Fractal Scaling or Scale-invariant Radar: A Breakthrough into the Future

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Abstract Results of application of theory of fractal and chaos, scaling effects and fractional operators in the fundamental issues of the radio location and radio physic are presented in this paper. The key point is detection and processing of super weak signals against the background of non-Gaussian intensive noises. The main ideas and strategic directions in synthesis of fundamentally new topological radar detectors of low-contrast targets / objects have been considered. The author has been investigating these issues for exactly 35 years and has obtained results of the big scientific and practical worth. The reader is invited to look at the fundamental problems with the synergetic point of view of non-Markovian micro- and macro- systems. The results of big practical and scientific importance obtained by the author were published in four summary reports of the Presidium of Russian academy of science (2008, 2010, 2012, and 2013) and in the report for the Government of Russian Federation (2012).

Keywords Fractal, Scaling, Texture, Chaos, Radar, Low-contrast Target, Signals Detector, Fractal Radio-systems

1. Introduction

Intensive development of modern radar technology establishes new demands to the radiolocation theory [1,2]. Some of these demands do not touch the theory basis and reduce to the precision increase, improvement and development of new calculation methods. Other ones are fundamental and related to the basis of the radiolocation theory. The last demands are the most important both in the theory and in practice.

Radar detection of unobtrusive and small objects near the ground and sea surface and also in meteorological precipitations is an extremely hard problem [1]. One should take into account that the noise from the sea surface and

vegetation has nonstationary and multi-scale behavior especially at high incidence angles of the sensing wave.

The entire current radio engineering is based on the classical theory of an integer measure and an integer calculation. Thus an extensive area of mathematical analysis which name is the fractional calculation and which deals with derivatives and integrals of a random (real or complex) order as well as the fractal theory has been historically turned out “outboard” (!) [2-14]. At the moment the integer measures (integrals and derivatives with integer order), Gaussian statistics, Markov processes etc. are mainly and habitually used everywhere in the radio physics, radio electronics and processing of multidimensional signals. It is worth noting that the Markov processes theory has already reached its satiation and researches are conducted at the level of abrupt complication of synthesized algorithms. Radar systems should be considered with relation to open dynamical systems. Improvement of classical radar detectors of signals and its mathematical support basically reached its saturation and limit. It forces to look for fundamentally new ways of solving of problem of increasing of sensitivity or range of coverage for various radio systems.

In the same time I'd like to point out that it often occurs in science that the mathematical apparatus play a part of “Procrustes bed” for an idea. The complicated mathematical symbolism and its meanings may conceal an absolutely simple idea. In particular the author put forward one of such ideas for the first time in the world in the end of seventies of XX century. To be exact he suggested introducing fractals, scaling and fractional calculation into the wide practice of radio physics, radio engineering and radio location [2,6-24]. Now after long intellectual battles my idea has shown its advantages and has been positively perceived by the majority of the thoughtful scientific community. For the moment the list of the author's and pupils works counts more than 800 papers including 23 monographs on the given fundamental direction. Nowadays it is absolutely clear that the application of ideas of scale invariance - “scaling” along with the set theory, fractional

measure theory, general topology, measure geometrical theory and dynamical systems theory reveals big opportunities and new prospects in processing of multi-dimensional signals in related scientific and engineering fields. In other words a full description of processes of modern signal and fields processing is impossible basing on formulas of the classical mathematics [2,6-24].

In this paper presented alternative solutions of actual problems of modern radiolocation are based on the ideas and methods of new scientific fundamental direction “Fractal Radio Physics and Fractal Radio Electronics: Designing of Fractal Radio Systems”. This direction was initiated by Professor A. Potapov since 1980 at the IREE RAS (Kotel’nikov Institute of Radio Engineering and Electronics of the Russian Academy of Sciences, Moscow, Russia) and currently it is widely developed in his works and acknowledged in the Scientific World.

2. Theory of Fractional Measure and Nonintegral Dimension

The main property of fractals is a nonintegral value of its dimension. Development of the dimension theory began with works of Poincare, Lebesgue, Brauer, Urysohn and Menger. Sets which are negligibly small and indistinguishable in terms of Lebesgue measure in one meaning or another appear in various fields of mathematics. To distinguish such sets with pathologically complex topological structure it is necessary to use non-traditional characteristics of smallness, for example capacity, potential, Hausdorff measures and dimension and so on. Application of Hausdorff fractional dimension which is tightly related to conceptions of entropy, fractals and strange attractors in the dynamic systems theory turned out to be the most fruitful [2,3,5-24].

Conception of Hausdorff measure and dimension is one of those required conceptions which must be mastered organically before every researcher can become the fractal specialist and deterministic chaos specialist. This fractional dimension is determined by p - dimensional measure with an arbitrary real positive number p which was introduced by Hausdorff in 1919. In general the measure conception is related neither with metrics nor with topology. However the Hausdorff measure can be built in a random metric space basing on its metrics and the Hausdorff dimension itself is related with the topological dimension.

Conceptions introduced by Hausdorff are based on the Caratheodory theory (1914). Let us assume that (M, ρ) is a metric space, F is a family of subsets of set M and f is such a function on F that $0 \leq f(G) \leq \infty$ when $C \in F$ and $f(\emptyset) = 0$.

Let us build auxiliary measures m_f^ε and then the main measure Λ_f in the following way. When $E \subset M$ and $\varepsilon > 0$ the value of m_f^ε is defined as the infimum of set of numbers

$$m_f^\varepsilon = \inf \sum_i f(G_i) \quad (1)$$

over every possible countable ε -coverings of $\{G_i\}$, $G_i \in F$.

It results from inequation $m_f^{\varepsilon_1}(E) \geq m_f^{\varepsilon_2}(E)$ for $\varepsilon_2 > \varepsilon_1$ that the limit exists

$$\Lambda(E) = \lim_{\varepsilon \rightarrow 0+} m_f^\varepsilon(E) = \sup m_f^\varepsilon(E). \quad (2)$$

It is clear that m_f^ε and $\Lambda(E)$ are *exterior measures* on M . Let $\rho(a, B) > \varepsilon > 0$. Let us consider a random ε -covering of $\{G_i\}$ set $A \cup B$ which consists of a certain number of sets. Then families $\{A \cap G_i\}$ and $\{B \cap G_i\}$ do not intersect and cover sets A and B respectively and so

$$m_f^\varepsilon(A \cup B) \geq m_f^\varepsilon(A) + m_f^\varepsilon(B) \quad (3)$$

or

$$\Lambda_f(A \cup B) = \Lambda_f(A) + \Lambda_f(B). \quad (4)$$

Class of Λ_f - measurable sets of space M forms a σ - ring which the exterior measure Λ_f is regular on. They also call measure Λ_f as the result of application of Caratheodory theory to function f and exterior measure m_f^ε as the approximating measure with order ε . Measure Λ_f represents properties of function f and family F quite fine although it is not an extension of f usually.

We point out two simple statements which describe behaviour of approximating measures at a decreasing sequence $C_1 \supset C_2 \supset \dots$ of compact subsets of space M . If elements of family F are open subsets of M then

$$\lim_{i \rightarrow \infty} m_f^\varepsilon(G_i) = m_f^\varepsilon\left(\bigcap_{i=1}^{\infty} C_i\right). \quad (5)$$

If $0 < \varepsilon_0 < \varepsilon$ and $f(S) = \inf\{f(T)\} : T \in F, S \subset \text{Int} T, d(T) \leq \varepsilon$ for every such $S \in F$ that

$$d(S) \leq \varepsilon_0 \lim_{i \rightarrow \infty} m_f^\varepsilon(G_i) \leq m_f^{\varepsilon_0}\left(\bigcap_{i=1}^{\infty} C_i\right), \quad (6)$$

where d is the diameter of the sets, Int is the set of all the internal points of set T .

