Sensorless Nonlinear Control Strategy of the Single Phase Active Power Filters via Two-time Scale Singular Perturbation Technique

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Abstract This paper focuses on the problem of controlling single-phase shunt active power filters (APFs) operating in the presence of nonlinear and uncertain loads. The main difficulties when controlling this kind of filter are the existence of a strong non-linearity of the system and state variables inaccessible to measurements. This paper proposes a double-loop cascade controller developed on the basis of the singular perturbation technique in order to meet two main control objectives: (i) the inner-loop is designed to compensate the harmonic and reactive currents absorbed by the nonlinear load enforcing power factor correction; (ii) the outer-loop is synthesized to regulate the inverter output capacitor voltage. The controller also includes two-time scale sliding mode observer to estimate the network voltage that is not accessible to measurement. In this work, the singular perturbation technique and averaging theory are used for a complete and rigorous formal analysis to describe the control system performances. The effectiveness of this approach has been successfully verified through computer simulations using the Matlab/Simulink environment.

Keywords Active Power Filters, Singular Perturbation, Two-time Scale Sliding Mode Observer, Averaging Theory, Nonlinear Controller

1. Introduction

The increasing use of power electronics-based devices in industrial applications (such as single-phase or three-phase rectifiers, motor drivers, switching power supplies, solar energy,...), is leading to more and more disturbance problems in power grids. Although these devices provide

flexibility and increased reliability with high efficiency, they behave as non-linear loads that cause overheating of the transformers, distortion of the supply voltage, malfunction of sensitive equipment, reactive power and low power factor leading to the distortion of the voltage waveform at the point of common coupling (PCC) and disturbing connected loads.

To reduce the impact of these harmonics on the power system, many solutions have been proposed. These solutions included modifications on the load itself for less harmonic emissions or the connection on the polluted power grids of other traditional or modern compensation systems like the case of active power filters, passive power filters and hybrid filters. The conventional solution is not only costly but also presents several short comings such as: highly dependent on the grid, load parameters and main frequency; and possibility of series and parallel resonance with the grid, which lead to create a problem with the loads and the network, causing voltage fluctuations.

Due to these drawbacks and the emergence of semiconductor devices such as GTO thyristors and IGBT transistors, as well as the revolution in the production of digital signal processing (DSP) and field programmable gate arrays (FPGAs), the active power filters (APF) [1] are designed to generate harmonic currents or voltages such that the mains current and voltage are sinusoidal and operate with a unit power factor.

In general, the shunt APFs can be divided into two categories [2]: single-phase and three-phase active filters, which represent the most important applications of APFs in the field. However, the implementation of a low-power single-phase APF at each single-phase nonlinear load works particularly well in some practical situations than the installation of a high-power three-phase APF at the PCC.

Recently, a great deal of interest has been devoted to the

problem of controlling energy systems involving shunt APFs and different techniques are proposed including linear and nonlinear controllers. In [3], a hybrid type of simple linear control, has been adopted and applied for APF where the performances have been demonstrated by experimental tests. In [4], a simple linear PID with experimental tests were proposed. These both solutions are limited and not guaranteed to the case of a wide range variation of the operation point. In contrast to linear control, there are some nonlinear control techniques can optimize the dynamic performance of the shunt APF, and they include sliding mode [5], Lyapunov design [6], feedback linearization [7], and backstepping technique [8]. However, in most of the proposed works, the obtained controllers are generally not backed by a formal stability analysis.

In this paper, we address the problem of controlling single-phase shunt APF in the presence of non-linear loads based on a model that takes into account the estimation of the network voltage value that is not assumed to be measurable. In the first step, the degree of difficulty is remedied by using a developed two-time-scale sliding mode observer [9]–[11] for the estimation of the network voltage value. Thus, the aim of the second step is to propose a nonlinear output feedback controller with high gain [12]–[15] based on the singular perturbation technique [16]–[18]. The control strategy involves a two-loop cascade nonlinear controller such as: (i) the inner-loop involves a current regulator designed to ensure the compensation of harmonic currents and reactive power absorbed by the nonlinear load. This is achieved by power factor correction (PFC); (ii) the outer-loop involves a voltage controller to regulate the DC bus voltage of the converter.

Compared to the previous works, the contribution of the new nonlinear controller enjoys several interesting features, including the following:

- The proposed control strategy consists of estimating the grid voltage unlike previous works [4]–[8], [19]. As a matter of fact, the reduction of the number of sensors entails a decrease in the cost of the control system and an increase of the controller reliability;
- A theoretical analysis will prove, making suitable use of different automatic control tools such as the singular perturbation technique and the averaging theory, that all control objectives are actually achieved in the mean. Such a formal analysis was missing in [4]–[6] ;
- The nonlinearity of the controlled system was preserved in the controller design in order to keep all the properties of the studied system, whereas it is partly or totally ignored in [7];
- The inclusion of two-time scale dynamics in the full-order closed-loop system can ensure optimal performance and insensitivity of the output transient behavior with respect to parameter variations and external disturbances, with accuracy better than [20].

The paper is organized as follows: the control problem formulation, including the APF modeling and control objectives, is presented in section 2; the design of the observer and the cascade nonlinear controller is dealt with in section 3; the performances are formally analyzed in section 4; performances controller are illustrated by simulation in section 5; a general conclusion ends the paper.

2. Control Problem Formulation

2.1. Active Power Filter Topology and Modeling

The components of the single-phase shunt APF are shown in Fig. 1. It includes a single-phase IGBT-based full-bridge inverter with an input filtering inductor L_f , a DC bus capacitor C_f , and a nonlinear load. The used inverter operates according to the well-known pulse width modulation (PWM) principle, where the binary input signals, defined as follows:

$$\mu = \begin{cases} 1 & \text{if } (s_1, s_4) \text{ is ON, } (s_2, s_3) \text{ is OFF} \\ -1 & \text{if } (s_1, s_4) \text{ is OFF, } (s_2, s_3) \text{ is ON} \end{cases} \quad (1)$$

Applying Kirchhoff's laws, the dynamic behavior of the single-phase shunt APF can be described by the following set of differential equations:

$$L_g \frac{di_g}{dt} = v_g - v_{pcc} - r_g i_g \quad (2a)$$

$$L_f \frac{di_f}{dt} = v_{pcc} - \mu v_o \quad (2b)$$

$$C_f \frac{dv_o}{dt} = \mu i_f \quad (2c)$$

Figure 1. The components of the single-phase shunt APF

The grid voltage (supposed sinusoidal) is not available for measurement. It is given by:

$$v_g(t) = E_g \sin(\omega_g t) \quad (3a)$$

E_g and ω_g denote, respectively, the amplitude and the angular frequency of $v_g(t)$. Then, the following set of differential equations (2a-c) is completed by internal state of the network grid voltage, which is defined by

$$\frac{d^2 v_g}{dt^2} = -\omega_g^2 v_g \quad (3b)$$

The resulting load current $i_L(t)$ is a periodic signal and its Fourier expansion is defined by

$$i_L(t) = \sum_{n=1}^{\infty} I_{Ln} \sin(n\omega_g t + \varphi_n) \quad (3c)$$

The instantaneous model (2a-c) et (3b) involves the binary control inputs, namely μ . It is therefore useful for building up an accurate simulator, but it is not suitable for controller design. To overcome this difficulty, it is usually resorted to model averaging over cutting period T_{PWM} (e.g., [21]) defined by

$$L_g \frac{dx_{1r}}{dt} = x_{2r} - v_{pcc} - r_g x_{1r} \quad (4a)$$

$$\frac{dx_{2r}}{dt} = x_{3r} \quad (4b)$$

$$\frac{dx_{3r}}{dt} = -\omega_g^2 x_{2r} \quad (4c)$$

$$L_f \frac{dx_{1f}}{dt} = v_{pcc} - u x_{2f} \quad (4d)$$

$$C_f \frac{dx_{2f}}{dt} = u x_{1f} \quad (4e)$$

where x_{1r} , x_{1f} , x_{2f} and u denote the average values, over cutting periods, of the signals i_g , i_f , v_o and μ . $x_{2r} = v_g$ and x_{3r} are an internal states of the network grid voltage which is supposed sinusoidal.

2.2. Control Objectives

For convenience, recall that we seek the achievement of the following control objectives.

- Canceling the harmonics and reactive currents generated by the nonlinear loads: This objective, commonly referred to power factor correction, means that the input current must be sinusoidal with the same frequency as the AC-line voltage source;

- ii. Regulating the DC bus voltage x_{2f} to maintain the capacitor charge at a suitable level so that the filter operates properly: the DC bus average voltage x_{2f} must be regulated to a desired level $x_{2f}^* = V_d$ always higher than the peak of the grid voltage v_g ;
- iii. Estimating of the internal states of the network grid voltage in order to reduce the sensors number: the proposed nonlinear controllers are designed assuming that the network internal impedance to be non-negligible where ($L_g \neq 0$ and $r_g \neq 0$), which implies that the sinusoidal network voltage v_g cannot be assumed accessible for measurement. This problem is presently solved by augmenting the controller with a two-time scale sliding mode observer, providing online estimates of the grid voltage v_g and ω_g , under the assumption that only the measurement of the network current i_g is available for the measurement.
- iv. Asymptotic stability of the closed loop system.

