An Introduction to Dynamic Optical Optimization

J. K. Kuipo¹, B. N. Tiwari^{1,2,*}, N. Gupta¹, M. Khosravy^{1,3}, S. Bellucci², N. Marina¹

¹University of Information Science and Technology, St. Paul the Apostle, Partizanska bb, Republic of Macedonia

²The INFN National Laboratory of Frascati, Italy

³Digital Signal Processing Laboratory, University of the Ryukyus, Japan

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Abstract In this paper, we introduce an optical objective function in order to obtain the optimized image of a dynamical object by an optical instrument having a variable zooming range. To be precise, about a given fixed point of the focal length of a single lens, mirror or an extended optical instrument, the local stability of the image thus formed is characterized by the positivity of pure correlation components of the fluctuation matrix. On the other hand, the corresponding global stability of the image is characterized by the positivity of the determinant of the fluctuation matrix. We also observed that converging and diverging lenses and mirrors show a clear cut distinction about the line of unit lateral magnification. Industrial applications of our proposed optical objective function are anticipated to enhance quality of the lenses, mirrors and their combinations.

Keywords Optics, Optimization, Instrument Designing, Stability Analysis, Fluctuation Model

1. Introduction

Optics as originating via photonics as the processes involving photon mediated decay reactions possesses a concrete foundation [1]. Such processes led down the fundamentals of geometrical and physical optics [2-4]. In the realism of the geometrical optics, light is considered as an assembly of rays, which travel in a straight line, see for example [3]. Because of this feature, we sometime call geometrical optics as the ray optics. Physical phenomena including reflection and refraction of light are studied with phase discontinuities [5]. Experiments pertaining to optical pulses find numerous applications in optical instruments, viz. lenses, mirrors, prisms and optical fibers [6]. Image formation via optical elements can be used individually or in their finite combinations. It is sufficient to form an image via lenses using three rays that obey the laws of refraction which originate from the object [2-7]. In the case of mirrors, it follows further that three rays of light obeying the laws of reflection are necessary to form the image.

An important feature of optical instruments, viz. lenses and mirrors or their combinations, is the focal length f , whose intrinsic optimization is the main focus of the present research. This is defined as the distance between the optical center of the optical instrument and its focal point. Images thus formed via lenses and mirrors can either be upright or inverted relative to the original objects, as well as real or virtual depending upon an actual or apparent intersection of refracted or reflected rays respectively [8, 9]. Another quantity concerning optical systems is the lateral magnification, viz. the ratio of the image distance to that of the object, or the ratio of the image height to that of the object [2], which plays an important role in this research.

As far as an optical image formation is concerned, its fundamentals lie in electrodynamics, where the visible light emerging as a portion of the total electromagnetic spectrum [1] is distinguishable to ordinary human eyes. Because of this qualification, optics finds importance in understanding the traveling properties of light. In this concern, it is worth mentioning that optics provides a comprehensive exploration of the image formation of a given object via an optical instrument. Herewith, optics finds daily-life applications involving mirrors, lenses, microscopes and telescopes in the light of geometrical optics [2-4]. Following the above outlines, this paper offers an apt designing of optical systems with an optimized aberration and tolerance, while the undermining optical system may be constrained in due course of the image formation, as well.

Under the above condition when a dynamical object is viewed by an optical instrument of variable zooming, we can independently control only two of the variables out of the object distance, image distance and lateral magnification. For a given dynamical object, since the lateral magnification is an intrinsic property of the instrument, therefore, we choose it to be one of the optimization variable. In this case, given the lateral magnification m of the instrument, the image distance v is apparently obtained as $v = mu$, where u is the distance of the dynamical object from the optical center of the instrument. For example, in the case of human eye, when an object is placed in front of it, its image is formed at the retina. When the eye

has defects, the same is achieved by introducing a suitable combination of lenses of variable lateral magnification. This observation concerning an optimized image formation continues to hold for various optical instruments including camera, microscope, telescope, laser instruments and smartphones in forming a stabilized image.

The rest of the paper is organized as follows: In Section 2, we offer needful fundamentals concerning physical and geometrical optics, and thereby describe the problem concerning a stable image formation via convex and concave lenses/ mirrors. In Section 3, we offer optimization analysis for such an optical system by defining the focal length of the lens/ mirror as the objective function, while the object distance and lateral magnification vary. Namely, we introduce an intrinsic stability characterization for an optical instrument with fluctuating lateral magnification m for a dynamical object situated at a position u from the optical center of the instrument. In Section 4, we discuss our results and research directions for future.

