About Fractal Geometry of the Glacial Cirques in Rila and Pirin Mountains (Southwest Bulgaria)

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Abstract
Rila Mountain and Pirin Mountain (N 42.1°, E 23.5° and N 41.7°, E 23.4°) are the highest morphounits in the Balkan Peninsula. During the last Ice Age (Wuerm), their highest parts were subjected to an alpine-type glaciation. This led to the formation of various glacial landforms. The most expressive of them are the cirques. The main goal of the present study is to examine and evaluate the possible fractal geometry of 77 glacial cirques in Rila and Pirin Mountains. For this purpose is used the number / area method for estimating of surface fractals. This is the first investigation of this type for the study lands at all. The results obtained unambiguously confirm the fractal geometry of glacial cirques in the Rila and Pirin Mountains. This in turn provides new guidance in the analysis and interpretation of the relief building processes within the research area.

Keywords Rila, Pirin, Mountains, Fractals, Glaciation, Cirques, Landforms, Bulgaria

Glacial activity in Rila and Pirin Mountains were subject to numerous studies. The first to describe glacial traces in Bulgaria was Jovan Cvijic (Cvijić, [1]), whose pioneer work in glacial geomorphology was done in Rila Mountain. He was the first to propose that during the Late Pleistocene (Wuermian) glaciation the ELA should have been at about 2200 m. (Cvijic, [1]) tried to search for evidence of older glaciations (morainic deposits in the valley of Iskar River above the town of Samokov), but his views were rejected by most of the scientists who studied glacial landforms in Rila Mountain (Glovnya, [2], Radev, [3], Annaheim, [4]). End moraines of the maximum glaciation in Rila were described by Glovnya [5] and Ivanov [6]. Later Kuhlemann et al. [7] made an inventory of terminal moraines throughout Rila Mountain, and sampled some of them for absolute dating (¹⁰Be method). Obtained ages between 24 and 16 ka BP are in support of the hypothesis for the young age of the maximum glaciation. Glacial landforms in Pirin Mountain were discussed in the works of Penck [8], Louis [9], Lilienberg [10] and many others. They considered traces of only one Ice age (the Wuermian) with three retreat stages. The maximum extent of glaciers during the last (Wuermian) Ice age (Mitkov, Gachev, [11]) Along with the configuration of valleys and valley shape, they indicate the position of former equilibrium line at altitudes between 2150 and 2250 m a. s. l.(Kuhlemann et al., [7], [12])

The main objective of the present study is to check and assess the possible fractal geometry of cirques in Rila and Pirin Mountains. This is the first study in which an attempt is made to assess the possible fractal properties of the glacial cirques within the Rila and Pirin Mountains. The study includes 37 cirques in Rila and 40 cirques in Pirin. Their areas are measured using interactive GIS software and the results obtained are depicted on special semi-logarithmic graphs. The fractal dimension of the cirques is calculated by the number / area method for surface fractals. The results obtained give a new insight into the landforms development in the research area.

1. Introduction
The activity of modern and Pleistocene glaciers is an important relief-building factor that has an impact on significant parts of the Earth’s surface. The contemporary relief of Rila and Pirin Mountains is formed during Pleistocene, when the mountains were subjected to an alpine type of glaciation. The glaciers have redesigned the existing relief, forming the characteristic cirques and expanding the river valleys, turning them into U-shaped valleys. At the end of Pleistocene, the climatic conditions have changed, which has led to a gradual withdrawal of the glaciers, postponing a series of frontal and stadialmorains. During Holocene, the glacial relief was reworked by modern morphogenetic processes - weathering, erosion and denudation.
2. Study Area

The object of the present study is the high parts (~2200-2300 meters above sea level) of the Rila (N 42.1°, E 23.5°) and Pirin (N 41.7°, E 23.4°) Mountains, where there were conditions for the formation of glacial landforms (cirques) (Fig.1). Rila and Pirin Mountains are situated in Southwestern Bulgaria and occupy a median position within the Balkan Peninsula (Fig.2). These are the highest and most impressive mountain morphounits in this part of Europe.

![Figure 1](image_url1)

**Figure 1.** Cirques and cirque thresholds in Rila and Pirin Mountains (white areas) (after Mitkov, Gachev, [11]; Mapping tool: GoogleEarth)

![Figure 2](image_url2)

**Figure 2.** Survey map of geographical position of Rila (1) and Pirin (2) Mountains within the eastern part of Balkan Peninsula (Mapping tool: GeoMapApp-http://www.geomapapp.org)
3. Methodology and Methods

The classical example of a fractal object is defined by Benoît Mandelbrot [13]. If the length of an object \( P \) is related to the measuring unit length \( l \) by the formula:

\[
P \sim l^{1-D}
\]

then \( P \) is a fractal and \( D \) is a parameter defined as the fractal dimension. This definition was given by B. Mandelbrot in the early 60-s of the 20-th century. His ideas support the view that many objects in nature can not be described by simple geometric forms, and linear dimensions, but they have different levels of geometric fragmentation. It is expressed into the irregularities of the different scales (sizes) – from very small to quite big ones.

