Sliding Mode Observer-based Actuator Fault Reconstruction for a Continuous Reactor

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Abstract The problem of fault detecting subject to external disturbances has been a topic of considerable interest. In this paper, a sliding mode observer for fault detection and isolation is applied to a continuous reactor. Additionally, a general review has been done on dynamic model of continuous reactor along with detailed study of the mathematical model of these kinds of systems. Then, sliding mode observer is investigated with detailed comment. In order to isolate and estimate the possible actuator faults a bank of Sliding Mode Observer (SMO) is designed. Also a simple canonical form for sliding mode observer is presented. A design procedure is described and linear simulation results are presented to demonstrate the approach.

Keywords Sliding Mode Observer, Fault Detection and Isolation, Continuous Reactor, Canonical Form for Sliding Mode Observer, CSTR Reactors

1. Introduction

In some systems like CSTR reactors and other complex systems, some sensors cannot be placed in desirable location. There is not any reason for that, expect fault detection and isolation. Fault in sensors or actuators can cause process degradation even in the chemical plants. For instance lower product quality. Addition Fault cause fatal accidents (e.g. temperature run-away). Furthermore, for a group of variables no sensor exists. Concentrations and moles belong to this group. So the accurately monitoring process variables and interpreting their variations increases quickly with the increase in the range of instrumentation. For operating a process in normal situation used a set of tools and methods, called supervision. Two main activities of supervision are real Fault Detection and Isolation (FDI) and Fault Tolerant Control (FTC). The parts of the supervision scheme are shown in Fig. 1. FDI and FTC achieve safe operation of the system in the presence of faults. This paper is focused on FDI, for FTC can refer to [1].

Researchers used existing approaches such the classic ones to develop their performance for new complex systems [2-4], or tried to find new approaches for performing fault diagnosis [5], [6], [7] and [8]. Applications of fault diagnosis approaches techniques to batch chemical processes are usually challenging, these depend on intrinsically unsteady operating conditions and their nonlinear dynamics. Generally, complete state is usually not available [9]. This is also honest about parameters measurements in a chemical plant (i.e. products composition). These approaches can be based on historical data or a mathematical model [10-13].

For Model-based methods we need to detect and isolate faults (i.e. the presence of one or more faults can be recognized, the faulty components are determined). In fact, the comparison between the measurements of variables set
characterizing the behavior of the monitored system and the corresponding estimates predicted via the mathematical model of system can be useful to consist of model-based methods. The deviations between measured and estimated process variables provide a group of residuals, sensitive to the occurrence of faults. Observer-based schemes that are kind of model-based analytical redundancy approaches, is successfully adopted in a variety of application fields [14, 16]. Diagnostic observer is operated in parallel to the process to compute estimated process variables to be compared to their measured values. Some of approaches are based on Kalman and/or Luenberger observer. Their applications to reactors diagnosis are designed by resorting to linearized models of reactor [17, 18].

Control problem of uncertain systems that have been exposed to an external perturbation has been an enabled field of study during the last decade. The majority of systems that we facing them in real terms are exposed to different uncertainties like nonlinearities, actuator faults, variations in parameters, and etc. In majority of proposed control strategies is assumed that all state variables are existing; this assumption is not always correct in real terms, therefore the state vector should be appraised for using in the control rules. The basic objective of a fault identification plan is to produce a warning when faults happen. Among research activities performed in this field can cited to Kalman filter [19], adaptive observers [20], high gain observers [21], sliding mode observers (SMO) [22-24], etc. See [25] to perform comparing.

A certain situation amongst observer based methods is taken by sliding mode observers. Largely, the sliding mode observer takes advantage from discontinuous control operation to move the observer fault direction toward a certain hyper-plane in the fault space, and then the direction is preserved to slide on this so long as the origin of the state space is attained. Basically, Observer generated the signals which are used to discover data associated with the fault. Specifically, ‘remaining generation’ statements, using as linear observers, have been extensively applied. In this method, difference among output of the system and output of the observer is calculated by a scaling matrix to create known attribution. However, they did not determine the most appropriate place for eigenvalues in the favorable district. Their procedure is partly complicated, although their procedure is obvious [33].

