Assessing the Impact of Modelling on the Expected Credit Loss (*ECL*) of a Portfolio of Small and Medium-sized Enterprisesⁱ

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Abstract This paper studies the impact of the internal modelling on the calculation of expected credit loss in the framework of the international standard *IFRS 9*. Indeed, the probability of default of counterparty depends on the model used for the conception of the internal rating system. The multitude of probabilistic models renders uncertain and imprecise, the calculation of the expected loss for the same *SMEs* portfolio of a Moroccan bank, as well as the comparison of losses over time due to the non-permanence of the rating system used. As a result, the regulator will be unable to guarantee an equitable and transparent system of provisioning of the losses, because of the absence of standardization of the elaboration process of the rating tool. To show this risk associated with the multitude of models, this paper studied the impact of choice of the model on the expected credit loss, by calculating of the probability of default for several types of modelling based respectively on the pure logistic regression and the logistic regression on the principal components.

Keywords Logistic Regression Model, Principal Components Analysis (*PCA*), Probability of Default (*PD*), *IRB* Foundation Approach, Expected Loss (*EL*)

1. Introduction

The banking regulation defines several approaches for calculating the capital required to cover the credit risk. This enriches the techniques available for determining the risk profile of banks.

These approaches are composed of simple approaches based on the application of a coefficient, on the credit exposure and of complex approaches based on the internal rating models (Internal Ratings-Based *IRB*).

The expected loss calculation is based on the probability of default of the corporate, the loss at the time of default and the exposure, in term of outstanding amount of credit, in the event of default. Indeed, various studies were conducted to define an approach for predicting the probability of default. These studies concern linear discriminant analysis, the intelligence techniques, Bayesian networks and probabilistic models, which we will summarize as follows:

– Multidimensional Linear Discriminant Analysis

The prediction of default by linear discriminant analysis was developed by Altman [2], by defining a linear relationship between default and financial ratios. Indeed, Altman has defined a score function (*Z*) that distinguishes between the healthy and the failing companies. The Altman's approach has been adopted by other research to determine ratios that predict failure such as those conducted by: Taffler [40], Bardos and al. [10] and Grover and Lauvin and al. [26].

– Intelligence Techniques

These techniques are based on different logics such as neural networks and genetic algorithms. The several studies have applied these techniques to predict the default of the corporates, such as those conducted by: Bel and al. [13], Back and al. [8], Liang and wu [32], Bose and Pal [16] and Oreski and al. [35].

– Bayesian Network

The Bayesian classifier (Friedman and al., [23]) is based on the calculation of a posterior probability of each observation belonging to a specific class. Indeed, he finds the posterior probability distribution $P(Y|X)$, where $Y = (Y_1, \dots, Y_n)$ is a random variable to be classified in *k* categories and $X = (X_1, \dots, X_n)$ is a set of *n* explanatory variables. The Bayesian classification of failing companies

a was studied by a set of researchers as Gemela [24], Das and al. [20], Dwyer and al. [21], Gôssl [25] and Tasch [42].

– Probabilistic Models

The probabilistic models are the Logit model based on the logistic distribution and the Probit model based on the gaussian distribution. The several studies have focused on discriminant logistic analysis to predict the default of companies such as those conducted by Ohlson [34], Hunter and al. [31], Hensher and al. [28] while the Probit model has been studied by other researchers such as Zmijewski [44], Grover and al. [26] and Bunn and al. [18].

Due to the multitude of models, the determination of the expected credit loss (ECL)¹ as defined by the Basel Committee on Banking Supervision [12] becomes dependent on the models and techniques chosen for the elaboration of the rating system, which renders uncertain the calculation of the expected credit loss (ECL), by the financial institutions, as a result of absence of standardization of the techniques used. In this case, the institutions can exploit the opportunities offered by the modelling to minimize the expected credit loss at the detriment of transparency and stability through the recourse of the arbitrage between the techniques and the models possible. This situation gives rise to a risk of models whose impact on the stability of banking system will have the same importance as the credit risk.

In this paper, we will show the uncertainty in the calculation of expected credit loss, generated by the multitude of models and the absence of standardization and it, by determining the expected loss of an SMEs portfolio of a Moroccan bank. Indeed, we will study the impact of the modelling of probability of default (PD) on the calculation of the expected credit loss and this, by using two versions of probabilistic models to predict the probability of default (PD).

The Modelling in our study is based on the logistic regression because we will use the pure logistic regression and the logistic regression on the principal components to determine the probability of default per the rating class.

The rest of this paper is as follows, Section 2 is devoted to the calculation of the expected credit loss. We first give a definition of the credit risk. We then define the approach to calculate the unexpected and expected credit loss. We finally present, in this section, the construction approach of the scoring system and a probabilistic model that will be used. The third section is reserved to the empirical study. In this section, we will first analyze and describe the database used. Then we present and interpret the empirical results of the probability of default for each chosen model. We finally compare the expected credit loss calculated by the

chosen models.

2. Calculation of the Expected Credit Loss (ECL)

2.1. Definition and Quantification of Credit Risk

2.1.1. Definition

The credit risk of the banking portfolio is defined as the risk that counterparty will default for a one-year period. The notion of default means that the counterparty is unable to honor these commitments towards the establishment of credit. The incidents causing the default are multiple, but the most recurrent are:

- The downgrading outstanding debts in the bad debts²,
- The chronic overruns on lines of credit granted to clients,
- The accounts that have few transactions and the applications that have lapsed and that have not been renewed,
- The unfavorable echoes from the market, the sectoral difficulties and a significant fall in the level of activity.

Each credit relationship is associated with an actual or potential credit risk situation.

2.1.1.1. The Credit Risk Situation

The credit risk situation is composed of the following elements:

- **Probability of default (PD)**: Probability that a counterparty falls at default in a horizon one year.
- **Loss given default (LGD)**: The share, expressed as a percentage of the amount a bank loses when a borrower falls at default on a credit.
- **Exposure at default (EAD)**: The total value to which a bank is exposed when a credit is at default.
- **Maturity (M)**: The effective maturity of credit.

2.1.1.2. The Losses Associated at the Risk of Credit

We distinguish two types of losses:

- The Expected Loss (EL)

The expected loss is a percentage equal to the multiplication of the PD and the LGD. The amount of the expected loss is equal to the multiplication of the expected loss (EL) and the exposure at default (EAD):

$$\begin{aligned} \text{Amount of the expected loss (EL}_M\text{)} \\ = PD \times LGD \times EAD \end{aligned} \quad (1)$$

$$EL_M = EL \times EAD$$

¹ Basel Committee on Banking Supervision, 2015, Guidance on credit risk and accounting for expected credit losses, Bank for International Settlements.

² In the Morocco, The downgrading outstanding debts in the bad debts is done in accordance with the provisions of BAM circular no. 19/G/2002 and Bank Al-Maghreb's instructions in the matter

– The Unexpected Loss (*UL*)

The unexpected loss is the total loss from to the expected loss (*EL*). It is calculated as a standard deviation from the mean at a certain confidence level $(1 - \alpha)$. It is also called *VaR* of credit.

2.1.2. The Quantification of Credit Risk

For the quantification of credit risk, the Basel Committee provides two categories of approaches for calculating the minimum capital requirements for credit risk (*K*).

2.1.2.1. The Standardized Approach

Under the standardized approach, the bank must calculate the weighted assets. Indeed, a bank's total risk-weighted on-balance sheet equal the sum of the risk-weighted amounts of each asset it holds on-balance sheet.

The risk-weighted amount of an on-balance sheet asset is determined by multiplying its current book value of credit exposures by the risk weight (α) specified by the regulator. Exposures should be risk-weighted net of specific provisions.

$$\begin{aligned} \text{Risk-weighted amount}_{STA} \\ &= \alpha \times \text{credit exposures on} \\ &\quad - \text{balance} \end{aligned}$$

The risk-weighting (α) is defined by type of claims (claims on sovereigns and central banks, claims on other official entities, claims on banks and securities firms, claims on corporates, claims included in the regulatory retail portfolios, claims secured by residential property, claims secured by commercial real estate, treatment of past due loans, higher-risk categories et other assets)³; and depending on the rating assigned to the counterparty by the external rating agencies.

The minimum capital requirements under the standard approach (K_{STA}) to cover the given counterparty credit risk exposure is defined as follows:

$$K_{STA} = \beta * \text{risk-weighted amount}_{STA} \quad (2)$$

Where : β , is a minimum coefficient of solvability⁴

2.1.2.2. The Internal Ratings-Based Approach(*IRB*)

The Internal Rating-Based Approach is based on the four determinants mentioned above, which are the *PD*, *EAD*, *LGD* and the Maturity *M* to calculate of the unexpected loss (*UL*) and expected loss (*EL*) according of the regulatory requirements of Basel committee.

If the expected loss (*EL*) is defined by the formula (1), the unexpected loss (*PU*) is calculated by multiplying a coefficient (α) by the exposure at default (*EAD*). The coefficient (α) is a function of the *PD*, *LGD* and Maturity (*M*). As a result:

$$UL = f(PD, LGD, M)5 \times EAD \quad (3)$$

Furthermore, the banking regulations provide for two categories of Internal Rating-Based⁶:

– **The Internal Ratings-Based- foundation approach**

Under the foundation approach, as a general rule, banks provide their own estimates of probability of default and rely on supervisory estimates for other risk components (*LGD*, *EAD* and *M*).

– **The Internal Ratings-Based- advanced approach**

Under the advanced approach, banks provide of their own estimates of *PD*, *LGD* and *EAD*, and their own calculation of *M*.

2.2. Calculation of the Expected Credit Loss

Under the *IRB* approach (foundation and advanced), the probability of default (*PD*) is an important component in calculating the expected loss (*EL*). Indeed, the aim is to determine the risk profile of the credit counterparty on the basis of qualitative and quantitative data through a discriminant analysis. The internal rating tool must enable credit counterparties to be classified as a function of their probability of default (*PD*) and must predict customer failure. For this reason, we will present the process of conception of this type of classifier (rating tool):

2.2.1. Presentation of the Process of Conception of a Rating Tool

The following schema can summarize the process of conception of rating tool:

3 Annex 11, Basel Committee on Banking Supervision, 2006, International Convergence of Capital Measurement and Capital Standards, Bank for International Settlements

4 For Morocco this coefficient is fixed at 12% as from January 2012

5 The formula of *f* for *SMEs* is defined by Committee on Banking Supervision, 2006, International Convergence of Capital Measurement and Capital Standards, Bank for International Settlements at the paragraph level 272-273.