In the field of Geosciences is accepted that definition of the different «fractals» as «real physical objects is most often connected to fragmentation» (Korvin, [14]). This reveals that each measurable object has a length, surface or volume, which depends on the measuring unit and the object’s form irregularity. The smaller the measuring unit is, the bigger is the total value for the linear (surface, volume) dimension of the object and vice versa. The same is valid for 2D and 3D objects (Rangelov, [15]).

Another definition of a fractal dimension is related to the serial number of measurement to each of the measuring units used and the object dimensions. If the number of the concrete measurement with a selected linear unit is bigger than \( r \), then it might be presented by (Turcotte, [16]):

\[
N \sim r^{-D}
\]

where \( D \) is the fractal dimension and \( N \) is the number of objects with a linear dimension \( r \) for a discrete distribution and the number with a linear dimension greater than \( r \) for a continuous distribution.

The present study methodology based on the correlation number-area is following the algorithm presented and effectively applied in a number of publications (Meyback, [17]; Rangelov et al., [18], [19]; Rangelov, [15], [20]):

- Calculation of the number of glacial cirques (\( N \)) with corresponding area (in \( \text{km}^2 \)) for the graphics.
- Presentation of the results on the graphics – on the X axis in logarithmic scale the areas of the glacial cirques are plotted, and on the Y axis in linear scale the number of the corresponding glacial cirques are plotted.
- The fractal dimensions (\( D \)) have been calculated using the data from the graphics and results discussed.

The proposed study examines the glacial cirques in Rila and Pirin Mountains as surface objects and applies the abovementioned methodology to them. The research is based on 77 glacial cirques in both mountains. The area of these cirques is calculated using free GIS software (Google Earth). This provides the necessary methodological basis for measuring the area of glacial cirques in other mountain regions with traces of glacial activity, which creates prerequisites in the future for different comparative studies on this basis.

4. Results and Discussion

The results from the analysis of the fractal geometry of the glacial cirques in Rila and Pirin Mountains are presented in graphical form in Figure 3. It is important to mention that dimensions have negative numbers, but for easier perception are presented as positive values in the graphics.

![Figure 3. Fractal analysis of the glacial cirques in Rila and Pirin Mountains](image)

On the basis of the results from the graphs, the following main conclusions can be drawn:

1) The glacial cirques in Rila and Pirin Mountains have fractal geometry (\( D=1.378 \) for Rila Mountain and \( D=1.358 \) for Pirin Mountain).

2) The level of fragmentation of the glacial cirques is low, indicating that the differences in the minimum and maximum area values are not so large.

3) The contemporary fractal geometry of the glacial cirques in Rila and Pirin Mountains shows that during the last 10 000 of years exogenous processes (wind activity, frost weathering, rainfalls and snowfalls, etc.) failed to destroy them and respectively the endogenous earthly forces could preserve the original mathematical configuration to this day.
4) Given the fractal properties of glacial cirques, there are all prerequisites to believing that landforms formation within Rila and Pirin Mountains is carried out in self-organizing environment which create natural structures with fractal geometry.

One of the most important results of the present investigation is the clarification of the proportion of the role of endogenous and exogenous earthly processes during the contemporary Rila-Pirin mountainous relief formation. The results obtained for the fractal geometry of the glacial cirques in the Rila and Pirin Mountains can serve as a strong argument in favor of the domination of the endogenous earthly processes for the relief development in study lands. Practically unchanging type of the glacial cirques configuration during the last 10 000 years is showing the obvious endogenous earthly processes predominance by the Earth’s superficial landscape modeling. This is also confirmed by the GPS data on the relatively rapid rise of the area (about 2 mm / per year) (Kotzev et al, [21]). That dispute the unsoundness allegations of some authors about the “equivalent” role of the endogenous and exogenous earthly processes for the contemporary mountainous topography development. The relative long endogenous processes predominance favors the effective application of the fractal analysis by the regional morphostructural research of the mountainous postglacial relief in Southwest Bulgaria.

5. Conclusions

This study confirms the fractal geometry of the glacial cirques within the Rila and Pirin Mountains. This unambiguously reflects the self-organizing nature of the relief building processes in the high parts of Rila and Pirin Mountains from last Ice Age to the present. In this regard, fractal analysis has proved to be an extremely useful tool in the field of regional morphostructural studies. This gives new directions to the study of the principles of creation and evolution of landforms with glacial origin and Quaternary morphostructural generations as a whole. On the other hand, the results obtained serve as an important argument regarding endogenous earthly processes predomination in the relief building in the Rila and Pirin Mountains. In the future current research gives grounds for different correlative studies on this basis with other mountainous regions in the Balkan Peninsula, where glacial landforms have developed.

REFERENCES

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