

OSeMOSYS: Introducing Elasticity

Denis Lavigne

Science Department, Royal Military College Saint-Jean (RMCSJ), Saint-Jean-sur-Richelieu, Quebec, Canada

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Abstract The OSeMOSYS project proposes an open-access energy modeling tool and its relative simplicity makes it appealing for academic research and governmental organizations to study the impacts of policy decisions on an energy system in the context of possible severe greenhouse gases (GHG) emissions constraints. In real life such limitations on emissions levels may result in a reduction of some end-use demands (for example for the transportation of passengers-kilometers by cars). The current OSeMOSYS model does not allow this type of behaviour and all end-use demands must be completely satisfied. This paper presents how an elasticity of demand capability can be embedded in an OSeMOSYS analysis in order to allow the possibility to endogenously reduce some end-use demands to satisfy GHG emissions objectives.

Keywords OSeMOSYS, Energy Modeling, Elasticity of Demand, Greenhouse Gases Emissions, Mathematical Modeling

1 Introduction

Energy models have been around for decades. Some models use a bottom-up techno-economic approach (see for example [1]). This means that the energy system of a particular region (or group of regions) is represented by a list of end-use demands which must be met using a typically extensive list of technologies. These technologies and their related energy fuels are highly detailed using technical and economic data. The basic goal of such models is to meet all end-use demands using optimal technological choices to minimize the long-term total system cost. This approach has been used in many parts of the world to perform long term prospective analysis of the energy sector.

Another tool using a similar approach is the OSeMOSYS (Open-Source energy MOdelling SYStem) optimization model [2]. As mentioned on OSeMOSYS' homepage, it "*potentially requires a less significant learning curve and time commitment to build and operate. Additionally, by*

not using proprietary software or commercial programming languages and solvers, OSeMOSYS requires no upfront financial investment. These two advantages extend the availability of energy modeling to the communities of students, business analysts, government specialists, and developing country energy researchers".

At its core, both of these tools must find a way (using technologies and fuels) to satisfy all end-use demands. In real life, meeting such demands may put a severe financial burden on a region if some difficult-to-achieve greenhouse gases (GHG) targets are imposed on the system. Introducing elasticity of demand within such models may greatly reduce the impact of GHG constraints. Instead of having to fully satisfy a particular demand, elasticity may result in an endogenous decrease of the demand if the cost to reduce it is less than the cost of having to meet the demand in its entirety. For example, the TIMES model has an elasticity of demand option and one can specify a specific approximation of the demand curve of a particular end-use demand. This paper builds on the existing Extended UTOPIA model [4] to propose the introduction of a similar capability within OSeMOSYS, allowing some flexibility that OSeMOSYS does not currently have.

Elasticity of demand being an economic issue unknown (or unclear) to energy modelers, section 2 presents a basic overview of what price elasticity of demand is for a linear demand curve. The topic is extended in section 3 to the analysis of what it means to have a constant elasticity of demand to its own price (this particular elasticity is the one used in TIMES). Section 4 proposes an approximation of the demand curve and a theoretical example is presented. Section 5 describes how to embed such a capability within the *Extended UTOPIA* OSeMOSYS problem while section 6 analyses the results of this problem using elasticity under emissions limitations constraints.

The goal of the paper is to help potential users to grasp the powerful yet simplicity of the approach. University professors may decide to include OSeMOSYS and its elastic capability in graduate classes on the subject and help develop highly trained professionals ready to use more complex models or develop real-life energy models using OSeMOSYS.

2 Price Elasticity of Demand

This section is largely influenced by Chapters 3 and 4 from [5]. It is standard material presented in any microeconomics introductory course. It is presented here for coherence, clarity and full understanding of the whole paper for energy modelers not used to economics issues. Engineers and researchers in the field of energy modeling are encouraged to take courses on micro- and macro-economics.