6 Basel Committee on Banking Supervision, 2006, International Convergence of Capital Measurement and Capital Standards, Bank for International Settlements at the paragraph level 245

Figure 1. The process of conception of rating tool

In the following, we will present the approach, which we have adopted for the conception of ratings models; it is based respectively on logistic regression and logistic regression on principal components.

2.2.1.1. Database Treatment

The treatment of the database and the choice of explanatory variables are made according to the following schema:

Figure 2. The treatment of the database and the choice of explanatory variables

The Constitution of the Database (Definition of Variables)

– The quantitative variables

The quantitative variables (V_j), $j = 1, \dots, 16$, are divided into 6 classes, defined as follows :

Table 1. The list of quantitative variables

Class	The quantitative variables ($V_j, 1 \leq j \leq 16$)
C_1 : Activity	$V_1 = \text{Sales/turnover (ST)}$ $V_2 = \text{Number of employees}$ $V_3 = \text{Profit growth}$ $V_4 = \text{Age of the company}$
C_2 : Return of investment	$V_5 = \frac{\text{Net Profit}}{\text{Equity}}$ $V_6 = \frac{\text{Net Profit}}{\text{Sales/turnover (ST)}}$
C_3 : Solvability	$V_7 = \frac{\text{Financial expenses}}{\text{Sales/turnover (ST)}}$ $V_8 = \frac{\text{DLMT (Debt long and medium term)}}{\text{Equity}}$ $V_9 = \frac{\text{Net Debt}}{\text{Equity}}$
C_4 : Liquidity	$V_{10} = \frac{(\text{Accounts Receivable} + \text{assets . Cash})}{(\text{Liabilities . Cash} + \text{working capital requirements} - \text{Accounts Receivable} - \text{inventory})}$
C_5 : financial structure	$V_{11} = \frac{\text{Working Capital}}{\text{Current Assets}}$ $V_{12} = \frac{\text{Equity}}{\text{Total Assets}}$
C_6 : Turnover	$V_{13} = \frac{\text{Sales/turnover (ST)}}{\text{Net fixed assets+WCR inventory}}$ where WCR is working capital requirements $V_{14} = \frac{\text{Sales/turnover (ST)}}{\text{Accounts Receivable}}$ $V_{15} = \frac{\text{Sales/turnover (ST)}}{\text{Accounts Payable}}$ $V_{16} = \frac{\text{Total purchase}}{\text{Total purchase}}$

– The qualitative variables

The qualitative variables ($q_m, 1 \leq m = 1, \dots, 19$) are grouped by theme $T_k, k = 1, \dots, 6$, as follows:

Table 2. The list of qualitative variables

Theme	The qualitative variables (q_m)
T_1 : The sector of activity	$(q_1 = q_{1T1})$: Sector fault rate $(q_2 = q_{2T1})$: Regulatory impact next year $(q_3 = q_{3T1})$: Exposure to natural risk
T_2 : The company's positioning and competition	$(q_4 = q_{1T2})$: Competitive position and intensity $(q_5 = q_{2T2})$: Barriers and new entrants $(q_6 = q_{3T2})$: International Competition
T_3 : The concentration and position of the counterparty vis-à-vis its suppliers and customers	$(q_7 = q_{1T3})$: Customer concentration $(q_8 = q_{2T3})$: Supplier concentration $(q_9 = q_{3T3})$: Positions vis-à-vis suppliers and customers
T_4 : Quality and management structure	$(q_{10} = q_{1T4})$: Succession planning and business continuity $(q_{11} = q_{2T4})$: Experience Chairman $(q_{12} = q_{3T4})$: Seniority of the principal operational staff $(q_{13} = q_{4T4})$: Capital distribution $(q_{14} = q_{5T4})$: Compliance with the accounting documents delivery schedule $(q_{15} = q_{6T4})$: Performance last crisis $(q_{16} = q_{7T4})$: Existence of agent's insurance.
T_5 : The company's history with banking	$(q_{17} = q_{1T5})$: Number of payment incidents in the last 12 months $(q_{18} = q_{2T5})$: Percentages of unpaid bills over the last 12 months
T_6 : Relations with banks	$(q_{19} = q_{1T6})$: Number of banks related to the company

Discretization of Qualitative Variables and Their Transformation into a Score

– Discretization of qualitative variables

The qualitative variables (q_m), $1 \leq m \leq 19$ are discretized into modalities. The number of modalities can be equal at 3 or 5 modalities. The rule of the modalities choice is based on the logical relationship between modalities and default.

– Transformation of quantitative variable into score

Let $(M_{q_m,l})$, $l = 1, \dots, l_{q_m}$, be the modalities of the qualitative variable (q_m) and (l_{q_m}) defines the number of modalities ($l_{q_m} \in \{3, 5\}$). For each modality, the score varies between 0 and 100 points with a jump of 50 points per modality for the variables at three modalities and a jump of 25 points for the variables at five-modalities. The score taken by the modalities is:

❖ Variables at three modalities : [0, 50, 100]

Example: the modalities relating to the sector default rate are : 1- below average, 2- equal to average, 3- above average. In this case, the scores given are respectively: 100, 50, 0.

❖ Variables at five modalities : [0, 25, 50, 75, 100]

Example: the modalities relating to natural risk are: 1- No risk, 2- Low risk and the adequate crisis plan, 3- High risk and the adequate crisis plan, 4- Low risk without crisis plan, 5- High risk without crisis plan. In this case, the scores given are respectively: 100, 75, 50, 25, 0.

The assessment of the logical relationship between the modalities of each variable and the default is determined on the basis of expert opinion.

Univariate analysis and the determination of explanatory variables

– Univariate analysis

The objective of univariate analysis is to determine the

relationship between a company's default and each of its quantitative and qualitative variables. The default is modeled by a binary variable Y defined as follows:

$$Y = \begin{cases} 1 & \text{if the company is healthy} \\ 0 & \text{if the company is in default} \end{cases} \quad (4)$$

The relationship between the variable Y to be explained and the explanatory variables V_j and q_m is determined by the logistic regression model.

Indeed, the definition (4) shows that the variable Y is a Bernoulli variable which takes the values 0 and 1 of parameter p with $P(Y = 1) = p$ and $P(Y = 0) = 1 - p$. Therefore, the formula (4) can be written in probability as follows:

$$P(Y = y_i) = p^{y_i}(1 - p)^{1-y_i} \quad (5)$$

The logistic regression relationship consists in defining a relationship between Y and a logistic probability (p) defined from the variables (V_j) and (q_m). Indeed, we need to make a logistical transformation (Logit) to bind the dependent variable Y to each variable among the quantitative variables (V_j), $j = 1, \dots, 16$ and qualitative variables (q_m), $m = 1, \dots, 19$.

For each quantitative variable V_j or qualitative q_m the relationship between the default and the variables studied is defined by the probability:

$$p_{0,j} = P(Y = 1/V_j) \text{ or } p_{0,q_m} = P(Y = 1/q_m)$$

Indeed, the modelling of Y conditionally to the quantitative variable V_j or qualitative variable q_m , is defined by the model:

$$Y = p_o + \varepsilon \quad (6)$$

With $p_o = P(Y = 1/V_j)$ (or $p_o = P(Y = 1/q_m)$)

❖ The modelling of the probability p_o

We will only present the univariate analysis of the

quantitative variables V_j since we will proceed in the same way for the analysis of the qualitative variables q_m .

Indeed, for each company (i), the value of the quantitative variable V_j is v_{ij} and let p_{0i} be the probability that the company (i) is healthy. Indeed, p_{0i} is defined by $P(Y_i = 1/V_j = v_{ij})$ with:

$$Y_i = \begin{cases} 1 & \text{if the company}(i) \text{ is healthy} \\ 0 & \text{if the company}(i) \text{ is in default} \end{cases}$$

Thus, the formula (6) for company (i) is written:

$$y_i = p_{0i} + \varepsilon_i$$

First, we will define the ratio $\frac{p_{0i}}{1-p_{0i}}$ noted the Odds of p_{0i} :

$$\begin{aligned} \text{Odds}(p_{0i}) &= \frac{p_{0i}}{1-p_{0i}} \\ &= \frac{P(Y_i = 1/V_j = v_{ij})}{1 - P(Y_i = 1/V_j = v_{ij})} \end{aligned} \quad (7)$$

Consequently, the logistic regression model establishes a linear regression relationship between $\ln\left(\frac{p_{0i}}{1-p_{0i}}\right)$ and v_{ij} as follows:

$$\text{Logit}(p_{0i}) = \ln\left(\frac{p_{0i}}{1-p_{0i}}\right) = \beta_0 + \beta_1 \times v_{ij} \quad (8)$$

where from $p_{0i} = \frac{e^{\beta_0 + \beta_1 \times v_{ij}}}{1 + e^{\beta_0 + \beta_1 \times v_{ij}}}$

Therefore, the formula (6) and the associated probability p_0 can be written:

$$\begin{cases} Y = p_0 + \varepsilon \\ p_0 = \frac{e^{\beta_0 + \beta_1 \times V_j}}{1 + e^{\beta_0 + \beta_1 \times V_j}} \end{cases}$$

❖ The estimation of parameter

Let Y_i be the random variable that models the company's default (i). The conditional variable ($Y_i/V_j = v_{ji}$) can be written in probability as follows:

$$\begin{aligned} P(Y_i/V_j = v_{ij}) &= P(Y_i = y_i/V_j = v_{ij}) = \\ &= p_{0i}^{y_i} \times (1 - p_{0i})^{(1-y_i)} \text{ with } y_i = 0 \text{ or } 1 \end{aligned} \quad (9)$$

❖ The likelihood function:

The likelihood function of n enterprise is defined by:

$$\begin{aligned} L &= \prod_{i=1}^n P(Y_i/V_j = v_{ij}) \\ &= \prod_{i=1}^n (1 - p_{0i})^{(1-y_i)} \times p_{0i}^{y_i} \end{aligned} \quad (10)$$

$$\begin{aligned} L &= \prod_{i=1}^n \left(\frac{e^{\beta_0 + \beta_1 \times v_{ij}}}{1 + e^{\beta_0 + \beta_1 \times v_{ij}}} \right)^{y_i} \\ &\times \left(1 - \frac{e^{\beta_0 + \beta_1 \times v_{ij}}}{1 + e^{\beta_0 + \beta_1 \times v_{ij}}} \right)^{(1-y_i)} \end{aligned}$$

❖ The Loglikelihood function (LL)

$$\begin{aligned} LL &= \sum_{i=1}^n y_i \times \ln\left(\frac{e^{\beta_0 + \beta_1 \times v_{ij}}}{1 + e^{\beta_0 + \beta_1 \times v_{ij}}}\right) \\ &+ \sum_{i=1}^n (1 - y_i) \times \ln\left(1 - \frac{e^{\beta_0 + \beta_1 \times v_{ij}}}{1 + e^{\beta_0 + \beta_1 \times v_{ij}}}\right) \end{aligned} \quad (11)$$

$$\begin{aligned} LL &= \sum_{i=1}^n \ln\left(\frac{1}{1 + e^{\beta_0 + \beta_1 \times v_{ij}}}\right) \\ &+ y_i \left(\ln\left(\frac{e^{\beta_0 + \beta_1 \times v_{ij}}}{1 + e^{\beta_0 + \beta_1 \times v_{ij}}}\right) - \ln\left(\frac{1}{1 + e^{\beta_0 + \beta_1 \times v_{ij}}}\right) \right) \\ LL &= \sum_{i=1}^n \ln\left(\frac{1}{1 + e^{\beta_0 + \beta_1 \times v_{ij}}}\right) + y_i \times (\beta_0 + \beta_1 \times v_{ij}) \end{aligned} \quad (12)$$

Let us note $\beta = (\beta_0, \beta_1)$ so the estimate of β is the determination of $\hat{\beta}$ which maximizes the likelihood function and consequently maximizes the Loglikelihood function (LL).

The maximization of LL will be done by solving the equation system determined by the following conditions:

$$\frac{\partial LL}{\partial \beta} = 0 \text{ and } \frac{\partial^2 LL}{\partial^2 \beta} < 0 \quad (13)$$

Three numerical algorithms are used to solve this problem, which are the Newton-Raphson algorithm, the score method and the Berndt-Hall-Hausman method.

❖ Testing of the significance of the coefficients (test de Wald)

The testing of the significance of the coefficient β_1 , is established by the Wald test. The Wald test is obtained by testing the hypothesis H_0 formulated as follows:

$$\begin{cases} H_0: \beta_1 = 0 \\ H_1: \beta_1 \neq 0 \end{cases} \quad (14)$$

The Wald test is based to the following ratio:

$$W_1 = \hat{\beta}_1^2 / \hat{\sigma}_{\beta_1}^2 \quad (15)$$

The ratio, under the hypothesis H_0 , will follow a χ^2 with one degree of freedom. We reject H_0 if $W_1 > \chi_1^2$.

❖ discriminatory power (power stat)

The discriminatory power represents the model's ability to predict future situations. We will use the *ROC* curve to determine the discriminatory power of each variable V_j and q_m . The determination of the *ROC* curve will be done from the classification table of the sample of estimation of the variable Y which is presented as follows:

Table 3. The classification table

	Healthy ($Y = 1$)	Default ($Y = 0$)
Healthy ($Y = 1$)	True Healthy (TH)	False Healthy (FH)
Default ($Y = 0$)	False Default (FD)	True Default (TD)

One indicates by sensitivity (SV), the proportion of the healthy companies classified well: $SV = \frac{TH}{TH+FH}$ and by specificity (SP), the proportion of the de companies is in default, classified well: $SP = \frac{FD}{FD+TD}$

If one varies the “probability threshold” from which it is considered that a company must be regarded as healthy, the sensitivity and specificity varies. The curve of the points $(1 - SP, SV)$ is the *ROC* curve.

– The determination of explanatory variables

To determine the explanatory variables to be retained for modelling, we will carry out a univariate logistic regression analysis for each variable in the chosen list. The choice of variables to be retained for the modelling is based on the discriminatory power of each variable.

The discriminatory power is determined by using the area under the *ROC* curve and the Accuracy ratio (*AR*).

❖ Definition of the area under the *ROC* curve (*AUC*) and the Accuracy ratio (*AR*)

✓ The area under the *ROC* curve (*AUC*)

The area under the *ROC* curve (*AUC*) provides an overall measure of model fit (Bewick, Cheek, & Ball,[45]). The *AUC* varies from 0,5 (predictive capacity absence for the model) to 1 (perfect predictive aptitude for the model).

✓ Accuracy ratio (*AR*)

The accuracy ratio is defined by the relationship:

$$AR = 2AUC - 1 \quad (16)$$

The *AR* takes values between 0 and 1.

❖ Determination of explanatory variables

✓ The decision rules

The variables that verify the following two characteristics will be retained:

- The Area Under the *ROC* Curve (*AUC*) is superior to 60% and accuracy ratio (*AR*) is superior to 20% ;
- The relationship established between the factor and the default rate must be logical. This qualification is based on expert opinion.

✓ Strongly correlated variables

After selecting the explanatory variables on the basis of the decision rules mentioned above, we will study the correlation between the selected variables. The study of correlations makes it possible to eliminate strongly correlated variables. Indeed, if two or more variables have a correlation coefficient superior to 0,5 ($\rho \geq 0,5$) then the variable that represents the greatest *AUC* will be selected.

2.2.1.2. Multivariate Discriminant Analysis and Definition of the Score Function

2.2.1.2.1. Multivariate Discriminant Analysis

In this paragraph, we will present the modeling of the relationship between quantitative and qualitative variables and defect, through pure logistic regression and logistic regression on main components and we will then present the tests for assessing the fit of the multivariate model.

– The pure logistic regression

Let $X_j, j = 1, \dots, J$ and $T_k, k = 1, \dots, 6$ be the respectively the quantitative and qualitative variables retained by the univariate analysis.

The objective of multivariate discriminant analysis by the pure logistic regression, is to study the relationship between the variable to be explained Y and the explanatory variables $X_j, j = 1, \dots, J$ and $T_k, k = 1, \dots, 6$. In this case, we will separate the modelling of the quantitative and qualitative variables. Indeed, the Models can be formulated by the following relationships:

✓ The quantitative model

$$Y = f_1(X_1, X_2, \dots, X_J) + \epsilon \quad (17)$$

✓ The qualitative model

$$Y = f_2(T_1, T_2, \dots, T_6) + \epsilon \quad (18)$$

where f_1 and f_2 are the logistics probabilities.

Let (n) be the size of the company sample used for modelling and $X = (x_{ij}), 1 \leq i \leq n, 1 \leq j \leq J$, the matrix whose columns are $X_j, 1 \leq j \leq J$ and let $T = (t_{ik}), 1 \leq i \leq n, 1 \leq k \leq 6$ be the matrix whose columns are $T_k, 1 \leq k \leq 6$.

❖ Presentation of the model

The objective of the modelling is to express the variable Y by the following model:

$$Y = p + \epsilon \quad (19)$$

With $p = P(Y = 1/X_1, \dots, X_J, T_1, \dots, T_6)$

Indeed, let $(x_{i1}, \dots, x_{iJ})$ and $(t_{i1}, \dots, t_{i6})$ be the quantitative and qualitative data, of the company (i) and y_i the realization of Y_i for this company then y_i is written:

$$y_i = p_i + \epsilon_i \quad (20)$$

Where $p_i = P(Y_i = 1/x_{i1}, \dots, x_{iJ}, t_{i1}, \dots, t_{i6})$

The Logit transformation has the following form:

$$\begin{aligned} \text{Logit}(p_i) &= \ln\left(\frac{p_i}{1-p_i}\right) \\ &= \ln\left(\frac{P(Y_i = 1/x_{i1}, \dots, x_{iJ}, t_{i1}, \dots, t_{i6})}{P(Y_i = 0/x_{i1}, \dots, x_{iJ}, t_{i1}, \dots, t_{i6})}\right) \\ &= \ln\left(\frac{P(Y_i = 1/x_{i1}, \dots, x_{iJ}, t_{i1}, \dots, t_{i6})}{1 - P(Y_i = 1/x_{i1}, \dots, x_{iJ}, t_{i1}, \dots, t_{i6})}\right) \end{aligned} \quad (21)$$

Thus, $Logit(p)$ can be written:

$$\begin{aligned} Logit(p) &= \ln\left(\frac{p}{1-p}\right) \\ &= \ln\left(\frac{P(Y=1/X_1, \dots, X_J, T_1, \dots, T_6)}{1-P(Y=1/X_1, \dots, X_J, T_1, \dots, T_6)}\right) \end{aligned}$$

❖ The quantitative modelling of Y by the probability p_1

The modelling of Y conditional only on quantitative data is defined by the model:

$$Y = p_1 + \varepsilon$$

where $p_1 = P(Y_i = 1/X_1, \dots, X_J)$

Let $(x_{i1}, \dots, x_{iJ})$ be the quantitative data of the company (i) which represent the realizations of the variables $(X_1, \dots, X_J)$ by the company (i) and y_i the realization of Y_i for this company then y_i is written:

$$y_i = p_{1i} + \varepsilon_i \quad (22)$$

Where $p_{1i} = P(Y_i = 1/x_{i1}, \dots, x_{iJ})$

To determine the expression of p_{1i} by the logistic regression model, we will first define the Odds represented by the ratio $\frac{p_{1i}}{1-p_{1i}}$. Hence:

$$Oddos(p_{1i}) = \frac{p_{1i}}{1-p_{1i}} = \frac{P(Y_i = 1/x_{i1}, \dots, x_{iJ})}{1-P(Y_i = 1/x_{i1}, \dots, x_{iJ})}$$

Therefore, we can write the model:

$$\begin{aligned} Logit(p_{1i}) &= \ln\left(\frac{p_{1i}}{1-p_{1i}}\right) \\ &= \beta_0 + \beta_1 \times x_{i1} + \beta_2 \times x_{i2} + \dots \\ &\quad + \beta_J \times x_{iJ} = \beta x_i' \end{aligned}$$