2. Formulation of the Problem

Concerning an optimized image formation, we concentrate hereby on the stability analysis of an image being formed for a dynamical object via an optical instrument with a variable zooming. As per the above introduction, we may prolong the aforementioned analysis for both the unconstrained and constrained optical systems. In the sequel, we begin our analysis by building a suitable objective function which can endure an optimized image formation via a lens, mirror or an optical system with their arbitrary combinations.

As far as an image formation via a lens/ mirror or their finite combinations is concerned, our proposal of optimized image formation finds high significance towards industrial applications. For instance, a concave lens which is thinner in center as compared to its edges [2-4], reflects incident rays away from its optical axis. This forms basically a virtual image because of the fact that it has an apparent intersection of refracted rays at its focal point. On the other hand, in the case of a convex lens which appears thicker at its center than that of its edges [2-4], the parallel incident rays emanating from the object converge at the focal point of the lens, whereby forming both the real and virtual images.

In contrast to the above, while we focus on an image formation via mirrors, it is worth emphasizing that spherical mirrors falls equally in two categories, viz. we have concave and convex mirrors. Notice that mirrors reflect and bend the light rays which incident to their surfaces. The concave mirrors function in a spirit similar to a converging lens, except the fact that the concave mirror has merely a single focal point [2-4]. On the other hand, it follows that convex mirrors bend the light rays having an incidence upon them away from their principal axis. Herewith, it leads to a virtual image formation behind the mirror under consideration.

It is worth mentioning that for both cases of the image

formation via lenses and mirrors, the respective focal length f , which is defined as the distance between mirror's/ lens's boundary surface to the corresponding focal point [2-4] is adopted in its thin approximation as $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$, where v denotes the image distance and u denotes the object distance from the optical point of the considered lens/ mirror. As mentioned in the introduction, it is worth realizing that v is beyond the control of the observer, as it is automatically formed via the optical instrument for a given object. Herewith, an optical instrument with nonzero zooming brings us into the notion of an important quantity in optical designing as the lateral magnification m , see for instance [3].

Notice in practice that the thin lens formula is slightly different from its above classical expression because of the spherical aberration of the lens as well as the effects emerging from dynamics of the environment where the object is situated, viz. the motion of the object plays a considerable hindrance in obtaining an optimized image. In the case of the spherical aberration, the resulting focal length f is determined statistically with all possible optical surfaces. With an effective consideration of an arbitrary optical surface, the focal length of the optical system swings around its mean value f_0 . In practice, under small fluctuation approximation, we consider the net focal length f of the system as $f = f_0 \pm \Delta f$, where Δf arises as noise in the system originating via background fluctuations and associated aberration effects amongst others.

From the perspective of spherical aberration, we observe that the lateral magnification of the lens/ mirror plays an important role in offering an optimized image by swinging the focal length of the optical system. Given a dynamical object, the optimal system is characterized by fluctuations of the focal length f around f_0 , forming an optimized image. In this paper, we concentrate on overcoming such problems concerning environmental dynamics and aberration effects jointly by introducing simultaneous variations in the object distance and lateral magnification of the optical system.

In this setup, an aberration introduces variations Δf in the mean focal length f_0 of the system, resulting a dynamical focal length f . As far as an optical designing in forming an optimized image is concerned, it is worth mentioning that the optimization of the focal length f as considered in this paper results in minimizing the noise Δf arising largely from aberration effects via dynamics of the object amongst others. Indeed, an inclusion of the aberration could introduce extra optimization variables into the system, e.g., refractive index, the corresponding optimization analysis and its industrial developments we leave open for a future research.

In short, given the object distance u and image distance v from the optical center of the lens/ mirror, it follows that the lateral magnification m arises as the real valued ratio

$$m = \frac{v}{u}$$

Physically, we are concentrating on optimized image

formation of a dynamic object via an optical instrument having variable zooming in a definite range. To do so, we wish optimizing the focal length f as a function of the zooming m of the optical instrument and position u of the object, while they both simultaneously fluctuate. Underneath, this is exploited from the framework of the saddle point based fluctuation theory analysis.