Let us assume that X is a limited compact metric space, F is the family of all the nonempty compact sets from X , function $f: F \rightarrow [0, +\infty]$ continuous in regard to Hausdorff metric and $f(C) > 0$ for all such $C \in F$ that $d(C) > 0$. If $A_1 \subset A_2 \subset A_3 \subset \dots$ form an increasing sequence of subsets of space X then

$$\lim_{k \rightarrow \infty} m_f^\varepsilon(A_k) = m_f^\varepsilon\left(\bigcup_{k=1}^{\infty} A_k\right) \quad (7)$$

Let us define h - Hausdorff measure. Let $h(r)$ be a continuous monotonic increasing function of r ($r \geq 0$)

which $h(0) = 0$ for. We indicate the class of such functions as H_0 . By applying the Caratheodory construction to function $f(E) = h[d(E)]$ where $E \neq \emptyset$ and $f(\emptyset) = 0$ (here $d(E)$ is the diameter of set E) we get Λ_h - the Caratheodory measure which is called as the Hausdorff h -measure. If at that $h(r) = \gamma(\alpha)r^\alpha$ where α is a fixed positive number which is not necessarily an integer and $\gamma(\alpha)$ is a positive constant which depends only on α then the Hausdorff h -measure is called as the α -dimensional measure or the Hausdorff α -measure H_α which is a Borel regular measure.

One can imagine the construction of Hausdorff h -measure in the following way. Let us cover α with a random sequence of circles C_v with radius $r_v \leq \varepsilon$ ($\varepsilon > 0; v = 1, 2, \dots$) and mark the infimum of respective sums $\sum_{v=1}^{\infty} h(r_v)$ as

$m_h^\varepsilon(\alpha, h) \geq 0$. This number increases with decrease of ε . By definition

$$\Lambda_h(E) = \lim_{\varepsilon \rightarrow 0} m_h^\varepsilon(\alpha, h) \quad (8)$$

so

$$0 \leq \Lambda_h(E) \leq +\infty \quad (9)$$

Limit (8) is the Hausdorff *exterior* h -measure which is a Borel regular measure at σ -ring of Λ_h -measureable sets of space M . By choosing various functions as $h(r)$ we get: linear measure ($h(r) = 2\pi r$), planar measure ($h(r) = \pi r^2$) and logarithmic measure ($h(r) = 1/\ln r$).

It follows from condition $E_1 \subset E_2$ that $\Lambda_h(E_1) \leq \Lambda_h(E_2)$ that is the Hausdorff h -measure is a monotonically increasing set function. With using an h -measure the dimension of set is defined in the following way. If $0 < \Lambda_h(A) < \infty$ then $\langle h \rangle$ is called as the metric dimension (the Hausdorff dimension) of set A . If $h(r) = cr^\alpha$ and $0 = \Lambda_h(A) < \infty$ then the dimension of set A is indicated as $\langle \alpha \rangle$, here c is a constant. Sets with a certain dimension have an h -measure equal to 0 for each exterior dimension and an h -measure equal to ∞ for each lowest dimension.

Next generalization of the dimension conception is a *Hausdorff-Besicovitch dimension*, which is introduced using nonnegative numbers $\alpha_0 = \alpha_0(E)$ in the form of equation

$$\alpha_0(E) = \sup\{\alpha : H_\alpha(E) \neq 0\} = \inf\{\alpha : H_\alpha(E) = 0\} \quad (10)$$

for set E . The Hausdorff-Besicovitch dimension of a set is defined by behaviour of $H_\alpha(E)$ not as function of E but as function of α .

Correctness of definition (10) confirms the following property of H_α -measure. If $H_\alpha(E) < \infty$ then $H_\alpha(E) = 0$ for every $\alpha_2 > \alpha_1$. If measure $H_{\alpha_2}(E)$ is a non-zero then

$H_{\alpha_1}(E) = \infty$ for every positive $\alpha_1 < \alpha_2$. Hence for set $E \subset M$ or $H_\alpha(E) = 0$ for every $\alpha > 0$, then $\alpha_0(E) = 0$ by definition or there is the point of "jumping" α_0 such that $H_\alpha(E) = \infty$ for $\alpha < \alpha_0$ and $H_\alpha(E) = 0$ for $\alpha > \alpha_0$. And given number α_0 is the Hausdorff-Besicovitch dimension.

If they use balls of the same size for covering during determination of the Hausdorff H_α -measure then such a measure is called as entropic. Then dimension (10) is called as entropic or a Kolmogorov dimension. For sets of positive k -dimensional Lebesgue measure both dimensions coincide and equal K . The Hausdorff-Besicovitch dimension describes the exterior property of a set. Therefore it is appropriate to introduce a conception of the Hausdorff-Besicovitch dimension at a point which would describe its internal structure.

In this case number

$$\alpha_E(x_0) = \lim_{n \rightarrow \infty} \alpha_0(E \cap O_n(x_0)) \quad (11)$$

is called a local Hausdorff-Besicovitch dimension of set E at point x_0 . Here $\{O_n(x_0)\}$ is a random sequence of contracting neighborhoods of point $x_0 \in M$.

Each limited closed set E of m -dimensional Euclidean space contains point $x_0 \in E$ such that

$$\alpha_E(x_0) = \alpha_0(E). \quad (12)$$

Function $\alpha_E(x)$ is called as a function of the local Hausdorff-Besicovitch dimension if

$$0 \leq \alpha_E(x) \leq \alpha_0(E) \text{ for every } x \in M,$$

$$\alpha_E(x) = 0, \text{ if set } E \text{ is closed and } x \notin E, \quad (13)$$

$$\alpha_E(x) = 0 \text{ for all the isolated points of set } E.$$

The Hausdorff-Besicovitch dimension is a metric conception but there is its fundamental association with the topological dimension $\dim E$ which was determined L.S. Pontryagin, L.G. Shnirelman. They introduced a conception of the metric order in 1932: the infimum of the Hausdorff-Besicovitch dimension for every metric of compact E equals its topological dimension $\dim E \leq \alpha(E)$. One of widely used methods for evaluating the sets Hausdorff dimension which is known as the principle of masses allocation was proposed by Frostman in 1935.

Sets which have the fractional Hausdorff-Besicovitch dimension are called fractals sets or fractals. More strictly, set E is fractal (a fractal) in a general sense (in the B. Mandelbrot sense) if its topological dimension does not coincide with the Hausdorff-Besicovitch dimension, to be exact $\alpha_0(E) > \dim E$. For example the set E of all the surd points $[0; 1]$ is fractal in a general sense since $\alpha_0(E) = 1$, $\dim E = 0$. Set E is called fractal (a fractal) in the narrow sense if $\alpha_0(E)$ is not an integer. A set which is fractal in the narrow sense is also fractal in a general sense.

As it was shown by A.S. Besicovitch for the first time in 1929 there were deep discrepancies between Lebesgue sets and fractals. First of all, these features concern densities. Geometric properties of fractal set E are determined by behaviour of function

$$D(x, \varepsilon) = \frac{H_\alpha(E \cap O(x, \varepsilon))}{\varepsilon^\alpha} \quad (14)$$

for small ε , where x is a random point of set E . The higher α which is the density of set E at point x is

$$\overline{D}_\alpha(E, x) = \lim_{\varepsilon \rightarrow 0} D(x, \varepsilon), \quad (15)$$

and the lower α which is the density of set E at point x is

$$\underline{D}_\alpha(E, x) = \lim_{\varepsilon \rightarrow 0} D(x, \varepsilon). \quad (16)$$

When $\overline{D}_\alpha(E, x) = \underline{D}_\alpha(E, x)$ then their generalized value is called as an α -density of set E at point x and it is identified as $D_\alpha(E, x)$. If $\varepsilon \rightarrow 0+$ then $\overline{D}_\alpha(E, x)$ and $\underline{D}_\alpha(E, x)$ are called as right-side, if $\varepsilon \rightarrow 0-$ then they are called as left-side, if $\varepsilon \rightarrow 0$ they are called two-side the upper and lower α -density respectively.

3. To the “Fractal” Conception in Radio Location

In general terms a radar image (RI) can always be presented as a set of elements X_k , whose values are proportional to the scattering cross-section (SCS) of a k -th element of resolution of the radar [6-10]. In Figure 1, *a* the RI of the terrain which was obtained at wavelength $\lambda = 8.6$ mm from a helicopter is shown. In Figure 1, *b* the RI of the same terrain region which was obtained by a radar at wavelength $\lambda \approx 30$ cm is shown. Both images are two-dimensional with gray level proportional to SCS.