3. Controller Design

3.1. Two-time scale Sliding Mode Observer Design for Grid Voltage

In the present paper, we propose an observer design based on the sliding mode approach as well as the singular perturbations theory that allows the separation of the two-time scales original model into two reduced order models (see [9]–[11]). The goal of the observer design is to have a state estimator for the slow variable from the measurement of the fast variables. To this effect, let us consider only the three first equations of the APF model (4a-c). Thus, the observer equations will now be expressed in function of (4a-c) as follows:

$$L_g \frac{d\hat{x}_{1r}}{dt} = -r_g x_{1r} + \hat{x}_{2r} - v_{pcc} + \Gamma_{1r} I_f \quad (5a)$$

$$\frac{d\hat{x}_{2r}}{dt} = \hat{x}_{3r} + \Gamma_{2r} I_f \quad (5b)$$

$$\frac{d\hat{x}_{3r}}{dt} = -\omega_g^2 \hat{x}_{2r} + \Gamma_{3r} I_f \quad (5c)$$

where Γ_{1r} , Γ_{2r} , and Γ_{3r} are the observer gains. By choosing the sliding surface as $S = \tilde{x}_{1r} = x_{1r} - \hat{x}_{1r}$, the switching term I_f is defined by:

$$I_f = \text{sign}(S) = \text{sign}(x_{1r} - \hat{x}_{1r}) \quad (6)$$

By defining $\tilde{x}_{2r} = x_{2r} - \hat{x}_{2r}$ and $\tilde{x}_{3r} = x_{3r} - \hat{x}_{3r}$, the estimation error dynamics are obtained by subtracting equations (5a-c) from (4a-c) as follows

$$L_g \frac{d\tilde{x}_{1r}}{dt} = \tilde{x}_{2r} - \Gamma_{1r} I_f \quad (7a)$$

$$\frac{d\tilde{x}_{2r}}{dt} = \tilde{x}_{3r} - \Gamma_{2r} I_f \quad (7b)$$

$$\frac{d\tilde{x}_{3r}}{dt} = -\omega_g^2 \tilde{x}_{2r} - \Gamma_{3r} I_f \quad (7c)$$

Based on the singular perturbation theory with $\varepsilon_r = L_g/\omega_g$ is the small parameter, $\tilde{x}_{1r}$ is the fast variable, and ($\tilde{x}_{2r}$, $\tilde{x}_{3r}$) are the slow variables, the corresponding standard singularly perturbed form can be written as:

$$\varepsilon_r \frac{d\tilde{x}_{1r}}{dt} = \frac{\tilde{x}_{2r}}{\omega_g} - \frac{\Gamma_{1r}}{\omega_g} I_f \quad (8a)$$

$$\frac{d\tilde{x}_{2r}}{dt} = \tilde{x}_{3r} - \Gamma_{2r} I_f \quad (8b)$$

$$\frac{d\tilde{x}_{3r}}{dt} = -\omega_g^2 \tilde{x}_{2r} - \Gamma_{3r} I_f \quad (8c)$$

The stability analysis of the above system consists, with the aid of the two-time-scale approach, to ensure the attractiveness of the sliding surface $S(\tau_r) = \tilde{x}_{1r} = 0$ in the fast time-scale ($\tau_r = t/\varepsilon_r$) by determining Γ_{1r} of the fast dynamic subsystem (FDS) of observation errors. Then, Γ_{2r} and Γ_{3r} of the slow dynamic subsystem (SDS) of observation errors is determined in order to guarantee the local stability of the reduced-order system obtained when $S(\tau_r) \equiv S(\tau_r) \equiv 0$.

First, the fast dynamic subsystem of observation errors is obtained by transforming the slow time-scale (t) into the fast time-scale defined by $\tau_r = t/\varepsilon_r$. By setting $\varepsilon_r = 0$, one gets

$$\frac{d\tilde{x}_{1r}}{d\tau_r} = \frac{\tilde{x}_{2r}}{\omega_g} - \frac{\Gamma_{1r}}{\omega_g} I_f \quad (9a)$$

$$\frac{d\tilde{x}_{2r}}{d\tau_r} = 0 \quad (9b)$$

$$\frac{d\tilde{x}_{3r}}{d\tau_r} = 0 \quad (9c)$$

Once the current converges towards the sliding surface $S = \tilde{x}_{1r} = 0$ which has the characteristic to slip on it. On this reduced manifold, the approximate dynamics can be formally derived from the so-called equivalent control method [9], which is equivalent to Fillipov's solution concept in the special case of linear input switching. When $S = \tilde{x}_{1r} = 0$ and since $\dot{\tilde{x}}_{1r} = 0$, the equivalent switching is defined by

$$I_f^{\text{eq}} = \tilde{x}_{2r}/\Gamma_{1r} \quad (10)$$

Substitution of (10) into (8b,c) yields the following SDS of observation errors

$$\frac{d\tilde{x}_{2r}}{dt} = \tilde{x}_{3r} - \frac{\Gamma_{2r}}{\Gamma_{1r}} \tilde{x}_{2r} \quad (11a)$$

$$\frac{d\tilde{x}_{3r}}{dt} = -\omega_g^2 \tilde{x}_{2r} - \frac{\Gamma_{3r}}{\Gamma_{1r}} \tilde{x}_{2r} \quad (11b)$$

The stability analysis of observer errors is the subject of the following proposition

Proposition 1. Consider the two time-scale sliding mode observer represented by the model (8a-c). For $0 < \varepsilon_r \ll 1$ this system takes the singular perturbation form, where the FDS of observation errors is given by (9a) and the SDS is defined by (11a,b). We assume that $\tilde{x}_{2r}$ is practically limited. Therefore, one gets the following properties

i. it is shown that for the sliding surface $S(\tau_r) = \tilde{x}_{1r} = 0$, the convergence condition given by $SdS/d\tau_r < 0$ is verified with the following inequality:

$$\Gamma_{1r} > \|\tilde{x}_{2r}\| \quad (12)$$

ii. Consider the SDS of observation errors described by (12a,b) together with the observer's gain of the current defined by (11), we can check that the stability of the reduced order observer is provided, if their gains meet the following conditions:

$$\Gamma_{2r} \gg 0 \quad (13a)$$

$$\Gamma_{3r} = \Gamma_{1r} (1 - \omega_g^2) \quad (13b)$$

Proof 1. (See Appendix A1).

3.2. Inner Loop: Power Factor Correction Objective

3.2.1. Control Law Design

The single-phase APF must operate with a power factor close to one. This means that, in steady-state, the current $i_g = i_L + i_f$ delivered by the supply network should be sinusoidal and in phase (or opposite-phase) with the supply net voltage v_g . Therefore, the current injected by active power filter x_{1f} must be enforced to track a reference signal x_{1f}^* of the form

$$x_{1f}^* = \beta \frac{\hat{x}_{2r}}{E_g} - i_L \quad (14a)$$

Indeed, it follows from the equality $x_{1f} = x_{1f}^*$ that i_g takes the following expression

$$i_g = \beta \frac{\hat{x}_{2r}}{E_g} \quad (14b)$$

To this end, let us consider the equation (4d) in the following form

$$\mathcal{X}_f = \Phi_1(X_f) - u\Phi_2(X_f) \quad (15)$$

where $X_f = (x_{1f} \ x_{2f})^T$, $\Phi_1 \stackrel{\text{def}}{=} v_{pcc}/L_f$ and $\Phi_2 \stackrel{\text{def}}{=} x_{2f}/L_f$

Remark 1. From a practical view point, the DC link voltage x_{2f} is assumed to be positive all the time (otherwise, from a practical view point, the power converter cannot operate).

Hence, it is assumed practically that x_{2f} is bounded $0 < x_{2f,\min} \leq x_{2f} \leq x_{2f,\max}$. Then, the conditions $0 < \Phi_{2,\min} \leq \Phi_2 \leq \Phi_{2,\max} < +\infty$ and $|\Phi_1| \leq \Phi_{1,\max} < +\infty$ are satisfied.

Let us introduce the tracking error on the current:

$$e_1 = x_{1f}^* - x_{1f} \quad (16a)$$

Then, let us also construct the reference model for current x_{1f} in the following form

$$\mathcal{X}_f = \frac{e_1}{T_1} \stackrel{\Delta}{=} D_1(x_{1f}^*, x_{1f}) \quad (16b)$$

where T_1 is the time constant of the desired dynamic for the current x_{1f} . Since (16b), the realization errors of the desired behaviors of $\mathcal{X}_f$, namely G_1 , is given by

$$G_1 = D_1(x_{1f}^*, x_{1f}) - \mathcal{X}_f \quad (17)$$

Thus, the control problem $\lim_{t \rightarrow \infty} e_1(t) = 0$ corresponds to the insensitivity condition given by

$$G_1 = 0 \quad (18)$$

Therefore, the behavior of $\mathcal{X}_f$ with prescribed dynamics of (16b) is fulfilled. Based on (15) and (17), the requirement (18) is reformulated as follows

$$D_1(x_{1f}^*, x_{1f}) - \Phi_1(X_f) + u\Phi_2(X_f) = 0 \quad (19)$$

The last equation has a unique isolated root, called the inverse dynamic solution, given by

$$u_{id} = -\frac{L_f}{x_{2f}} \left[D_1(x_{1f}^*, x_{1f}) - \frac{v_{pcc}}{L_f} \right] \quad (20)$$

