A Numerical Solution for a Frictional Contact Problem between an Orthotropic Strip and Punch System

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Abstract In this study, a numerical solution of elasticity problem is examined. This problem is a plane contact problem. The frictional contact problem for an elastic strip under a rigid punch system is considered. The frictional contact problem is related to infinite length elastic strip in contact with N punches under the influence of horizontal and vertical forces. The lower boundary of the strip is hinged. The solution of contact problems is often reduced to the solution of an integral equation. This integral equation system can be derived from contact problem by using the basic equations of elasticity theory and the given boundary conditions. The singular integral equation system is solved with the help of Gauss Jacobi Quadrature Collocation Method. The frictional contact problem for a homogenous and orthotropic elastic layer are investigated numerically the pressure distribution under the punch system due to the geometrical and mechanical properties of elastic layer are examined and the results are shown in the graphics and tabular form.

Keywords Singular Integral Equation, Plane Contact Problem, Cauchy Kernel, Gauss-Jacobi Quadrature, a System of Linear Algebraic Equation

Many problems related to the mechanic of elastic bodies can be converted into the singular integral equations. For this reason, studies on the solution of singular integral equations hold an important place in mathematic. The first important studies on this subject were made by Muskhelishvili. [1-2]. Many methods have been developed to obtain an analytic solution of integral equations. [3-5].

The numerical solution methods have been developed since it is difficult to solve integral equations analytically. There are many important studies. Erdogan and Gupta has obtained approximate solution of the system of singular integral equations by using the properties of the orthogonal polynomials.[6-7]. Plane contact problems with mixed boundary conditions is studied by Alexandrov [8,9]. Chebakov investigated the asymptotic solution of contact problems for a relatively thick elastic layer when there are friction forces in the contact area.[10]. Yusufoglu E. and his friends [11] investigated a plane contact problem for an elastic orthotropic strip by using a functional method for thick strips and an asymptotic method for thin strips. There are many important studies in addition to this studies.

In this study, a numerical solution of elasticity problem is investigated. The frictional contact problem is related to infinite length elastic strip in contact with N punches under the influence of horizontal and vertical forces.

1. Introduction

The elements of many structural and mechanical systems are in contact with each other. The shape of this contact plays an important role in the deformation of the objects and in the behavior of stress distribution along the contact zone. The elementary theory did not meet all needs in the solution of the contact problems. In recent years, the rapid development of computer technology has triggered the development of numerical solution methods. In addition to this, the solution of contact problems has gained speed with the aid of elasticity theory and important studies have been done.

2. Formulation of the Problem

Upper boundary of the infinite length elastic strip is in contact with a punch system consisting of N punches under the influence of horizontal and vertical forces.

It is assumed that the surface forces do not affect the area outside the contact zone. The lower boundary of the strip is hinged. (Figure 2.1). In addition, when $|x| \rightarrow \infty$, the stresses will be zero. The stress on the strip will be regarded as the plane stress under these conditions.

Figure 2.1. The Geometry of the Problem

Suppose that the thickness of the elastic strip is h and the coefficient of friction is ε . The bottom surfaces of the punches are given the equations $\delta_i(x)$. The vertical and horizontal forces which are affected the punches are P_i and Q_i respectively, where $i = 1, 2, \dots, N$.

The solution of the plane contact problem investigated in study [1]. The solution of the contact problem is reduced to the solution of the following integral equation system by using the basic equations of the elasticity theory.

$$\varepsilon p_i(x) + \frac{1}{\pi} \int_{a_i}^{b_i} \frac{p_i(\tau)}{\tau - x} d\tau + \sum_{j=1, j \neq i}^N \frac{1}{\pi} \int_{a_j}^{b_j} \frac{p_j(\tau)}{\tau - x} d\tau \quad (1)$$

$$+ \frac{1}{h} \sum_{j=1}^N \int_{a_j}^{b_j} M_{ij} \left(\frac{\tau - x}{h} \right) p_j(\tau) d\tau = \theta \delta'_i(x) \quad (2)$$

$$\int_{a_i}^{b_i} p_i(\tau) d\tau = P_i \quad i = 1, 2, \dots, N$$

$[a_i, b_i]$ is the contact interval of the punch for each i , and the pressure of the punches along the contact areas is $p_i(x)$ ($i = 1, 2, \dots, N$). θ is a constant which can be calculated with the help of the constants indicating the mechanical properties of the strip.