Let us suppose that for every RI a surface (Figure 1, *c*) with height h which is proportional to the gray level is built. Let us suppose that we need to measure the square of the resulting surface. On RI which corresponds $\lambda \approx 30$ cm the square will be less than for RI on $\lambda \approx 8.6$ mm since the smaller wavelength the more terrain details can be recognized.

A probing electromagnetic wave is some kind of a “ruler” in this case. At that an increasingly finer structure of time-spatial signals or wave fields begins to have an effect.

If we have a RI which was obtained at even shorter waves then its square will be bigger and so on. By decreasing the wavelength λ we will get increasing values of the squares. Then the question arises: and what is the square of the surface which the RI was obtained from in reality? If the surface is covered with simple objects, for example a rectangular eminence (Figure 1, *d*), and sizes of this eminence are much higher than the wave length then the squares of objects on the RI will be approximately equal for short and long waves. Then we would answer the mentioned

question by calculating the number of resolution elements covering the object. Surface area S in this case would be equal to:

$$S \equiv S(\lambda) = N(\lambda)\delta(\lambda) \quad (17)$$

where $\delta(\lambda)$ - the square of a resolution element of the radar; $N(\lambda)$ - number of resolution elements required to cover the object, λ - the wavelength of the radar, as it was already noted for a simple object (Figure 1, *d*) value $S(\lambda) = \text{const}$.

For the RI on Figure 1, *a* and 1, *b* one can build dependence $S(\lambda) = f(\lambda)$ and assuming that $\delta(\lambda) = K(\lambda)$, where K is a known function then one can build dependence $S(\lambda) = f(\delta)$. It happens that the measured square S is described well by formula

$$S(\lambda) = k\lambda^{-D} \quad (18)$$

Then just taking the logarithm we can calculate parameter D . Dependence which determines a fractal signature $D(t, f, \bar{r})$ of a RI by itself is shown on Figure 1, *e*. This dependence describes a space fractal cepstrum of an image (this conception was introduced by the author in nineties of XX century).

Figure 1. Examples explaining the matter of fractal processing (*a - d*) and a fractal space signature (*e*)

The fractional parameter D is called *the Hausdorff-Besicovitch dimension* or *the fractal dimension* [3,5,7,8]. For RIs of objects with simple geometric form (rectangles, circles, smooth curves) this dimension coincides with the topological one that is it equals 2 for two-dimensional RIs and it is determined by the slope of straight lines (18) in binary logarithmic coordinates. However the value of D for majority of images of *real coverings* and *meteorological formations* turns out to be higher than the topological dimension $D_0 = 2$ that emphasizes its complexity and random nature.

4. Textural and Fractal Measures in Radio Physics and Radar

A radar along with observation objects and radio waves propagation medium forms a space-time radar-location

probing channel. During the process of radio location the useful signal from target is a part of the general wave field which is created by all reflecting elements of observed fragments of the target surrounding background, that is why in practice signals from these elements form the interfering component. It is worthwhile to use the texture conceptions to create radio systems for the landscape real inhomogeneous images automatic detecting [6-8]. A texture describes spatial properties of earth covering images regions with locally homogenous statistical characteristics. Target detecting and identification occurs in the case when the target shades the background region at those changing integral parameters of the texture.

Many natural objects such as a soil, flora, clouds and so on reveal fractal properties in certain scales [5-10]. Today analysis of natural textures is undergone by significant changes due to use of metrics taken from the fractal geometry. After a texture they introduced the conception of fractals that is signs based on the fractional measure theory for fundamentally different approach of solving modern radio location problems. The fractal dimension D or its signature in different regions of the surface image is a measure of texture i.e. properties of spatial correlation of radio waves scattering from the corresponding surface regions. At already far first steps the author initiated a detailed research of the texture conception during the process of radio location of the earth coverings and objects against its background. Further on a particular attention was paid to development of textural methods of objects detecting against the earth coverings background with low ratios of signal/background (see for example [6-10,14,20,24] and references).

5. Textural Measures and Textural Signatures

Regions of background reflections which are united in a general texture conception are always presented around a detectable target. It allows proposing new approaches to detecting extensive low-contrast targets against the background of earth coverings in obtained radar images (RI) or multidimensional signals. Analysis of experimentally obtained extensive data bases in aggregate with visual research of degree of complexity of profiles of isolines of scattered radiation which was fixed on optical and radio images brought the author to ideas of synergetic developments of ensembles of fractal signs based on synthesis of scaling invariants with fractional measure properties in eighties of XX century [6-8].

Unlike tone and colour which relate to image separate fragments a texture relates to more than one fragment. We think that the texture is a matrix or a fragment of space properties of images regions with homogenous statistical characteristics. Textural signs are based on statistical characteristics of levels of intensity of image elements and relate to probabilistic signs whose random values are

distributed over all classes of natural objects. A decision on texture belonging to one or another class can be made only basing on specific values of signs of the given texture. In this case it is usual to say about a texture signature. Classic radar signatures include time, spectrum and polarized features of the reflected signal. In our view the texture signature is a distribution of general totality of dimensions for the given texture in scenes of the same kind as the given one.

When it is possible to decompose a texture two main factors are revealed. The first one correlates a texture with non-derivative elements which form the entire image and the second one serves for describing a spatial dependence between them. Tone non-derivative elements by itself represent image fields which are characterized by certain values of brightness proportional to the intensity of the reflected signal which in turn depends on values of the normalized effective cross-section σ_* of the earth surface. Since a conception of normalized effective cross-section is meaningful only for a spatially homogenous object then consequently the texture of an image of the real earth surface is determined by space changes of σ_* .

Everything pointed above allows setting mutual relationships between conceptions of normalized effective cross-sections of underlying surface and its texture. When a small part of the image is characterized by a minor change of typical non-derivative elements then the dominant property of this part is the value of the normalized effective cross-section. At a visible change of the brightness of these elements the dominant property is put in the texture. In other words when decreasing the number of distinguishable typical non-derivative elements in an image the part of energy signs (in particular σ_*) increases. In fact for one resolution element the energy signs are the only signs. If the number of distinguishable typical non-derivative elements increases then textural signs begin dominating.

It turned out that use of textural signs is extremely useful during detecting of low-contrast targets on images of any nature. Application of optical and radar images of the earth surface allows supplementing conventional signs with new quite significant ones which allow decreasing the signatures overlapping. The space organization of a texture can be structural, functional and probabilistic [6,25]. Texture signs describe representative properties general for the given class of textures.

During the process of the statistical analysis of textures they use statistics of the first or second order. When using statistics of the second order the textural signs are directly extracted using matrixes of distribution of probability of space dependence of brightness gradation P which is also called as a matrix of gradients distribution. This method was proposed in [25]. It was experimentally shown in [6,25] that signs based on parameters of correlation functions do not estimate an image texture so good as the signs determined over the gradient matrix P do.

Let us briefly consider the classical approach to obtaining textural signs [25]. Also let us assume that the image under consideration is rectangular and has N_x resolution elements

horizontally and N_y elements vertically. At that $G = \{1, 2, \dots, N\}$ is a set of N quantized brightness values. Then image I is described by a function of brightness values from set G that is $I: L_x \times L_y \rightarrow G$, где $L_x = \{1, 2, \dots, N_x\}$ and $L_y = \{1, 2, \dots, N_y\}$ are horizontal and vertical space zones respectively. The collection of N_x and N_y is a collection of resolution elements in a scan pattern. Matrix of gradients distribution $\mathbf{P}$ contains relative frequencies p_{ij} of presence of image neighbor elements which are placed at distance d from each other with brightness $i, j \in G$. Usually they distinguish horizontal ($\alpha = 0^\circ$), vertical ($\alpha = 90^\circ$) and transversally diagonal ($\alpha = 45^\circ$ and $\alpha = 135^\circ$) elements pairs.

