True Navier–Stokes Vector PDE

Alexandr Kozachok

Kiev, Ukraine

Copyright © 2015 Horizon Research Publishing All rights reserved.

Abstract The additional equalities (additional differential equations) for the Navier – Stokes and other vector PDE are established in this paper. These equations are obligatory requirements (properties) of three functions forming a vector field on Euclidean space. Therefore all solutions of the vector PDE should satisfy these requirements. Without these equalities the Navier–Stokes equations with so called a continuity equation are underdetermined as vector system and any “exact solution” is not solution of the true Navier – Stokes vector equation.

Keywords Vector Field, Vector Lines Equations, Vector Functions, Vector Equations, Functions Forming Vector Field, Incompressible Fluid, Navier – Stokes Equations, Acceleration Divergence, Partial Derivatives, Composite Function

Here, $\vec{F} = \vec{F}_1 + \vec{F}_2 + \dots$ - vectors sum of the given, externally applied forces (e.g. gravity $\vec{F}_1$, magnetic $\vec{F}_2$ and other), p - pressure, $\vec{u}$ - velocity vector, $\vec{\ddot{u}} = d\vec{\dot{u}}/dt$ - acceleration vector, ρ - density, μ - viscosity, ∇^2 - Laplace operator.

The component form of (1) can be written so

$$\begin{aligned} \rho F_x - \frac{\partial p}{\partial x} + \mu \nabla^2 \dot{u}_x &= \rho \left(\frac{\partial \dot{u}_x}{\partial t} + \dot{u}_x \frac{\partial \dot{u}_x}{\partial x} + \dot{u}_y \frac{\partial \dot{u}_x}{\partial y} + \dot{u}_z \frac{\partial \dot{u}_x}{\partial z} \right), \\ \rho F_y - \frac{\partial p}{\partial y} + \mu \nabla^2 \dot{u}_y &= \rho \left(\frac{\partial \dot{u}_y}{\partial t} + \dot{u}_x \frac{\partial \dot{u}_y}{\partial x} + \dot{u}_y \frac{\partial \dot{u}_y}{\partial y} + \dot{u}_z \frac{\partial \dot{u}_y}{\partial z} \right), \\ \rho F_z - \frac{\partial p}{\partial z} + \mu \nabla^2 \dot{u}_z &= \rho \left(\frac{\partial \dot{u}_z}{\partial t} + \dot{u}_x \frac{\partial \dot{u}_z}{\partial x} + \dot{u}_y \frac{\partial \dot{u}_z}{\partial y} + \dot{u}_z \frac{\partial \dot{u}_z}{\partial z} \right). \end{aligned} \quad (3)$$

The NSE in this form are so called scalar equations for a fixed time $t = \bar{t}$. The form of these equations do not depends on any displacement and rotation of the Cartesian coordinates x, y, z . In these equations each unknown of four $p, \dot{u}_x, \dot{u}_y, \dot{u}_z$ is only the mathematical symbol of different variables. The unknown p can depend of the coordinate system displacement and should not depend of any system rotation. It means that unknown variable $p = p(x, y, z, \bar{t})$ constructs the so called scalar fields. The unknowns $\dot{u}_x, \dot{u}_y, \dot{u}_z$ can depend of any displacement and also of the coordinate system rotation. It means that unknown variables $\dot{u}_x, \dot{u}_y, \dot{u}_z$ construct the vector function or so called vector field. Therefore we should impose any requirements on the variables $\dot{u}_x = f_x(x, y, z, \bar{t}), \dot{u}_y = f_y(x, y, z, \bar{t}), \dot{u}_z = f_z(x, y, z, \bar{t})$

because “...not any three functions $f_i(x, y, z)$ can construct the vector field” [2, p. 46, only in ed. 1970]. In this university textbook [3, p. 142] we can read “it is other kind of the tensor sometimes called the covector...” and also “under co-ordinate change the gradient of function transforms differently from a vector” [3, p.30]. Therefore the gradient of scalar function does not construct a true vector field. Also, in previous article [4] we have established that $\text{rot}A$ (also known curl A) is not a true vector function. It is known [1, p. 183; 2, p.50; 3, p. 220] that $\text{rot}A$ (sometime called pseudovector) is equivalent to the antisymmetric

1. Introduction

1.1. Article’s Aim

The main aim of this paper is to prove that the well-known Navier–Stokes equations (NSE) with so called a continuity equation are underdetermined as vector system. To identify vector’s components the additional conditions are needed. Without additional conditions the velocity vector components can be considered as usual scalar functions. Therefore the NSE in component form is usually called “scalar equations” and three velocity components are called “scalar velocities”.