The real control variable u has appeared in the insensitivity condition. Therefore, the control law with the first derivative in feedback of the current controller must now be determined using Lyapunov approach so that the e_{1f} -system is made asymptotically stable. For this purpose, we consider the following Lyapunov function candidate

$$V_3(u) = \frac{1}{2} (G_1(u))^2 \quad (21a)$$

the time-derivative of V_3 would imply:

$$\frac{dV_3(u)}{dt} = \Phi_2 G_1 \frac{du}{dt} \quad (21b)$$

Since the expression (21b), one sees that the asymptotical convergence is guaranteed if

$$\frac{du}{dt} = kG_1 \quad (22)$$

Taking into account that $\Phi_2 < 0$ and selecting the negative value of the gain $k < 0$, it is shown from the equation (22) and (21b) that

$$\frac{dV_3(u)}{dt} = \Phi_2 k G_1^2 < 0 \quad \text{for all } G_1 \neq 0 \quad (23)$$

In order to set the system in the standard form of singular perturbation, let us consider the high gain k in the form of $k = k_1/\varepsilon_{1f}$. Therefore, the discussed output feedback controller with high gain is defined as follows

$$\varepsilon_{1f} \frac{du}{dt} = k_1 \left(\frac{e_1}{T_1} - \frac{dx_{1f}}{dt} \right) \quad (24)$$

3.2.2. Singular Perturbation System of Inner Loop

Consider the inner loop system composed of the main equations (4d,e) and the control law (24). The replacement of x_f in (26) by the right member of (6d) yields

$$\varepsilon_{1f} \frac{dz}{dt} = h(X_f, z, t) \quad (25a)$$

$$\frac{dX_f}{dt} = f'(X_f, z, t) \quad (25b)$$

where $z = u$, $h(X_f, z, t) = \frac{k_1 x_{2f}}{L_f} z + k_1 \left(\frac{e_1}{T_1} - \frac{v_{pcc}}{L_f} \right)$,

$$f'(X_f, z, t) = \begin{pmatrix} \frac{v_{pcc}}{L_f} - \frac{x_2}{L_f} z \\ \frac{x_1}{C_f} z \end{pmatrix}.$$

The above equation takes the singularly perturbed differential equations for $0 < \varepsilon_{1f} \ll 1$ where the FDS is obtained by transforming the slow time-scale (t) into the fast time scale $\tau_{1f} = t/\varepsilon_{1f}$. Then, by setting $\varepsilon_{1f} = 0$, one has

$$\frac{dz}{d\tau_{1f}} = h(X_f, z, t) \quad (26a)$$

$$\frac{dX_f}{d\tau_{1f}} = 0 \quad (26b)$$

Remark 2. During the fast transient in (26a), the variables X_f are treated as the frozen parameters.

After the rapid decay of transients in (26a) the steady state (more precisely, quasi-steady state) of FDS (26a) tends toward an equilibrium z^s defined as follows

$$z^s = h^s(X_f) = -\frac{L_f}{x_{2f}} \left(\frac{e_1}{T_1} - \frac{v_{pcc}}{L_f} \right) \quad (27)$$

Since remark 1, it seems that the division by the DC link voltage x_{2f} in (27), which is a common fact in most power converter control problems, is not an issue.

By substituting the expression of z^s given by (27) into (25b), the SDS of inner loop takes place on the slow manifold, according to the following equation

$$\frac{dX_f}{dt} = \begin{pmatrix} \frac{e_1}{T_1} \\ -\frac{x_{1f}}{x_{2f}} \left(\frac{L_f e_1}{C_f T_1} - \frac{v_{pcc}}{C_f} \right) \end{pmatrix} \quad (28)$$

The results are thus summarized in the following proposition.

Proposition 2. Consider the inner loop system composed of equations (25a) and (25b). For $0 < \varepsilon_{1f} \ll 1$, the system takes the singular perturbation form where the FDS is given by (26a) and SDS is given by (28). Under the assumptions given by Remark 1 and 2, one has

- If the gain k_1 is negative such that $k_1 x_{2f}/L_f < 0$, the FDS (26a) will be exponentially stable and z converges exponentially fast to z^s .
- The behaviour of x_{1f} is prescribed by a stable reference equation of the form $dx_{1f}/dt = (x_{1f}^* - x_{1f})/T_1$. Then, the requirement $\lim_{t \rightarrow \infty} e_1 = \lim_{t \rightarrow \infty} (x_{1f}^* - x_{1f}) = 0$ is maintained.

3.3. Outer Loop: DC Bus Voltage Regulation Objective

3.3.1. Control Law Design

For outer loop, the control objective is to generate the signal (a tuning law for the ratio) β so that the DC bus voltage x_{2f} must be regulated to a given reference value $x_{2f}^* = V_d$. According to the two-time-scale singular perturbation methodology, it follows that the outer loop of the voltage is assumed to be slow in comparison to the transients of the inner control loop (this can be guaranteed by choosing $\varepsilon_{1f} < \varepsilon_{2f}$ and $T_1 < T_2$, where (ε_{2f}, T_2) are the parameters of the outer loop that will be designed in this subsection. Therefore, the steady-state for the current x_{1f} yields $x_{1f} = x_{1f}^*$. Then, the SDS given by (28) will be reduced to

$$\frac{dx_{2f}}{dt} = \frac{v_{pcc} x_{1f}^*}{C_f x_{2f}} \quad (29)$$

The aim is to design a new control law for the outer loop so that the ratio β stands as a new control input, while the output voltage x_{2f} stands as a new output variable. To this end, it is necessary to establish the relationship

between the ratio β (the control signal) and the output voltage x_{2f} , which is the subject of the following proposition.

Proposition 3. Consider the result equation described by (29) with the PFC requirement defined by (14) and the observer (8). Then, the voltage x_{2f} varies, in response to the tuning ratio β , according to the following first-order time-varying nonlinear equation:

$$\frac{dx_{2f}}{dt} = \varepsilon_r \omega_g (\psi_1(X, t) + \psi_2(X, t)\beta) \mathfrak{E} + \psi_3(X, \varepsilon_r, t)\beta + \psi_4(X, \varepsilon_r, t)\beta^2 + \psi_5(X, t) \quad (30)$$

with $X = (x_{1f} \quad x_{2f} \quad \tilde{x}_{2r} \quad \tilde{x}_{3r})^T$,

$$\begin{aligned} \psi_1(X, t) &= \frac{I_{L1} \cos(\varphi_1)}{2x_{2f} C_f} - \frac{I_{L1} \cos(2\omega_g t + \varphi_1)}{2x_{2f} C_f} \\ &\quad - \frac{i_L \tilde{x}_{2r}}{x_{2f} E_g C_f} - \frac{x_{2r}}{x_{2f} E_g C_f} \sum_{n=2}^{\infty} I_{Ln} \sin(n\omega_g t + \varphi_n), \\ \psi_2(X, t) &= \frac{2\tilde{x}_{2r} x_{2r}}{E_g^2 x_{2f} C_f} + \frac{\tilde{x}_{2r}^2}{E_g^2 x_{2f} C_f} + \frac{\cos(2\omega_g t)}{2x_{2f} C_f} - \frac{1}{x_{2f} C_f}, \\ \psi_3(X, t) &= \frac{E_g}{2x_{2f} C_f} (2 - \cos(2\omega_g t)) + \frac{I_{L1}}{2x_{2f} C_f} \\ &\quad \times [\varepsilon_r \omega_g^2 (\sin(2\omega_g t + \varphi_1) + \sin(\varphi_1)) - r_g (\cos(2\omega_g t + \varphi_1) \\ &\quad - \cos(\varphi_1))] - \frac{(\varepsilon_r \omega_g x_{3r} + r_g x_{2r})}{E_g x_{2f} C_f} \sum_{n=2}^{\infty} I_{Ln} \sin(n\omega_g t + \varphi_n) \\ &\quad + \frac{(\varepsilon_r \omega_g i_L \Gamma_{2r} I_f - \varepsilon_r \omega_g i_L \tilde{x}_{3r} - (x_{2r} + i_L r_g) \tilde{x}_{2r})}{x_{2f} E_g C_f}, \\ \psi_4(X, \varepsilon_r, t) &= \frac{r_g (E_g \cos(2\omega_g t) - 2)}{2x_{2f} C_f} + \frac{\tilde{x}_{2r} (r_g x_{2r} + \varepsilon_r \omega_g x_{3r})}{x_{2f} E_g^2 C_f} \\ &\quad - \frac{\varepsilon_r \omega_g x_{2r} x_{3r}}{x_{2f} C_f E_g^2} + \frac{(\tilde{x}_{2r} + x_{2r})}{x_{2f} E_g^2 C_f} (r_g \tilde{x}_{2r} + \varepsilon_r \omega_g (\tilde{x}_{3r} - \Gamma_{2r} I_f)), \\ \psi_5(X, t) &= \frac{E_g I_{L1}}{x_{2f} C_f} \cos(2\omega_g t + \varphi_1) - \frac{E_g I_{L1}}{2x_{2f} C_f} \cos(\varphi_1) \\ &\quad + \frac{x_{2r}}{x_{2f} C_f} \sum_{n=2}^{\infty} I_{Ln} \sin(n\omega_g t + \varphi_n). \end{aligned}$$

Proof 2. Follows from (29) replacing there v_{pcc} and x_{1f}^* by their expressions given by (4a) and (14a) (see Appendix A2).