Where

$$\beta = (\beta_0, \dots, \beta_J)$$

and $x_i' = Transpose(1, x_{i1}, \dots, x_{iJ})$, so:

$$p_{1i} = \frac{e^{\beta x_i'}}{1 + e^{\beta x_i'}} \quad (23)$$

Let be $X' = Transpose(1, x_1, \dots, x_J)$ then p_1 and Y can be written

$$\begin{cases} p_1 = \frac{e^{\beta X'}}{1 + e^{\beta X'}} \\ Y = p_1 + \varepsilon \end{cases}$$

❖ The estimation of parameters

Let y_i be the realization of Y_i related the company (i) , the conditional variable $(Y_i/x_{i1}, \dots, x_{iJ})$ can be written in probability as follows:

$$\begin{aligned} P(Y_i = y_i/x_{i1}, \dots, x_{iJ}) &= p_{1i}^{y_i} \times \\ (1 - p_{1i})^{(1-y_i)} &\text{ where } y_i = 1 \text{ or } y_i = 0 \end{aligned} \quad (24)$$

❖ The likelihood function

The likelihood function of n enterprise is defined by:

$$\begin{aligned} L &= \prod_{i=1}^n P(Y_i = y_i/x_{i1}, \dots, x_{iJ}) \\ &= \prod_{i=1}^n (1 - p_{1i})^{(1-y_i)} \\ &\quad \times p_{1i}^{y_i} \end{aligned} \quad (25)$$

$$L = \prod_{i=1}^n \left(\frac{e^{\beta x_i'}}{1 + e^{\beta x_i'}} \right)^{y_i} \times \left(1 - \frac{e^{\beta x_i'}}{1 + e^{\beta x_i'}} \right)^{(1-y_i)} \quad (26)$$

❖ The loglikelihood function (LL)

$$\begin{aligned} LL &= \sum_{i=1}^n y_i \times \ln\left(\frac{e^{\beta x_i'}}{1 + e^{\beta x_i'}}\right) \\ &\quad + \sum_{i=1}^n (1 - y_i) \times \ln\left(1 - \frac{e^{\beta x_i'}}{1 + e^{\beta x_i'}}\right) \end{aligned} \quad (27)$$

$$LL = \sum_{i=1}^n \ln\left(\frac{1}{1 + e^{\beta x_i'}}\right) + y_i \beta x_i' \quad (28)$$

Let us note $\beta = (\beta_0, \beta_1, \dots, \beta_J)$ so the estimate of β is to determine $\hat{\beta}$ which maximizes the likelihood function and consequently maximizes the loglikelihood function(LL).

The maximization of LL will be done by solving the equation system determined by the following conditions:

$$\frac{\partial LL}{\partial \beta} = 0 \text{ and } \frac{\partial^2 LL}{\partial^2 \beta} < 0$$

Also, this problem is solved numerically, by the Newton-Raphson algorithm, the score method or the Berndt-Hall-Hausman method.

❖ The quantitative score function :

The estimation of β allows determining the quantitative score function S_1 that is written:

$$S_1 = \hat{\beta} X'$$

Hence, the probability p_1 is estimated by:

$$p_1 = \frac{e^{S_1}}{1 + e^{S_1}}$$

❖ The qualitative modelling of Y by the probability p_2

The objective of qualitative modelling is to determine the relationship between the variable Y and the themes T_i . The values taken by the theme T_i is a linear weighting of the qualitative variables q_m that compose it, noted, q_{mTi} , $1 \leq m \leq p$. Indeed, T_i is defined as follows:

$$T_i = \alpha_1 q_{1Ti} + \dots + \alpha_p q_{pTi} \quad (29)$$

where

- ✓ $\alpha_m = 0$ if the variable q_{mTi} is not explanatory
- ✓ For explanatory variables q_{mTi} , the associated α_m weights are determined by weight simulations that maximize the discriminatory power of the theme T_i (max AUC) by a logistic regression between the variable Y and the them T_i .

The modelling of Y conditional on qualitative data only, is defined by the model:

$$Y = p_2 + \varepsilon$$

with $p_2 = P(Y_i = 1/T_1, \dots, T_6)$

Let $(t_{i1}, \dots, t_{i6})$ be the qualitative data of the company (i) and y_i the realization of Y_i for this company then y_i is written:

$$y_i = p_{2i} + \varepsilon_i \quad (30)$$

Where $p_{2i} = P(Y_i = 1/t_{i1}, \dots, t_{i6})$

In this case, the odds of the probability p_{2i} are written:

$$\text{Oddos}(p_{2i}) = \frac{p_{2i}}{1 - p_{2i}} = \frac{P(Y_i = 1/t_{i1}, \dots, t_{i6})}{1 - P(Y_i = 1/t_{i1}, \dots, t_{i6})}$$

Therefore, we can write the model:

$$\begin{aligned} \text{Logit}(p_{2i}) &= \ln\left(\frac{p_{2i}}{1 - p_{2i}}\right) \\ &= \tau_0 + \tau_1 \times t_{i1} + \dots + \tau_6 \times t_{i6} \\ &= \tau t_i' \end{aligned}$$

where $\tau = (\tau_0, \dots, \tau_n)$ and $t_i' = \text{Transpose}(1, t_{i1}, \dots, t_{i6})$ thus:

$$p_{2i} = \frac{e^{\tau t_i'}}{1 + e^{\tau t_i'}}$$

Let be $T' = \text{Transposé}(1, t_1, \dots, t_6)$ then p_2 and Y can be written :

$$\begin{cases} p_2 = \frac{e^{\tau T'}}{1 + e^{\tau T'}} \\ Y = p_2 + \varepsilon \end{cases}$$

For the estimation of $\hat{\tau} = (\hat{\tau}_0, \dots, \hat{\tau}_n)$, the same procedure presented for the quantitative model will be used.

❖ The qualitative score function :

The estimation of τ allows determining the qualitative score function S_2 that is written:

$$S_2 = \hat{\tau} T'$$

Hence, the probability p_2 is estimated by:

$$p_2 = \frac{e^{S_2}}{1 + e^{S_2}}$$

– The logistic regression on the principal components

The principal components analysis (PCA) is a technique based on the reduced of the explanatory variables by

transforming the correlated variables into uncorrelated variable, which explain the maximum amount of variance, called the principal components (PCS), this technique was introduced by Karl Pearson in the early 20th century, and developed by Harold Hötelling in 1933.

❖ The principal components analysis (CPA)

Let $(Z_j, 1 \leq j \leq p)$ be the p explanatory variables (quantitative and qualitative) and let $(z_{ij})_{n \times p}$ be the np realization of these variables. Let $Z = (z_{ij})_{n \times p}$ be the matrix of these realizations. The column vectors of the matrix Z are $z_{.1}, z_{.2}, z_{.3}, \dots, z_{.p}$ who represent the realization of each variable Z_j .

Let us note $S = (s_{jk})_{p \times p}$ the covariance matrix of the variables $(Z_j, 1 \leq j \leq p)$ and let $(\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_p)$ and $(F_1, F_2, \dots, F_p)$ be, respectively, the eigenvalues and the eigenvectors of the matrix S . We designated by L the matrix whose columns $L_i, i = 1, \dots, p$ are the principal components defined by $L = ZF$ with $F = (f_{jk})_{p \times p}$ being the matrix which has as columns the eigenvectors (F_i) of the matrix S .

The decomposition into principal component makes it possible to express the vectors Z_j into reduced number of principal components $Z_j = \sum_{k=1}^s L_k f_{jk}, j=1, \dots, p$, which represents a high percentage of cumulative variation (CV) given by the following formula:

$$CV = \left[\frac{\sum_{j=1}^s \lambda_j}{\sum_{j=1}^p \lambda_j} \times 100 \right], s \leq p \quad (31)$$

The average variation V_m is defined by:

$$V_m = \left[\frac{\sum_{j=1}^p \lambda_j}{p} \times 100 \right] \quad (32)$$

❖ The logistic regression on the principal components

The logistic regression on the p principal components L_i is equivalent to the logistic regression on the explanatory variables $Z_j, j = 1, \dots, p$. Indeed, let $z_{ji}, i = 1, \dots, n$ be the realizations of the variables Z_j hence $z_{ji} = \sum_{k=1}^p L_{ki} f_{jk}$ so

$$p_i = \frac{e^{(\beta_0 + \sum_{j=1}^p \beta_j z_{ji})}}{1 + e^{(\beta_0 + \sum_{j=1}^p \beta_j z_{ji})}} = \frac{e^{(\beta_0 + \sum_{j=1}^p \sum_{k=1}^p L_{ki} f_{jk} \beta_j)}}{1 + e^{(\beta_0 + \sum_{j=1}^p \sum_{k=1}^p L_{ki} f_{jk} \beta_j)}} = \frac{e^{(\beta_0 + \sum_{k=1}^p L_{ki} \gamma_k)}}{1 + e^{(\beta_0 + \sum_{k=1}^p L_{ki} \gamma_k)}}$$

with $\gamma_k = \sum_{j=1}^p f_{jk} \beta_j, (k = 1, \dots, p)$

In order to improve the quality of the model in the case of collinearity, we will use a reduced number of (s) principal components for modelling the variable Y . Indeed, the formula (20) becomes:

$$y_i = p_{i(s)} + \varepsilon_{i(s)} \quad (33)$$

Hence

$$p_{i(s)} = \frac{e^{(\beta_0 + \sum_{k=1}^s L_{ki} \gamma_k)}}{1 + e^{(\beta_0 + \sum_{k=1}^s L_{ki} \gamma_k)}} \quad (34)$$

In our study, the number (s) is defined as the number of components whose cumulative variation is superior to 70%.