2.1 Demand

The demand of a particular economic good is the relationship between the quantity customers are willing to buy of that good for a wide range of prices. The shape of the curve representing the relationship between the price of an item and the quantity wanted by customers is typically given by a decreasing function such as the one below.

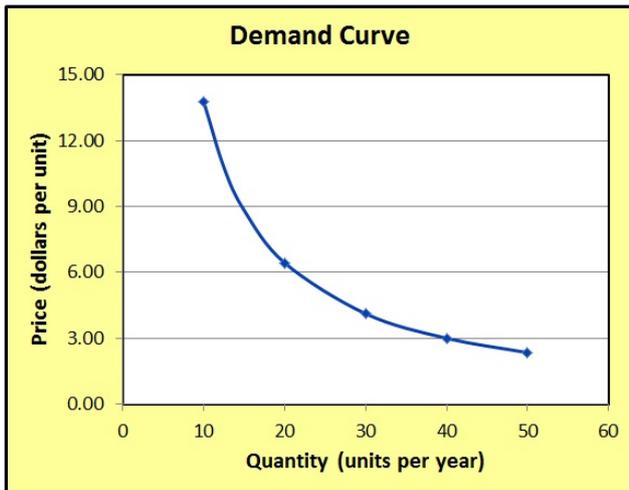


Figure 1. Demand

For example, the demand curve shows that customers are willing to buy a quantity of 40 units per year if the price is set to \$3 per unit. As expected, quantity decreases as price increases. The demand of an item is thus related to its own price. Parkin and Bade [5] mention that shifts of the demand curve could be induced by six main factors other than the own price of an economic good: (1) the prices of related goods, (2) expected future prices, (3) income, (4) expected future income, (5) population and (6) preferences. This paper does not take into account these six factors. It rather supposes that the demand is only influenced by its own price, as shown in the demand curve represented by Figure 1.

2.2 Price Elasticity

Figure 1 shows that the demand varies according to the price. Since the demand is not fixed, we say that it is *elastic*. We can determine the value of the elasticity at any point of the demand curve. Each point has its own elasticity value. To make things simpler, suppose that the demand curve is linear such as Figure 2.

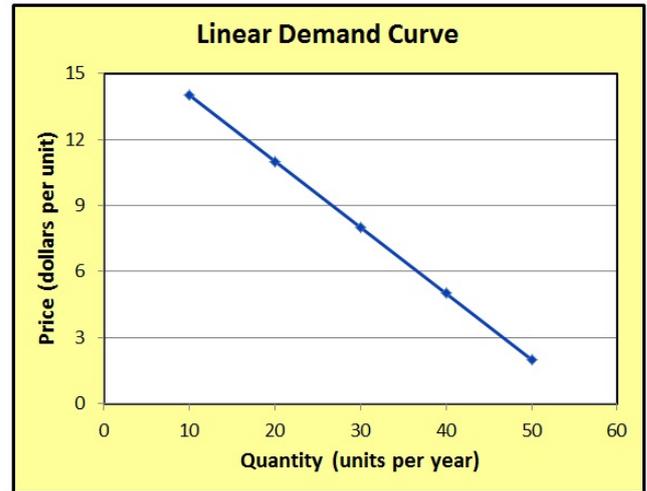


Figure 2. Linear Demand

The *price elasticity of demand* is calculated using the following formula:

$$\eta_{(Quantity, Price)} = \text{Price elasticity of demand} =$$

$$\frac{\text{Percentage change in quantity demanded}}{\text{Percentage change in price}}$$

Note that this formula implies that price elasticity of demand is a unit-free measure. Using a linear demand made it easy to calculate the elasticity at a given point. For example, to calculate the elasticity at point (Quantity, Price) = (Q, P) = (30, 8), one can use points (20, 11) and (40, 5) which are at equal distance from (30, 8).

$$\eta_{(30,8)} = \frac{\Delta Q}{Q_{average}} = \frac{(40-20)}{\frac{30}{2}} = \frac{20}{15} = \frac{4}{3} = 1.33$$

Since a decrease in price leads to an increase in quantity, the elasticity value is typically negative. In practice the positive value is used and it is said that the elasticity at point (30, 8) is equal to $\frac{8}{9}$. It shows that an increase of 1% in price induces a decrease of $\frac{8}{9}\%$ in quantity demanded.