In this study, a simple SMO is proposed for a special class of class of linear systems in the presence of faults/unknown inputs. All the parametric uncertainties/disturbances present in the system are modeled in the form of unknown inputs/faults. The unknown input can be a combination of un-measurable or unmeasured disturbances, unknown control actions, or un-modeled system dynamics. The novelty of this study lies in the choice of robust terms to deal with faults/unknown inputs. The methods of the researches use all the output information to deal with unknown inputs, and so require the reduced order system itself to be stable in the sliding model. Moreover, the robust terms are applied to ‘reconstruct’ all the faults/unknown inputs from the sliding mode. Finally, a Continuous Reactor system example is given to illustrate the efficiency of the proposed approach.

2. Study about Continuous Reactor

2.1. Description of the Continuous Reactor

The continuous reactor with heat exchange is used in manufacturing plants for many process operations such as fermentation, chemical synthesis, polymerization, crystallization … etc.

Reaction vessel, a jacket vessel, an entry and exit feeding pipes, a coolant and products, valves, a stirring system and a heat exchange surface make an important group of the systems. In a continuous reactor many process depend on this group (e.g. the process to be supervised). However, the reaction takes place within the reactor but jacket is fitted to the reactor vessel by using an external heated transfer coil wrapped around the vessel surface. A system alike washer machine keep in good condition the mixture among the reactants and products with a good homogeneous degree of physical and chemical properties.

Function of the location does not consist of concentration and temperature variables. They just represent average values for all the reactor volume.

Some irreversible and exothermic reactions happen in the reactor vessel [15], the oxidation of sodium thiosulfate. By hydrogen peroxide is given by:

\[
Na_2S_2O_3 + 2H_2O_2 \rightarrow \frac{1}{2}Na_2S_2O_6 + \frac{1}{2}Na_2SO_4 + 2H_2O
\]  

(1)

The kinetic law is written as following:

\[
r_A = -(k + \Delta k)\exp\left(-\frac{E_a\Delta E_a}{RT}\right)P_A P_B
\]  

(2)
Where \( P_A \) and \( P_B \) are concentrations of components A and B (B is the \( H_2O_2 \) and A is the \( Na_2SO_3 \)), \( R \) is the perfect gas constant, \( E_a \) is the activation energy, \( k \) is the pre-exponential factor, \( T_r \) is the reactor temperature and \( \Delta \) represent uncertainty.

Energy balances and A mole balance for species A and the cooling jacket result in the following nonlinear process model with \( (P_A = P_B) \):

\[
\dot{P}_A = \frac{F_r}{V} (P_{A\text{in}} - P_A) - 2k(t)P_A^2 \\
\dot{T}_r = \frac{F_r}{V} (T_{\text{in}} - T_r) + \frac{2(-\Delta H_r) + \Delta(-\Delta H_r)}{\rho P_r} k(t)P_A^2 \\
\dot{T}_j = \frac{F_o}{V_o} (T_{\text{jin}} - T_j) + \frac{UA + \Delta Ua}{\rho P_r P_{pa} V_o} (T_r - T_j)
\]

Where the new parameter \( (T_j) \) is the cooling jacket temperature. The three equations represent of three states. State vector can be defined as:

\[
x(t) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} P_A \\ T_r \\ T_j \end{bmatrix}
\]

And the initial state vector is:

\[
x(0) = \begin{bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{bmatrix} = \begin{bmatrix} P_{A\text{in}} \\ T_{\text{rin}} \\ T_{\text{jin}} \end{bmatrix}
\]

### 2.2. Model Linearization

For \( P_A = 0.0192076 \text{ mol/l}, T_r = 384.005 \text{ K} \) and \( T_j = 271.272 \text{ K}, \) the nominal nonlinear system is stable and chosen as a normal operating point. The linear model around the chosen steady state is:

\[
\begin{cases}
\dot{x} = Ax + Bu \\
y = Cx
\end{cases}
\]

\[
x = \begin{bmatrix} \Delta P_A \\ \Delta T_r \\ \Delta T_j \end{bmatrix}; y = \begin{bmatrix} \Delta T_r \\ \Delta T_j \end{bmatrix};
\]

\[
A = \begin{bmatrix} -125.8815 & -0.0747 & 0 \\ 1.7711e + 004 & 6.5538 & 2.8571 \\ 0 & 28.5714 & -31.5714 \end{bmatrix};
\]

\[
B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; u = T_{\text{jin}}
\]