Let us formulate conceptions of adjacent or neighbor elements [25]. Consider Figure 2 and the central pixel on it which painted as a dark small circle with eight neighbor pixels around it. Resolution elements 1 and 5 are the nearest neighbor elements and the angle between them equals zero. Resolution elements 2 and 6 are the nearest neighbor elements which angle $\alpha = 135^\circ$. Consequently elements 3 and 7 are the nearest neighbor elements which angle 90° and elements 4 and 8 are the nearest neighbor which angle 45° with regard to the central pixel.

Figure 2. Diagram of formation of gradients distribution matrix $\mathbf{P}$

It should be noted that this information is purely spatial and does not relate to brightness levels. Then we assume that the information about textural signs is properly determined by matrix $\mathbf{P}$ of relative frequencies which two neighbor elements separated with distance d appear on the image with. At that one element has brightness i and other elements has brightness j .

In case of need the respective normalization of frequencies for matrixes of gradients distribution can be easily done. For $d=1$, $\alpha=0$ we have $2N_y(N_x-1)$ pairs of adjoining horizontally to each other resolutions elements. For $d=1$, $\alpha=45^\circ$ we get only $2(N_y-1)(N_x-1)$ pairs of adjoining diagonally to each other resolutions elements. After getting M pairs of adjoining to each other resolution elements matrix of gradients distribution $\mathbf{P}$ is normalized by dividing every element by M .

Number of arithmetic operations which are needed for processing images using this method is directly proportional to $N_x N_y$. Frequently used linear integral Fourier and Adamar transforms require $N_x N_y \log(N_x N_y)$ operations. Besides saving of time during processing big data arrays we need to keep just two strings of data about the image in the operational computer memory during calculation matrixes $\mathbf{P}$.

The first calculation of the full ensemble of 28 textural signs and a detailed synchronous analysis of textural signatures for real (optical and radar in the range of millimeter waves (MW) at wave 8.6 mm) and synthesized textures as well was performed in IREE RAS in 1985 and fully presented in [6]. The full-sized experiments were carried out in co-operation with Central Design Bureau "Almaz". At that the task of calculation of textural signs taking into account the signatures drift at the season change was formulated and solved. We also note that in [25] questions of informativity of all 28 textural signs were not considered and there is no estimation of windows size impact to accuracy of determination of textural signs. Choice of window sizes is caused by the fact that a texture is determined by the neighborhood of the image point.

It turned out that for windows with size 3×3 or 5×5 pixels statistical textural measures act more as detectors of brightness drops than as texture meters though at that the calculation time is reduced [6]. Too big windows sizes may distort the results due to impact of structures margins and images edges. However the big window allows reaching a high statistical confidence. Windows 20×20 pixels are the most effective for textural processing of aerospace photos of farming lands, pastures, woodlands and other similar objects. When changing the window sizes from 80×80 to 20×20 pixels the numeric values of textural signs changed by 5...10 %. Further change of windows size resulted in considerable distortion of textural signs.

Compactness of areas of textural signs existence for RI textures gives us a possibility to guess that classification of earth coverings and targets detection sometimes is carried out more precisely using RI. However, interconnecting of optical and radio engineering systems mutually complements their main advantages and increases general informativity. The scale invariance and the rotation invariance is reached by selecting a particular step of discretization while digitization of texture (usually it is about an autocorrelation interval) and operation of averaging signs values on four scanning directions during computer processing.

Earlier the author proposed for the first time and implemented with his colleagues the following nontraditional effective methods of signals detection at small ratios signal - background q_0^2 : the dispersion method on the basis of f -statistics [6], method of detection using the linearly simulated standards [6] and the method of direct use of ensemble of textural signs or textural signatures [6]. The most complete description of performance potential of textural methods of processing of optical and radar images was presented in [6, 10] where for the first time the prospects of using textural signatures when detecting of weak radar

signals while the ratio signal/background q_0^2 is about unity or less was proved.

As a result of theoretical and experimental researches it was also shown that determination of textural signs reduces the effect of passive interferences from the earth surface and improves extraction and detection of weak signals.

Moreover the important advantage of textural methods of processing is a capability of neutralization of speckles on coherent images of the earth surface which were obtained by synthetic-aperture radar.

6. Methods of Determination of Fractal Dimension D and Fractal Signatures

When using the fractal approach it is natural to focus attention on description and also processing of radio physical signals and fields exceptionally in the fractional measure space with application of hypothesis of the physical scaling and distributions with heavy tails or stable distributions. Fractal and scaling methods of processing of signals, wave fields and images are in the wide sense based on that part of information which was irretrievably lost when using the classical processing methods. In other words the classical methods of signals processing basically select only that information component which is related to the integer-valued measure.

Fractal methods can function at all signal levels: amplitude, frequency, phase and polarized. Nothing of the kind exists in the world literature before the author's researches and works.

The absolute worth of Hausdorff-Besicovitch dimension is the possibility of its experimental determining [6-8]. Some set can be measured with d -dimensional (d is an integer) samples with side l_1 . Then number of samples N_1 covering the set will be: $N_1 = A/l_1^d$. Value d must be based on preliminary information about the set's dimension. Theoretically, if d is less than the topological dimension then $N_1 \rightarrow \infty$, and if $d > R^n$ where R^n is the Euclidean space then $N_1 \rightarrow 0$. The sample with size l_2 will give estimation $N_2 = A/l_2^d$, then the similarity dimension will be:

$$D = -\log_{l_2/l_1} N_2 / N_1. \quad (19)$$

Let us define the Hausdorff dimension in the following way. Let's consider some set of points N_0 in a d -dimensional space. If there are $N(\varepsilon)$ - dimensional sample bodies (cube, sphere) with typical size ε needed to cover that set, at that

$$N(\varepsilon) \approx 1/\varepsilon^D, \text{ when } \varepsilon \rightarrow 0 \quad (20)$$

is determined by the similarity law.

The practical implementation of the method described above faces the difficulties related to the big volume of calculations. It is due to the fact that one must measure not just the ratio but the upper bound of that ratio to calculate the Hausdorff-Besicovitch dimension. Indeed, by choosing a

finite scale which is larger than two discretizes of the temporal series or one image element we make it possible to "miss" some peculiarities of the fractal.

Building of the fractal signature $D(t, f, \vec{r})$ [6-8,26] or dependence of estimates of kind (19) and (20) on the observation scale often helps to solve this problem Figure 1,e. Also the fractal signature describes the spatial fractal cepstrum of the image. In V.A. Kotelnikov IREE RAS besides the classical correlation dimension we developed various original methods of measuring the fractal dimension including methods: dispersing, singularities accounting, on functionals, triad, basing on the Hausdorff metric, samplings subtraction, basing on the operation "Exclusive OR" and so on [7,8,10,11]. During the process of adjustment and algorithms mathematical modeling our own data were used: air photography (AP) and radar images (RI) at long millimeter waves [6]. Enduring season measurements of scattering characteristics of the earth coverings were already naturally conducted at wavelength 8.6 mm by the author from board of a flying laboratory located in helicopter in the eighties of XX.

A significant advantage of dispersing dimension is its implementation simplicity, operation speed and calculations efficiency. In 1998 we proposed to calculate the fractal dimension using the locally dispersing method (see for example [2,7-11,15,17,22,26-28]). Parameters of the algorithms which measure fractal signatures D affect measurements errors strongly enough. In the developed algorithms they use two typical windows: a scale one and a measuring one. The unbiased measurements can be carried out when using the scale windows which exceed sizes of the measuring window. One selects the necessary measurements scale using the scale window. This window defines the minimum and maximum values of scales which the scaling is observed in. That is why the scale window serves for selection of the object to be recognized and its following description in the framework of fractal theory. An image brightness local variance or image intensity is determined by the measuring window using common statistical methods. The locally dispersing method of measurements of the fractal dimension D is based on measuring a variance of the image fragments intensity/brightness at two spatial scales:

$$D \approx \frac{\ln \sigma_2^2 - \ln \sigma_1^2}{\ln \delta_2 - \ln \delta_1}. \quad (21)$$

In formula (21) σ_1, σ_2 are root-mean-squares at the first δ_1 and second δ_2 scales of image fragment, respectively. Accuracy characteristics of the locally dispersing method were investigated in [15,17,26-28]. Determination of one-dimensional fractal signatures over the area of images under investigation in different directions gives the new technique of measuring the *anisotropy* of surface images. It should be noted that the proposed locally dispersing method of measuring the fractal signatures allows direct obtaining of empirical distributions of fractal dimensions D .