1.2. General Data

The introduction in this problem we will consider on an example of well-known Navier–Stokes equations (NSE). The NSE vector form are given by

$$\rho \vec{F} - \text{grad } p + \mu \nabla^2 \vec{\dot{u}} = \rho \vec{\ddot{u}} \quad (1)$$

with a continuity equation for incompressible fluids [1, p. 174]

$$\text{div } \vec{\dot{u}} = \frac{\partial \dot{u}_x}{\partial x} + \frac{\partial \dot{u}_y}{\partial y} + \frac{\partial \dot{u}_z}{\partial z} = 0. \quad (2)$$

tensor (more strictly, the components of pseudovector rotA are equivalent to the antisymmetric tensor components). Note, these little-known representations in university textbooks [2, 3] contradict to long-established traditional representations in other textbooks. For instance in [1, p. 120], we can read "...any three quantities P, Q, R can be treated as vector components" and probably therefore vector field can be constructed out of scalar fields using the gradient operator [5, p. 316]. As follows from [2, p. 46; 3, p. 30] these classical representations are doubtful. Later we will strictly prove that these representations are wrong.

2. Unknown Properties of Vector Function

2.1. Proofs of Main Results

To prove that not any three functions can construct true vector field we use the vector lines (streamlines) equations [1, p. 41; 2, p. 57; 5, p. 318]

$$\frac{dx}{\dot{u}_x} = \frac{dy}{\dot{u}_y} = \frac{dz}{\dot{u}_z} = d\zeta \Rightarrow \frac{dx}{\dot{u}_x} = d\zeta, \frac{dy}{\dot{u}_y} = d\zeta, \frac{dz}{\dot{u}_z} = d\zeta \quad (4)$$

$$\Rightarrow \frac{dx}{\dot{u}_x} = \frac{dy}{\dot{u}_y}, \frac{dy}{\dot{u}_y} = \frac{dz}{\dot{u}_z}$$

Here, ζ - scalar parameter, d - differential of x, y, z for fixed time $t = \bar{t}$.

Note that equations (4) have sense if and only if the variables $\dot{u}_x, \dot{u}_y, \dot{u}_z$ are the true vector components. Therefore we can say that these equations identify the vector components in NSE (3) and other PDE of mathematical physics. Without equations (4) the NSE system (3) are not true vector system because the variables $\dot{u}_x, \dot{u}_y, \dot{u}_z$ can be considered as usual scalar functions. Also, equations (4) are not independent. As we can see $d\zeta$ can be eliminated and we obtain only two equations. This fact allows suppose that $\dot{u}_x, \dot{u}_y, \dot{u}_z$ can not be any independent functions.

Equations (4) can now be written in the form

$$\begin{aligned} \zeta &= \int \frac{1}{\dot{u}_x} dx + C_x \Rightarrow \zeta = F_x(x, y, z, \bar{t}), \\ \zeta &= \int \frac{1}{\dot{u}_y} dy + C_y \Rightarrow \zeta = F_y(x, y, z, \bar{t}), \\ \zeta &= \int \frac{1}{\dot{u}_z} dz + C_z \Rightarrow \zeta = F_z(x, y, z, \bar{t}). \end{aligned} \quad (5)$$

Three expressions $\zeta = F_i(x, y, z, \bar{t})$ can be considered as any algebraic equations system. This system can be solved in such form $x = x(\zeta, \bar{t})$, $y = y(\zeta, \bar{t})$, $z = z(\zeta, \bar{t})$. After substitution of these expressions into $\vec{u} = \vec{u}(x, y, z, \bar{t})$ we will obtain $\dot{u}_i = \dot{u}_i(\zeta, \bar{t})$.