$M_{ij}(t)$ ($i = 1, 2, \dots, N$, $j = 1, 2, \dots, N$) are the regular kernels including the geometrical and mechanical constants of elastic strip.

3. Discretization of the Integral Equation

In this section, the equation (1) will be discretized. Suppose that

$$x = \eta(t) = r_i t + s_i \quad \text{for } x \in [a_i, b_i] \quad (3)$$

$$\tau = \eta(z) = r_j z + s_j \quad \text{for } \tau \in [a_j, b_j] \quad (4)$$

$$r_i = \frac{b_i - a_i}{2}, \quad s_i = \frac{b_i + a_i}{2} \quad (5)$$

Substituting Eq. (3), (4) and (7) into Eq. (1), integral equation becomes the following form,

$$\varepsilon p_i(\eta(t)) + \frac{1}{\pi} \int_{-1}^1 \frac{p_i(\eta(z))}{z - t} dz + \sum_{j=1, j \neq i}^N \frac{1}{\pi} \int_{-1}^1 \frac{p_j(\eta(z))}{z - \frac{r_i}{r_j} t + \frac{s_j - s_i}{r_j}} dz \quad (6)$$

$$+ \frac{1}{\lambda_j} \sum_{j=1}^N \int_{-1}^1 M_{ij} \left(\frac{z - \frac{r_i}{r_j} t + \frac{s_j - s_i}{r_j}}{\lambda_j} \right) p_j(\eta(z)) dz = \theta \delta'_i(\eta(t))$$

Where,

$$\frac{h}{r_j} = \lambda_j. \quad (7)$$

We assume that the following equations,

$$p_i(\eta(t)) = \varphi_i(t) \quad (8)$$

$$\theta \delta'_i(\eta(t)) = f_i(t) \quad i = 1, 2, \dots, N \quad (9)$$

Thus, Integral equation (6) will be converted the following form.

$$\varepsilon \varphi_i(t) + \frac{1}{\pi} \int_{-1}^1 \frac{\varphi_i(z)}{z - t} dz + \sum_{j=1, j \neq i}^N \frac{1}{\pi} \int_{-1}^1 \frac{\varphi_j(z)}{z - \frac{r_i}{r_j} t + \frac{s_j - s_i}{r_j}} dz \quad (10)$$

$$+ \frac{1}{\lambda_j} \sum_{j=1}^N \int_{-1}^1 M_{ij} \left(\frac{z - \frac{r_i}{r_j} t + \frac{s_j - s_i}{r_j}}{\lambda_j} \right) \varphi_j(z) dz = f_i(t)$$

The additional condition (2) may be expressed as

$$\int_{-1}^1 \varphi_j(z) dz = \frac{P_j}{r_j} \quad (11)$$

Where $i = 1, 2, \dots, N$, $j = 1, 2, \dots, N$.

4. The Numerical Solution Method

In this section, a numerical solution method of the equation system (10) and (11) is developed. We will suppose that the punches are of equal width, for convenience in practice. In this situation, the width of

punches are $r_1 = r_2 = \dots = r_N = r$ and $\frac{h}{r} = \lambda$.

The solution of the system is found as;

$$\varphi_j(t) = \omega(t) \cdot g_j(t), \quad \omega(t) = (1-t)^\alpha (1+t)^\beta \quad (12)$$

$$j = 1, 2, \dots, N.$$

As is known from the singular integral equations theory, index is equal to

$$-(\alpha + \beta) = \kappa, \quad (-1 < \alpha, \beta < 0). \quad (13)$$

The pressures created by these punches must be infinite at the edge, since both ends of the punches are assumed to enter the elastic strip. In this case value of κ is equal to 1, according to index theory.

The values of α and β is chosen to from $\varepsilon + \cot \pi \alpha = 0$ and $\beta = -1 - \alpha$ respectively, [1].