Using similar calculations, it is possible to show that $\eta_{(20,11)} = 1\frac{5}{6}$ and $\eta_{(40,5)} = \frac{5}{12}$ (remember that these real values are negative), which shows that a linear demand curve induces decreasing positive values for elasticities as one moves along the x-axis (increasing quantities). An elasticity equal to 1 is said to be *unit elastic*. An elasticity greater than 1 is considered *elastic* and a value lower than 1 is said to be *inelastic*.

The previous discussion implies that *constant elasticity* η can exist when the demand curve is looking somewhat like Figure 1. It would imply that an increase of 1% in price reduces by $\eta\%$ the quantity demanded and this ought to be true at any point on the demand curve.

Techno-economic energy models must satisfy many end-use demands. An example of an end-use demand is the quantity of passengers-kilometers travelling by car. The demand

does not take into account which type of car will be selected to satisfy its needs (the optimal solution dictates this decision; many types of cars could be used to satisfy a particular demand). Instead of using a demand curve, the OSeMOSYS approach only use a single number to identify a demand. It is fixed and is not influenced by any factor, including its own price. It can be computed for a given time period, say for the year 2020. The demand is thus represented as in Figure 3. In such a case elasticity is null.

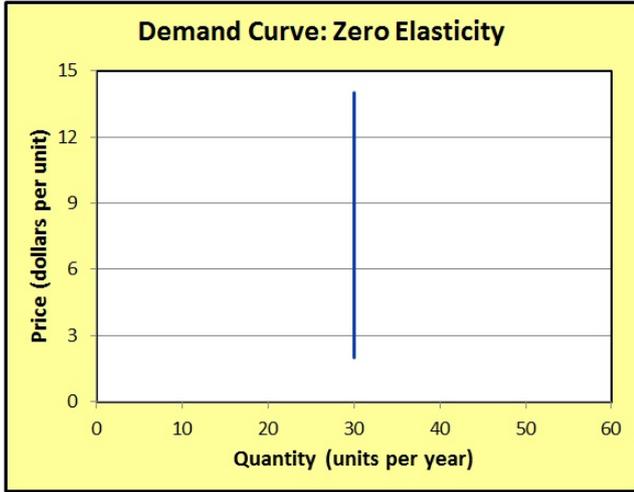


Figure 3. Zero Elasticity

3 Constant Elasticity

In the constant elasticity, the demand curve is similar to the one presented on Figure 1. The formula to compute η must consider the fact that the demand curve is not linear and that the previous formula is not valid anymore. One must then turn to differential calculus to find the general formula for elasticity which, for constant elasticity, does not depend on the quantity Q and the price P (keep in mind that η is a negative real number).

$$\eta_{(Q,P)} = \eta = \frac{\frac{dQ}{Q}}{\frac{dP}{P}} = \frac{dQ}{Q} \cdot \frac{P}{dP} = \frac{dQ}{dP} \cdot \frac{P}{Q}$$

It will now be shown that, under constant elasticity, $P(Q) = Q^{1/\eta} e^{k/\eta}$ where k is a constant.