It is proved in [15,26,28] that in the Gaussian case the dispersing dimension of a random sequence converges to the

Hausdorff dimension of a corresponding stochastic process. The essential problem is that any numerical method includes a discretization (or a discrete approximation) of the process or object under analysis and the discretization destroys fractal features. Development of a special theory based on the methods of fractal interpolation and approximation is needed to fix this contradiction. Various topological and dimensional effects during the process of fractal and scaling detecting and processing of multidimensional signals were studied by the author in [2,7-11,14-24,26-28].

7. Fractal Processing of Signals and Images against the Background of High-intensity Interferences and Noises

The author was the first who shows that the fractal processing excellently does for solving modern of processing the low-contrast images and detecting superweak signals in high-intensity noise when the modern radars do not practically function [2,6-11,14,16]. When using the fractal approach, as it was pointed out above, it is natural to focus attention on description and also on processing of radio physical signals (fields) exceptionally in the fractional measure space with use the hypothesis of the scaling and universal distributions with “heavy tails” or stable distributions [7,8,29].

The author's developed fractal classification was personally approved by B. Mandelbrot [9,10] in USA in 2005. It is presented on Figure 3 where the fractal properties are described on the assumption that D_0 is the topological dimension of the space of embeddings.

The textural and fractal digital methods under author's and his pupils development (Figure 4) allow to partially overcome a prior uncertainty in radar problems using the geometry or the sampling topology (one- or multidimensional) [6,16]. At that topological peculiarities of the sampling get very important as opposed to the average realizations which have different behavior.

It turns out that the concepts of fractal signatures and fractal spectra are very helpful for measurements. For example, these concepts are effectively applied to solve problems of detection of low-contrast targets and weak signals in the presence of intense non-Gaussian (!) interference. The methods of fractal processing should take into account the scaling effect of real radio signals and electromagnetic fields. The introduction of a fractional measure and scaling invariants necessitates the predominant use of power-series probabilistic distributions. These distributions result from feedback that amplifies events. Note that, for distributions with heavy tails, sample means are unstable and carry little information because the law of large numbers cannot be applied in this situation.

Figure 3. The author's classification of fractal sets and signatures

Figure 4. Textural and fractal methods of processing of low-contrast images and superweak signals in high-intensity noise

Figure 5 shows the general view of distributions with fractal dimension D . At fractal processing of realizations of signals in noise it is shown, that at the relations a $\text{SNR } q_0^2 = +10 \text{ dB}$ we precisely measure statistics of a signal. With reduction of value aside negative values (for example, $q_0^2 = -3 \text{ dB}$) there is a displacement of a maximum final fractal distributions aside values fractal dimensions of noise or a

handicap. Thus always in a vicinity of value fractal dimensions of a useful component there is “a heavy tail” fractal distribution, reaching stable size, about 20 %. The given tendency is kept and at much smaller values, equal SNR -10 dB and -20 dB, as shown in Figure 5.

Figure 5. Empirical fractal distributions with heavy tails for images observed in the presence of an intense Gaussian noise: (1 and 3) scene A, (2 and 4) scene B, (1 and 2) $q_0^2 = -10$ dB, and (3 and 4) $q_0^2 = -20$ dB

Thus, the algorithms of fractal pattern recognition based on the paradigm «topology of targets is their fractal dimension» [7,8]. The methodological basis of the fractal pattern recognition algorithms is the rejection of topological constants and a description of the types of targets using features of fractal dimensions D or fractal target signature.

High sensitivity of estimation of functionals of non-integral dimension to the presence of a continuous contour in images suggests a large potential of fractal filtering of the contours of objects in strong interference (Figure 6). The observation was made using ground-based telescope, the distance between it and objects was about 800 km. These data are presented in the book [11]. None of the modern methods of digital processing can provide comparable objects resolution!

Figures 7-9 show selected results of fractal nonparametric filtering of low-contrast objects. Aircraft images were masked by an additive Gaussian noise. In this case, the SNR ratio was -3 dB. It is seen in the figures that all desired

information is hidden q_0^2 in the noise. The optimum mode of filtering of necessary contours or objects is chosen by the operator using the spatial distribution of fractal dimensions D of a scene. This distribution is determined automatically and is shown in the right panel of the computer display [8-11,13,28].

This concept can be widely applied to solve modern problems of radar, correlation-extremely navigation, artificial intelligence, and dynamic systems. The algorithms developed by the authors for calculating fractal signatures are efficient over an extremely wide range of physical sizes of characteristic image details and provide detection estimation for scaling effects, including even those masked by noise.

Figure 6. The initial image of space complex at the time of joining «Shuttle» – «Peace» (a) and the results of (b – d) the fractal processing (targets clustering) for various values of the threshold D of topological fractal nonparametric detector

Figure 7. Source image

Figure 8. Source image and noise SNR ratio was -3 dB

Figure 9. Results of fractal filtration image on Figure 7

Many other examples of fractal scaling processing of low-contrast images and real weak signals from the practice of radar, telecommunications, astronomy, materials science, medicine and many other disciplines are given in [2,7-11,28].

8. Designed Breakthrough Technologies and Fractal Radio Systems

A critical distinction between the author's proposed fractal and scaling methods and classical ones is due to fundamentally different (fractional) approach to the main components of a physical signal. It allowed us to come to the new level of informational structure of the real non-Markov signals and fields. Thus this is *the fundamentally new* radio engineering.

For 35 years of scientific researches the global fractal and scaling method designed by author has justified itself in many applications. This is a challenge to time in a way. I

labeled all of this briefly and expressively – “*The Fractal Paradigm*” [18,22,23,30-37].

The fractal geometry is a huge and of genius merit of mathematician B. Mandelbrot. But its radio physical/radio engineering implementation is a merit of the Russian (now it is international) scientific school of fractal methods and fractional operators under the supervision of Professor A.A. Potapov (V.A. Kotelnikov IREE RAS, see also the author's web page www.potapov-fractal.com).

In modern situation all the attempts to belittle their meaning and to reckoning only on the classical knowledge is endures the intellectual fiasco. Union of specified problem triad in common “Fractal Analysis and Synthesis” therefore creates base of “Fractal Radio Systems” (2005) – Figure 10, “Global Fractal-Scaling Method” (2006), and “Fractal Paradigm” (2011) [7-14,18,19,24,30,34-37].

The work obtained in Kotelnikov IREE of RAS by author and his apprentices is based on the theoretical and experimental results in scheduled introduction of the fractals, fractional integration-differentiation and the scaling effects in radio physics, radio engineering, and some contiguous scientific directions (Figure 11). We have published a sufficiently large number of works for each direction from the data from Figure 11 in Russia and abroad.

Figure 10. Author conception of fractal radio systems and devices

Figure 11. Sketch of author's development of new information technologies based on fractals, fractional operators and scaling effects

9. Conception of Fractal Radio Elements (Fractal Capacitor), Fractal Antennas and Fractal Radio Systems

As it follows from above, significantly positive results in area of justification and development of different methods of digital fractal filtering of weak multidimensional stochastic signals are obtained. *The third* stage of the work on creation and development of breakthrough informational technologies for solving modern problems of radio physics and radio electronics, which was begun in IREE RAS in 2005, is characterized by transformation to *design of fractal element base of fractal radio systems* on the whole. Creation of the first reference dictionary of fractal signs of targets classes and permanent improvement of algorithmic supply were the main points during the development and prototyping of the fractal nonparametric detector of radar signals (FNDRS) in the form of a back-end processor. Basing on the obtained results we can speak about design of not only *fractal blocks (devices)* but also about design of a *fractal radio system* itself [7-14,18,19,24,31-37]. Such fractal radio systems (Figure 10) which structurally include (beginning with the input) fractal antennas and digital fractal detectors are based on the fractal methods of information

processing and they can use fractal methods of modulation and demodulation of radio signals in the long view [7-10,33-37].