Substituting Eq. (12) into Eq.(10), integral equation becomes the following form,

$$\begin{aligned} \varepsilon \omega(t) g_i(t) + \frac{1}{\pi} \int_{-1}^1 \frac{\omega(z) g_i(z)}{z-t} dz \\ + \sum_{j=1}^N \frac{\omega(z) g_j(z)}{z-t + \frac{s_i - s_j}{r}} dz + \\ \frac{1}{\lambda} \int_{-1}^1 \sum_{j=1}^N M_{ij} \left(\frac{z-t + \frac{s_i - s_j}{r}}{\lambda} \right) \omega(z) g_j(z) dz = f_i(t) \end{aligned} \quad (14)$$

We will try to find the corresponding Lagrange interpolation polynomials instead of the unknown functions $g_i(t)$, where $i = 1, 2, \dots, N$

We choose the interpolation node points z_m , $m = 1, 2, \dots, n$ as the roots of Jacobi polynomials $P_n^{(\alpha, \beta)}(t)$. Lagrange interpolation polynomials is considered as the following form,

$$g_i(t) \approx g_{in}(t) = \sum_{m=1}^n \frac{g_{inm} P_n^{(\alpha, \beta)}(t)}{(t - z_m) P_n^{(\alpha, \beta)}(z_m)} \quad (15)$$

$$i = 1, 2, \dots, N$$

$$g_{inm} = g_{in}(z_m), \quad m = 1, 2, \dots, n. \quad (16)$$

As is known, the following equation is true. [12]

$$\begin{aligned} \frac{1}{\pi} \int_{-1}^1 \frac{\omega(z) P_n^{(\alpha, \beta)}(z)}{z-t} dz \\ = -\varepsilon \omega(t) P_n^{(\alpha, \beta)}(t) - \frac{1}{2 \sin(\alpha \pi)} P_{n-1}^{(-\alpha, -\beta)}(t) \end{aligned} \quad (17)$$

Substituting the Eq.(15) and (17) into Eq.(14), it is obtained Eq.(18)

$$\begin{aligned} \sum_{m=1}^n \frac{g_{inm}}{P_n^{(\alpha, \beta)}(z_m)} \frac{1}{t - z_m} \\ \left[\begin{aligned} & \varepsilon \omega(t) P_n^{(\alpha, \beta)}(t) - \varepsilon \omega(t) P_n^{(\alpha, \beta)}(t) \\ & - \frac{1}{2 \sin(\alpha \pi)} P_{n-1}^{(-\alpha, -\beta)}(t) \\ & + \varepsilon \omega(z_m) P_n^{(\alpha, \beta)}(z_m) + \frac{1}{2 \sin(\alpha \pi)} P_{n-1}^{(-\alpha, -\beta)}(z_m) \end{aligned} \right] \\ + \sum_{j=1}^N \sum_{m=1}^n \frac{g_{jnm}}{P_n^{(\alpha, \beta)}(z_m)} \frac{1}{\pi} \int_{-1}^1 \frac{\omega(z)}{z-t + \frac{s_i - s_j}{r}} \frac{P_n^{(\alpha, \beta)}(z)}{z - z_m} dz \\ + \sum_{j=1}^N \frac{1}{\lambda_j} \sum_{m=1}^n \frac{g_{jnm}}{P_n^{(\alpha, \beta)}(z_m)} \int_{-1}^1 M_{ij} \left(\frac{z-t + \frac{s_i - s_j}{r}}{\lambda_j} \right) \omega(z) \frac{P_n^{(\alpha, \beta)}(z)}{z - z_m} dz = f_i(t) \end{aligned} \quad (18)$$