$$\eta = \frac{dQ}{dP} \cdot \frac{P}{Q}$$

$$\Rightarrow \frac{\eta}{P} = \frac{1}{Q} \cdot \frac{dQ}{dP}$$

$$\Rightarrow \int \frac{\eta}{P} dP = \int \frac{1}{Q} \cdot \frac{dQ}{dP} dP$$

$$\Rightarrow \eta \ln P + k_1 = \ln Q + k_2 \quad (k_1 \in \mathbb{R}, k_2 \in \mathbb{R})$$

$$\Rightarrow \eta \ln P = \ln Q + k \quad (k \in \mathbb{R})$$

$$\Rightarrow \ln(P^\eta) = \ln Q + k \quad (k \in \mathbb{R})$$

$$\Rightarrow P^\eta = Q \cdot e^k \quad (k \in \mathbb{R})$$

$$\Rightarrow P(Q) = Q^{1/n} \cdot e^{k/n} \quad (k \in \mathbb{R})$$

The conclusion is that constant elasticity leads to an analytical expression of the demand curve. This expression is generic in the sense that the value of k is unknown. Finding a point (Q_0, P_0) on the demand curve would fix the value of the constant k as shown below.

$$P_0 = Q_0^{1/n} \cdot e^{k/n}$$

$$\Rightarrow e^{k/n} = \frac{P_0}{Q_0^{1/n}}$$

$$\Rightarrow \frac{k}{n} = \ln \left(\frac{P_0}{Q_0^{1/n}} \right)$$

$$\Rightarrow k = n \cdot \ln \left(\frac{P_0}{Q_0^{1/n}} \right) = \ln \left[\left(\frac{P_0}{Q_0^{1/n}} \right)^\eta \right] = \ln \left(\frac{P_0^\eta}{Q_0} \right)$$

This value of k can be used in the formula for $P(Q)$ given previously as follows:

$$P(Q) = Q^{1/n} \cdot e^{\frac{1}{n} \cdot \ln \left(\frac{P_0^\eta}{Q_0} \right)}$$

$$\Rightarrow P(Q) = Q^{1/n} \cdot e^{\ln \left[\left(\frac{P_0^\eta}{Q_0} \right)^{1/n} \right]} = Q^{1/n} \cdot e^{\ln \left(\frac{P_0}{Q_0^{1/\eta}} \right)}$$

$$\Rightarrow P(Q) = Q^{1/n} \cdot \frac{P_0}{Q_0^{1/\eta}}$$

It is thus possible to have an exact analytical expression of the demand curve under the assumptions of knowing a point on such a demand curve with constant elasticity. For example, if we suppose $Q_0 = 5$, $P_0 = 2$ and $\eta = -1/2$, the demand curve is given by $P(Q) = Q^{-2} \cdot \frac{2}{5^{-2}} = 50 Q^{-2}$. Using the given point (5, 2) it is easy to find other points on the demand curve. For example, $P(2) = 12\frac{1}{2}$, $P(3) = 4\frac{5}{9}$, $P(4) = 3\frac{1}{8}$ and $P(6) = 1\frac{7}{18}$ (see Figure 4).

Determining such an exact demand curve for a particular end-use demand of an energy system thus require (1) its η value and (2) a point on its demand curve. Some real-world price elasticities of demand are mentioned in [5] and economics specialists can propose such values for particular end-use demands included in techno-economic energy models. A point on the demand curve can be obtained by minimizing

the total cost of the energy system for a given value of a particular end-use demand and then find the marginal value of meeting such a demand. For example, a demand could be fixed to 5 units and the marginal value resulting from the optimization can be evaluated as 2. This initial (and usually sole) optimization of the energy system is the first step of the process of including elasticity within a techno-economic energy model such as OSeMOSYS. This initial step does not include any GHG limitation target and simply optimize the *reference* (or *base-case*) scenario of the energy model.



Figure 4. Exact Demand

4 Approximation of the Demand

It can be argued that constant elasticity is a very strong assumption which cannot be used to propose an exact demand curve based on a single initial point. It may be more acceptable to say that constant elasticity may be used in an interval *close* to the given point (in a reasonable *neighborhood* of the point). Such interval is particular to each individual end-use demand of the energy sector. Moreover, it is supposed in this paper that imposing GHG targets may either lead to (1) a stable demand value or (2) a decrease of the demand from its initial value.

