Then the system with coefficient matrix

$$(TAT^{-1}, TB, CT^{-1})$$

Now the system is balanced which is partitioned as:

$$G_{Bal}(s) = \left[\begin{array}{c|c} TAT^{-1} & TB \\ \hline CT^{-1} & D \end{array} \right]$$

$$\begin{aligned}
 TAT^{-1} &= \underbrace{\begin{bmatrix} -0.0063 & 0.0073 & 0.0036 \\ -0.0119 & -0.0222 & -0.0660 \\ 0.0182 & -0.1106 & 0.9224 \end{bmatrix}}_{\tilde{T}} \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -106.2 & -42.46 & -15.88 \end{bmatrix}}_A \\
 &= \underbrace{\begin{bmatrix} -109.8679 & -28.0578 & -1.5790 \\ 38.7612 & -23.2863 & -1.8163 \\ 6.8173 & -2.2381 & 0.8976 \end{bmatrix}}_{\tilde{T}^{-1}}
 \end{aligned}$$

$$TAT^{-1} = \begin{bmatrix} -0.6024 & 2.5000 & 0.6751 \\ -2.5000 & -1.7885 & -1.8269 \\ 0.6751 & 1.8269 & -13.4891 \end{bmatrix} = A_{Bal}$$

$$TB = \underbrace{\begin{bmatrix} -0.0063 & 0.0073 & 0.0036 \\ -0.0119 & -0.0222 & -0.0660 \\ 0.0182 & -0.1106 & 0.9224 \end{bmatrix}}_{\tilde{T}} \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_B$$

$$TB = \begin{bmatrix} -1.579 \\ -1.816 \\ 0.8976 \end{bmatrix} = B_{Bal}$$

$$CT^{-1} = \underbrace{\begin{bmatrix} 250 & 0 & 0 \end{bmatrix}}_{\tilde{C}} \underbrace{\begin{bmatrix} -109.8679 & -28.0578 & -1.5790 \\ 38.7612 & -23.2863 & -1.8163 \\ 6.8173 & -2.2381 & 0.8976 \end{bmatrix}}_{\tilde{T}^{-1}}$$

$$CT^{-1} = \underbrace{\begin{bmatrix} -1.579 & 1.816 & 0.8976 \end{bmatrix}}_{CT^{-1}} = C_{Bal}$$

And

$$D = D_{Bal}$$

$$\begin{aligned}
 G_{Bal}(s) &= \left[\begin{array}{c|c} A_{Bal} & B_{Bal} \\ \hline C_{Bal} & D_{Bal} \end{array} \right] = \left[\begin{array}{ccc|c} -0.6024 & 2.5 & 0.6751 & -1.579 \\ -2.5 & -1.789 & -1.827 & -1.816 \\ 0.6751 & 1.827 & -13.49 & 0.8976 \\ -1.579 & 1.816 & 0.8976 & 0 \end{array} \right] \\
 &= \underbrace{\begin{bmatrix} A_{11} & B_1 \\ C_1 & D \end{bmatrix}}_{\text{Strong-Subsystem}} + \underbrace{\begin{bmatrix} A_{22} & B_2 \\ C_2 & 0 \end{bmatrix}}_{\text{Weak-Subsystem}} \Leftrightarrow \Sigma : \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix}
 \end{aligned}$$

Where A_{11} and Σ_1 are $r \times r (r < n)$ matrices.

$$\Sigma_1 = \begin{bmatrix} -0.6024 & 2.5 & -1.579 \\ -2.5 & -1.789 & -1.816 \\ -1.579 & 1.816 & 0 \end{bmatrix}, \quad \Sigma_2 = \begin{bmatrix} -13.489 & 0.8976 \\ 0.8976 & 0 \end{bmatrix}$$

Strong-Subsystem *Weak Subsystem*

So the states of the balanced system corresponding to singular values σ_3 are truncated and the reduced model (A_{11}, B_1, C_1) or $G_r(s)$ is obtained as

$$G_r(s) = \frac{-0.8056s + 16.81}{s^2 + 2.391s + 7.327} = \Sigma_1,$$

Strong Subsystem

$$G_{ws}(s) = \frac{0.8056}{s + 13.49} = \Sigma_2$$

(σ_3) Weak Subsystem

The modified BTM is used to calculate the gain factor K using the equation (14) as obtained.

$$K = 1.02604$$

$$\hat{G}_r(s) = \frac{-0.8056s + 17.2477}{s^2 + 2.391s + 7.327}$$

We obtained the final reduced-order model after inserting K into the transfer function $G_r(s)$ of (A_{11}, B_1, C_1) to gain $\hat{G}_r(s)$, making the minimal state-space Realization $\hat{G}_r(s)$ to gain $(\hat{A}_{11}, \hat{B}_1, \hat{C}_1)$ and resulting from the order of $(\hat{A}_{11}, \hat{B}_1, \hat{C}_1)$, we obtained the final reduced-order model (A_r, B_r, C_r) as follows

$$\left. \begin{aligned} A_r &= \begin{bmatrix} -2.391 & -3.663 \\ 2 & 0 \end{bmatrix} \\ B_r &= \begin{bmatrix} 4 \\ 0 \end{bmatrix}, C_r = [-0.2064 \quad 2.156], D = 0 \end{aligned} \right\} = ROM$$