Eq.(18) reduced to Eq.(19) at the collocation point $t = t_1, t_2, \dots, t_{n-1}$ points for each $i = 1, 2, \dots, N$,

$$\begin{aligned} \frac{1}{2 \sin(\alpha \pi)} \sum_{m=1}^n \frac{g_{inm}}{P_n^{(\alpha, \beta)}(z_m)} \frac{1}{t_k - z_m} \left[P_{n-1}^{(-\alpha, -\beta)}(z_m) \right] + \\ \sum_{m=1}^n \sum_{j=1}^N \frac{g_{jnm}}{P_n^{(\alpha, \beta)}(z_m)} \frac{1}{\pi} \int_{-1}^1 \frac{\omega(z)}{z-t_k + \frac{s_i - s_j}{r}} \frac{P_n^{(\alpha, \beta)}(z)}{z - z_m} dz + \\ \sum_{j=1}^N \sum_{m=1}^n \frac{g_{jnm}}{P_n^{(\alpha, \beta)}(z_m)} \frac{1}{\lambda} \int_{-1}^1 M_{ij} \left(\frac{z-t_k + \frac{s_i - s_j}{r}}{\lambda} \right) \omega(z) \frac{P_n^{(\alpha, \beta)}(z)}{z - z_m} dz = f_i(t_k) \end{aligned} \quad (19)$$

We choose the t_k points as roots of $P_{n-1}^{(-\alpha, -\beta)}(t_k) = 0$ where $k = 1, 2, \dots, n-1$.

As is known, the following equation is true.

$$a_{nm} = \frac{1}{P_n^{(\alpha, \beta)}(z_m)} \int_{-1}^1 \frac{\omega(z) P_n^{(\alpha, \beta)}(z)}{z - z_m} dz \quad (20)$$

and it's obvious that from [7]

$$a_{nm} = -\frac{\pi}{2 \sin \alpha \pi} \frac{P_{n-1}^{(-\alpha, -\beta)}(z_m)}{P_n^{(\alpha, \beta)}(z_m)} \quad (21)$$

Substituting Eq. (20) and (21) into Eq. (19), we obtain

$$-\frac{1}{\pi} \sum_{m=1}^n \frac{g_{im} a_{nm}}{t_k - z_m} + \frac{1}{\pi} \sum_{m=1}^n \sum_{j=1}^N \frac{g_{jnm} a_{nm}}{z_m - \frac{r_i}{r_j} t_k + \frac{s_j - s_i}{r_j}} + \quad (22)$$

$$\sum_{m=1}^n \sum_{j=1}^N g_{jnm} a_{nm} M_{ij}(z_m, t_k) = f_i(t_k)$$

Where

$$M_{ij}(z_m, t_k) = \frac{1}{\lambda} M_{ij} \left[\frac{z_m - t_k + \frac{s_j - s_i}{r}}{\lambda} \right] \quad (23)$$

Eq.(22) can be written more compactly as follows.

$$\sum_{j=1}^N \sum_{m=1}^n \left[\frac{1}{z_m - t_k + \frac{s_j - s_i}{r}} + \pi M_{ij}(z_m, t_k) \right] a_{nm} g_{jnm} = \pi f_i(t_k) \quad (24)$$

$$i = 1, 2, \dots, N \quad k = 1, 2, \dots, n-1$$

As can be seen from equation Eq. (24), number of equations are $N(n-1)$, the number of unknowns are Nn . and the number of missing equations are N . The missing equations will be completed by using the static condition. The additional condition (25) may be expressed as

$$\sum_{m=1}^n a_{nm} g_{jnm} = P_j^* \quad (25)$$

Where $j = 1, 2, \dots, N$.

There are N additional conditions in total. Thereby, the number of unknowns and equations are equal to N .

Unknown g_{jnm} functions where $(j = 1, 2, \dots, N, m = 1, 2, \dots, n)$ can be obtained from the linear equation system. $g_1(t), g_2(t), \dots, g_N(t)$ functions can be found by using the Eq. (15), and the Eq. (12) can be used to find the $\varphi_1(t), \varphi_2(t), \dots, \varphi_N(t)$ functions, so that the solution of the integral equation system is obtained.

5. Numerical Results and Conclusions

In this section, the system of equations (24) and (25) which are obtained by applying of Gauss Jacobi Quadrature and collocation method are examined. The punch system corresponding to the case $N=1$ and

$N=2$ will be examined.

We consider the flat-bottomed punches. In this case, according to Eq.(9), the right side of Eq.(1) will be equal to zero.