For the CPA, the quantitative variables and the qualitative variables are modeled simultaneously to determine the probability $p_{i(s)}$

– **Assessing the fit of the multivariate model:**

To assess the fit of the multivariate quantitative and qualitative models, we will proceed as follows:

❖ Testing of the significance of the coefficients

Let be the logistics model defined by:

$$y_i = p_i + \varepsilon_i$$

With $p_i = \frac{e^{\beta H'}}{1 + e^{\beta H'}}$, $\beta = (\beta_0, \dots, \beta_m)$ and $H' = tr(1, H_1, \dots, H_m)$

The significance of the regression parameters $(\beta_1, \dots, \beta_m)$ consists in testing the individual and overall significance of the model parameters:

✓ Testing of the individual significance (Test de Wald)

To assess individual significance, the hypothesis to be tested is:

$$\begin{cases} H_0: \beta_i = 0 \\ H_1: \beta_i \neq 0 \end{cases}$$

The testing for the significance of the coefficient β_i , is established by the Wald test based to the following ratio :

$$W_i = \frac{\hat{\beta}_i^2}{\hat{\sigma}(\hat{\beta}_i)^2}$$

The ratio, under the hypothesis H_0 , will follow a χ^2 with one degree of freedom. We reject H_0 if $W_i > \chi_1^2$

✓ Testing of the overall significance (the likelihood ratio test)

The overall significance test is based on the likelihood ratio between the model without the explanatory variables and the model with the explanatory variables. Indeed, the test can be expressed as follows:

$$\begin{cases} H_0: \text{logit}(P(Y = 1)) = \beta_0 \\ H_1: \text{logit}(P(Y = 1)) = \beta H' \end{cases} \quad (35)$$

The test ratio is defined by:

$$\begin{aligned} ST &= -2 \left[\frac{\text{Maximum loglikelihood of the model } M_1}{\text{Maximum loglikelihood of the model } M_2} \right] \\ &= -2 \left[\frac{LL(M_1)}{LL(M_2)} \right] \end{aligned}$$

where:

- M_1 the model defined by $\text{logit}(P(Y = 1)) = \beta_0$
- Maximum loglikelihood of the model M_1 :

$$LL(M_1) = \sum_{i=1}^n \ln \left(\frac{1}{1 + e^{\beta_0}} \right) + y_i \beta_0$$

- Maximum loglikelihood of the model M_2 :

$$LL(M_2) = \sum_{i=1}^n \ln \left(\frac{1}{1 + e^{\beta H'}} \right) + y_i \beta H'$$

The ratio, under the hypothesis H_0 , will follow a χ^2 with n degree of freedom. We reject H_0 if $ST > \chi_n^2$ and model M_2 is better than model M_1

❖ The Hosmer-Lemeshow test

The objective of this test, defined in Hosmer et al [29], is to assess the concordance between the predicted and the observed values. Indeed, the data are arranged in ascending order of the probabilities calculated by using the model, then divided into 10 groups.

The test hypotheses are:

$$\begin{cases} H_0: \text{the model is correct} \\ H_1: \text{the model is not correct} \end{cases}$$

The test statistic (HL), is defined by:

$$HL = \sum_{i=1}^{10} \frac{(O_i - n_i \hat{p}_i)^2}{n_i \hat{p}_i (1 - \hat{p}_i)} \quad (36)$$

Where $O_i = \sum_{j \in i} y_j$ is the number of observed outcomes, events, in group i , n_i is the number of observations in group i , $\hat{p}_i$ is the average predicted probability by the model in group i . HL , under the hypothesis H_0 , will follow a χ^2 with 8 degree of freedom. We reject H_0 if $HL > \chi_8^2$

❖ The performance of the model in the classification of the entrepreuse

The assessment of the performance of the model is necessary to determine the discriminatory power of the model, and to compare several models. To measure the performance of the model we will use the area under the ROC curve (AUC) presented previously.

A higher AUC means that the discriminatory power of the model is excellent. Hosmer and al.[29] defines the general rules for classification of the models based on the AUC . Consequently, in our study, the model will be retained if the AUC is superior or equal to 0,7, which means that the model has an acceptable discriminatory power.

Determination of the Overall Score Function

To determine the model that combines the quantitative explanatory variables $X_j, j = 1, \dots, J$, and qualitative explanatory variables $T_i, i = 1, \dots, 6$, we have to determine the function f such that :

$$y = f(X_1, \dots, X_J, T_1, \dots, T_6) + \varepsilon \quad (37)$$

where f is a logistic probability

– **The pure logistic regression model.**

Let S_1 and S_2 be respectively the quantitative and qualitative score of the company (i), where $S_1 = \hat{\beta} X'$ and $S_2 = \hat{t} T'$. The conditional probabilities p_1 and p_2 related to quantitative and qualitative data, are respectively

$$p_1 = \frac{e^{S_1}}{1+e^{S_1}} \text{ and } p_2 = \frac{e^{S_2}}{1+e^{S_2}}.$$

The final score S_i of company (i) is defined by the weighted average of the two scores S_{i1} and S_{i2} and the probability $p_i = P(Y = 1/x_{i1}, \dots, x_{ij}, t_{i1}, \dots, t_{i6})$ is defined by :

$$p_i = \frac{e^{\alpha S_{i1} + (1-\alpha)S_{i2}}}{1+e^{\alpha S_{i1} + (1-\alpha)S_{i2}}} = \frac{e^{S_i}}{1+e^{S_i}} \text{ where } S_i = \alpha S_{i1} + (1-\alpha)S_{i2} \quad (38)$$

The score function S and the probability p of the model can be formulated as follows:

$$p = \frac{e^{\alpha S_1 + (1-\alpha)S_2}}{1+e^{\alpha S_1 + (1-\alpha)S_2}} = \frac{e^S}{1+e^S} \text{ où } S = \alpha S_1 + (1-\alpha)S_2$$

The weighting α is determined according to the discriminatory power of the model. Indeed, the weighting which will be retained is those which maximizes the curve of *ROC* among all the values 10%, 20%, 30%, ..., 80% and 90%.

– The principal components of the logistic regression model.

In this case, we will simultaneously treat the quantitative and qualitative variables. As a result, the probability $p_i = P(Y = 1/x_{i1}, \dots, x_{ij}, t_{i1}, \dots, t_{i6})$ of the company (i)

will be a logistic probability determined directly as a function of the eigenvectors and the cumulative variation chosen.

The formulation of the S score function and the probability p are determined directly by multivariate analysis of the logistic regression between the default and the principal components.

Determination of the Score Grid, and Calculation of the Probability of Default by Class.

A company (i) is considered healthy if $p_i \geq \frac{1}{2}$, so:

$$p_i = \frac{e^{S_i}}{1+e^{S_i}} \geq \frac{1}{2} \Leftrightarrow e^{S_i} \geq \frac{1}{2}(1+e^{S_i}) \Leftrightarrow e^{S_i} \geq 1 \Leftrightarrow S_i \geq 0 \quad (39)$$

Consequently, the company is healthy if the score is positive.

2.2.1.2.2. The Rating Grid

The classification of healthy companies is based on the score function. Indeed, this classification gives rise to the rating grid composed of 8 rating classes.

Each company (i) is classified into a rating class, the classes vary between A and H , and are defined as follows:

Table 4. The rating grid

Class	A	B	C	D	E	F	G	H
Rating score	[86-100]	[76-86]	[65-76]	[55-65]	[46-55]	[40-46]	[30-40]	<30

The rating score SN_i of the company (i) is defined by :

$$SN_i = \frac{S_i}{\max(S)} \times 100 \quad (40)$$

where S_i is a score fonction of the company (i).

2.2.1.2.3. Calculation of the Probability of Default per Rating Class

The probability of default of the class K (PD_K) is defined by the probability of default of the company (i) knowing that the company (i) belonging to the class K . Consequently:

$$PD_K = \frac{\text{Number of companies in default belonging to the class } K}{\text{Total number of companies belonging to the class } K} \quad (41)$$

To define this probability of default per rating class, we will distribute the sample of healthy and default companies as follows:

Table 5. The probability of default by class

Class	A	B	C	D	E	F	G	H
Rating score	[86-100]	[76-86[	[65-76[	[55-65[	[46-55[	[40-46[	[30-40[	<30
Number $\{Y = 0\}$	n_{A0}	n_{B0}	n_{C0}	n_{D0}	n_{E0}	n_{F0}	n_{G0}	n_{H0}
Number $\{Y = 1\}$	n_{A1}	n_{B1}	n_{C1}	n_{D1}	n_{E1}	n_{F1}	n_{G1}	n_{H1}
Total number	n_A	n_B	n_C	n_D	n_E	n_F	n_G	n_H
Probability of Default (PD)	$\frac{n_{A0}}{n_A}$	$\frac{n_{B0}}{n_B}$	$\frac{n_{C0}}{n_C}$	$\frac{n_{D0}}{n_D}$	$\frac{n_{E0}}{n_E}$	$\frac{n_{F0}}{n_F}$	$\frac{n_{G0}}{n_G}$	$\frac{n_{H0}}{n_H}$

2.2.2. Determination of Expected Credit Losses

The expected loss is calculated by the formula (1). It depends on three components which are the *PD*, *LGD* and *EAD*. The process for calculating the *PD* was discussed in the previous paragraphs while the calculation of the *LGD* and *EAD* will be presented in the following paragraphs.

2.2.2.1. Determination of the Loss Given Default (*LGD*)

The calculation of the loss given default (*LGD*) consists in constructing regression models, which associated the *LGD* with relevant factors such as the seniority of the debt, the seniority of the relationship, the date of the transfer to the banking litigation, the activity sector the company, the collection process adopted by the bank, the time required for the completion of litigation procedures in justice, the rate of coverage by the first-ranking guarantees, the economic conditions and the probability at default (*PD*).

These variables were analyzed and used for the calibration of the model *LGD* in numerous publications. Indeed, Chalupka and Kopecsni [19] present a case study on *LGD* modelling of bank loans, in which the principal factors that have been identified are the period of issuance of the loan, the quality of the guarantees, the amount of the loan and the duration of the relationship with the debtor. The relationship between the default rate and the recovery rate was investigated in Altman and al ([3],[4], [6], [7]). The study of the dependence between default probabilities and recovery rates by Bade and al. [9], has shown some improvement in the *LGD*. While Gurtler et Hibbeln [27] has studied the influence of the length of the recovery process (training) on the *LGD* level estimated.

Different regression models have been used by researchers to model loss given default (*LGD*) as the Tobit model found in McDonald and al.[33], the Beta regression in Huang and al. [30], the inflated Beta regression in Pereira and al. [36], the censored gamma regression in Sigrist and al.[38] and [39] and a mixture of distributions into the model used, by Altman et al. [5].

In this paper, we will use the estimate provided by the Basel Accord for the loss given default (*LGD*), under the *IRB* foundation approach, fixed at 45%⁷. Indeed, we will ignore the guarantees associated with the credit to determine the *LGD*.