The OSeMOSYS techno-economic model uses linear programming. One cannot represent the demand curve see in Figure 4 within OSeMOSYS and a linear approximation has to be implemented. A step-function is proposed to perform this task. Figure 5 shows a 3-step approximation on a given interval.

Point $(P_0, Q_0) = (5, 2)$ is the result of the initial optimization problem under no GHG constraint. Suppose that imposing such a constraint has the impact of changing the marginal value of satisfying the end-used demand represented by Figure 5 by increasing it to \$2.50. Instead of satisfying the full demand, the optimization of this new problem including this elasticity approximation will result in a decrease of the demand from 5 units to 4.5 units since the marginal value of reducing the demand is only \$2.00 (which is smaller than

\$2.50). The model thus have the possibility to react by modifying the end-use demand. If a more severe GHG limitation is imposed and induces a marginal value of meeting this demand equal to \$4.00, the optimization will further decrease the demand to a value of 3.5 units. Using this 3-step approximation, no GHG target could result in a demand lower than 2.5 units which has been set by the modeler as a minimal demand value reflecting a reality that some demand may always have to be met. Note that the number of steps (and thus the width of each step) in the approximation of the demand curve is also under the control of the modeler. Each step will be modeled as a fictive technology related to its particular end-use demand. It has an impact on the number of constraints and variables created in the optimization process. It is thus suggested to use this capacity wisely to avoid an undesired excess for these numbers.

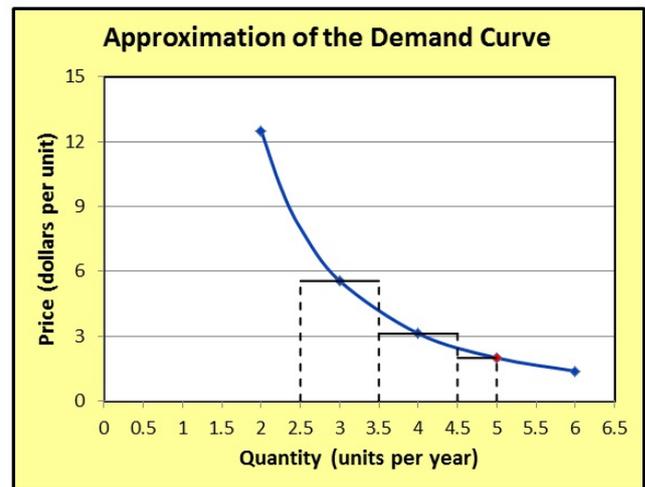


Figure 5. Approximation of the Demand

5 Introducing Elasticity within OSeMOSYS

This section presents how to introduce elasticity within OSeMOSYS. The *Extended UTOPIA* example described in [4] is used. In this problem, a passengers-kilometers end-use demand called TZ can be met using two distinct technologies: (1) gasoline cars (TZ1) and (2) electric cars (TZ2). For simplicity, the particular demand for the year 2005 will be used.

5.1 Finding (Q_0, P_0)

The first step is to solve the original *Extended UTOPIA* problem with no CO₂ constraint and no elasticity ($\eta = 0$). This is the *Reference Scenario*. The TZ end-use demand in 2005 is set exogenously to 3.483 Gpkm (giga passengers-kilometer). We thus have $Q_0 = 3.483$. The optimal objective value corresponds to the minimal value of the total discounted cost of the system and is equal

to \$38,397 (all monetary values are in millions of dollars). The marginal value associated to the constraint of satisfying demand TZ for 2005 is needed to find the value of P_0 . The name of this constraint in OSeMOSYS is `EBb4_EnergyBalanceEachYear4[UTOPIA, TZ, 2005]` and the value found is \$84.178.

This value of \$84.178 corresponds to the discounted value of satisfying the demand TZ in 2005. Using a 5% discount and a starting year of 1990 (as is the case in the Extended UTOPIA problem) implies that the value of P_0 in 2005 is $1.05^{15} \times 84.178 = 175.00$. Point $(Q_0, P_0) = (3.483, 175.00)$ will be used for the approximation of the demand curve.