Fractal antennas are extremely effective during development of two-frequency or multi-frequency radio location and telecommunication systems. The structures form of such antennas is invariant to certain scale transformations that is an *electrodynamics similarity* is observed. As it is known, spiral and log periodic antennas are the most obvious examples of frequency-independent antennas. Fractal antennas were the next step in building of new ultra broadband and multiband antennas. The scaling of fractal structures gives them multiband properties in an electromagnetic sense [7-9,38-42]. Multi-frequency radio measurements along with fractal processing of the obtained information are a serious alternative to existing methods of enhancing the signal-to-noise ratio. Since every target has its own typical scales then one can directly determine a new signs class (except for the pointed above) in the form of *fractal-and-frequency signatures* by selecting the search frequency grid [11,28,31].

Unlike the classical methods when smooth antenna diagrams (AD) are synthesized an idea of realization of radiation characteristics with a repetitive structure at

arbitrary scales initially underlies the fractal synthesis theory. It gives a possibility to design new regimes in the fractal radio dynamics, to obtain fundamentally new properties and fractal radio elements as well (for example a *fractal capacitor*) [39].

Application of a recursive process theoretically allows to create a self-similar hierarchical structure up to separate conductive tracks in a microchip and in nanostructures. In practice the sum of random values converges not to Gaussian distributions but to Levi stable distributions with heavy tails (i.e. fractal distributions - paretians) quite frequently. Simulation of Levi distributed random values can lead to processes of anomalous diffusion which is described with *fractional derivatives* on space and/or time variables. In substance, equations with fractional derivatives describe non-Markov processes with memory.

Physical simulation of fractional integral and differential operators allows creating radio elements with passive elements, simulating nonlinear impedances $Z(\omega)$ with frequency scaling

$$Z(\omega) = A(j\omega)^{-\eta}, \quad (22)$$

where $0 \leq \eta \leq 1$, $A - \text{const}$, ω - angular frequency basing on the modern nanotechnologies. For that purpose the model of impedance $Z(\omega)$ was created in the form of an unlimited chain (continuous) fraction. In case of a finite stage of building the equivalent circuit for RC chains with using the n -th matching fraction for the given continuous fraction one can adjust frequency ranges which the necessary power law of impedance of the form $\omega^{-\eta}$ will be observed in. In this particular case we will for the first time realize a "non-linear" *fractal capacitor* [8,10,39].

Thus, independently of the [4], our model of the impedance $Z(\omega)$ in the form of an endless chain (continuous) fraction was created. In the case of the final stage of construction of the equivalent electrical circuit for RC chains when the corresponding n -th fraction of the considered continued fraction is used, we can adjust the frequency bands in which there will be a power-law dependence of the impedance $\omega^{-\eta}$ (Figure 12). In this case, we first implement in practice nonlinear «fractal capacitor» or the fractal impedance.

Figure 12. An example of the implementation of the fractional operator $d^{-1/2}/dt^{-1/2}$ or fractal capacitor

Basing on nanophase materials one can also create planar and volume nanostructures which simulate the considered above "fractal" radio elements and radio devices of microelectronics i.e. the question is about building *an element base of new generation*. In particular, an elementary

generalization of Cantor set at physical level allows to proceed to so called *Cantor blocks* in the planar technology of molecular nanostructures.

Application of fractal structures also allows to create media which show complex reflecting and transmitting properties in a wide frequency range and able to simulate three-dimensional *photon* and *magnon crystals* which are the new media of information transfer (for more details see [7,8,10]). One can select a configuration and sizes of fractal structures and check such unusual properties for a frequency range on the scheme on Figure 13. The pickup antenna (is not shown) was placed closely to the fractal plates.

Figure 13. Scheme of "fractal" experiments

On the right on Figure 13 pictures of a secondary electromagnetic field for fractal and copper plates are shown. One can see that the "superwave" fractal structure slows down the directional radiation while a metallic plate does not reveal this function. Such "superwave" properties mean that a fractal plate can act as a *compact reflector*.

Thus fractal structures always have a self-similar series of resonances which lead to logarithmic periodicity of working zones. The related topologic fractal structure allows to modulate the electromagnetic waves transmission coefficient. The lowest frequency of weakening corresponds to wave lengths which can significantly enhance the outer sizes of the fractal plate and makes such fractal structures be *the superwave reflectors*. The obtained results allow to extend the applied above calculation method on the basis of algorithms of a numerical solution of hyper-singular integral equations to a wide class of electrodynamic problems which appear during researches of fractal magnon crystals, fractal resonators, fractal screens, fractal radar barriers and also other fractal frequency-selective surfaces and volumes which are required for realizing the fractal radio systems.

The fractal radio systems proposed by the author reveal new opportunities in the modern radio electronics and can have the widest outlooks of practical application.

Promising elements of fractal radio electronics include also functional elements fractal impedances which are implemented based on the fractal geometry of the conductors on the surface (fractal nano-structures) and in space (the fractal antenna), the fractal geometry of micro-relief surface of substrates or fractal structure of polymer composites, etc.

10. Principles of Fractal-scaling or Scale-invariant Radio Location

At the moment world investigations on the fractal radio location are conducted exceptionally in V.A. Kotelnikov IREE RAS [2,6-24,26-28,30-37,39-52].

In accordance with requirements to the promising radars let us consider a generalized functional scheme of the classical system - Figure 14. On the one hand it is quite simple and on the other hand it contains all the basically necessary elements.

Also the case in point here can be both single-channel radar station (RS) and a multi-channel RS. A synchronizing device provides work coordination for every element of an RS scheme.

Electromagnetic energy is generated and radiated by means of a transmitting device which consists of a modulator, a high-frequency generator and a transmitting antenna. Reflected signals arrive to a receiving antenna. A receiving device performs all the necessary transformations of arriving signals related to their separation, amplification, extraction from noise.

From the information of Figure 14 one can directly proceed to fractal radar. On Figure 15 there are almost all points of application of hypothetical or now projectable fractal algorithms, elements, nodes and processes which can be introduced into the scheme on Figure 14. Ideology of a fractal radar [7-9,24,30,36,37,43,46,48,51,52] is based on conception of fractal radio systems - Figure 10.

11. Postulates of Fractal Radar

Fractal radar defined in [7-9,24,30,36,46,48,52] is based on four main postulates:

- 1) intelligent signal processing based on the theory of fractional measure, scaling effects and fractional operator's theory;
- 2) Hausdorff dimension or fractal dimension D of a signal or a radar image (RI) is directly connected with the topological dimension;
- 3) robust non-Gaussian probability distributions of the fractal dimension of the processed signal;
- 4) "Maximum topology with a minimum of energy" for the received signal. It allows to take advantages of fractal scaling information processing more effectively.

The key point of fractal approach is to focus on describing and processing of radar signal (fields) exclusively in the space of fractional measure with the use of the scaling hypothesis and distributions with heavy-tailed or stable distributions (non-Gaussian). Fractal-scaling processing methods of signals, wave fields and images are in a broad sense based on the pieces of information, which isn't usually taken into account and irretrievably lost if classical methods of processing are applied.

This work is concerned with the main radio physical area –

radiolocation and it aims to ascertain what's done and things to do in this field on the basis of the fractal theory. Investigations carried out showed the correctness of the path chosen by the author (since 1980) to improve the radiolocation technique.

Figure 14. The generalized functional scheme of classical radar

Figure 15. The points of application of fractals, scaling and fractional operators in a classical radar for proceeding to a fractal radar

It is necessary to think about the processing of the input signals with a low threshold at high levels of false alarm and then a transition to a low level of false alarms. Moreover, the false alarm probability is never measured in real time. In principle [7-9,24,36,37,52], we need a new metric, and the new parameters of radar detection.

12. Strategic Directions in Synthesis of New Topological Radar Detectors of Low-Contrast Targets

Intensive development of modern radar technology establishes new demands to the radiolocation theory. Some of these demands do not touch the theory basis and reduce to the precision increase, improvement and development of new calculation methods. Other ones are fundamental and

related to the basis of the radiolocation theory. The last demands are the most important both in the theory and in practice.

Radar detection of unobtrusive and small objects near the ground and sea surface and also in meteorological precipitations is an extremely hard problem [1,2,6-9,48,52]. One should take into account that the noise from the sea surface and vegetation has nonstationary and multi-scale behavior especially at high incidence angles of the sensing wave.