2.2.2.2. Determination Exposure at Default (*EAD*)

The exposure at default *EAD* is defined as the sum of:

- The value accounted for in the balance sheet (VCB_0).
- The value of the unused funding commitment, accounted for off-balance sheet ($VCHB_0$) multiplied by a credit conversion factor (*CCF*).

The credit conversion factor(*CCF*) is defined between 0

et 1 ($FCC \in [0,1]$) and the mathematical formulation of the *EAD* , is given by the following relationship:

$$EAD = VCB_0 + CCF \times VCHB_0 \quad (42)$$

The calculation of the exposure at default (*EAD*) will be done in two different ways. The first, consists in modelling the credit conversion factor (*CCF*) by using regression models that associated the *CCF* with pertinent factors, whereas the second, consists in modelling directly the exposure at default (*EAD*).

The models used to model the credit conversion factor (*CCF*) such as the ordinary least squares (*OLS*), the Tobit model, the fractional response regression and the utilization change model, are detailed in Brown [17], Bellotti and al. [14] and Bijak and al. [15]. While the models used for the direct modelling of the *EAD* as a zero-adjusted gamma model and others, are detailed in E.N.C. Tong and al.[22].

In this paper, we will use the estimate provided by the Basel Accord for credit conversion factor (*CCF*) at the paragraph level 366. Indeed, under the *IRB* foundation approach, the *CCF* is fixed at 75%. The exposure at default (*EAD*) of the line of credit is determined as follows :

$$\begin{aligned} EAD &= VCB_0 + FCC \times VCHB_0 \\ EAD &= VCB_0 + 75\% \times VCHB_0 \\ EAD &= VCB_0 + 75\% \times (M_{efa} - VCB_0) \\ EAD &= 25\% VCB_0 + 75\% \times M_{efa} \end{aligned} \quad (43)$$

where M_{efa} the amount of the financing authorization granted by the bank to the customer.

3. Empirical Results

3.1. Description of the Database

In this study, we used a database of small and medium enterprises (SMEs) of a Moroccan bank composed of 1447 enterprises. For the definition of a *SME* we have based on the definition of the Central Bank which considers that an enterprise is an *SME* if it realizes a turnover between 10 and 175 million *MAD*. In terms of default, the portfolio structure is as follows:

Table 6. Default Portfolio Structure

	Number	percentage
Healthy Enterprises	1333	92.12%
Enterprises in default	114	7.88%
Total	1447	100%

⁷ Committee on Banking Supervision, 2006, International Convergence of Capital Measurement and Capital Standards, Bank for International Settlements at the paragraph level 366

3.2. Choice of Explanatory Variables

3.2.1. Choice of Quantitative Variables

The univariate analysis of the quantitative variables has permitted to determine the quantitative variables which explain the failure. The choice of quantitative variables, is based on the Wald test and the discriminatory power (power stat), determined from the *AUC* and the *AR*. Indeed, the Wald test and the discriminatory power made have permitted to determine the seven explanatory variables follows:

Table 7. univariate analysis and choice of quantitative variables

Title	variable	Khi ² (Wald)	$pv = Pr(x > Wald)$	<i>AUC</i>	<i>AR</i>
Profit growth	$X_1 = V_3$	4,782	0,029	61%	22%
$\frac{Net\ Profit}{Equity}$	$X_2 = V_5$	9,427	0,002	65%	30%
$\frac{Financial\ expenses}{Sales/turnove\ (ST)}$	$X_3 = V_7$	23,912	< 0,0001	69.2%	38,4%
$\frac{Net\ Debt}{Equity}$	$X_4 = V_9$	23,683	< 0,0001	69%	38%
Liquidity	$X_5 = V_{10}$	6,136	0,013	63.3%	26.6%
$\frac{Equity}{Total\ Assets}$	$X_6 = V_{12}$	6,590	0,010	64%	28%
$\frac{Sales/turnove\ (ST)}{Net\ fixed\ assets + WCR}$	$X_7 = V_{13}$	10,289	0,001	65%	30%

For the variables listed in the table 7, the Wald's test shows that the selected variables are significant because the p-value(*pv*) is inferior to 0,05 with a satisfactory discriminatory power because the *AUC* is superior to 0,6 and the *AR* is superior to 0,2.

The correlation between the selected quantitative variables is presented as follows:

Table 8. The correlation matrix of quantitative variables.

Variables	X_1	X_2	X_3	X_4	X_5	X_6	X_7
X_1	1,000	0,128	0,076	0,060	0,067	0,109	0,008
X_2	0,128	1,000	-0,021	0,069	0,087	0,213	-0,084
X_3	0,076	-0,021	1,000	0,415	0,195	0,085	0,440
X_4	0,060	0,069	0,415	1,000	0,297	0,357	0,358
X_5	0,067	0,087	0,195	0,297	1,000	0,371	-0,027
X_6	0,109	0,213	0,085	0,357	0,371	1,000	-0,159
X_7	0,008	-0,084	0,440	0,358	-0,027	-0,159	1,000

The previous matrix shows that the variables are not strongly correlated because the correlation coefficients do not exceed 0,5.

3.2.2. Choice of Qualitative Variables

Similarly, for the qualitative variables, the choice is based on Wald's test and on the discriminatory power (power stat). The following table summarizes the results.

Table 9. univariate analysis and choice of qualitative variables.

(q_m)	theme	qualitative variables	Khi ² (Wald)	$pv = P(x > Wald)$	<i>AUC</i>	<i>AR</i>
q_1	T_1	Sector fault rate	7,378	0,007	61,5%	23%
q_6	T_2	International Competition	3,875	0,049	60,5%	21%
q_{12}	T_4	Seniority of the principal operational staff	3,840	0,050	60%	20%
q_{17}	T_5	Number of payment incidents in the last 12 months	141,290	< 0,0001	79,9%	59,8%
q_{19}	T_6	Number of banks related to the company	11,227	0,001	65,4%	30,8%

For each theme, we found that only one variable is significant except for theme T_3 where the variables are not significant. Consequently, the modelling of the themes is equivalent to the modelling of the selected variables.

As with the quantitative variables, the Wald's test shows that the selected variables are significant because the *p*-value (*pv*) is inferior to 0,05. For the discriminatory power, the variables have a satisfying discriminatory power because the *AUC* is superior to 0,6 and the *AR* is superior to 0,2.

The correlation between the chosen qualitative variables is presented as follows:

Table 10. The correlation matrix of quantitative variables.

Variables	T_1	T_2	T_4	T_5	T_6
T_1	1,000	0,072	-0,079	-0,012	0,053
T_2	0,072	1,000	0,029	-0,039	0,078
T_4	-0,079	0,029	1,000	-0,035	-0,040
T_5	-0,012	-0,039	-0,035	1,000	0,043
T_6	0,053	0,078	-0,040	0,043	1,000

The previous matrix shows that the variables are not strongly correlated because the correlation coefficients do not exceed 0,5.

3.3. Calculation of the Expected Loss from the Rating Model based on Pure Logistic Regression.

We have previously presented the process of construction of the rating tool. Indeed, we will determine in this paragraph the rating grid and the probability of default by class by using pure logistic regression.

3.3.1. Multivariate Analysis and Determination of the Score Function.

3.3.1.1. Multivariate Analysis and Determination of the Score Function of Quantitative Variables.

– Parameter estimation and testing of significance.

The parameters estimation and the tests of significance of the coefficients are presented as follows:

❖ The parameters estimations and the Wald's test.

The maximization of the likelihood function is done by the Newton-Raphson algorithm. The results of the estimation of the parameters and Wald's significance test are presented in the following table:

Table 11. The parameters estimations and the Wald's test

	β_i	Standard deviation	Khi ² (Wald)	$pv = P(x > Wald)$
Cte	$\beta_0=0,706$	0,356	3,942	0,047
X_1	$\beta_1=0,004$	0,003	4,050	0,044
X_2	$\beta_2=0,009$	0,004	5,223	0,022
X_3	$\beta_3=0,009$	0,003	9,104	0,003
X_4	$\beta_4=0,007$	0,003	4,431	0,035
X_5	$\beta_5=0,002$	0,001	3,896	0,048
X_6	$\beta_6=0,003$	0,001	4,355	0,037
X_7	$\beta_7=0,003$	0,002	4,225	0,040

Wald's test confirms the significance of the coefficients β_i because the *p*-value (*pv*) of all variables is inferior to 0,05.

❖ The testing of the overall significance (the likelihood ratio test)

The likelihood ratio test shows that the value of the *ST* statistic, previously defined, is equal to 46,883, this value is superior to 14,067 (χ^2 of degree 7 at the 0,05 threshold). Consequently, we reject H_0 and the variables are totally significant and determine a better model.

– Hosmer-Lemeshow test and the assessment of the performance of the model.

❖ The Hosmer-Lemeshow test

The Hosmer-Lemeshow test shows that the model fits with the sample data. Indeed, the *HL* statistic is equal to 4,42 with a *p*-value equal to 0,817, superior to the threshold of 0,05.

❖ The assessment of the performance of the model

The *ROC* curve shows that the model offers an acceptable classification of companies because the *AUC* is equal to 0,705.