It is assumed that the width of punches is same and vertical forces acting on the punches are equal to each other. ($P_1 = P_2 = \dots = P_N = P$ and $r_1 = r_2 = \dots = r_N = r$). By using Eq. (26)

$$\varphi_i(t) = \frac{r}{P} P_i(\eta(t)) \quad (26)$$

It will be obtained the following equations instead of the system of Eq. (24) and Eq. (25).

$$\sum_{j=1}^N \sum_{m=1}^n \left[\frac{1}{z_m - t_k + \frac{s_j - s_i}{r}} + \pi M_{ij}(z_m, t_k) \right] a_{nm} g_{jnm} = 0 \quad (27)$$

$$\sum_{m=1}^n a_{nm} g_{jnm} = 1 \quad (28)$$

5.1. $N=1$ State

When $N=1$ is examined, Eq. (27) and Eq. (28) can be expressed as

$$\sum_{m=1}^n \left[\frac{1}{z_m - t_k} + \pi M(z_m, t_k) \right] a_{nm} g_{jnm} = 0 \quad (29)$$

$$k = 1, 2, \dots, n-1$$

$$\sum_{m=1}^n a_{nm} g_{nm} = 1. \quad (30)$$

It is assumed that the elastic strip is made of orthotropic material. E_1 and E_2 are constant which characterized the orthotropic properties in the Ox and Oy directions. The relative thickness (λ) will refer to the geometrical property of the elastic strip. The minimum pressure value and its apses are $\varphi(t_*)$ and t_* respectively in the pressure distribution under the punches. The pressure values q_{11} and q_{12} corresponding to the $t = -0.95$ and $t = 0.95$ values are given in table 5.1.

Table 5.1. The pressure values in the different points corresponding to the orthotropic properties, relative thickness and friction coefficient.

E_1/E_2	λ	$\mathcal{E}$	t_*	$\varphi(t_*)$	q_{11}	q_{12}
0.25	2	0	0	0,330572	0,988396	0,988396
		0.1	0,132304	0,321649	1,244413	0,767013
		0.2	0,317430	0,300283	1,521653	0,575754
0.25	8	0	0	0,318752	1,018282	1,018282
		0.1	0,109738	0,311573	1,272946	0,790084
		0.2	0,232840	0,291778	1,537710	0,598278
1	2	0	0	0,326948	0,998089	0,998089
		0.1	0,121185	0,317943	1,258037	0,774986
		0.2	0,299810	0,297900	1,541526	0,580285
1	8	0	0	0,318523	1,018859	1,018859
		0.1	0,110156	0,311247	1,274161	0,789961
		0.2	0,230824	0,291444	1,539046	0,597714
5	2	0	0	0,322507	1,008899	1,008899
		0.1	0,109340	0,313569	1,271468	0,784673
		0.2	0,271140	0,295113	1,559161	0,587085
5	8	0	0	0,318388	1,019204	1,019204
		0.1	0,110507	0,311038	1,274936	0,789822
		0.2	0,229376	0,291231	1,539851	0,597277
20	2	0	0	0,319647	1,015982	1,015982
		0.1	0,102392	0,310775	1,279655	0,791283
		0.2	0,253063	0,293228	1,569088	0,592178
20	8	0	0	0,318331	1,019351	1,019351
		0.1	0,110675	0,310947	1,275273	0,789750
		0.2	0,228710	0,2911390	1,540189	0,597073

Figure 5.1. The changing of pressure along contact zone for $\lambda = 1, \mathcal{E} = 0$ and different values of E_1/E_2

Figure 5.2. The changing of pressure along contact zone for $\lambda = 1, \varepsilon = 0.2$ and different values of E_1/E_2

Figure 5.3. The changing of pressure along contact zone for $\lambda = 2, \varepsilon = 0$ and different values of E_1/E_2

Figure 5.4. The changing of pressure along contact zone for $\lambda = 2, \varepsilon = 0.2$ and different values of E_1/E_2

Figure 5.5. The changing of pressure along contact zone for $E_1/E_2 = 5, \varepsilon = 0.2$ and different values of λ

Figure 5.6. The changing of pressure along contact zone for $E_1/E_2 = 20, \varepsilon = 0$ and different values of λ

5.1.1. Conclusions in Case of $N=1$

E_1 and E_2 constants are Young modules that show the orthotropic properties of the strip in O_x and O_y directions. ε is friction coefficient and λ is the relative thickness of strip.