5.2 Creating the step-function

The construction of a 3-step-function to approximate the demand curve involves the creation of three *fictive* technologies (TZA, TZB and TZC) that can satisfy the end-use demand TZ (which is usually met using *real* technologies TZ1 and TZ2). Using a fictive technology is equivalent to a reduction of the actual demand.

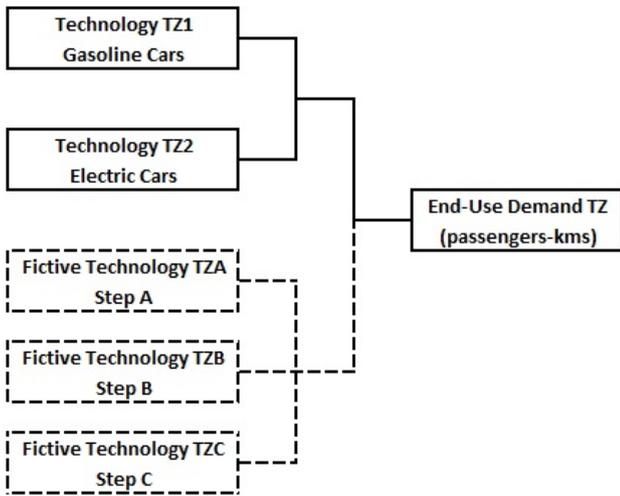


Figure 6. Technologies to Satisfy Demand TZ

A fictive technology is used to represent a particular step of the step-function used to approximate the demand. The upper bound and price must be given to represent such a technology. Suppose that the user accepts a possible total reduction of 25% of the real demand and that this percentage is broken into three steps with upper bounds of 5%, 10% and 10% respectively for technologies TZA, TZB and TZC (these technologies are named from right to left starting from point Q_0).

Table 1. Upper Bounds for Fictive Technologies

Technologies	Upper Bounds
TZA	$0.05 \times 3.483 = 0.17415$
TZB	$0.10 \times 3.483 = 0.34830$
TZC	$0.10 \times 3.483 = 0.34830$

The price related to TZA is set to P_0 . The middle point of the two other steps must be found to calculate the price of TZB and TZC.

Table 2. Middle points for TZB and TZC

Technologies	Middle Points
TZB	$Q_B = 3.13470$
TZC	$Q_C = 2.78640$

Prices for TZB and TZC are found using the price function defined in section 3 with quantities Q_B and Q_C presented in Table 2. Using a user-defined constant elasticity of $\eta = -1$, the values presented in Table 3 are used to create the 3-step approximation of the demand curve.

Table 3. Prices for Fictive Technologies

Technologies	Prices
TZA	175.00
TZB	194.44
TZC	218.75

Figure 7 shows the approximation of the end-use demand curve for TZ using values from Tables 1 and 3.

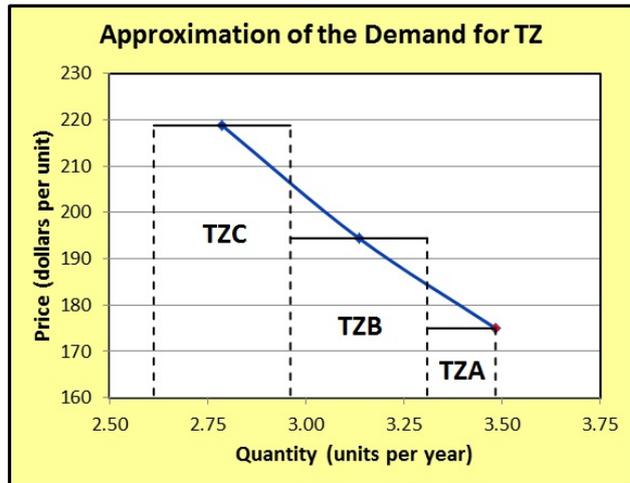


Figure 7. Approximation of End-Use Demand TZ

5.3 Changes to the Data File

Here is the detailed description of all the changes that must be applied to the *Extended UTOPIA* data file in order to model the approximation of the demand for TZ.