From equations (5) these obvious equalities can be

written

$$\begin{aligned} \int \frac{1}{\dot{u}_x} dx + C_x &= \int \frac{1}{\dot{u}_y} dy + C_y, \quad \int \frac{1}{\dot{u}_x} dx + C_x = \int \frac{1}{\dot{u}_z} dz + C_z, \\ \int \frac{1}{\dot{u}_y} dy + C_y &= \int \frac{1}{\dot{u}_z} dz + C_z \end{aligned}$$

From these equalities we can see that $\dot{u}_i = \dot{u}_i(x, y, z, \bar{t})$ can not be any functions. This result confirms that "...not any three functions $f_i(x, y, z)$ can construct the vector field" [2, p. 46].

From (4) represented in the form

$$\begin{aligned} \frac{dx}{d\zeta} &= \dot{u}_x(x, y, z, \bar{t}), \\ \frac{dy}{d\zeta} &= \dot{u}_y(x, y, z, \bar{t}), \\ \frac{dz}{d\zeta} &= \dot{u}_z(x, y, z, \bar{t}) \end{aligned} \quad (6)$$

we can receive without any proof the same result $\dot{u}_i = \dot{u}_i(\zeta, \bar{t})$ or simply $\dot{u}_i = \dot{u}_i(\zeta)$. As we well know from [5, p. 65-73], the solutions of such equations can be written so $x = x(\zeta, \bar{t})$, $y = y(\zeta, \bar{t})$, $z = z(\zeta, \bar{t})$. This representation is well known as vector function of scalar argument [6, p. 514] or vector-valued function.

We can now state the more detailed properties of functions forming the vector field. To define these properties we use partial derivatives of a composite function, which can be given by only one auxiliary variable [6, p. 644]. The velocity vector $\vec{u} = \vec{u}(x, y, z, \bar{t})$ is such composite function $\vec{u} = \vec{u}(\zeta(x, y, z, \bar{t}), \bar{t})$. We take into account that basic properties of the derivatives are maintained for the vectors [6, p. 516; 7, p. 79]. Then

$$\frac{\partial \vec{u}}{\partial x_i} = \frac{\partial \vec{u}}{\partial \zeta} \frac{\partial \zeta}{\partial x_i}, (x_i = x, y, z). \quad (7)$$

Note that formulas (7) can be written explicitly concerning a common factor $\frac{\partial \vec{u}}{\partial \zeta}$. Therefore this factor can be eliminated. As a result we have

$$\frac{\partial \vec{u}}{\partial x_i} = \frac{\partial \vec{u}}{\partial x_j} \frac{\partial \zeta / \partial x_i}{\partial \zeta / \partial x_j}. \quad (8)$$

In component form formulas (8) look like

$$\begin{aligned} \frac{\partial \dot{u}_x}{\partial x_i} &= \frac{\partial \dot{u}_x}{\partial x_j} \frac{\partial \zeta / \partial x_i}{\partial \zeta / \partial x_j}, \quad \frac{\partial \dot{u}_y}{\partial x_i} = \frac{\partial \dot{u}_y}{\partial x_j} \frac{\partial \zeta / \partial x_i}{\partial \zeta / \partial x_j}, \\ \frac{\partial \dot{u}_z}{\partial x_i} &= \frac{\partial \dot{u}_z}{\partial x_j} \frac{\partial \zeta / \partial x_i}{\partial \zeta / \partial x_j} \end{aligned} \quad (9)$$

Note that relations (9) can be written explicitly concerning common factor $\frac{\partial \zeta / \partial x_i}{\partial \zeta / \partial x_j}$. Therefore this common

factor can be eliminated if we will equate right-hand sides of transformed expressions. Thus, we obtain

$$\begin{aligned} \frac{\partial \dot{u}_x}{\partial x_i} \frac{\partial \dot{u}_y}{\partial x_j} &= \frac{\partial \dot{u}_x}{\partial x_j} \frac{\partial \dot{u}_y}{\partial x_i}, \quad \frac{\partial \dot{u}_x}{\partial x_i} \frac{\partial \dot{u}_z}{\partial x_j} = \frac{\partial \dot{u}_x}{\partial x_j} \frac{\partial \dot{u}_z}{\partial x_i}, \\ \frac{\partial \dot{u}_y}{\partial x_i} \frac{\partial \dot{u}_z}{\partial x_j} &= \frac{\partial \dot{u}_y}{\partial x_j} \frac{\partial \dot{u}_z}{\partial x_i} \end{aligned} \quad (9^*)$$