Figure 3. The AUC of quantitatives variables

– **Determination of the score function.**

For each company (i) the score of company $S_1(i)$ is written as follows:

$$S_1(i) = \beta_0 + \beta_1 X_{i,1} + \beta_2 X_{i,2} + \beta_3 X_{i,3} + \beta_4 X_{i,4} + \beta_5 X_{i,5} + \beta_6 X_{i,6} + \beta_7 X_{i,7}$$

As a result, the variable Y_i can be estimated by the probability:

$$p_{1i} = \frac{e^{\beta_0 + \beta_1 X_{i,1} + \beta_2 X_{i,2} + \beta_3 X_{i,3} + \beta_4 X_{i,4} + \beta_5 X_{i,5} + \beta_6 X_{i,6} + \beta_7 X_{i,7}}}{1 + e^{\beta_0 + \beta_1 X_{i,1} + \beta_2 X_{i,2} + \beta_3 X_{i,3} + \beta_4 X_{i,4} + \beta_5 X_{i,5} + \beta_6 X_{i,6} + \beta_7 X_{i,7}}}$$

$$p_{1i} = \frac{e^{0,706 + 0,04X_{i,1} + 0,09X_{i,2} + 0,09X_{i,3} + 0,07X_{i,4} + 0,02X_{i,5} + 0,03X_{i,6} + 0,03X_{i,7}}}{1 + e^{0,706 + 0,04X_{i,1} + 0,09X_{i,2} + 0,09X_{i,3} + 0,07X_{i,4} + 0,02X_{i,5} + 0,03X_{i,6} + 0,03X_{i,7}}}$$

Hence, the variable Y can be estimated by the probability:

$$p_1 = \frac{e^{0,706 + 0,04X_1 + 0,09X_2 + 0,09X_3 + 0,07X_4 + 0,02X_5 + 0,03X_6 + 0,03X_7}}{1 + e^{0,706 + 0,04X_1 + 0,09X_2 + 0,09X_3 + 0,07X_4 + 0,02X_5 + 0,03X_6 + 0,03X_7}}$$

3.3.1.2. Multivariate Analysis and Determination of the Score Function of Qualitative Variables.

– **Parameter estimation and testing of significance.**

The parameters estimation and the tests of significance of the coefficients are presented as follows:

❖ The parameters estimations and the Wald's test.

The maximization of the likelihood function is done by the Newton-Raphson algorithm. The results of the estimation of the parameters and Wald's test are presented as follows:

Table 12. The estimation of the parameters and the Wald's test

	τ_i	Standard deviation	Khi ² (Wald)	$pv = P(x > Wald)$
Constant	$\tau_0 = -5,650$	0,755	55,954	< 0,0001
T_1	$\tau_1 = 0,009$	0,003	9,195	0,002
T_2	$\tau_2 = 0,007$	0,004	4,221	0,040
T_4	$\tau_3 = 0,012$	0,005	7,685	0,006
T_5	$\tau_4 = 0,083$	0,007	138,743	< 0,0001
T_6	$\tau_5 = 0,012$	0,005	6,642	0,010

Wald's test confirms the significance of the coefficients τ_i because the p-value (pv) of all variables is inferior to 0,05.

❖ The testing of the overall significance (the likelihood ratio test)

The likelihood ratio test shows that the value of the ST statistic, previously defined, is equal to 178,364, this value is superior to 14,067 (χ^2 of degree 7 at the 0,05 threshold). Consequently, we reject H_0 and the variables are totally significant and determine a better model.

– **Hosmer-Lemeshow test and the assessment of the performance of the model.**

❖ The Hosmer-Lemeshow test

The Hosmer-Lemeshow test shows that the model fits with the sample data. Indeed, the HL statistic is equal to 12,265 with a p-value equal to 0,092, superior to the threshold of 0,05.

❖ The assessment of the performance of the model

The ROC curve shows that the model offers an acceptable classification of companies because the AUC is equal to 0,784.

Figure 4. The AUC of quantitatives variables

– **Determination of the score function.**

For each company (i) the score of company $S_2(i)$ is written as follows:

$$S_2(i) = \tau_0 + \tau_1 T_{i,1} + \tau_2 T_{i,2} + \tau_3 T_{i,4} + \tau_4 T_{i,5} + \tau_5 T_{i,6}$$

As a result, the variable Y_i can be estimated by the probability:

$$p_{2i} = \frac{e^{\tau_0 + \tau_1 T_{i,1} + \tau_2 T_{i,2} + \tau_3 T_{i,4} + \tau_4 T_{i,5} + \tau_5 T_{i,6}}}{1 + e^{\tau_0 + \tau_1 T_{i,1} + \tau_2 T_{i,2} + \tau_3 T_{i,4} + \tau_4 T_{i,5} + \tau_5 T_{i,6}}}$$

$$p_{2i} = \frac{e^{-5,650 + 0,009 * T_{i,1} + 0,007 * T_{i,2} + 0,012 * T_{i,4} + 0,083 * T_{i,5} + 0,012 * T_{i,6}}}{1 + e^{-5,650 + 0,009 * T_{i,1} + 0,007 * T_{i,2} + 0,012 * T_{i,4} + 0,083 * T_{i,5} + 0,012 * T_{i,6}}}$$

Hence, the variable Y can be estimated by the probability:

$$p_2 = \frac{e^{-5,650 + 0,009 * T_1 + 0,007 * T_2 + 0,012 * T_4 + 0,083 * T_5 + 0,012 * T_6}}{1 + e^{-5,650 + 0,009 * T_1 + 0,007 * T_2 + 0,012 * T_4 + 0,083 * T_5 + 0,012 * T_6}}$$

3.3.2. The Function of Overall Score and the Construction of the Rating Grid.

3.3.2.1. Determination of the Function of Overall Score.

The combination of the quantitative and qualitative score to determine the overall score is based on the maximization of the discriminatory power of the model. Therefore, we will retain the combination $\alpha S_1 + (1 - \alpha) S_2$ that maximizes the AUC . To achieve this, we have varied α and we calculated the AUC . The results are presented below:

Table 13. Variation in the weighting (α) and the *AUC*

α	10%	20%	30%	40%	50%	60%	70%	80%	90%
<i>AUC</i>	0,79	0,795	0,799	0,8016	0,80338	0,80226	0,798	0,78	0,744

The value of (α) equal to 50% is the one that maximizes the *AUC*. Therefore, we have retained the model that gives the same weight to the quantitative and qualitative score.

3.3.2.2. The Construction of the Rating Grid.

The score grid and the rating classes defined per score and the distribution of the enterprises of the sample per the rating classes, are as follows:

Table 14. The score grid, the rating classes and the distribution of the enterprises

Rating classes	Score	Distribution of the companies	
		Number	%
A	[86-100]	19	1,3%
B	[76-86[	279	19,3%
C	[65-76[	460	31,8%
D	[55-65[	372	25,7%
E	[46-55[	164	11,3%
F	[40-46[	55	3,8%
G	[30-40[	50	3,5%
H	<30	48	3,3%

The distribution of the portfolio is presented in the following graph:

Figure 5. The distribution of the portfolio enterprises

3.3.3. Determination of the Probability of Default per Class and Calculation of the Expected Credit Loss (*ECL*).

3.3.3.1. Determination of the Probability of Default per Rating Class.

For this first model, the probability of default is determined according to the approach detailed above. Therefore, the *PD* per rating class is presented as follows:

Table 15. Model 1, probability of default by rating class

Rating class	Number of healthy enterprises	Number of enterprises in default	PD
A	19	0	0,03%
B	276	3	1,08%
C	445	15	3,26%
D	345	27	7,26%
E	150	14	8,54%
F	50	5	9,09%
G	36	14	28,00%
H	12	36	75,00%
Total	1333	114	

3.3.3.2. Calculation of Expected Credit Loss (*ECL*).

The portfolio studied represents a use of outstanding credits (VCB_0) of 7.496,370 Million *MAD* distributed per rating class as follows:

Table 16. Distribution of the VCB_0 per rating class (in Million *MAD*)

Rating Class	VCB_0 (amount)	%
A	45	0,6%
B	598	8,0%
C	1 379	18,4%
D	2 543	33,9%
E	1 471	19,6%
F	483	6,4%
G	572	7,6%
H	406	5,4%
Total	7 496	100,0%

The amount of the expected loss per rating class is determined by the formula (1). As a result, for a loss given default (*LGD*) equal to 45% and a *FCC* equal to 75%, the total amount of the expected loss (EL_M) is distributed per rating class as follows :

Table 17. Distribution of expected loss amount (in Million *MAD*)

Rating class	VCB_0	M_{efa}	<i>PD</i>	<i>LGD</i>	<i>EAD</i>	EL_M
A	45	50	0,03%	45%	54	0,01
B	598	748	1,08%	45%	648	3,15
C	1 379	1622	3,26%	45%	1 494	21,92
D	2 543	2992	7,26%	45%	2 803	91,58
E	1 471	1961	8,54%	45%	1 990	76,48
F	483	537	9,09%	45%	574	23,46
G	572	572	28,00%	45%	572	72,07
H	406	406	75,00%	45%	406	137,03
Total	7 496	8 888			8 540	425,69

3.4. Calculation of the Expected Loss from the Rating Models based on Logistic Regression on Principal Components.

3.4.1. Determination of Eigenvectors.

3.4.1.1. Determination of the Eigenvalues and the Cumulative Variation(*CV*).

The principal components analysis makes it possible to determine the eigenvalues as well as the cumulative variation. The results are summarized below:

Table 18. The eigenvalues and variation

	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}
Eigenvalues	2,16	1,68	1,12	1,06	0,98	0,93	0,89	0,87	0,77	0,63	0,52	0,40
Variation (%)	18,02	13,96	9,33	8,82	8,19	7,76	7,42	7,23	6,42	5,22	4,30	3,33
Cumulative variation	18,02	31,98	41,31	50,13	58,32	66,08	73,50	80,73	87,15	92,37	96,67	100,00

The vectors from F_1 to F_7 offer a cumulative variation superior to 70%, the vectors from F_1 to F_8 offer a cumulative variation superior to 80% and the vectors from F_1 to F_{10} offer a cumulative variation superior to 90%. Therefore, we will use them to calculate the expected loss corresponding to the cumulative variation of 70%, 80% and 90%.