Often, variety of subjacent coverings, conditions of radar observation and maintenance of the objects mentioned above leads to the fact that almost always signal-to-noise ratio q_0^2 for these tasks fills in the area of negative (in decibels) values, that is $q_0^2 < 0$ dB. It makes the classical radar methods and algorithms of detection non-applicable in most cases that are use of energy detectors (when likelihood ratio is exclusively defined by the energy of an input signal) is impossible.

Detection of low-contrast objects against the background of natural high-intensity noise mentioned above inevitably requires us to be able to propose and then to calculate some fundamentally new property which differs from the functionals related to the noise and signal energy.

We think that the initial information comes from different radio systems in the form of a one-dimensional signal and a radar image - Figure 16 [9,36,37,51,52]. The system of initial radio systems and consideration of a radar image and a one-dimensional signal in the millimeter waves (MMW) range was already presented by the author in [6]. Now a fractal radar, a MIMO - radar and unmanned aerial vehicles (UAV) are included into the pattern of Figure 16.

The fractal radar conception is presented in [9, 48] in detail, the MIMO-radar conception is considered in [9,37,48,52]. The main idea of fractal MIMO-radars is use of fractal antennas and fractal detectors [7,8,15,31,48]. An ability of fractal antennas of simultaneous operating at several frequencies or radiating a wideband sensing signal drastically increases the number of degrees of freedom. It determines many important advantages of such a kind of radio location and sufficiently broadens opportunities of adaptation.

All the currently existing methods and signs of detection of unobtrusive objects against the background of high-intensity reflections from the sea, ground and meteorological formations are presented on the Figure 17 [9,13,48,51,52].

As compared with usual detection methods, the fractal-scaling or scale-invariant methods proposed by Professor A. Potapov, can effectively improve the signal/interference relation and considerably increase the probability of target detection. Methods under consideration are suitable both for usual radars and for SAR, and also for MIMO systems for multi-positioning radiolocation.

Figure 16. Radio systems of the initial information

Figure 17. Signs and methods of detection of low-contrast objects against the background of high-intensity ground noise

Also in terms of Weierstrass function for one-dimensional fractal scattering surface we obtained scattering field absolute value dependences on incident angle and surface fractal dimension D . In subsequent computer calculations, we used the above expression for the coherence function (CF) Ψ_k - Figure 17:

$$\Psi_k = \langle E_s(k_1) E_s(k_2) \rangle, \quad (23)$$

of the fields $E_s(k)$ scattered by the fractal surface [6-9,47].

We can show that the tail intensity of signals reflected by a fractal surface is described by power functions:

$$I(t) \sim 1/(t')^{3-D}. \quad (24)$$

Result (24) is very important because, for standard cases, the intensity of a reflected quasi-monochromatic signal decreases exponentially. Thus, the shape of a signal scattered by a fractal statistically rough surface substantially differs from the shape of a scattered signal obtained with allowance for classical effects of diffraction by smoothed surfaces [7-9,20,21,24,36,47,49].

Fractal (Hausdorff) dimension D or its signature at different surface-mapping parts are simultaneously and texture measure [6-8] i. e. properties of spatial correlation of radio scattering by corresponding surface patches.

Fractal signatures including spectra of fractal dimensions and fractal cepstrum represent the attribute vector uniquely determining wide class of targets and objects than the use of fractal dimension values. Thus, we can specify the propose structure of the fractal radiolocation detector of target classes consisting of edge detector and fractal signature calculator. Obtained signatures are compared with the signature database and the decision concerning the presence or absence of the object is made in accordance with some criterion.

The general conception of the textural or fractal detector is presented on Figure 18. The set of textural or fractal signs is determined basing on the received radio signal or image. Then a decision of signal presence H_1 or its absence H_0 is done in the threshold device at threshold value Π and certain level of probability of a false alarm F .

Figure 18. Conception of textural or fractal detector

Values of fractal dimension D , Hurst exponents H for multi-scale surfaces, Holder exponents, values of lacunarity and so on can be also used as signs. Hurst exponent

$$H = 3 - D \quad (25)$$

for a radar image and

$$H = 2 - D \quad (26)$$

for a one-dimensional signal.

Some original variants of generalized structures of radar fractal detectors are presented in Figure 19. One can

synthesize all kinds of fractal detectors from these schemes.

The structure aggregated scheme of the fractal detector of radar signals is presented in Figure 19, a. It includes the contour filter and the fractal cepstrum calculator. After comparison with the database of standard fractal cepstrum one makes a decision at the compare facility. Further concretization of the FNDRS structure scheme is presented in Figure 19, b. Input signal (radar image, 1-D sampling) comes into the input transducer. It is intended for preliminary preparation of analyzed sampling. This preparation includes either compulsory noise (in the case when radar low-resolution analog-digital converter is used) or, for example, contrast compression (in the case of sampling with high dynamic range).

Figure 19. Initial (a) and detailed (b) structures of fractal signal detectors

Prepared input sampling comes into the edge detector. Operation of this facility is based on the measurement of the local fractal dimension over all sampling elements. It is important that one can execute the preliminary detection in accordance with the empiric distribution of local fractal dimensions obtained at the edge detector output over all sampling elements [15,28,31].

After the edge detector the input sampling actually represents a binary array. "Units" in this array mean the belonging of corresponding sampling element to the contour of some object. If several objects are present in the sampling, the question about their division arises. In our facility the cluster selector realizes this task.

Obtained subset of initial sampling containing the contours of one object comes to the input of the signature calculator. This facility creates several "smoothed" samplings in accordance with the expected observation scales and calculates the "length" and "area". The detection process is realized by means of the comparison of the fractal sampling signature with the database fractal signatures (or data bank).

A detector on the basis of the Hurst exponent (Figure 20)

works with using one or several search frequencies of radar. The Hurst exponent H reflects irregularity of a fractal object – (25) and (26). The less exponent H the more irregular a fractal object. So, the Hurst exponent gets higher when an object appears.

Figure 20. Fractal detector basing on the Hurst exponent

On Figure 21 there is the scheme of fractal detector with autoregressive estimation of the power spectrum of the interference from the Earth surface.

Figure 21. Fractal detector with autoregressive estimation of the spectrum of the interference and the Hurst exponent

The autoregressive model represents a linear model of prediction which estimates the power spectrum of the interference from the surface and forms its autocorrelation matrix. The autoregressive equation describes relation between current and preceding counts of a sampled stochastic process. Earlier, in the eighties of XX we were resolving the problem of autoregression on the basis of canonical system of Yule-Walker equations with transform of brightness histograms [6]. Thus in the detector on Figure 16 real fractal properties of the power spectrum on the basis of autoregressive spectral estimation which are applied for detection of low-contrast objects are used. We used much the same detectors during the textural processing of APG and RI as early as the eighties of XX.

I should note that the correlation dimension which requires a big size sampling cannot be considered as detection statistics (see Figure 17) and this is impossible in radiolocation

13. Strange Attractors in the Phase Space of Reflected Radar Signals in Millimeter Wave Range

A deterministic chaos mode was discovered during radio location of plant covering at wave length 2.2mm [11]. Estimations of fractal dimension D , nest dimension m , maximum Lyapunov exponent λ_1 and prediction time $\tau_{\max}$ were used to measure and reconstruct the strange attractor.

Calculation of the correlation integral $C(r)$ was conducted using the F. Takens theorem on a sampling out of 50 000 counts which corresponds to the angle of incidence of an electromagnetic wave $\theta=50^\circ$. The following values were obtained: $D=1+1,84 \approx 2,8$; $m=7$; $\lambda_1 \geq 0.6$ bit/s; $\tau_{\max} \approx 1700\text{ms}$ when the reflected signal intensity correlation time is $\tau \approx 210$ ms and the wind velocity is 3 m/s (Figure 22).