Let's substitute $x_i = x, y, z$, ($x_j \neq x_i$) into (9*). Finally, we obtain

$$\begin{aligned} \frac{\partial \dot{u}_x}{\partial x} \frac{\partial \dot{u}_y}{\partial y} &= \frac{\partial \dot{u}_x}{\partial y} \frac{\partial \dot{u}_y}{\partial x}, \quad \frac{\partial \dot{u}_x}{\partial x} \frac{\partial \dot{u}_z}{\partial z} = \frac{\partial \dot{u}_x}{\partial z} \frac{\partial \dot{u}_z}{\partial x}, \\ \frac{\partial \dot{u}_y}{\partial y} \frac{\partial \dot{u}_z}{\partial z} &= \frac{\partial \dot{u}_y}{\partial z} \frac{\partial \dot{u}_z}{\partial y} \end{aligned} \quad (9^{**})$$

At first this result has been received in [8]. As you can see equalities (9**) are not independent, rather third equality implies from first and second equalities. The same result we have seen for vector lines equations (4). Below (section 4. Example) we will show that equalities (9**) can be obtained more simply and visibly (though not so strictly as it is here).

2.2. Important Notes and Links

As we can see, equalities (9*) are additional differential equations. These equations are properties (obligatory requirements) of three functions which can construct the true vector field. Therefore all exact vector PDE solutions should satisfy these requirements. Otherwise without requirements (9*), all “exact solutions” are not solutions of the vector equations. For instance, NSE solutions can make sense only if $\text{rot } \vec{u} \neq 0$ because from (9*) we have

$$\frac{\partial \dot{u}_i}{\partial x_i} \frac{\partial \dot{u}_j}{\partial x_j} = \frac{\partial \dot{u}_i}{\partial x_j} \frac{\partial \dot{u}_j}{\partial x_i}$$

In the case $\text{rot } \vec{u} = 0$ we can see that

$$\frac{\partial \dot{u}_i}{\partial x_j} = \frac{\partial \dot{u}_j}{\partial x_i} \Rightarrow \frac{\partial \dot{u}_i}{\partial x_i} \frac{\partial \dot{u}_j}{\partial x_j} = \left(\frac{\partial \dot{u}_i}{\partial x_j} \right)^2 \Rightarrow \frac{\partial \dot{u}_i}{\partial x_i} \frac{\partial \dot{u}_j}{\partial x_j} \geq 0$$

Let's consider these two cases. For first case we have

$$\begin{aligned} \frac{\partial \dot{u}_i}{\partial x_i} \frac{\partial \dot{u}_j}{\partial x_j} > 0 &\Rightarrow \frac{\partial \dot{u}_x}{\partial x} > 0, \quad \frac{\partial \dot{u}_y}{\partial y} > 0, \\ \frac{\partial \dot{u}_z}{\partial z} > 0 \text{ or } \frac{\partial \dot{u}_x}{\partial x} < 0, \quad \frac{\partial \dot{u}_y}{\partial y} < 0, \quad \frac{\partial \dot{u}_z}{\partial z} < 0 \end{aligned}$$

As follows from continuity equation (2), such case is impossible.

Let's consider next case

$$\frac{\partial \dot{u}_i}{\partial x_i} \frac{\partial \dot{u}_j}{\partial x_j} = 0 \Rightarrow \frac{\partial \dot{u}_x}{\partial x} = 0, \quad \frac{\partial \dot{u}_y}{\partial y} = 0, \quad \frac{\partial \dot{u}_z}{\partial z} = 0$$

For all directions of coordinates, this case is impossible. Thus, the requirements $\text{rot } \vec{u} = 0$, $\text{div } \vec{u} = 0$ are inconsistent for true vector function. Therefore vector fields can not be constructed out of scalar fields using the gradient operator [9, p. 149-153], and so-called Laplacian field [9, p. 214] is

not a true vector field. This result confirms the conclusions “it is other kind of the tensor sometimes called the covector...” [3, p. 142] and the little-known proof in other university textbook [10, p. 100-101] about impossibility of an irrotational velocity field for a viscous incompressible fluid.