The linear model with fault is obtained as follows:

\[
\begin{cases}
\dot{x} = Ax + Bu + Ff_a \\
y = Cx
\end{cases}
\]

Where \( f_s \) is sensor fault, \( f_a \) is actuator fault, \( F \) is fault matrix in state expression.

### 3. FDI Using Sliding Mode Observer

Notice the following dynamical system

\[
\dot{x}(t) = Ax(t) + Bu(t) + Df(t) \\
y(t) = Cx(t) + f_0(t)
\]

where \( A \in R^{nxn}, B \in R^{nxm}, C \in R^{p\times n}, D \in R^{p\times q} \) with \( q \leq p \leq n. \ A \in R^n \) is the state vector, \( u \in R^m \) is the input vector and \( y \in R^p \) is the output vector. Also \( f_1(t) \) and \( f_0(t) \) show the actuator and sensor faults, respectively. The matrices \( C \) and \( D \) are supposed to be full rank. Here, \( u(t) \) and \( y(t) \) are accessible.

Suppose the observer of the form:

\[
\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) - G_1e_y(t) + G_nev(t) \\
\dot{\hat{y}}(t) = C\hat{x}(t)
\]

where \( G_1, G_n \in R^{pxp} \) are proper gain matrices. The discontinuous vector \( v \) is presented by:

\[
v = \begin{cases} -\rho\|D_2\|\frac{P_2e_y(t)}{\|P_2e_y(t)\|} & e_y \neq 0 \\
0 & \text{otherwise} \end{cases}
\]

Where \( e_y(t) = \hat{y}(t) - y(t) \) and \( P_2 \in R^{p\times p} \) is symmetric positive definite. The definitions of the matrices \( P_2 \) and \( D_2 \) and the scalar can \( \rho \) are presented in the later part. The discontinuous term is designed to drive the \( D_2 \) trajectories of the observer in a way that the state prediction error vector is forced into and as a result stays on a surface in the error space.

Now consider the dynamical system offered in (5) and (6) and suppose that:

* rank(\( CD \)) = \( q \)

* Invariable zeros of (A, D, C) must lie in the open LHP

First, suppose the case that \( f_0(t) = 0 \). It can be demonstrated that under these hypotheses, there exists a resemblance transformation as \( T \) (The result expressed in [30, 35] confirms the existence of a nonsingular transform matrix to have this structure, and two ways to gain it is presented in the Appendix) such a way that the system will be appear in the following form

\[
\begin{cases}
x_1(t) = A_11x_1(t) + A_12y(t) + B_1u(t) \\
y(t) = A_21x_1(t) + A_22y(t) + B_2u(t) + D_2f_1(t)
\end{cases}
\]

Where, \( x_1 \in R^{n-p} \) and the matrix \( A_{11} \) has stable eigenvalues [35]. The above transformation is applied to gain the following form of observer (7):

\[
\begin{cases}
\dot{x}_1(t) = A_{11}\dot{x}_1(t) + A_{12}\dot{y}(t) + B_1u(t) - A_{12}e_y(t) \\
\dot{y}(t) = A_{21}\dot{x}_1(t) + A_{22}\dot{y}(t) + B_2u(t) - (A_{22} - M)e_y(t) + v
\end{cases}
\]

where \( M \) is a stable design matrix that must be specified by designer. Also, \( P_2 \) is a Lyapunov Matrix for \( M \) and the scalar \( \rho \) is selected in such that

\[
\|f_1(t)\| < \rho(t)
\]

From (10) and (11)
\[
\dot{e}_1(t) = A_{11}e_1(t) \\
e_y(t) = A_{21}e_1(t) + Me_y(t) + \nu - D_2f_1(t)
\]

(15)