3. Optical Optimization

In the light of modern optimization, by considering the focal length f of a given lens or mirror as the objective function f , we study optimization properties of the optical system while f is varied as a function of the object distance u and lateral magnification m . Explicitly, by considering $\{u, m\}$ as a given set of input variables, the objective function f can be expressed as

$$f(u, m) = \frac{um}{1 - m}$$

While varying f with respect to $\{u, m\}$, we find the following pair of flow components

$$\begin{aligned} \frac{\partial f}{\partial u} &= \frac{m}{(1 - m)} \\ \frac{\partial f}{\partial m} &= \frac{u}{(1 - m)^2} \end{aligned}$$

We see that the finite solution of corresponding flow equations $\frac{\partial f}{\partial u} = 0$ and $\frac{\partial f}{\partial m} = 0$ is $m = 0$ and $u = 0$. Herewith, it follows that the origin $(0, 0)$ is the only critical point of the optical objective function f . In the sequel, we examine (in)stabilities undermining the optical objective function f . This is determined via the nature of the extrema of f , which is implemented via the Hessian matrix H defined below, viz. the symmetric matrix of the second order partial derivatives of f .

In this setup, we analyze stability properties of the image thus formed via lenses and mirrors by invoking the notion of stability analysis [10] and associated critical points and concerning scaling hypothesis [11]. Interesting fluctuation theory applications include notions of thermodynamic Ruppeiner geometry [12,13], black branes thermodynamic geometry [14], power networks [15], network reliability and voltage stability [16] and equilibrium thermodynamic configurations and chemical equilibria [17, 18].

In this regard, for a given objective function as the focal length of an optical instrument having zooming while forming the image of a dynamical object, we investigate the stability structures of the image under fluctuations of $\{u, m\}$. Inserting the values of the pure optical capacities $\{\frac{\partial^2 f}{\partial u^2}, \frac{\partial^2 f}{\partial m^2}\}$ and mixed optical capacity $\frac{\partial^2 f}{\partial u \partial m}$ with the help of the above flow components $\{\frac{\partial f}{\partial u}, \frac{\partial f}{\partial m}\}$, the Hessian matrix H [10-18] corresponding to f reads as

$$H = \begin{pmatrix} 0 & \frac{1}{(1 - m)^2} \\ \frac{1}{(1 - m)^2} & \frac{2u}{(1 - m)^3} \end{pmatrix}$$

Under the fixed point analysis, we need to determine the signature of the determinant Δ of H at the stationary point $(u, m) = (0, 0)$ of our optical objective function $f(u, m)$. Having a unique stationary point, we can herewith classify the convexity and concavity of $f(u, m)$ at the fixed point $(0, 0)$ by determining the eigenvalues $\{\lambda_1, \lambda_2\}$ of H , as their product [10] yields the determinant Δ , under joint fluctuations of $\{u, m\}$.

In order to do so, let's consider an eigenvalue λ of H with the corresponding eigenvector $\underline{x}$. Thence, we may express the eigenvalue equation as $H\underline{x} = \lambda\underline{x}$. Thus, given a real eigenvalue λ , the preceding equation is satisfied if for a nonzero two dimensional column vector $\underline{x} \in \mathbb{R}^2$, we have $(H - \lambda I)\underline{x} = 0$, where I is a 2×2 identity matrix. In this concern, the eigenvalues $\{\lambda_1, \lambda_2\}$ are obtained by evaluating the determinant of the above equation, namely, as the solutions of the characteristic equation: $|H - \lambda I| = 0$ for all eigenvectors $\underline{x} \in \mathbb{R}^2$. Thus, with the above H , we see that the eigenvalues $\{\lambda_1, \lambda_2\}$ arise as per the following expressions

$$\begin{aligned} \lambda_1(u, m) &= \frac{u + \sqrt{u^2 + (1 - m)^2}}{(1 - m)^3}, \\ \lambda_2(u, m) &= \frac{u - \sqrt{u^2 + (1 - m)^2}}{(1 - m)^3}. \end{aligned}$$

Herewith, we observe that the optical system possesses a saddle point behavior. This is because of the fact that $\{\lambda_1, \lambda_2\}$ at critical points $(0, 0)$ leads to the criteria: $\lambda_1(0, 0) > 0$ and $\lambda_2(0, 0) < 0$. It is worth mentioning that the global stability of the system is determined by the sign of the determinant Δ of H . Given the local stability factors $\{\lambda_1, \lambda_2\}$, it is not difficult to see that Δ could be expressed as the product $\Delta := \lambda_1 \lambda_2 = -1/(1 - m)^4 < 0$. In short, we find that the optical system remains globally unstable while the input variables $\{u, m\}$ have simultaneous variations. For a real image formation, we require further that both the eigenvalues $\lambda_1, \lambda_2 \in \mathbb{R}$, viz. the inequality $u^2 + m^2 + 1 > 2m$ must be satisfied.