Now, let us introduce the tracking error on the output voltage:

$$e_2 = x_{2f}^* - x_{2f} \quad (31a)$$

then, the desired behavior of x_{2f} be assigned by

$$\mathfrak{E} = \frac{e_2}{T_2} = D_2(x_{2f}^*, x_{2f}) \quad (31b)$$

The error of the desired dynamic realization is as follows

$$G_2 = D_2(x_{2f}^*, x_{2f}) - \mathfrak{E} \quad (31c)$$

Similarly to the discussed inner-loop design and bearing

in mind the fact that β and their first and second derivative must be available, it follows that in order to meet the requirement $G_2 = 0$, the control law for the ratio β must take the following structure (see [12], [13])

$$\varepsilon_{2f}^2 \frac{d^2 \beta}{dt^2} + a \varepsilon_{2f} \frac{d\beta}{dt} = k_2 \left(\frac{e_2}{T_2} - \frac{dx_{2f}}{dt} \right) \quad (32)$$

3.3.2. Singular Perturbation System of Outer Loop

In accordance to (30) and (32), the replacement of $\mathfrak{E}_f$ in (32) by the right member of (30), the reduced closed-loop system becomes

$$\begin{aligned} \varepsilon_{2f}^2 \frac{d^2 \beta}{dt^2} + a \varepsilon_{2f} \frac{d\beta}{dt} &= k_2 \psi_5(X, t) \\ &\quad + k_2 \left(\frac{e_2}{T_2} - \varepsilon_r \omega_g (\psi_1(X, t) + \psi_2(X, t)\beta) \mathfrak{E} \right) \\ &\quad + k_2 (\psi_3(X, \varepsilon_r, t)\beta + \psi_4(X, \varepsilon_r, t)\beta^2) \end{aligned} \quad (33a)$$

$$\frac{dx_{2f}}{dt} = \varepsilon_r \omega_g (\psi_1(X, t) + \psi_2(X, t)\beta) \mathfrak{E} + \psi_3(X, \varepsilon_r, t)\beta + \psi_4(X, \varepsilon_r, t)\beta^2 + \psi_5(X, t) \quad (33b)$$

In order to enable us to use the standard technique of two time-scale motion analysis for (33a,b), we must normalize the ε_r to ε_{2f} . To this end, we consider $\varepsilon_{2f} = \varepsilon_r \omega_g \mathcal{G}$ with $\mathcal{G} \in [\mathcal{G}_{\min}, \mathcal{G}_{\max}]$. By setting $\beta = y_1$ and $\varepsilon_{2f} d\beta/dt = y_2$, the equations (33a,b) can be rewritten in the following form:

$$\varepsilon_{2f} \frac{dY}{dt} = g(X, Y, \varepsilon_{2f}, t) \quad (34a)$$

$$\frac{dx_{2f}}{dt} = f_2(X, Y, \varepsilon_{2f}, t) \quad (34b)$$

where

$$Y = (y_1 \quad y_2)^T,$$

$$g(X, Y, \varepsilon_{2f}, t) = \begin{pmatrix} y_2 \\ k_2 \left(\frac{e_2}{T_2} - f_2(X, Y, \varepsilon_{2f}, t) \right) - a y_2 \end{pmatrix},$$

$$f_2(X, Y, \varepsilon_{2f}, t) = \mathcal{G} (\psi_1(X, t) + \psi_2(X, t)y_1) y_2 + \psi_5(X, t) \psi_3(X, \varepsilon_{2f}, t)y_1 + \psi_4(X, \varepsilon_{2f}, t)y_1^2.$$

Let us introduce the new fast time-scale $\tau_{2f} = t/\varepsilon_{2f}$ into the equations of the above system. By letting $\varepsilon_{2f} = 0$, the FDS of the outerloop is given by

$$\frac{dY}{d\tau_{2f}} = \begin{pmatrix} y_2 \\ k_2 \left(\frac{e_2}{T_2} - f_2(X, Y, 0, t) \right) - a y_2 \end{pmatrix} \quad (35a)$$

$$\frac{dx_{2f}}{d\tau_{2f}} = 0 \quad (35b)$$

Remark 3. During the fast transient in (35a), the variables X are treated as the frozen parameters.

After the rapid decay of transients in (35a), the steady

state (more precisely, quasi-steady state) tends toward a stable equilibrium $Y^s = g^s(X, 0, t) = (y_1^s \ y_2^s)^T$, which is given by

$$y_1^s = \frac{-\psi_3(X, 0, t)}{2\psi_4(X, 0, t)} + \frac{\sqrt{\psi_3^2(X, 0, t) - 4\psi_4(X, 0, t)\left(\psi_5(X, t) + \frac{e_2}{T_2}\right)}}{2\psi_4(X, 0, t)} \quad (36a)$$

$$y_2^s = 0 \quad (36b)$$

The FDS (35a) of the outer loop is nonlinear. Thus, the stability properties of its equilibrium can be checked through the analysis of the Jacobian matrix of the linearized version defined as follows

$$A_F = \begin{bmatrix} 0 & 1 \\ -k_2 A_3(X, 0, t) & -k_2 A_4(X, 0, t) \end{bmatrix} \quad (37a)$$

With

$$A_4(X, t) = (a/k_2 + \mathcal{G}\psi_1(X, t) + \mathcal{G}y_1^s \psi_2(X, t)) \quad (37b)$$

$$A_3(X, 0, t) = (\psi_3(X, 0, t) + 2y_1^s \psi_4(X, 0, t)) \quad (37c)$$

Then, let us check that A_F is Hurwitz by determining the eigenvalues of the matrix

$$\lambda_{1,2} = \frac{-(a + k_2 \mathcal{G}(\psi_1 + \psi_2 y_1^s))}{2} \pm \frac{\sqrt{(a + k_2 \mathcal{G}(\psi_1 + \psi_2 y_1^s))^2 - 4k_2(\psi_3 + 2\psi_4 y_1^s)}}{2} \quad (38)$$

Taking into account that y_1^e is positive, the design parameters k_2 and a must be in the sense that

$$a > |k_2 \mathcal{G}(\psi_1(X, t) + \psi_2(X, t)y_1^e)| \quad (39a)$$

$$k_2(\psi_3(X, 0, t) + 2\psi_4(X, 0, t)y_1^e) > 0 \quad (39b)$$

Thus, we conclude that both eigenvalues of A_F are negatives. Hence, A_F is Hurwitz matrix. By substituting of the equilibrium Y^s into (34b), the SDS of outer loops takes place on the slow manifold according to

$$\dot{\mathcal{X}}_{2f} = \frac{e_2}{T_2} \quad (40)$$

The results are thus summarized in the following proposition.

Proposition 4. Consider the system of outer loop composed of (34a) and (34b). This system takes the singular perturbation form for $0 < \varepsilon_{2f} \ll 1$ where the FDS is given by (35a) and SDS is given by (40). One has

- i. If the gain k_2 and the parameter a are chosen in such a way that the inequalities (39a) and (39b) hold, the FDS (35a) will be exponentially stable and (y_1, y_2) converges exponentially towards (y_1^s, y_2^s) .

- ii. The behavior of x_{2f} is prescribed by the stable reference equation of the form $dx_{2f}/dt = (x_{2f}^* - x_{2f})/T_2$. Then, the requirement $\lim_{t \rightarrow \infty} e_2 = \lim_{t \rightarrow \infty} (x_{2f}^* - x_{2f}) = 0$ is maintained.

4. Stability Analysis

The global stability of closed loop system can be analyzed in the following theorem. It is shown that the control objectives are achieved (in the mean) with an accuracy that depends on $\rho = 1/\omega_g$ and small parameter ε

Theorem. (Main result) Consider the shunt active power filter, shown by Fig.1, represented by its average model (6a-e), where the states (x_{1r}, x_{1f}, x_{2f}) are accessible to measurements but (x_{2r}, x_{3r}) are not, together with the cascade controller consisting of:

- The observer (8a-c), where the observer gains Γ_{1r} and Γ_{2r} are positive and Γ_{3r} is sufficiently large negative, and ε_{1r} is a small parameter;
- The inner regulator (25), where $(\varepsilon_{1f}, k_1, T_1)$ are the design parameters;
- The outer regulator (34), where $(a, \varepsilon_{2f}, k_2, T_2)$ are the design parameters and the reference signal $(x_{2f}^* = V_d)$ with $(E_g < V_d)$.