1. ε is equal to zero in frictionless contact problem. In this situation, the smallest pressure value occurs in the middle point of the contact area $t = 0$. the pressure graph is also a symmetrical curve.
2. In the case of frictional contact where the coefficient of friction is constant, while the thickness of the strip increases, the strip resistance against the pressure of the punch is decreasing.
3. E_1/E_2 is a parameter indicating where the orthotropic property of the material is greater. While the friction coefficient increases and E_1/E_2 is constant, t_* apse corresponding to the smallest punch pressure shifts to the right from the axis of symmetry and smallest punch pressure $\phi(t_*)$ decreases. While the pressure value q_{11} near the left end of the contact increases, the pressure values q_{12} near the right end of the contact decrease.
4. For the same friction coefficient values, the minimum pressure value decreases as the punches width increases.

5. As the orthotropic properties E_1/E_2 in the x-axis direction increase, t_* value corresponding to the smallest pressure approaches the origin from the right and the smallest pressure values corresponding to these values are also decreases.

5.2. $N=2$ State

When $N = 2$ is examined, Eq.(27) and Eq.(28) can be expressed as

$$\sum_{j=1}^2 \sum_{m=1}^n \left[\frac{1}{z_m - t_k + \frac{s_j - s_i}{r}} + \pi M_{ij}(z_m, t_k) \right] a_{nm} g_{jnm} = 0 \quad (31)$$

$$\sum_{m=1}^n a_{nm} g_{jnm} = 1 \quad j = 1, 2 \quad (32)$$

It is assumed that the punches are in the range of $[-3, -1] \cup [1, 3]$ for the examined condition. The minimum pressure values and it's apses are $\phi(t_1^*), \phi(t_2^*)$ and t_{1*}, t_{2*} respectively, in the pressure distributions under the punches for different values of $E_1/E_2, \lambda$ and ε . The pressure values q_{11}, q_{12}, q_{21} and q_{22} corresponding to the $t_{1L} = -2.95, t_{1R} = -1.05$ and $t_{2L} = 1.05, t_{2R} = 2.95$, are given in table 5.2.

Table 5.2. The pressure values for two punches in the different points corresponding to the orthotropic properties, relative thickness and friction coefficient.

E_1/E_2	λ	$\mathcal{E}$	t_{1*} t_{2*}	$\varphi_1(t_*)$ $\varphi_2(t_*)$	q_{11} q_{12}	q_{21} q_{22}
0.25	1	0	-1,733968	0,273517	1,554785	0,832148
			1,773962	0,273517	0,832148	1,554785
		0.1	-1,820201	0,258770	1,893576	1,115467
			1,790128	0,271955	0,578257	1,205693
		0.2	-1,416172	0,222889	2,134976	1,354012
			1,792504	0,277015	0,460855	1,042480
0.25	4	0	-1,646090	0,277001	1,491133	0,634961
			1,646090	0,277001	0,634961	1,491133
		0.1	-1,462164	0,232469	1,773244	0,875070
			1,803773	0,303383	0,448188	1,227263
		0.2	-1,258671	0,159541	2,1512248	1,230667
			1,886943	0,304686	0,247503	0,888217
1	1	0	-1,818119	0,261090	1,556680	0,797078
			1,818119	0,261090	0,797078	1,556680
		0.1	1,907087	0,242588	1,903758	1,080881
			1,823269	0,256766	0,539479	1,189994
		0.2	-1,509647	0,224582	2,122941	1,300079
			1,826076	0,264138	0,449887	1,073329
1	4	0	-1,603309	0,284973	1,450982	0,617960
			1,603309	0,284973	0,617960	1,450982
		0.1	-1,433975	0,236162	1,709568	0,838564
			1,776409	0,315600	0,452292	1,213520
		0.2	-1,246815	0,155265	2,100502	1,202710
			1,878307	0,318256	0,245256	0,862913
10	1	0	-1,695294	0,271127	1,513258	0,663500
			1,695294	0,271127	0,663500	1,513258
		0.1	-1,766075	0,245686	1,877313	0,939460
			1,771926	0,274039	0,415765	1,122653
		0.2	-1,368939	0,208317	2,088640	1,151709
			1,758219	0,278192	0,356363	1,059078
10	4	0	-1,659281	0,304044	1,337678	0,679678
			1,659281	0,304044	0,679678	1,337678
		0.1	-1,478211	0,259974	1,566810	0,882252
			1,882214	0,328829	0,531814	1,135209
		0.2	-1,273876	0,183366	1,964869	1,258262
			2,005012	0,327459	0,304243	0,787841