Where
\[
e_1(t) = \hat{x}_1(t) - \bar{x}_1(t) \\
e_y(t) = \hat{y}(t) - y(t)
\]

(16)

It is demonstrated in [35] that the nonlinear error system in (12) is stable and a sliding motion takes place forcing \(e_y(t) = 0\) in limited time. The dynamical system in (12) may so be considered as an observer for the system in (5) and (6). It follows that if
\[
G_l = T^{-1} \begin{bmatrix} A_{12} \\ A_{22} - M \end{bmatrix}, G_n = T^{-1} \begin{bmatrix} 0 \\ I_P \end{bmatrix}
\]

(17)

then the observer presented in (12) can be written in the basis of the main coordinates in the form of (7).

It is demonstrated that, offered a sliding motion can be achieved, the state of the system can be restored and also, estimates of \(f(t)\) and \(f_j(t)\) can be calculated as:
\[
f_1(t) \approx -\rho(t)\|D_2\| (D_2^T D_2)^{-1} D_2^T \frac{P_2 e_y(t)}{\|P_2 e_y(t)\| + \delta}
\]

(18)

And
\[
f_j(t) \approx -(A_{22} - A_{21}A_1^{-1}A_{12})^{-1} \frac{P_2 e_y(t)}{\|P_2 e_y(t)\| + \delta}
\]

(19)

where \(\delta\) is a small positive scalar and \(f_0(t)\) is supposed to be a slowly changing fault [35].

4. Simulation Results

Simulation 1: Until now, many controllers have been proposed for industrial systems [36-91]. However, none of them cannot show a good performance when faults accrue in these systems. So, in this section by choosing a chemical system, we try to show the superiority of our proposed method than conventional ones. Consider the continuous reactor equations (6). This results in the triple system below

\[
\dot{x} = \begin{bmatrix} -125.8815 & -0.0747 & 0 \\ 1.7711e + 004 & 6.5538 & 2.8571 \\ 0 & 28.5714 & -31.5714 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} u
\]

\[
y = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} x
\]

With supposed the fault distribution matrix \(D = B\). Using an algorithm like that suggested in [1] it can be demonstrated that in the canonical form of the system will be appeared in the following form:

\[
A = \begin{bmatrix} -125.8815 & -0.0747 & 0 \\ 1.7711e + 004 & 6.5538 & 2.8571 \\ 0 & 28.5714 & -31.5714 \end{bmatrix};
\]

\[
D = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}
\]

\[
C = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}
\]

Using the suggested procedure:
\[
G_l = \begin{bmatrix} 22.4197 & 1.6205 \\ 2.6723 & 1.8571 \\ 27.5714 & -32.5714 \end{bmatrix}
\]

\[
G_n = \begin{bmatrix} -4.8614 & 0 \\ 3.0000 & 0 \\ 0 & 3.0000 \end{bmatrix}
\]

In this specific design the scalar function \(\rho = 35\) and design of the observer is perfect. General scheme of the designed Sliding Mode Observers is shown in Figs. 2, 3.

Figure 2. SMO with fault estimation

Figure 3. Simulation design in Matlab

Simulation results for fault estimation and determination are shown in Figs. 4, 5. You can see that the robust term was able to track the fault occurred, the reconstructed fault from the Sliding mode.
In this stage, a fault estimator based on the proposed sliding mode observer of Edwards et al. (1999) is designed for an HIRM aircraft system [89]. Then the performance of the proposed method is compared with that of the sliding mode observer presented by Edwards et al. (1999) to show the superiority of the proposed method. It should be noted that the design parameters of the proposed method are the same as those in the first stage of simulation. It can be concluded from Fig. 5 that the proposed observer has estimated the fault efficiently. But the sliding mode observer of Yan et al. (2007) shows chattering and is not highly accurate. By changing the fault, as shown in Fig. 6 to verify the robustness of the suggested procedure, we selected $6\sin (0.6\pi t) - 3\cos (0.35\pi t)$ as a fault signal. It can be observed that the performance of the proposed method is still acceptable. Although the sliding mode observer of Edwards et al. (1999) does not have chattering, it has a high estimation error making its implementation problematic.