Then, the resulting closed-loop system enjoys the following properties:

- i. The state observation errors $(\tilde{x}_{1r}, \tilde{x}_{2r}, \tilde{x}_{3r})$ related to the power network observer, converge asymptotically to zero.
- ii. The augmented state vector $Z = (\tilde{z}_1 \ \tilde{z}_2 \ \tilde{z}_3 \ \tilde{z}_4)^T = (z \ y_1 \ y_2 \ \tilde{x}_{1r})^T$, and $X = (x_1 \ x_2 \ x_3 \ x_4)^T = (x_{1f} \ x_{2f} \ \tilde{x}_{2r} \ \tilde{x}_{3r})^T$ undergoes the following state equations

$$\dot{\mathcal{Z}} = DH(X, Z, \varepsilon, t) = D \begin{pmatrix} k_1 \left(\frac{(x_1^* - x_1)}{T_1} - \frac{v_{pcc}}{L_f} + \frac{x_2}{L_f} \tilde{z}_1 \right) \\ \tilde{z}_3 \\ k_2 \left(\frac{e_2}{T_2} - f_2(X, Z, \varepsilon, t) \right) - a \tilde{z}_3 \\ \frac{x_3 - \Gamma_{1r} I_f}{\omega_g} \end{pmatrix} \quad (41a)$$

$$\mathcal{X} = F(X, Z, \varepsilon, t) = \begin{pmatrix} \frac{v_{pcc}}{L_f} - \frac{x_2}{L_f} \tilde{z}_1 \\ f_2(X, Z, \varepsilon, t) \\ x_4 - \Gamma_{2r} I_f \\ -\omega_g^2 x_3 - \Gamma_{3r} I_f \end{pmatrix} \quad (41b)$$

Where $\varepsilon = (\varepsilon_{1f} \quad \varepsilon_{1r} \quad \varepsilon_{2f})^T = (\varepsilon_1 \quad \varepsilon_2 \quad \varepsilon_3)^T$
 $\varsigma = \max(\varepsilon_1, \varepsilon_2, \varepsilon_3) \quad D = (\varsigma/\varepsilon_1 \quad \varsigma/\varepsilon_2 \quad \varsigma/\varepsilon_2 \quad \varsigma/\varepsilon_3)$.

- i. Let the control design parameters be selected such that the following inequalities hold
 $0 < \varepsilon_1 < \varepsilon_2 \ll 1$, $0 < \varepsilon_3 \ll 1$ $T_1 < T_2$, $k_1 < 0$,
 $k_2 > 0$, and $a > 0$, $\Gamma_{1r} > \|\tilde{x}_{2r}\|$, $\Gamma_{2r} \gg 0$,
 $\Gamma_{3r} = \Gamma_{1r}(1 - \omega_g^2)$.

Then, there exist positive constants ρ^* , ς^* , and λ such that the system (34a,b) has a unique asymptotically stable $(2\pi/\omega_g)$ periodic solution $X_p(\rho, \varsigma, t)$, $Z_p(\rho, \varsigma, t)$ with the properties

- $\|X_p(\rho, \varsigma, t) - X_0(\varsigma)\| \leq \lambda\rho$,
 $\|Z_p(\rho, \varsigma, t) - Z_0(\varsigma)\| \leq \lambda\rho$ for all $0 < \rho < \rho^*$;
- $X_0(\varsigma) - X_0^s = O(\varsigma)$, $Z_0(\varsigma) - Z_0^s = O(\varsigma)$ for all $0 < \varsigma < \varsigma^*$.

Where $X_0^s = (x_{1,0}^* \quad V_d \quad 0 \quad 0)^T$ and

$$Z_0^s = (\tilde{x}_{1,0}^s \quad \tilde{x}_{2,0}^s \quad 0 \quad 0)^T, \quad \tilde{x}_{1,0}^s = \frac{V_{pcc,0}}{V_d} \text{ and}$$

$$\tilde{x}_{1,0}^s = \frac{(2E_g + r_g I_{L1} \cos(\varphi_1))}{4r_g} - \frac{\sqrt{(2E_g + r_g I_{L1} \cos(\varphi_1))^2 - 8r_g E_g I_{L1} \cos(\varphi_1)}}{4r_g}$$

Proof 3. See Appendix B.

5. Numerical Simulation

The experimental setup is described by Fig. 1 and simulated within the MATLAB/ SIMULINK environment. The controlled APF-nonlinear load includes a single-phase shunt active power filters with the characteristic of Fig. 1, while the chosen nonlinear load is a single-phase full bridge rectifier supplying an inductive load which consists of a resistor R in series with an inductor L with the characteristics of Table 1.

Table 1. System and APF Characteristics

Parameters	Symbol	Values
Network	E_g / f_g	$220\sqrt{2}V/50Hz$
	$L_g ; r_g$	$10^{-4}H ; 0.05\Omega$
APF	$L_f ; C_f$	$1.8mH ; 1500\mu F$
Rectifier	L_o	$4mH$
	$L ; R$	$100mH ; 5\Omega$

As a matter of fact, the controller implementation entails the proper choice of the numerical values given to the controller parameters. It is worth noting that the suitable

choices for these values consist in proceeding with singular perturbation approach Doing so, the numerical values are summarized in Table 2.

Table 2. Controller Parameters

Parameters	Symbol	Values
Two time-scale S.M Observer	$\Gamma_{1r} ;$ Γ_{2r} Γ_{3r}	$4 ; 8 \times 10^5$ -3.9478×10^5
Inner Loop	$k_1 ; T_1$ ε_{1f}	$-4.1 \times 10^{-5} ; 10^{-5}s$ 1.5×10^{-5}
Outer Loop	$k_2 ;$ T_2 $\varepsilon_{2f} ;$ a	$1.1 \times 10^{-5} ; 9.4 \times 10^{-3}s$ $1.2 \times 10^{-4} ; 17.31 \times 10^{-2}$

5.1. Control Performance in Presence of Constant Load

The objective of the simulation is to illustrate the performances and design aspects of the controller in response to progressive changes of the voltage reference V_d and the load resistance R . According to Tables 1, 2, and the simulation protocol described by Fig. 2, it is seen that voltage reference goes from 600V to 1000V while the load resistance is kept constant equal to 5 Ω .

Figures (2 to 8) illustrate the resulting controller performance. As expected (Theorem 1) the output voltage v_o is tightly regulated: it quickly converges in the mean to its reference value (see Fig. 2), furthermore, it is checked that the output voltage is subjected to small amplitude ripples oscillating at the frequency $2\omega_g$. Fig. 3 shows that the ratio β always takes a constant value after the transient periods that results from the reference voltage steps. This confirms the achievement (in the mean) of the unit power factor. This is also illustrated by Fig. 4 which shows that the network current i_g is remains sinusoidal all the time (except during short transient periods) and in phase with the voltage network ensuring a unit power factor. The performance of the observer is illustrated by this figure which shows that the estimated signals $(\hat{x}_{1r}, \hat{x}_{2r})$ converge to their true values (i_g, v_g) ensuring the efficiency of the observer. In order to better appreciate the controller performances, a zoom is made in Fig. 5 on power network current i_g . Fig. 6 illustrates the corresponding input filter current i_f that follows its reference. The compensation current delivered by the active filter is highly infected with harmful harmonics generated by the load. This is further illustrated by Fig. 7 which shows that the waveform of the load current i_c presents a harmonic distortion and phase shift with respect to v_g . Fig. 8 shows the grid current spectrogram, where the THD value of this current after compensation is very low (1.79%).

Figure 2. Output voltage v_o with its reference V_d

Figure 3. Control signal β

(a)

(b)

Figure 4. (a) Network current i_g and voltage v_g . (b) Zoom on (a)

Figure 5. Network current i_g

Figure 6. Input filter current i_f with its reference i_f^*

Figure 7. Load current i_c

Figure 8. Load current i_c

5.2. Control Performance in Presence of Energy Level Change

The robustness of the nonlinear cascade controller against uncertainties on system parameters is progressively checked in the present subsection. It consists in changing the load resistance from 5Ω to 1Ω at $0.35s$, except for this change, the rest of the converter characteristics are the same as previously but the DC bus voltage reference V_d is kept constant equal to $1000V$. The resistance change that is not accounted for in the controller design, entails a change of the involved energy level.

It is seen (from Figs 9, 10) that the control performance deterioration remains very limited despite an 80% uncertainty on the load resistance. In particular Fig. 9 shows that the disturbing effect due to load changes is well compensated by the controller. Therefore, it is seen that the controller still performs well despite the present uncertainty, except for a small change in the signals ripple; these grow up with energy level. Specifically, the load undergoes a deviation of 80% from its nominal value (Fig. 10). Nevertheless, the ripple signal ratio, which presently is nearly 4% (Fig.9) is very small.

Figure 9. Output voltage V_o .

Figure 10. Load current i_c .

5.3. Control Performance Robustness to System Parameter Uncertainty

Presently, it is of interest to evaluate the performances of the proposed controller in the presence of $L_f C_f$ -filter uncertainty (Fig11 and Fig 12). This mainly concerns the inductance and capacitance that may see their values vary according to the following progression: • a 50% decrease at time 0.3s; • a 100% increase at time 0.45s.

Such variation is entirely ignored in the controller where the nominal value is used all time. The corresponding control performances are partly illustrated by Fig. 11 focusing on capacitance uncertainty, and partly illustrated by Fig. 12 focusing on inductance uncertainty. It is seen that the control performance deterioration remains very small compared to the 50% and 100% uncertainty on the $L_f C_f$ - filter components.

Figure 11. Illustration of controller robustness against capacitance uncertainty; (a) control signal β , (b) Output voltage v_o , (c) power factor checking

It is also seem that the ripple voltage grow up after each transient periods following decrease capacitance or in ductance (uncertain value) and vice-versa.