2.3. Digital Illustration

To deny first work [8,] and formulas (9**) the counterexamples are suggested by some anonymous reviewers and mathematicians (on Web-pages). Let's consider one of these counterexamples. Suppose $\vec{u} = (y, x, 0)$. It means $\dot{u}_x = y$, $\dot{u}_y = x$, $\dot{u}_z = 0$. From first equality (9**) we have $0=1$. Also, note $\text{rot } \vec{u} = 0$, $\text{div } \vec{u} = 0$. These results contradict to above example (considered in section 2.2) and show $\vec{u} = (y, x, 0)$ is not the true vector function.

To confirm formulas (9**) the counterexamples are also suggested by mathematicians (on Web-pages). One of these alternative counterexamples look like $\dot{u}_x = F(z)$, $\dot{u}_y = G(x)$, $\dot{u}_z = H(y)$. In this counterexample equalities (9**) are satisfied. Also $\text{rot } \vec{u} \neq 0$, $\text{div } \vec{u} = 0$.

3. True Vector Equations System

The complete system (as system for vector field completion) should contain together three equations systems (2), (3), (4).

Without vector lines equations (4) this system is not vector one. After above transformation of the last equations this system can be represented so

$$\begin{aligned} \rho F_x - \frac{\partial p}{\partial x} + \mu \nabla^2 \dot{u}_x &= \rho \left(\frac{\partial \dot{u}_x}{\partial t} + \dot{u}_x \frac{\partial \dot{u}_x}{\partial x} + \dot{u}_y \frac{\partial \dot{u}_x}{\partial y} + \dot{u}_z \frac{\partial \dot{u}_x}{\partial z} \right), \\ \rho F_y - \frac{\partial p}{\partial y} + \mu \nabla^2 \dot{u}_y &= \rho \left(\frac{\partial \dot{u}_y}{\partial t} + \dot{u}_x \frac{\partial \dot{u}_y}{\partial x} + \dot{u}_y \frac{\partial \dot{u}_y}{\partial y} + \dot{u}_z \frac{\partial \dot{u}_y}{\partial z} \right), \\ \rho F_z - \frac{\partial p}{\partial z} + \mu \nabla^2 \dot{u}_z &= \rho \left(\frac{\partial \dot{u}_z}{\partial t} + \dot{u}_x \frac{\partial \dot{u}_z}{\partial x} + \dot{u}_y \frac{\partial \dot{u}_z}{\partial y} + \dot{u}_z \frac{\partial \dot{u}_z}{\partial z} \right), \\ \frac{\partial \dot{u}_x}{\partial x} + \frac{\partial \dot{u}_y}{\partial y} + \frac{\partial \dot{u}_z}{\partial z} &= 0, \\ \frac{\partial \dot{u}_x}{\partial x} \frac{\partial \dot{u}_y}{\partial y} &= \frac{\partial \dot{u}_x}{\partial y} \frac{\partial \dot{u}_y}{\partial x}, \\ \frac{\partial \dot{u}_x}{\partial x} \frac{\partial \dot{u}_z}{\partial z} &= \frac{\partial \dot{u}_x}{\partial z} \frac{\partial \dot{u}_z}{\partial x}, \\ \frac{\partial \dot{u}_y}{\partial y} \frac{\partial \dot{u}_z}{\partial z} &= \frac{\partial \dot{u}_y}{\partial z} \frac{\partial \dot{u}_z}{\partial y} \end{aligned} \quad (10)$$

Note that last equality in (10) follows from two above. However the true utility of this overdetermined formulation is seen when we try to guess the form of NSE solutions. Let's recollect that without last equalities we can not receive the true solutions of vector equations. Therefore all so called NSE exact solutions need attentive check. As we can see from last equality in (10), for example, the well-known Couette flow (with or without pressure gradient) and

probably some other simple solutions are exact solutions.