In order to calculate the eigenvectors $\underline{x} \in \mathbb{R}^2$ of the Hessian matrix H , we may in particular consider $\underline{x}$ as the pair $\underline{x} := \begin{pmatrix} u \\ m \end{pmatrix}$, and thereby solve the eigenvalue equation

$$H\underline{x} = \lambda\underline{x}$$

Let $\underline{x} = \underline{x}_1$ be the eigenvector corresponding to the eigenvalue $\lambda = \lambda_1$ and $\underline{x} = \underline{x}_2$ be that of the case for $\lambda = \lambda_2$, by using the above expression of H and eigenvector $\underline{x}$ as a 2×1 vector with the first entry as the object distance u and the second as the lateral magnification m , it follows that we have the following pair of normalized eigenvectors

$$\hat{x}_1 := \frac{x_1}{\|x_1\|} = \frac{1}{\sqrt{1 + \left(\frac{1}{(1-m)^2 \lambda_1}\right)^2}} \begin{pmatrix} \frac{1}{(1-m)^2 \lambda_1} \\ 1 \end{pmatrix},$$

$$\hat{x}_2 := \frac{x_2}{\|x_2\|} = \frac{1}{\sqrt{1 + \left(\frac{1}{(1-m)^2 \lambda_2}\right)^2}} \begin{pmatrix} \frac{1}{(1-m)^2 \lambda_2} \\ 1 \end{pmatrix}.$$

This represents the undermining normalized fluctuation basis vectors as a two dimensional vector of the object distance u and lateral magnification m . Physically, such a vector signifies an optimal amount of the zooming that is required for a dynamical object facing the optical instrument while one wishes to obtain an optimized stable image of the object. Perspective applications of this optimization could yield an improved understanding of the growth of bacteria by analyzing the image of paths of motion via electron microscopes, see [19, 20] for instance.

As a future work of this research, viz. as an application of our optimization model to Newtonian telescope, we wish emphasizing the following illustration concerning its apt optical designing in offering an optimized image. In this regard, our model arises as a special case of the Newtonian telescope when only one of its components amongst the eyepiece and reflecting mirrors is kept dynamical. Indeed, anyone of the optical components amongst the focusers, and primary and secondary mirrors could be chosen to be a dynamical component. Concerning practical applications of this proposal, a generic improvement including the Newtonian telescope when all of their components are dynamical is in future scope. It is worth highlighting that our proposed optimization model continues to hold in an apt material designing of optical systems concerning lenses, mirrors, glass plates and their arbitrary combinations. The detailed analysis of the same is hereby left open for a future research investigation.

4. Result and Conclusion

Optimization plays a vital role in designing optical instruments such as lenses, mirrors and optical fibers in understanding an apt image formation via the ray model of optics. In this paper, we introduce an optical objective function as the focal length of the lens/ mirror while expressed in terms of an administrable set of input variables, viz. the distance of the object from its optical center and the corresponding zooming views as the lateral magnification of the instrument. By means of the multivariable calculus, we thus examine stability qualifications of the image of an arbitrary dynamic object under variation of zooming in the setup of optimization theory.

Under this consideration, our results demonstrate that images formed via concave and convex type lenses/ mirrors lie in a separate domain of optimization variables. We further find that the saddle point based fluctuation theory analysis indicates instabilities at a discrete set of the values of the optimization variables. Namely, the local and global stabilities are respectively determined by the positivity of

the pure fluctuation components and the product of eigenvalues of the fluctuation matrix of the optical objective function. It follows further that our conclusion thus achieved for unconstrained optical systems holds additionally for both the constantly and linearly constrained optical systems with the exception of a translation in the extrema of the underlying optical objective function for the linearly constrained case.

As addressed in this paper, following the notion of our optimization procedure of the focal length of an optical instrument as a function of the object distance and lateral magnification, it appears worth extending our analysis towards optical optimizations with constraints. Practical applications involving multi-lens optical systems are subject matter of our future research. Indeed, our formulation also extends for lenses, mirrors, and their combinations in offering an optimized power of an arbitrary optical system with multiple components, as well. Optimization of such nonlinear optical systems is of extraordinary industrial importance in order for offering ramifications in designing of modern optical instruments. Further engineering applications of our proposed optical objective function are anticipated in enhancing the quality of lenses, mirrors and systems with their finite combinations. Some examples include an optimal focusing of cold neutrons with multiple biconcave lenses in the limit of small-angle scattering [19], an exact modeling of Leica microsystems [20] and supervision of vehicle exterior mirror with signal light [21]. Such subject matters of investigations, we leave open for a future research development.

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