1. Set Technology: add TZA, TZB and TZC;
2. Parameter `OutActivityRatio`: add a value of 1 for TZA, TZB and TZC under the demand TZ;
3. Parameter `CapitalCost`: for 2005, add the respective prices for TZA, TZB and TZC (see Table 3); add default value 99999 for other years;

4. Parameter `OperationalLife`: add a value of 1 for TZA, TZB and TZC;
5. Parameter `TotalAnnualMaxCapacity`: for 2005, add the respective upper bounds for TZA, TZB and TZC (see Table 1); add default value of 0 for other years.

6 Analysing Results

Results for the *Reference Scenario* show that non-limited CO₂ emissions are equal to 220.70 units (million tonnes). The approximation of the end-use demand TZ is introduced for scenarios limiting CO₂ emissions to values ranging from 200 units to the lowest possible emissions level of 64.25 units. As more pressure is imposed on the system to respect restrictive CO₂ levels, optimal decisions may require to either invest in clean technologies such as TZ2 (electric cars) to replace existing technologies such as TZ1 (gasoline cars) or rather choose to lower the demand using fictive technologies TZA, TZB and TZC. The system can of course select a mix of such strategies leading to minimal objective values of the total system cost. Results for two different constant elasticities values (inelasticity $\eta = -0.40$ and high elasticity $\eta = -5$) are presented in Figure 8 and Figure 9.

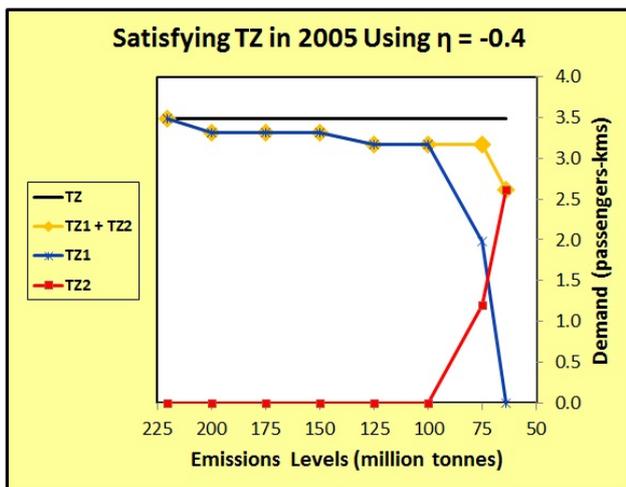


Figure 8. Satisfying TZ Using $\eta = -0.4$

Results show that the end-use demand TZ is fully satisfied only for the non-limited CO₂ scenario. For all other scenarios limiting CO₂ emissions it can be seen that elasticity plays its role as the optimal solutions suggest to decrease the demand. The imposed limitation on emissions may lead to a maximal demand reduction or to some non-maximal degree of demand reduction coupled with some mix of technological choices. For example, a limit of 75 million tonnes of CO₂ emissions for the $\eta = -0.4$ scenario induces less demand reduction than for the $\eta = -5$ scenario. In the former scenario gasoline cars (TZ1) are used to a higher level than in the latter scenario, resulting in more demand reduction when greater elasticity is used.

Figure 10 shows the impact of a higher elasticity value. Results suggest that steps TZA (5%), TZB (10%) and TZC

(10%) are more heavily used in the $\eta = -5$ scenario than in the $\eta = -0.4$ scenario. Figure 10 presents results confirming that TZA is fully used in all scenarios, that TZA and TZB are fully used when the emissions limit is set to 100 million tonnes in the more elastic scenario and that a target of 75 million tonnes in the more elastic scenario uses elasticity to its full capacity (25%).