4. Examples of Alternative Proofs

As we well know the vector field divergence on Euclidean space is the scalar function (or scalar field). Therefore, let's calculate the divergence of velocity vector acceleration $\text{div } \ddot{\vec{u}}$. Then after some transformations

$$\begin{aligned} \text{div } \ddot{\vec{u}} &= \frac{\partial \ddot{u}_x}{\partial x} + \frac{\partial \ddot{u}_y}{\partial y} + \frac{\partial \ddot{u}_z}{\partial z} = \frac{\partial}{\partial t} \text{div } \dot{\vec{u}} + \dot{u}_x \frac{\partial}{\partial x} \text{div } \dot{\vec{u}} + \\ &+ \dot{u}_y \frac{\partial}{\partial y} \text{div } \dot{\vec{u}} + \dot{u}_z \frac{\partial}{\partial z} \text{div } \dot{\vec{u}} + \left(\frac{\partial \dot{u}_x}{\partial x} \right)^2 + \left(\frac{\partial \dot{u}_y}{\partial y} \right)^2 + \left(\frac{\partial \dot{u}_z}{\partial z} \right)^2 + \\ &+ 2 \left(\frac{\partial \dot{u}_x}{\partial y} \frac{\partial \dot{u}_y}{\partial x} + \frac{\partial \dot{u}_x}{\partial z} \frac{\partial \dot{u}_z}{\partial y} + \frac{\partial \dot{u}_y}{\partial z} \frac{\partial \dot{u}_z}{\partial x} \right). \end{aligned} \quad (11)$$

Now we restrict attention to the right hand side of (11). This formula can be written so

$$\text{div } \ddot{\vec{u}} = \frac{d}{dt} \text{div } \dot{\vec{u}} + (\text{div } \dot{\vec{u}})^2 \quad (12)$$

if such equality is true

$$\begin{aligned} &\left(\frac{\partial \dot{u}_x}{\partial x} \right)^2 + \left(\frac{\partial \dot{u}_y}{\partial y} \right)^2 + \left(\frac{\partial \dot{u}_z}{\partial z} \right)^2 + \\ &+ 2 \left(\frac{\partial \dot{u}_x}{\partial y} \frac{\partial \dot{u}_y}{\partial x} + \frac{\partial \dot{u}_x}{\partial z} \frac{\partial \dot{u}_z}{\partial y} + \frac{\partial \dot{u}_y}{\partial z} \frac{\partial \dot{u}_z}{\partial x} \right) = (\text{div } \dot{\vec{u}})^2 \end{aligned} \quad (13)$$

As we can see equality (13) is possible if

$$\begin{aligned} \frac{\partial \dot{u}_x}{\partial x} \frac{\partial \dot{u}_y}{\partial y} &= \frac{\partial \dot{u}_x}{\partial y} \frac{\partial \dot{u}_y}{\partial x}, \quad \frac{\partial \dot{u}_x}{\partial x} \frac{\partial \dot{u}_z}{\partial z} = \frac{\partial \dot{u}_x}{\partial z} \frac{\partial \dot{u}_z}{\partial x}, \\ \frac{\partial \dot{u}_y}{\partial y} \frac{\partial \dot{u}_z}{\partial z} &= \frac{\partial \dot{u}_y}{\partial z} \frac{\partial \dot{u}_z}{\partial y} \end{aligned} \quad (14)$$

In this case we really have formula (12). Therefore $\text{div } \ddot{\vec{u}}$ is the scalar function if equalities (14) are true. Other words expression $\text{div } \ddot{\vec{u}}$ (11) is the scalar function (12) if $\dot{u}_x, \dot{u}_y, \dot{u}_z$ are true vector components and satisfy equalities (14). Note that equalities (14) really are equalities (9**) but we have obtained (14) more simply by an intuition way. Also, in [8, section 3.3] we can see the alternative proof of equation (12) from main statements of Continuum Mechanics. Therefore these three independent proofs confirm that equalities (14) and complete NSE representation (10) are correct.

5. Conclusions

In this paper we have established that the well-known Navier–Stokes equations (NSE) with so called a continuity equation are underdetermined as vector system. Without vector lines (streamlines) equations (4) NSE systems (2), (3) are determined incompletely. Therefore we can say that the

vector lines equations identify the vector components in NSE (3) and other PDE of mathematical physics. Without equations (4) NSE system (3) are not true vector system because the variables $\dot{u}_x, \dot{u}_y, \dot{u}_z$ are not identified as vector components and can be considered as usual scalar functions. From vector lines equations (4) we can strongly find that any vector function on Euclidean space $\vec{u} = \vec{u}(x, y, z)$ can be represented as $\vec{u} = \vec{u}(\zeta)$, $\zeta = \zeta(x, y, z)$. This representation is well known as a vector function of scalar argument (also called vector-valued function) [6, p. 514]. According this representation we obtain equalities (9**).