Figure 12. Illustration of controller robustness against inductance uncertainty; (a) control signal β , (c) Output voltage v_o , (d) power factor checking

In comparison with reference [8] where the authors use a cascade control structure with two overlapping loops using backstepping technique in the inner loop and a PI regulator in the outer loop, we have almost the same results (Figures 2,3,5-7,9,10) with the exception of two things:

- In our case, the grid voltage is estimated by means of a state observer based on two-time scale sliding mode using singular perturbations theory (Figure 4a,b). Indeed, the grid voltage is actually inaccessible to measurement because of the presence of grid impedance. This is completely ignored in the reference [8] where the grid is simply represented by a perfect voltage source;
- A robustness test has been added (figures 11 and 12). This shows that our control approach is robust with regard to certain parametric variations such as the change in capacitance C_f and inductance L_f .

6. Conclusions

In this paper we have considered the problem of controlling a single-phase shunt active power filters. The control objectives are compensation of harmonic and reactive current in the presence of nonlinear and uncertain loads, as well as tight voltage regulation at the inverter output capacitor. The converter dynamics have been described by the average 5th order nonlinear state space model. A sliding mode observer design in order to estimate the non-measurable state x_{2r} based on the partial model (4a-b), has been presented using two-time-scale approach. Based on (4a-e), a cascade nonlinear controller has been designed via the singular perturbation technique. It has been formally established, with aid of singular perturbation technique and averaging theory, that the obtained controller meets its objectives. The theoretically proved performances of the controller are confirmed by several simulations that also show its robustness against load change and uncertainty.

Appendix

Appendix A1: (Proof 1)

Part 1. In order to occur the sliding mode in equation (9a) along the manifold $S(\tau_r) = \tilde{x}_{1r} = 0$, it is natural to choose the following function $V_1 = 0.5S^2$. In this time-scale and Since (6), the time-derivative of V_1 can be expressed by $dV_1/d\tau_r = -(S/\omega_g)(\Gamma_{1r} \text{sign}(S) - \tilde{x}_{2r})$. Therefore, for $S(\tau_r) = \tilde{x}_{1r} = 0$ to be asymptotically stable, it is sufficient to choose a positive number Γ_{1r} satisfying condition of equation (10) such that $S(dS/d\tau_r) < 0$.

Part 2. Consider the equations consisting of the SDS represented by (11a,b). To analyze the system stability, the following Lyapunov function candidate is considered $V_2 = 0.5\tilde{x}_{2r}^2 + 0.5\tilde{x}_{3r}^2$. Its time derivative along the trajectory of the errors can be expressed as $\dot{V}_2 = \tilde{x}_{2r}\dot{\tilde{x}}_{2r} + \tilde{x}_{3r}\dot{\tilde{x}}_{3r}$. Since (11a,b), one has $\dot{V}_2 = -(\tilde{x}_{2r}^2 \Gamma_{2r}/\Gamma_{1r}) - \tilde{x}_{2r}\tilde{x}_{3r}(-1 + \omega_g^2 + \Gamma_{3r}/\Gamma_{1r})$. For $\tilde{x}_{2r} = \tilde{x}_{3r} = 0$ to be asymptotically stable, it is sufficient to choose Γ_{2r} and Γ_{3r} satisfying the conditions of equation (13a,b).

Appendix A2: (Proof 2)

The equations of interest are defined by (29) and (14a). Then, time derivative of x_{2f} is expressed as follows

$$\frac{dx_{2f}}{dt} = \frac{v_{pcc}\beta\hat{x}_{2r}}{E_g C_f x_{2f}} - \frac{v_{pcc}i_L}{C_f x_{2f}} \quad (A1)$$

Let us determine the expression of v_{pcc} . To this end, the following equations are considered (given by (4a))

$$v_{pcc} = x_{2r} - r_g x_{1r} - \varepsilon_r \omega_g \frac{dx_{1r}}{dt} \quad (A2)$$

Since $x_{1r} = x_{1f} + i_L$ and $x_{1f}^* = \beta(\hat{x}_{2r}/E_g) - i_L$, and according to proposition 3 where $x_{1f} = x_{1f}^*$, one gets the following expression

$$x_{1r} = \beta \frac{\hat{x}_{2r}}{E_g} \quad (A3)$$

On the other hand, the substitution of $\tilde{x}_{2r} = x_{2r} - \hat{x}_{2r}$ yields

$$x_{1r} = \frac{\beta}{E_g} (x_{2r} - \tilde{x}_{2r}) \quad (A4)$$

The expression of v_{pcc} becomes, using (A2) and (A4):

$$v_{pcc} = \left(1 - \beta \frac{r_g}{E_g}\right) x_{2r} - \frac{\varepsilon_r \omega_g (x_{2r} \beta + \tilde{x}_{2r} \beta)}{E_g} + \beta \frac{\varepsilon_r \omega_g}{E_g} \tilde{x}_{2r} + \frac{(\varepsilon_r \omega_g \beta + \beta r_g)}{E_g} \tilde{x}_{2r} \quad (A5)$$

Replacing $\hat{x}_{2r}$ and $\tilde{x}_{2r}$ in (A5) by their expressions $\hat{x}_{2r} = \tilde{x}_{3r} - \Gamma_{2r} I_f$ and $\tilde{x}_{2r} = x_{3r}$ yields

$$v_{pcc} = \left(1 - \beta \frac{r_g}{E_g}\right) x_{2r} - \frac{\varepsilon_r \omega_g}{E_g} (x_{2r} \beta + x_{3r} \beta) + \left(\beta \frac{\varepsilon_r \omega_g}{E_g} + \beta \frac{r_g}{E_g}\right) \tilde{x}_{2r} + \beta \frac{\varepsilon_r \omega_g}{E_g} \tilde{x}_{3r} - \beta \frac{\varepsilon_r \omega_g}{E_g} \Gamma_{2r} I_f \quad (A6)$$

By introducing the expression of v_{pcc} (A6) and since (2b), the equation (A1) can be rewritten in the form of (30).

Appendix B: (Proof 3)

Part 1: This has already been established by Proposition 1.

Part 2: Equations (41a,b) are immediately obtained from (8a-c), (25a,b) and (34a,b).

Part 3: The stability of the time-varying system (41a,b) will now be performed in two steps using the averaging theory (e.g [20], chapter 10 in [17]) and the singular perturbation theory (e.g [16], [18] and chapter 11 in [17]).

Step 1. The first step consists in using the averaging theory. To this end, let us introduce the time-scale change $t' = \omega_s t$. Using this time, one gets that $\bar{\mathcal{X}}(t') = \rho D \bar{H}(\bar{X}, \bar{Z}, t')$ and $\bar{\mathcal{X}}(t') = \rho \bar{F}(\bar{X}, \bar{Z}, t')$. In view of (41a,b), it is seen that these functions are periodic with period 2π . Let us introduce the averaged functions:

$$\bar{\mathcal{X}}_0 = \rho D \lim_{\rho \rightarrow 0} \frac{1}{2\pi} \int_0^{2\pi} \bar{H}(\bar{X}, \bar{Z}, t') dt' = \rho D H_0(X_0, Z_0) \quad (B1a)$$

$$\bar{\mathcal{X}}_0 = \rho \lim_{\rho \rightarrow 0} \frac{1}{2\pi} \int_0^{2\pi} \bar{F}(\bar{X}, \bar{Z}, t') dt' = \rho F_0(X_0, Z_0) \quad (B1b)$$

Since the systems here studied present equilibrium different from zero, in order to satisfy this requirement, a change of variables is introduced such that defines the new system in terms of its error-dynamics. Therefore, the error dynamics are defined by introducing: $\tilde{Z}_0 = Z_0 - Z_0^*$ and $\tilde{X}_0 = X_0 - X_0^*$, where the constants Z_0^* et X_0^* represent the desired average values of the state variables. Then, since (41a,b) and according to (B1a,b), the system can be rewritten into its error dynamics formulation thus defining the closed-loop error dynamics as:

$$\bar{\mathcal{X}}_0 = \rho D \tilde{H}_0(\tilde{X}_0, \tilde{Z}_0, \varepsilon) \quad (B2a)$$

$$\bar{\mathcal{X}}_0 = \rho \tilde{F}_0(\tilde{X}_0, \tilde{Z}_0, \varepsilon) = \rho \begin{pmatrix} \frac{v_{pcc,0}}{L_f} - \frac{(\tilde{x}_{2,0} + x_{2,0}^*)}{L_f} (\tilde{z}_{1,0} + z_{1,0}^*) \\ \tilde{f}_{2,0}(\tilde{X}_0, \tilde{Z}_0, \varepsilon) \\ \tilde{x}_{4,0} - \Gamma_{2r} I_{f,0} \\ -\omega_s^2 \tilde{x}_{3,0} - \Gamma_{3r} I_{f,0} \end{pmatrix} \quad (B2b)$$

In order to get stability results regarding the system of interest (41a,b), it is sufficient (thanks to averaging theory) to analyze the averaged system (B2a,b).

Step 2: The asymptotic stability analysis formulation for the general two-time-scale singular perturbation system (B2a,b), follows the well known theory of asymptotic stability analysis for multi-parameter Singularly Perturbed Systems, based on the boundary-layer system and reduced system [16].