Figure 22. A kind of the screen of a computer with dependences D for radar-tracking signals

Hence, if the current conditions are measured within the accuracy of 1 bit then the whole predictive power in time will be lost for about 1.7 c. At that the interval of prediction of radar signal intensity is about 8 times the correlation time. The obtained results show that a correct description of the process of radio waves scattering requires not more than 5 independent variables. The correlation integral $C(r)$ can also be used as a mean of separation of modes of the deterministic chaos and white noise – Figure 23. Calculation of the classical Henon attractor (Figure 24) was conducted with the purpose of verification of adequacy of the created algorithms.

Figure 23. A kind of the screen of a computer with dependences D for Gaussian noise

Figure 24. Cross section: Henon's attractor

Dependences of fractal dimensions D and correlation integrals of radar processes under examination of millimeter waves scattering by a birch (1) and spruce (2 and 3) forests with $D \approx 2.6$ are given on Figure 25.

Figure 25. Scattering of millimeter waves by a birch (1) and spruce (2 and 3) forests

The obtained results along with a family of fractal distributions underlie the new dynamical model of signals scattered by plant coverings. The proposed model of electromagnetic waves scattering by earth coverings has a fundamental difference from existing classical models [6-8].

14. A New Direction in the Theory of Statistical Solutions

Fast development of the fractal theory in radar and radio physics led to establishing of the new theoretical direction in modern radar. It can be described as «*Statistical theory of fractal radar*». This direction includes (at least at the initial stage) the following fundamental questions:

1. The theory of the integer and fractional measure.
2. Caratheodory construction in the measure theory.
3. Hausdorff measure and Hausdorff-Besicovitch dimension.
4. The theory of topological spaces.
5. The dimension theory.
6. The line from the point of view of mathematician.
7. Non-differentiable functions and sets.
8. Fundamentals of the theory of probability.
9. Stable probability distributions.
10. The theory of fractional calculus.
11. The classical Brownian motion.
12. Generalized Brownian motion.
13. Fractal sets.
14. Anomalous diffusion.
15. The main criteria for statistical decision theory in radar.
16. Wave propagation in fractal random media.
17. Wave scattering generalized Brownian surface.
18. Wave scattering surface on the basis of non-differentiable functions.
19. Diffractals.
20. Cluster analysis.
21. Theory and circuitry of fractal detectors.

22. Fractal-scaling or scale-invariant radar.
23. The multi-radar.
24. MIMO radar.
25. Cognitive radar.

This list of studied questions, of course, is supposed to be expanded and refined in the future. The author has been dealing with it for nearly 40 years of his scientific career.

15. Officially Admitted Results of the Fractal Researches

Results of our scientific activity on fractal-and-scaling processing of information in the presence of high intensity noise and fractal radio systems and radio elements as well in V.A. Kotelnikov IREE RAS are published in four summary reports of *The Presidium of the Russian Academy of Science* (Scientific achievements of RAS. - M: Nauka, 2008, 2010, 2012, 2013) and in *The Report For the Russian Federation Government*. About results of realization of the 2008 - 2012 Program of fundamental scientific researches of the state academies of sciences in 2011. In three volumes, - M.: Nauka, 2012. We'll briefly cite the text of these officially admitted achievements.

- In book "The summary report of the Presidium of the Russian Academy of Sciences. Scientific achievements of the Russian Academy of Science in 2007" (M.: Nauka, 2008, pp. 204) in subsection "Location systems" of section "Informational technologies and computational systems" (p. 41) there is the following text: "A reference dictionary of fractal signs of optical and radio images which is necessary for realization of fundamentally new fractal methods of processing of radar information and synthesis of high-informative devices of detection and recognition of weak signals against the background of high intensity non-Gaussian noise was created. It was determined that for effective solving of radar problems and designing of fractal detectors of multidimensional radio signals, the fractional dimension, fractal signatures, fractal cepstrum and also textural signatures of the place noise are essential (IREE RAS)" - 2007, published in 2008.
- In book "The summary report of the Presidium of the Russian Academy of Sciences. Scientific achievements of the Russian Academy of Science in 2009" (M.: Nauka, 2010, pp. 616) in subsection "Location systems. Geo-information technologies and systems" of section "Nanotechnologies and informational technologies" (p. 24) there is the following text: "The principles of designing of new, fractal adaptive radio systems and fractal radio elements for modern problems of radio engineering and radio location are proposed and shown by experiments for the first time in the world. The operating principle of such systems and elements is based on introduction of fractional transformations of radiated and received signals in a fractional dimension

space with taking into account its scaling effects and non-Gaussian statistics. It allows to get the new level of informational structure of the real non-Markov signals and fields (IREE RAS)” - 2009, published in 2010.

- In book “The summary report of the Presidium of the Russian Academy of Sciences. Scientific achievements of the Russian Academy of Science in 2011” (M.: Nauka, 2012, pp. 620) in subsection “Location systems. Geo-information technologies and systems” of section “Informatics and informational technologies” (p. 199 - 200) and in book “The Report for the Russian Federation Government. On the results of realization of the fundamental scientific researches 2008-2012 Program of national academies of sciences in 2011. In three volumes” (M.: Nauka, 2012, pp. 1016) on p. 242 there is the following text: “A systematic research of electrodynamic properties of fractal antennas was carried out basing on the fractal analysis. Broadband and multiband properties of fractal antennas and dependence of resonances number on the number of fractal iteration are confirmed. It is shown that an effective realization of frequency-selective media and protective screens which deform a radar portrait of the target is possible using miniature fractal antennas. Fractal frequency-selective 3D-media or fractal “sandwiches” (engineering radio electronic micro- and Nano constructions) are studied. (IREE RAS)” - 2011, published in 2012.
- In book “The summary report of the Presidium of the Russian Academy of Sciences. Scientific achievements of the Russian Academy of Science in 2012” (M.: Nauka, 2013, pp. 616) in subsection “Elemental base of microelectronics, Nano electronics and quantum computers. Materials for micro- and Nano electronics. Nano- and microsystem engineering. Solid-state electronics” of section “Nanotechnologies and informational technologies” (p. 195) there is the following text: “It is determined that the integer-valued quantum Hall effect is a physical base of memristor functioning. The relations between the current and the voltage for a random memristor type have been obtained. The results are addressed to practical realization of memristors as new elements of electronic circuits. (Research Institute Of Applied Mathematics And Automation Of Kabardino-Balkaria Research Centre RAS, IREE RAS)” - 2012, published in 2013.

16. Conclusions

This work is concerned with the main radio physical area – radiolocation and it aims to ascertain what's done and things to do in this field on the basis of the fractal theory. Investigations carried out showed the correctness of the path chosen by the author (since 1980) to improve the radiolocation technique.

In particular, over period of thirty-five years this resulted

in invention, creation and development of the new kind and method of radiolocation, namely, fractal-scaling or scale-invariant radiolocation. This implies radical changes in the structure of theoretical radiolocation itself and in its mathematical apparatus also.

Earlier fractals made up the thin amalgam on the strong science frame of the XX century ending. In the modern situation attempts to humiliate their significance and rely only on the classical knowledge suffered an intellectual fiasco.

The detailed analysis of all works published by the author's is not an aim of this chapter. Nevertheless, the acquaintance with the author's investigations in this area should substantially help to large group of experts and more accurately determine the practical application ways of the fractal theory to solve the radio physical and radiolocation problems. I consider that the “sampling topology” problem [6-10,16] is one of the most important in radio electronics, and I am also convinced that without fractals and scaling all signal-detection theory loses its causal meaning for the signal and noise conceptions.

The functional principle “*Topology maximum at energy minimum*” for receivable signal permitting effective application of advantages of the fractal-scaling information processing was introduced by the author. This refers to the adaptive target signal processing. Application of the fractal principle results in the soul-searching in the detection field of movable and immovable objects at the intensive disturbance and noise background.

In this chapter, the author touched upon only the most important problems connected with the application of the fractals and scaling effects in statistical radio physics and radiolocation. In the development of fractal directions many important periods have already passed including the establishment period of this field of science. However we will have to solve many problems. It is the solution technique (approach) is the most valuable but not results and implementations. This method was created by Professor A. Potapov [2,6-24,26-28,30-37,39-52]. Scientific results obtained in recent years are the initial material for further development and substantiation of practical application of fractal methods in modern fields of radio physics, radio engineering, radiolocation, electronics and information controlling systems.

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