**Simulation 2 [89]:** In this stage, a fault estimator based on the proposed sliding mode observer of Edwards et al. (1999) is designed for an HIRM aircraft system [89]. Then the performance of the proposed method is compared with that of the sliding mode observer presented by Edwards et al. (1999) to show the superiority of the proposed method. It should be noted that the design parameters of the proposed method are the same as those in the first stage of simulation. It can be concluded from Fig. 5 that the proposed observer has estimated the fault efficiently. But the sliding mode observer of Yan et al. (2007) shows chattering and is not highly accurate. By changing the fault, as shown in Fig. 6 to verify the robustness of the suggested procedure, we selected $6\sin (0.6\pi t) - 3\cos (0.35\pi t)$ as a fault signal. It can be observed that the performance of the proposed method is still acceptable. Although the sliding mode observer of Edwards et al. (1999) does not have chattering, it has a high estimation error making its implementation problematic.
5. Conclusions

This paper demonstrates how Sliding Mode Observers can be used to detect, estimate and isolate of the faults in actuators. In addition, the proposed method is simple and is of a relatively lower complexity compared to existing methods. Robust terms are designed in a way that faults can be reconstructed directly from the sliding surfaces. Our proposed method does not need a nonlinear transformation. The stability of the reduced-order error system in the sliding mode is established. Moreover, conditions for the existence of a sliding mode linear functional observer are given. Then is demonstrated once the existing circumstances are satisfactory, how to discover parameters of the observer. The proposed FDI (Fault Detection and Isolation) design is easy to implement and can be applied to a reasonably wide class of systems. The numerical example for a continuous reactor illustrates that this approach is effective and easy to implement.

6. Appendix

Let state description of the system (1), (2) with \( m = p \) is
\[
\dot{x}(t) = Ax(t) + Bu(t) \quad (A.1)
\]
\[
y(t) = Cx(t) \quad (A.2)
\]
\[
C_1 = CT_1 = [0 \quad I_p], T_1^{-1} = \begin{bmatrix} n-p & 0 \\ C & 0 \end{bmatrix} \quad (A.3)
\]
Then
\[
B_1 = T_1^{-1}B = T_1^{-1} \begin{bmatrix} B_{01} \\ B_{02} \end{bmatrix} = \begin{bmatrix} B_{01} \\ CB \end{bmatrix} = \begin{bmatrix} B_{11} \\ B_{12} \end{bmatrix} \quad (A.4)
\]
If \( CB = B_{12} \) is a regular matrix (in opposite case the pseudoinvers os \( B_{12} \) is possible to use), then the second transform matrix \( T_2^{-1} \) can be defined as follows
\[
T_2^{-1} = \begin{bmatrix} I_{n-p} & -B_{11}B_{12}^{-1} \\ 0 & I_p \end{bmatrix}, T_2 = \begin{bmatrix} I_{n-p} & -B_{11}B_{12}^{-1} \\ 0 & I_p \end{bmatrix} \quad (A.5)
\]
These results in
\[
B = T_2^{-1}B_1 = \begin{bmatrix} I_{n-p} & -B_{11}B_{12}^{-1} \\ 0 & I_p \end{bmatrix} \begin{bmatrix} B_{11} \\ B_{12} \end{bmatrix} = \begin{bmatrix} 0 \\ B_2 \end{bmatrix} \quad (A.6)
\]
Where
\[
B_{11} = B_{01}, B_2 = B_{12} = CB \quad (A.7)
\]
and
\[
C = C_1T_2 = [0 \quad I_p] \begin{bmatrix} I_{n-p} & -B_{11}B_{12}^{-1} \\ 0 & I_p \end{bmatrix} = [0 \quad I_p] \quad (A.8)
\]
Finally, with \( T_{con}^{-1} = T_2^{-1}T_1^{-1} \) it yields
\[
A = T_{con}^{-1}AT_{con} \quad (A.9)
\]
Thus, (A.6), (A.8) and (A.9) represent the system canonical model.

Note

The structure of \( T_1^{-1} \) is not unique and others can be obtained by permutations of the first \( n - p \) rows in the structure defined in (A.3) (Please check example in [35]).
REFERENCES


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