For the boundary-layer system, it is necessary to ensure that its dynamic does not shift from the following quasi-steady-state equilibrium

$$\tilde{Z}_0^s = \tilde{H}_0^s(\tilde{X}_0) = \begin{pmatrix} \frac{L_f}{(\tilde{x}_{2,0} + x_{2,0}^*)} \left(\frac{\tilde{x}_{1,0}}{T_1} + \frac{v_{pcc,0}}{L_f} \right) - \tilde{z}_{1,0}^* \\ \tilde{H}_{2,0}^s \\ 0 \\ 0 \end{pmatrix} \quad (B3a)$$

Where

$$\tilde{H}_{2,0}^s = -\tilde{z}_{2,0}^* - \frac{\tilde{\psi}_{3,0}(\tilde{X}_0, 0)}{2\tilde{\psi}_{4,0}(\tilde{X}_0, 0)} + \frac{\sqrt{\tilde{\psi}_{3,0}^2(\tilde{X}_0, 0) - 4\tilde{\psi}_{4,0}(\tilde{X}_0, 0) \left(\tilde{\psi}_{5,0}(\tilde{X}_0) + \frac{\tilde{x}_{2,0}}{T_2} \right)}}{2\tilde{\psi}_{4,0}(\tilde{X}_0, 0)} \quad (B3b)$$

Then, the Lyapunov function candidate for this subsystem is obtained by introducing a change of variables $\tilde{Z}_0 = \tilde{Z}_0 - \tilde{H}_0^s(\tilde{X}_0)$ so that its equilibrium is centred at zero. First, by letting $\tau = t'/\varepsilon$, the boundary-layer system is defined by

$$\frac{d\tilde{Z}_0}{d\tau} = \rho D \tilde{H}_0(\tilde{X}_0, \tilde{Z}_0, 0) = \rho D \begin{pmatrix} k_1 \frac{(\tilde{x}_{2,0} + x_{2,0}^*)}{L_f} \tilde{z}_{1,0} \\ \tilde{H}_{3,0}(\tilde{X}_0, \tilde{Z}_0, 0) \\ (\tilde{x}_{3,0} - \Gamma_{1r} I_{f,0})/\omega_s \end{pmatrix} \quad (B3c)$$

With

$$\begin{aligned} \tilde{H}_{3,0}(\tilde{X}_0, \tilde{Z}_0, 0) &= -k_2 \left[\mathcal{G} \tilde{\psi}_{2,0}(\tilde{X}_0) \tilde{z}_{2,0} \tilde{z}_{3,0} + \tilde{\psi}_{4,0}(\tilde{X}_0, 0) \right. \\ &\times \tilde{z}_{2,0}^* + \left(\frac{a}{k_2} + \mathcal{G} \tilde{\psi}_{1,0}(\tilde{X}_0) + \mathcal{G} \tilde{\psi}_{2,0}(\tilde{X}_0) \tilde{H}_{2,0}^s \right) \tilde{z}_{3,0} \\ &\left. + (\tilde{\psi}_{3,0}(\tilde{X}_0, 0) + 2\tilde{H}_{2,0}^s \tilde{\psi}_{4,0}(\tilde{X}_0, 0)) \tilde{z}_{2,0} \right] \end{aligned} \quad (B3d)$$

in which $\tilde{X}_0$ is treated as a fixed parameter. Thus, the associated Lyapunov function can be defined by

$$V_{UH}(\tilde{X}_0, \tilde{Z}_0) = V_U(\tilde{X}_0, \tilde{z}_{1,0}) + V_F(\tilde{X}_0, \tilde{z}_{2,0}, \tilde{z}_{3,0}) + V_I(\tilde{z}_{4,0}) \quad (B4)$$

$$\begin{cases} V_U = \frac{1}{2\rho} p_U \tilde{z}_{1,0}^2 \\ V_F = \frac{1}{2\rho} (p_{F1} \tilde{z}_{2,0}^2 + p_{F2} \tilde{z}_{3,0}^2 + 2p_{F3} \tilde{z}_{3,0} \tilde{z}_{2,0}) \\ V_I = \frac{1}{2\rho} \tilde{z}_{4,0}^2 \end{cases} \quad (B5)$$

According to the proposition 2, it is shown that, for $k_1 < 0$, the solution p_U can be chosen as

$$p_U = \frac{L_i}{2\rho k_1 (\tilde{x}_{2,0} + x_{2,0}^*)} q_U \quad (B6)$$

with q_U is positive.

According to the proposition 4, and comparing the average value of

$$\tilde{\psi}_{3,0}(\tilde{X}_0, 0) = \frac{2E_g + I_{L1}r_g \cos(\varphi_1)}{2(\tilde{x}_{2,0} + x_{2,0}^*)C_f} \quad (B7a)$$

$$\tilde{\psi}_{4,0}(\tilde{X}_0, 0) = \frac{r_g \tilde{x}_{3,0}^2 - r_g E_g^2}{(\tilde{x}_{2,0} + x_{2,0}^*)E_g^2 C_f} \quad (B7b)$$

it is shown that, for $k_2 > 0$, the solutions (p_{F1} , p_{F2} and p_{F3}) to the associated Lyapunov V_F can be chosen as

$$p_{F1} = \left(\frac{\tilde{A}_{4,0}(\tilde{X}_0, 0)}{\tilde{A}_{3,0}(\tilde{X}_0, 0)} + \frac{1}{k_2 \tilde{A}_{4,0}(\tilde{X}_0, 0)} \right) q_{F1} + \frac{\tilde{A}_{3,0}(\tilde{X}_0, 0)}{\tilde{A}_{4,0}(\tilde{X}_0, 0)} q_{F2} \quad (B7c)$$

$$p_{F2} = \frac{q_{F1}}{k_2^2 \tilde{A}_{3,0}(\tilde{X}_0, 0) \tilde{A}_{4,0}(\tilde{X}_0, 0)} + \frac{q_{F2}}{k_2 \tilde{A}_{4,0}(\tilde{X}_0, 0)} \quad (B7d)$$

$$p_{F3} = \frac{1}{k_2 \tilde{A}_{3,0}(\tilde{X}_0, 0)} q_{F1} \quad (B7e)$$

Now, the reduced system is obtained by substituting of the equilibrium $\tilde{Z}_0^s = \tilde{H}_0^s(\tilde{X}_0)$ into (B2b) with $I_{f,0}^{eq} = \tilde{x}_{3,0}/\Gamma_{1r}$, one gets

$$\begin{aligned} \dot{\tilde{X}}_0 &= \rho \tilde{f}_0(\tilde{X}_0, \tilde{H}_0^s(\tilde{X}_0), 0) = \rho \left(-\frac{\tilde{x}_{1,0}}{T_1} - \frac{\tilde{x}_{2,0}}{T_2} \right. \\ &\quad \left. \left(\tilde{x}_{4,0} - \frac{\Gamma_{2r}}{\Gamma_{1r}} \tilde{x}_{3,0} \right) \left(-a^2 \tilde{x}_{3,0} - \frac{\Gamma_{3r}}{\Gamma_{1r}} \tilde{x}_{3,0} \right) \right)^T \end{aligned} \quad (B8a)$$

It is easy to define the associated Lyapunov function for the slow dynamic subsystems as follows

$$V_s(\tilde{X}_0) = \frac{1}{2\rho} \tilde{X}_0^T P_s \tilde{X}_0 = \frac{1}{2\rho} (p_{s1} x_{1,0}^2 + p_{s2} x_{2,0}^2 + \tilde{x}_{3,0}^2 + \tilde{x}_{4,0}^2) \quad (B8b)$$

where $p_{s1} = T_1 q_{s1}/2$ and $p_{s2} = T_2 q_{s2}/2$ are positive constants.

From the Lyapunov functions V_s , and V_{UFI} , one can determine the condition on the value of the parameter ζ in order to guarantee the stability of the entire system. These functions must satisfy certain inequality

- Isolated Equilibrium of the Origin:

The origin ($\tilde{X}_0 = 0, \tilde{Z}_0 = 0$) is an isolated equilibrium for the system (A2a,b), i.e. $\tilde{F}_0(0,0) = 0$ and $\tilde{H}_0(0,0) = 0$. Moreover, $\tilde{Z}_0 = \tilde{H}_0^s(\tilde{X}_0)$ are the root of $0 = \tilde{H}_0(\tilde{X}_0, \tilde{Z}_0)$, which vanishes at $\tilde{X}_0 = 0$. And $\|\tilde{H}_0^s(\tilde{X}_0)\| < \Theta(\|\tilde{X}_0\|)$ where $\Theta(\cdot)$ is a κ function

- Reduced System Condition:

Using (B8b) and (B8a), and since proposition 1, one gets

$$\left(\frac{\partial V_s(\tilde{X}_0)}{\partial \tilde{X}_0} \right)^T \rho \tilde{f}_0(\tilde{X}_0, \tilde{H}_0(\tilde{X}_0)) \leq -\alpha_1 \phi^2(\tilde{X}_0) \quad (B9a)$$

Where $\alpha_1 \leq 1$ and

$$\phi(\tilde{X}_0) = \sqrt{\frac{q_{s1}}{2} \tilde{x}_{1,0}^2 + \frac{q_{s2}}{2} \tilde{x}_{2,0}^2 + \frac{\Gamma_{2r}}{\Gamma_{1r}} \tilde{x}_{3,0}^2} \quad (B9b)$$

- Boundary-Layer System Condition:

Using the following Lyapunov function $V_{UFI}(\tilde{X}_0, \tilde{Z}_0)$, given by (B4,5) and substituting $\rho \dot{\tilde{H}}_0(\tilde{X}_0, \tilde{Z}_0)$ given by (B3c,d), one obtains

$$\begin{aligned} &\left(\frac{\partial V_{UFI}(\tilde{X}_0, \tilde{Z}_0)}{\partial \tilde{Z}_0} \right)^T \rho \dot{\tilde{H}}_0(\tilde{X}_0, \tilde{Z}_0) \leq -\frac{q_U}{2} \zeta_{1,0}^2 \\ &- \zeta_{3,0}^2 \left[\frac{q_{F2}}{2} - \frac{g \tilde{\psi}_{2,0}(\tilde{X}_0)}{\tilde{A}_{3,0}(\tilde{X}_0, 0)} \zeta_{\epsilon,0} \right] q_{F1} \\ &- \zeta_{\epsilon,0}^2 \left[\frac{q_{F1}}{2} - \frac{g \tilde{\psi}_{2,0}(\tilde{X}_0) \zeta_{3,0} + \tilde{\psi}_{4,0}(\tilde{X}_0, 0) \zeta_{2,0}}{k_2 \tilde{A}_{3,0}(\tilde{X}_0, 0) \tilde{A}_{4,0}(\tilde{X}_0, 0)} \right] \\ &\times \left[\frac{(q_{F1} + k_2 \tilde{A}_{3,0}(\tilde{X}_0, 0) q_{F2})}{k_2 \tilde{A}_{3,0}(\tilde{X}_0, 0) \tilde{A}_{4,0}(\tilde{X}_0, 0)} \right] \\ &+ \frac{\tilde{\psi}_{4,0}(\tilde{X}_0, 0)}{\tilde{A}_{3,0}(\tilde{X}_0, 0)} \zeta_{3,0} \left[q_{F1} \zeta_{2,0}^2 - \frac{1}{2} c |\zeta_{4,0}| \right] \end{aligned} \quad (B10a)$$

with $c = \Gamma_{1r} - \|\tilde{x}_{3,0}\| > 0$. Practically, it is assumed that $\tilde{Z}_{0,\min} \leq \tilde{Z}_0 \leq \tilde{Z}_{0,\max}$ and $\tilde{X}_{0,\min} \leq \tilde{X}_0 \leq \tilde{X}_{0,\max}$. By ensuring the appropriate choice of q_U , q_I , and q_{F2} such as

$$-\frac{q_U \zeta_{1,0}^2}{2} - \frac{q_I}{2} c |\zeta_{4,0}| + \left| \frac{\tilde{\psi}_{4,0}(\tilde{X}_0, 0) \zeta_{3,0}}{\tilde{A}_{3,0}(\tilde{X}_0, 0)} \right|_{\max} q_{F1} \zeta_{2,0}^2 < 0 \quad (B10b)$$

$$\frac{q_{F2}}{2} > \left| \frac{g \tilde{\psi}_{2,0}(\tilde{X}_0)}{\tilde{A}_{3,0}(\tilde{X}_0, 0)} \zeta_{\epsilon,0} \right|_{\max} q_{F1} \quad (B10c)$$

And by ensuring the appropriate choice of k_2 , a , (where $0 < k_2 \ll a$) and q_{F1} such as

$$\begin{aligned} \frac{q_{F1}}{2} &> \left| \frac{g \tilde{\psi}_{2,0}(\tilde{X}_0) \zeta_{3,0} + \tilde{\psi}_{4,0}(\tilde{X}_0, 0) \zeta_{2,0}}{k_2 \tilde{A}_{3,0}(\tilde{X}_0, 0) \tilde{A}_{4,0}(\tilde{X}_0, 0)} \right|_{\max} \\ &\times \left[\frac{(q_{F1} + k_2 \tilde{A}_{3,0}(\tilde{X}_0, 0) q_{F2})}{k_2 \tilde{A}_{3,0}(\tilde{X}_0, 0) \tilde{A}_{4,0}(\tilde{X}_0, 0)} \right]_{\max} \end{aligned} \quad (B10d)$$

So, it is shown that

$$\left(\frac{\partial V_{UFI}(\tilde{X}_0, \tilde{Z}_0)}{\partial \tilde{Z}_0} \right)^T \rho \dot{\tilde{H}}_0(\tilde{X}_0, \tilde{Z}_0) \leq \alpha_2 \phi^2(\tilde{Z}_0) \quad (B10e)$$

Where $\alpha_2 \leq 1$ and

$$\begin{aligned} \phi^2(\tilde{Z}_0) = & \frac{q_U}{2} \left(\tilde{z}_{1,0}^2 + \tilde{z}_{3,0}^2 \right) \left[\frac{q_{F2}}{2} - \left| \frac{g\tilde{\psi}_{2,0}(\tilde{X}_0)}{\tilde{A}_{3,0}(\tilde{X}_0, 0)} \tilde{z}_{\epsilon,0} \right|_{Max} q_{F1} \right] \\ & + \tilde{z}_{\epsilon,0}^2 \left[\frac{q_{F1}}{2} - \left| \frac{(g\tilde{\psi}_{2,0}(\tilde{X}_0)\tilde{z}_{3,0} + \tilde{\psi}_{4,0}(\tilde{X}_0, 0)\tilde{z}_{2,0})}{k_2 \tilde{A}_{3,0}(\tilde{X}_0, 0) \tilde{A}_{4,0}(\tilde{X}_0, 0)} \right|_{Max} \right] \\ & \times \left[\frac{(q_{F1} + k_2 \tilde{A}_{3,0}(\tilde{X}_0, 0) q_{F2})}{k_2 \tilde{A}_{3,0}(\tilde{X}_0, 0) \tilde{A}_{4,0}(\tilde{X}_0, 0)} \right]_{Max} \\ & - \left| \frac{\tilde{\psi}_{4,0}(\tilde{X}_0, 0)}{\tilde{A}_{3,0}(\tilde{X}_0, 0)} \tilde{z}_{3,0} \right|_{Max} q_{F1} \tilde{z}_{2,0}^2 + \frac{1}{2} c |\tilde{z}_{4,0}| \end{aligned} \quad (B10f)$$

Applying Theorem 11.1 in [17], one concludes that there exists a ς^* such that for $0 < \varsigma < \varsigma^*$, the differential equation (A2a,b) has a unique solution $\tilde{Z}_0(\varsigma)$ and $\tilde{X}_0(\varsigma)$

$$\begin{cases} \tilde{Z}_0(\varsigma) - \tilde{H}_0^s(\tilde{X}_0^s) = O(\varsigma) \\ \tilde{X}_0(\varsigma) - \tilde{X}_0^s = O(\varsigma) \end{cases} \quad (B11)$$

Where

$$\tilde{X}_0^s = (\tilde{x}_{1,0}^s \quad \tilde{x}_{2,0}^s \quad \tilde{x}_{3,0}^s \quad \tilde{x}_{4,0}^s)^T = (v_{pcc,0}/V_d \quad V_d \quad 0 \quad 0)^T.$$

- Assumption: (Interconnection conditions)

To lighten the presentation of this paper, we assume the following inequality

$$\begin{aligned} \left(\frac{\partial V_s(\tilde{X}_0)}{\partial \tilde{X}_0} \right)^T \rho[\tilde{f}_0(\tilde{X}_0, \tilde{Z}_0, \epsilon) - \tilde{f}_0(\tilde{X}_0, \tilde{H}_0^s(\tilde{X}_0), 0)] \\ \leq \theta_1 \phi(\tilde{Z}_0) \rho(\tilde{X}_0) + \varsigma \gamma_1 \phi^2(\tilde{X}_0) \end{aligned} \quad (B12a)$$

$$\begin{aligned} \left(\frac{\partial V_{UFI}(\tilde{X}_0, \tilde{Z}_0)}{\partial \tilde{Z}_0} \right)^T \rho[\tilde{H}_0(\tilde{X}_0, \tilde{Z}_0, \epsilon) \\ - \tilde{H}_0(\tilde{X}_0, \tilde{Z}_0, 0)] \leq \varsigma \gamma_2' \phi^2(\tilde{Z}_0) + \varsigma \theta_2' \phi(\tilde{Z}_0) \rho(\tilde{X}_0) \end{aligned} \quad (B12b)$$

$$\begin{aligned} \left(\frac{\partial V_{UFI}(\tilde{X}_0, \tilde{Z}_0)}{\partial \tilde{X}_0} \right)^T \rho f_0(\tilde{X}_0, \tilde{Z}_0, \epsilon) \leq \\ \theta_2'' \phi(\tilde{Z}_0) \rho(\tilde{X}_0) + \gamma_2'' \phi^2(\tilde{Z}_0) \end{aligned} \quad (B12c)$$

The constants θ_1 , θ_2' , θ_2'' , γ_1 , γ_2' and γ_2'' are nonnegative.

Finally, from the singular perturbation theory (e.g. [16] and Theorem 11.3 in [17]), it follows that the equilibrium X_0^s , and $\tilde{Z}_0^s$ is asymptotically stable for all $0 < \varsigma < \varsigma^*$ where

$$\varsigma^* = \frac{\alpha_1 \alpha_2}{\alpha_1 \gamma_2 + \alpha_2 \gamma_1 + \theta_1 \theta_2} \quad (B13)$$

where $\theta_2 = \theta_2' + \theta_2''$ and $\gamma_2 = \gamma_2' + \gamma_2''$. Therefore, Part 3 is immediately follows from the averaging theory (e.g.

Theorem 10.4 of [17]). Theorem 1 is established.

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