Table 1. A Comparison of the proposed method with Other Existing MOR Method

MOR Approaches	ROM	H_∞ Norm
Proposed Approach	$\frac{-0.8056s + 17.2477}{s^2 + 2.391s + 7.327}$	0.013490
Balanced Truncation [57]	$\frac{-0.8056s + 16.810}{s^2 + 2.391s + 7.3270}$	0.05979408
Enhanced Balanced Realization [62]	$\frac{-1.26750s + 17.24801}{s^2 + 2.3910s + 7.327}$	0.263998061
Balanced Realization (BR) Method and Factor Division Method (FDM) [63]	$\frac{-1.26720s + 17.24860}{s^2 + 2.3901s + 7.327}$	0.263869610
Improved Routh Stability Method [64]	$\frac{250.00}{15.880s^2 + 35.772s + 106.200}$	0.24653080
Modified Routh Approximation[64], Routh Approximation and Factor Division [51]	$\frac{24.0220s + 8.6880}{s^2 + 13.3799s + 7.9399}$	3.21066260
Singular Perturbation Approximation [65][66]	$\frac{15.740}{s^2 + 2.674s + 6.880}$	0.48349860
Stability Equation Method [63]and Pade Approximation Method[67]	$\frac{-0.00460s + 249.995}{15.880s^2 + 42.460s + 106.2}$	0.47710100
Pade Approximation [1]	$\frac{-1.19100s + 18.9200}{s^2 + 2.7080s + 8.0403}$	0.12595607
Routh approximation [1]	$\frac{18.680}{s^2 + 3.173s + 7.94}$	0.64210960
Routh Stability method and Factor Division Method [68]	$\frac{39.36600s + 3344.701}{35.80s^2 + 584.80s + 1420.80}$	1.90855030
Differential Method[69]	$\frac{250.00}{5.29300s^2 + 28.3007s + 106.200}$	2.223737203
Routh- Pade Approximation [70]	$\frac{24.0220 + 8.6880}{s^2 + 13.3790s + 7.939}$	3.210662604
Factor Division Method [71]	$\frac{18.8170}{s^2 + 2.5940s + 7.9940}$	0.534206074
Routh Stability Method and Pade Approximation[72]	$\frac{-15.748s + 249.9950}{15.88s^2 + 35.772s + 106.20}$	0.440232056
SEM [73], FDM and Stability Method[22][49] Truncation Method[74],	$\frac{250.00}{15.88s^2 + 42.467s + 106.200}$	0.477107207
Pade- Approximation and Differentiation Method[75]	$\frac{-33.3210s + 249.9950}{2.2930s^2 + 28.3070s + 106.20}$	1.815180029
Routh Stability Method[76]	$\frac{250}{15.88s^2 + 35.772s + 106.20}$	0.246530806

Figure 1. Step Response of The ROM with Original Order System of Continuous System

The step response of the original system, ROM found by a proposed method based upon modified and other methods existing in the literature review are depicted in Fig 1. It has been seen that the ROM so obtained is its closed approximation.

Example 2. We are now considering a discrete SISO system. For comparative purposes, we simply used the bilinear transformation mapping to discretize the system in example 1.

$$A_D = \begin{bmatrix} 1.907 & -1.118 & 0.3124 \\ 1 & 0 & 0 \\ 0 & 0.5 & 0 \end{bmatrix}; B_D = \begin{bmatrix} 0.5 \\ 0 \\ 0 \end{bmatrix};$$

$$C_D = [0.1603 \quad 0.06147 \quad 0.07553], D_D = [0.01633]$$

Applying transformation $G(z)$ is transformed to $G(s)$ in state-space form as

$$A_C = \begin{bmatrix} 10.2 & -11.77 & 2.617 \\ 8.377 & -5.771 & -4.812 \\ -7.702 & 18.87 & -23 \end{bmatrix}; B_C = \begin{bmatrix} 2.385 \\ -1.804 \\ 2.949 \end{bmatrix};$$

$$C_C = [0.1603 \quad 0.06147 \quad 0.07553], D_C = [0.01633]$$

Corresponding Transfer function

$$G_C(s) = \frac{0.01633s^3 + 0.7973s^2 + 10.42s + 298.9}{s^3 + 18.57s^2 + 48.87s + 127}$$