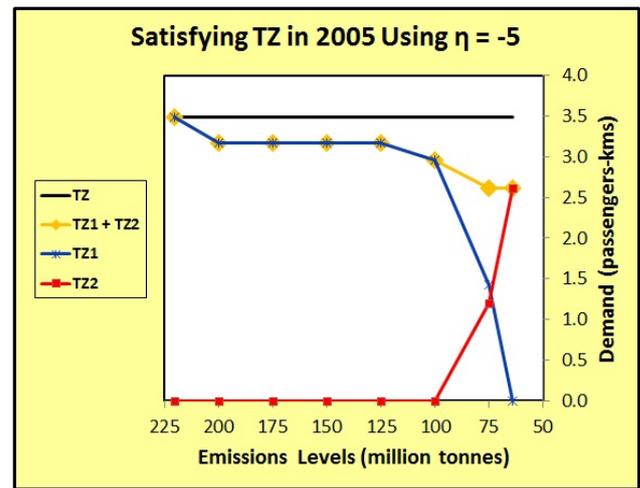


Figure 9. Satisfying TZ Using $\eta = -5$

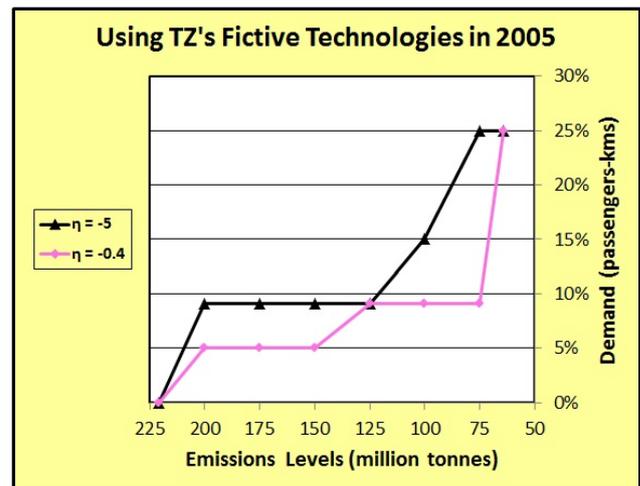


Figure 10. Using TZ's Fictive Technologies

7 Conclusion

Techno-economic energy models of the MARKAL-family typically use deterministic end-use demands. This is an issue raised by researchers and professionals since it does not allow the end-use demands to react to some severe greenhouse gases emissions constraints on the energy system under study. In real life such limitations on emissions levels may induce changes in society's behaviour which could lead to a reduction of some end-use demands. This paper presents

an overview on how to introduce a demand's own price elasticity within OSeMOSYS to allow the possibility to endogenously reduce end-use demands to satisfy some given GHG emissions objectives.

The *Extended UTOPIA* problem is used as an example. A step-by-step approach describes all the details on how to model a 3-step approximation of a demand curve within OSeMOSYS. Results show the impact of the approach as severe emissions limitations are imposed on a system. Using a similar approach would allow OSeMOSYS users to better analyze the impacts of severe GHG emissions targets on their own energy system, helping them identify which behaviour change for some end-use demands shall be encouraged.

Additional work could include the use of the approach over all the horizon (and not only for a particular year such as 2005 used in the example). It could also benefit from an automation of the approach by using some demand's parameters such as the number of steps used in the approximation and the width of each step (such parameters exist within TIMES).

It is argued that the paper responds to the goals of the OSeMOSYS modeling community by offering to modelers an easy-to-use and powerful tool to understand and better represent the reality of an energy system under severe emissions limitations. Users will rapidly gain knowledge on the issue and be able to use it within other tools. Moreover, future (and present) TIMES users may find in the paper all which is necessary to understand the approach before using it in their current work.

Acknowledgements

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Military College of Canada.

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