3.4.1.2. Determination of Eigenvectors.

The eigenvectors F_i are determined by the combination of quantitative and qualitative explanatory variables as follows:

Table 19. The eigenvectors

	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}
T_1	0,160	-0,167	0,362	0,512	-0,129	-0,552	-0,010	-0,016	-0,454	0,148	0,037	0,056
T_2	0,064	0,130	0,676	0,051	0,166	0,155	-0,525	0,292	0,293	-0,142	-0,003	0,012
T_4	-0,092	0,367	0,021	-0,246	-0,005	-0,393	0,340	0,692	0,078	0,193	0,036	-0,055
T_5	0,166	-0,097	-0,369	-0,177	0,656	-0,287	-0,441	0,119	-0,248	-0,016	-0,112	-0,004
T_6	0,227	-0,042	0,295	0,057	0,561	0,369	0,581	0,030	-0,229	-0,006	-0,100	0,070
X_1	0,119	0,152	-0,373	0,526	-0,171	0,438	-0,145	0,493	-0,228	-0,029	-0,085	0,034
X_2	0,102	0,327	-0,183	0,526	0,309	-0,194	0,093	-0,237	0,582	0,188	0,011	0,029
X_3	0,495	-0,223	-0,073	-0,098	-0,032	0,088	-0,014	0,103	0,117	0,191	0,787	0,036
X_4	0,526	0,060	-0,054	-0,148	-0,217	-0,158	0,076	0,008	0,138	-0,366	-0,288	0,617
X_5	0,333	0,371	0,083	-0,233	-0,146	0,167	-0,178	-0,216	-0,170	0,681	-0,263	-0,033
X_6	0,297	0,520	0,008	-0,031	-0,049	-0,078	0,030	-0,194	-0,213	-0,500	0,166	-0,520
X_7	0,371	-0,467	-0,020	0,017	-0,121	-0,052	0,089	0,162	0,301	0,032	-0,410	-0,577

3.4.2. Univariate Analysis of Variables F_i and Study of the Correlation between F_i and Y .

The objective of the univariate analysis is to verify if the correlation between the failure and the main components is maintained as in the case of the initial variables, or the transformation generates a loss of information. The results of this analysis are as follows:

Table 20. Univariate analysis of variables of the eigenvectors

variable	Khi ² (Wald)	$pv = Pr(x > Wald)$	AUC	AR
F_1	61,738	< 0,0001	72,2%	44,4%
F_2	0,015	0,901	50%	0%
F_3	15,082	0,000	58,6%	17,2%
F_4	0,278	0,598	51,5%	3%
F_5	88,887	< 0,0001	69,2%	38,4%
F_6	26,163	< 0,0001	62,8%	25,6%
F_7	25,725	< 0,0001	62,5%	25%
F_8	19,627	< 0,0001	62,2%	24,4%
F_9	8,036	0,005	55,8%	11,6%
F_{10}	0,305	0,581	52,3%	4,6%
F_{11}	1,938	0,164	54,3%	8,6%
F_{12}	0,338	0,561	50,7	1,4%

The Table 20 shows that the correlation between Y and the F_i is lost for some eigenvectors by transforming the initial variables into the principal components. However, in the modelling we will retain all components including those whose significance is not justified.

3.4.3. Multivariate Analysis and Determination of the Score Function

3.4.3.1. Multivariate Analysis per Level of Cumulative Variation (CV)

Multivariate analysis will be done for the three levels of cumulative variation, respectively 70%, 80% and 90%. The results of this analysis are as follows:

Table 21. Multivariate analysis per level of cumulative variation

	Model 2 : <i>CV</i> is equal to 70%				Model 3 : <i>CV</i> is equal to 80%				Model 4 : <i>CV</i> is equal to 90%			
	Value	ST. Dev.	Khi ² Wald	$Pv = P(x > Khi^2)$	Value	ST. Dev.	Khi ² Wald	$Pv = P(x > Khi^2)$	Value	ST. Dev.	Khi ² Wald	$Pv = P(x > Khi^2)$
Cte	3,02	0,14	468,74	< 0,0001	3,08	0,14	457,69	< 0,0001	3,10	0,15	451,07	3,10
F_1	0,52	0,08	44,14	< 0,0001	0,53	0,08	44,61	< 0,0001	0,54	0,08	45,11	0,54
F_2	0,04	0,09	0,17	0,682	0,02	0,09	0,05	0,820	0,04	0,09	0,19	0,04
F_3	-0,14	0,10	2,14	0,143	-0,14	0,10	2,13	0,145	-0,13	0,10	1,70	-0,13
F_4	0,01	0,10	0,00	0,959	0,02	0,10	0,04	0,832	0,03	0,10	0,11	0,03
F_5	0,66	0,10	46,51	< 0,0001	0,65	0,10	43,27	< 0,0001	0,64	0,10	41,82	0,64
F_6	-0,37	0,11	10,80	0,001	-0,35	0,11	9,98	0,002	-0,34	0,11	9,44	-0,34
F_7	-0,32	0,11	8,13	0,004	-0,31	0,11	7,53	0,006	-0,29	0,11	6,72	-0,29
F_8					0,46	0,12	14,73	0,000	0,45	0,12	14,28	0,45
F_9									-0,24	0,12	4,24	-0,24
F_{10}									0,09	0,14	0,44	0,09

3.4.3.2. The Assessment of the Performance of the Model

The discriminatory power of the three previous *CPA* models determined by the *AUC* is presented as follows:

Table 22. The *AUC* as a function of the model

	Model 2	Model 3	Model 4
<i>AUC</i>	78,4	79,8	80,2

The three models have a discriminatory power that varies between acceptable and good. As a result, they can be used to classify companies and determine the probability of default.

3.4.3.3. The Score Function per Level of Cumulative Variation

The score function is defined per level of cumulative variation as follows:

– The score function of the model 2

$$S_1 = 3,02 + 0,52 * F_1 + 0,04 * F_2 - 0,14 * F_3 + 0,01 * F_4 + 0,66 * F_5 - 0,37 * F_6 - 0,32 * F_7$$

– The score function of the model 3

$$S_2 = 3,08 + 0,53 * F_1 + 0,02 * F_2 - 0,14 * F_3 + 0,02 * F_4 + 0,65 * F_5 - 0,35 * F_6 - 0,31 * F_7 + 0,46 * F_8$$

– The score function of the model 4

$$S_3 = 3,10 + 0,54 * F_1 + 0,04 * F_2 - 0,13 * F_3 + 0,03 * F_4 + 0,64 * F_5 - 0,34 * F_6 - 0,29 * F_7 + 0,45 * F_8 - 0,24 * F_9 + 0,09 * F_{10}$$

3.4.4. Calculation of the Expected Credit Loss per Model

3.4.4.1. The Probability of Default per Rating Class and per Model

The probability of default per rating class for the three models based of the *CPA* is represented as follows:

Table 2. The *PD* per rating class for the three models based of the *CPA*

Rating class	PD per model		
	Model 2	Model 3	Model 4
A	0,03%	0,03%	0,03%
B	1,15%	1,19%	1,20%
C	3,11%	3,15%	3,50%
D	6,45%	6,54%	6,77%
E	8,21%	8,42%	8,61%
F	10,53%	11,00%	9,80%
G	28,86%	30,29%	22,92%
H	72,05%	81,40%	74,50%

3.4.4.2. The Expected Credit Loss per Model

The expected credit loss per model is determined on the basis of the *EAD* and the *PD* per rating class.

Table 24. The expected credit loss per model (Million MAD)

	Model 2			Model 3			Model 4		
rating	EAD	PD	EL _M	EAD	PD	EL _M	EAD	PD	EL _M
A	51	0,03%	0,00	53	0,03%	0,01	50	0,03%	0,00
B	938	1,15%	3,38	995	1,19%	5,15	936	1,20%	3,63
C	1 993	3,11%	27,47	2 086	3,15%	29,57	1 978	3,50%	33,32
D	2 679	6,45%	84,04	2 546	6,54%	71,89	2 565	6,77%	82,22
E	1 454	8,21%	59,41	1 293	8,42%	58,79	1 339	8,61%	55,95
F	702	10,53%	27,68	668	11,00%	34,18	665	9,80%	27,85
G	234	28,86%	42,58	410	30,29%	34,57	518	22,92%	46,89
H	489	72,05%	156,74	489	81,40%	179,27	489	74,50%	172,32
Total	8 540		401,30	8 540		413,43	8 540		422,18

3.5. Comparison of Expected Credit Losses per Model.

The comparison of the models, based on their performances, show that the models resulting from the principal components analysis, are more performing in terms of discriminatory power. Indeed:

Table 25. The comparison of the performance of the four models.

Models	Model 1	Model 2	Model 3	Model 4
AUC(%)	70,5	78,4	79,8	80,2

For the expected credit loss, we noticed that it increases in parallel with the increase in cumulative variation to approach the expected credit loss determined by the pure logistic regression.

Table 26. The comparison of expected credit losses of the four models.

Models	Model 1	Model 2	Model 3	Model 4
VC(%)	100	70	80	90
EL _M (Million MAD)	425,69	393,59	401,69	413,03
Variation	0%	5,73%	2,88%	0,82%

4. Conclusions

Credit risk is the most important aspect of the banking risk management. Indeed, banks must continuously monitor the adequacy of their capital with the risks taken because they must cover the unexpected loss and must constitute a provision to cover expected losses.

The expected losses are deducted from the result, this, impact directly the level of core capital. Therefore, their calculation must be meticulous to avoid regulatory penalties on the one hand, and not to generate a lack to ganger for the bank on the other hand.

The rating system impact directly the calculation of expected losses because it provides the different components which are the probability of default (PD), the loss given default (LGD) and the exposure at default (EAD) per rating class and per counterparty for the calculation of

the loss amount. As a result, the expected loss amounts depend directly on the modelling approach used.

In this article, we have constructed four rating models that have an acceptable discriminatory power that allow them to predict counterparty default. Indeed, we constructed the first model by using the pure logistic regression approach and the other three models from the principal components on the basis of the cumulative variation, which must be superior to 70%.

The passage from the initial variables to the principal components to model the failure can generate a loss of information, particularly in terms of correlation between the eigenvectors and the default.

The expected loss amounts calculated by the four models are different. We conclude that those determined from the logistic regression on the principal components, they increase in parallel with the increase in cumulative variation and are lower of the expected loss calculated by the pure logistic regression model.

This study shows that there is multitude of powerful models of failure prediction, which offer several structures of probability of default per rating class, enable to calculate of various expected loss amounts. As a result, the amount of expected losses becomes a random variable, depends of the model.

In our study, the uncertainty for the second model reaches the threshold of 5.73%. Probably, it will be greater if one opts for other types of models, such as Bayesian modelling or genetic algorithms or fuzzy numbers. In this context, the research focusing on the modelling of the probability of default by the three previous logics can help to determine the degree of uncertainty.

For this reason, the regulator must standardize the used techniques for the construction of the internal models in order to reduce uncertainty and enable a comparison of the credit risk profiles of banks. Indeed, the regulator must provide a standard approach for calculating the expected loss similar to that provided for the unexpected loss (weighted assets) and requires, as a result, that the expected losses calculated by the internal models must not be below

a fixed floor in relation to the standard measurement.

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