This is then reduced to 2nd order Model using BTM

$$A_{Cr} = \begin{bmatrix} -0.6961 & -2.499 \\ 2.499 & -1.725 \end{bmatrix}; B_{Cr} = \begin{bmatrix} -1.671 \\ 1.718 \end{bmatrix};$$

$$C_{Cr} = [-1.671 \quad -1.718], D_{Cr} = [0.01633]$$

The corresponding reduced Transfer function

$$G_{Cr}(s) = \frac{0.0163s^2 - 0.1229s + 17.23}{s^2 + 2.421s + 7.448}$$

We can calculate the gain factor K, by the Modified BTM scheme

$$K = 1.017377$$

After introducing K into the transfer function $G_{Cr}(s)$ of $(A_{C11}, B_{C1}, C_{C1}, D_{C1})$ to gain $\hat{G}_{Cr}(s)$, making the minimal state -space Realization $\hat{G}_{Cr}(s)$ to gain $(\hat{A}_{C11}, \hat{B}_{C1}, \hat{C}_{C1}, \hat{D}_{C1})$ and resulting from the order of $(\hat{A}_{C11}, \hat{B}_{C1}, \hat{C}_{C1}, D_{C1})$, we obtained the final reduced-order model (A_r, B_r, C_r, D_r) as follows

$$A_r = \begin{bmatrix} -2.421 & -3.724 \\ 2 & 0 \end{bmatrix}; B_r = \begin{bmatrix} 4 \\ 0 \end{bmatrix};$$

$$C_r = [-0.04061 \quad 2.176]; D_r = [0.01633]$$

Corresponding reduced Transfer function with inserting

$$\text{gain Factor } K = \frac{G_C(s)}{G_{Cr}(s)}$$

$$G_r(s) = \frac{0.0163s^2 - 0.1229s + 17.53}{s^2 + 2.421s + 7.448}$$

Retransformation into z-domain leads to a steady-state error which after steady-state correction is given by

$$A_{Dr} = \begin{bmatrix} 1.722 & -0.7876 \\ 1 & 0 \end{bmatrix}; B_{Dr} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix};$$

$$C_{Dr} = [0.2597 \quad 0.0417]; D_{Dr} = [0.04739]$$

The corresponding discrete-time reduced Transfer function

$$G_{Dr}(z) = \frac{0.04739z^2 + 0.04825z + 0.05817}{z^2 - 1.722z + 0.7876}$$

Figure 2. Step Response of The ROM with The Original System of Discrete System

Table 2. A Comparative Summary of Frequency Domain Computations for Single Area Power System

Computations	H_{∞} Norm					
	Example 1- 3 rd Power system model in continuous time system			Example 2- 3 rd Power system model in Discrete-time system		
	BTM	BR & FDM Mixed	Modified BTM	BTM	BR & FDM Mixed	Modified BTM
$\ G(s)\ _{\infty}$ Infinity norm (H_{∞})		2.84965550			2.84965550	
Theoretical Infinity norm: $\leq 2 \sum_{i=r+1}^2 \sigma_i$ For n=2		0.0598			0.0598	
A Priori Error Bound						
Actual Infinity norm						
$\ Original\ System(G(s)) - Reduced\ System(G_r(s))\ _{\infty}$	0.05979480	0.26386960	0.01349	0.2871	0.0181	0.03027
Error Bound in Simulation						
Actual Relative Error Bound:						
$\frac{\ G(s) - G_r(s)\ _{\infty}}{\ G(s)\ _{\infty}}$	0.020983160	0.09264240	0.0047	0.1008	0.004641	0.0106

The comparison of step response of the original system, ROM found by the proposed method based upon modified BTM and other methods existing in the literature review are depicted in Fig.2. It has been seen that the ROM so obtained is its close approximation. Further, Measured the accuracy of the proposed method by H_{∞} Norm value are shown in Table 2. It is observed that the proposed method shows the all error bound value much lesser than the value obtained by other methods available in the literature.

6. Discussion

This article presents the step-by-step response of the reduced model and the original system is shown in the figure above. All numerical and Simulation experiments results have been performed on Intel ® CoreTM i7-8700 CPU @ 3.20 GHz and 8 GB memory using MATLAB R2019a (Academic Use) at the place of EED, MMMUT,

Gorakhpur. The step responses of the original system and ROM depicted in the figures of both examples are taken from the literature search. This figure shows that the reduced model is very close to the original system. This proposed method's excellence in comparison to the use of the BT method and literature review has been justified through both examples. The H_{∞} modelling error has been also computed and results are depicted in Tables 1 and Table 2. It is seen to be an excellent precise approximation with a minor error between the original system and ROM. It is observed that the results obtained by the proposed method are far superior. It is clearly stated that the approximation to the Reduced Model of the original system with the lesser error must be closed. It has already been shown that some bilinear mapping not only maintains a balanced structure between a continuous-time system and a discrete time method, but also the Modified BTM structure between a reduced continuous-time order system and a

reduced discreet time system. The result of continuous-time, stable and balanced system on Modified BTM can easily be extended to a discreet time system.

7. Conclusion

A new reduced-order model approach to the reduction of the power system has been proposed in this article. The proposed modified BTM procedure is superior to any conventional method or other mixed methods. This paper has updated the traditional method (BT) to narrow deviations, assuming that dynamic behaviour is maintained. This method drawback has been eliminated by the modified BT of the BTM. The modified BTM is used to effectively display an example of an LTI system and has been easily applied to a discreet time system to support this approach. Also, the time response comparison shows that the ROM calculated using the proposed method provides a close approximation of the HOS. The accuracy, justification and superior performance of the submitted method are also demonstrated by comparing the error standard with the existing literature. The method will become more advantageous when it comes to large-scale continuous/discrete-time systems.

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