Equalities (9**) and systems (2), (3) written together are determined as vector system and looks like (10). Let's recollect that without last equalities (9**) we can not receive the true NSE solutions. Therefore any well-known “3D NSE exact solutions” need attentive check. However even any true exact solutions of the vector equations (10) can appear wrong in case of not co-ordinated or not smoothed initial and boundary conditions (about this important problem for the wave equation read [11, p. 63-83]).

REFERENCES

- [1] L.I. Sedov. Mechanics of Continuous Media, v. 1. Textbook, Science, Moscow, 1970 http://eqworld.ipmnet.ru/ru/library/books/Sedov_MSS_t1_1970ru.djvu. (Russian) ISBN-13: 978-9971507282 (English)
- [2] L.G. Lojtsjansky. Fluid and Gas Mechanics. Manual, Science, Moscow, (only in edition 1970year) <http://eqworld.ipmnet.ru/ru/library/books/Lojcyanskij1950ru.djvu> (Russian)
- [3] B.A. Dubrovin, S. P. Novikov, A.T. Fomenko Modern geometry. Science, Moscow, 1986 [http://www.newlibrary.ru/download/dubrovin_b_a_novikov_s_p_fomenko_a_t_sovremennaja_geometrija\(metod_y_i_prilozhenija\).html](http://www.newlibrary.ru/download/dubrovin_b_a_novikov_s_p_fomenko_a_t_sovremennaja_geometrija(metod_y_i_prilozhenija).html). (Russian) Modern Geometry--methods and Applications: The geometry of surfaces, transformation groups, and fields (2nd ed.). Springer. 1992 ISBN 0387976639 http://books.google.com.ua/books?id=FC0QFlx12pwC&pg=PA15&redir_esc=y#v=onepage&q&f=false (English)
- [4] Alexandr Kozachok. Navier – Stokes Second Exact Transformation, Universal Journal of Applied Mathematics Vol. 2(3), pp.136-140. DOI:10.13189/ujam.2014.020303 <http://www.hrpub.org/download/20140205/UJAM3-12601996.pdf> (English)
- [5] V.I. Smirnov. Higher Mathematics Course, v. 2 (21th edition). Textbook, Science, Moscow, 1974 http://eqworld.ipmnet.ru/ru/library/books/Smirnov_t2_1974ru.djvu (Russian)
- [6] M.J. Vygodsky. Manual on Higher Mathematics, (12th edition). Science, 1977 (Russian) <http://eqworld.ipmnet.ru/ru/library/books/Vygodskij1977ru.djvu>. ASIN: B001U5VF9O (English)

- [7] N.E. Kochin Vector Calculus and beginnings of Tensors Calculus (9th edition). Science, Moscow, 1965 <http://eqworld.ipmnet.ru/ru/library/books/Kochin1965ru.djvu> (Russian)
- [8] Alexandr Kozachok. Navier –Stokes First Exact Transformation, Universal Journal of Applied Mathematics, 1, 157 - 159. doi: 10.13189/ujam.2013.010301, 2013 <http://www.hrpub.org/download/20131107/UJAM1-12600416.pdf> (English)
- [9] A.I. Borisenko, I.E. Tarapov. Vector Analysis and beginnings of Tensors Calculus (3th edition). Manual, Higher school, Moscow. 1966 (Russian) <http://eqworld.ipmnet.ru/ru/library/books/BorisenkoTarapov1966ru.djvu> . ISBN-13: 978-0486638331(English)
- [10] N.A. Slezkin. Dynamics of the Viscous Incompressible Fluids. Textbook, State publishing house, Moscow, 1955 <http://eqworld.ipmnet.ru/ru/library/books/Slezkin1955ru.djvu> (Russian)
- [11] Alexandr Kozachok. (2009) Paradoxes of Continuum Mechanics and contiguous fields of knowledge. Section 1. New approaches to statements and solutions of some classical Mathematical Physics problems . Manual, VDM Verlag Dr. Muller, ISBN 978-3639135954 <http://www.shelfari.com/books/13880604/PARADOXES-of-CONTINUUM-MECHANICS-and-contiguous-fields-of-knowledge>