Figure 5.7. The changing of pressure along contact zone for $\lambda = 1, \varepsilon = 0$ and different values of E_1/E_2

Figure 5.9. The changing of pressure along contact zone for $\lambda = 8, \varepsilon = 0$ and different values of E_1/E_2

Figure 5.8. The changing of pressure along contact zone for $\lambda = 1, \varepsilon = 0.2$ and different values of E_1/E_2

Figure 5.10. The changing of pressure along contact zone for $\lambda = 8, \varepsilon = 0.2$ and different values of E_1/E_2

Figure 5.11. The changing of pressure along contact zone for $E_1/E_2 = 0.25$, $\lambda = 1$ and different values of ε

Figure 5.12. The changing of pressure along contact zone for $E_1/E_2 = 20$, $\lambda = 0.2$ and different values of ε

5.2.1. Conclusions for N=2 Case

1. The pressure distribution curves along the contact zone are symmetrically with respect to the y axis, in the case of frictionless contact. As the relative thickness increases ($\lambda > 1$), the minimum pressure value also increases. The minimum pressure

increases as the orthotropic property in the x axis direction increases for the strips of the same thickness.

2. In the case of frictional contact, the amount of pressure in the area under the punch on the right is naturally greater than the amount of pressure on the left punch.

The above examinations are made for flat-bottomed and equal width punches. It's assumed that the vertical and horizontal forces are equal. But the proposed algorithm is valid for all contact problems involving non-flat punches with different width.

REFERENCES

- [1] Muskhelishvili, N.I. (1958), Singular Integral Equations, Wolters-Noordhoff Publishing, Groningen
- [2] Muskhelishvili, N.I., (1953), Some Basic Problems of mathematical Theory of Elasticity, Noordhoff, Groningen.
- [3] Green, C.D., (1969), Integral Equation Methods, Thomas Nelson & Sons Ltd. London.
- [4] Popov, G.Y., (1982), Contact Problems for a Linearly Deformable Base, Vishcha Shkola, Kiev-Odessa (Russian).
- [5] Tricomi, F. G., (1985), Integral Equations, Dover Publications Inc., New York.
- [6] Erdoğan, F., (1969), Approximate Solutions of Systems of Singular Integral Equations, Society for Industrial and Applied Mathematics, 17, 1041-1059.
- [7] F. Erdogan and G.D. Gupta, Cook T.S, The numerical solution of singular integral equations, Q. J. Appl. Math. 29 (1973), pp. 525-534.
- [8] Aleksandrov, V. M., Two problems with mixed boundary conditions for an elastic orthotropic strip. Journal of Applied Mathematics and Mechanics vol. 70, (2006), p.128-138
- [9] Aleksandrov, V. M., On Plane contact problems of the theory of elasticity in the presence of adhesion and friction. Journal of Applied Mathematics and Mechanics vol. 34/2, (1970), p.232-243.
- [10] Chebakov M. I. Asymptotic solution of contact problems for a relatively thick elastic layer when there are friction forces in the contact area, Journal of Applied Mathematics and Mechanics 69 (2005) 296-304
- [11] Erbaş B, Yusufoglu E, Kaplunov J. A plane contact problem for an elastic orthotropic strip. J. Engrg. Math. 70: 399-409, 2011.
- [12] Gradshteyn, I.S. and Ryzhik. I.M. Table of Integrals, Sums, Series and Products. Nauka. Moscow 1971.
- [13] Oguz H. The Contact Problem for an Elastic Strip and Layer. Ph.D. Thesis. Kutahya 2016