Developing a Mathematical Model to Predict the Optimum Friction Phase Parameters for Friction Welding of High Speed Steel to Medium Carbon Steel

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Abstract  This work was carried out in order to optimize the friction phase parameters, of friction welding of M2 high speed steel, to AISI 1040 medium carbon steel, namely; rotational speed, friction pressure and friction time. The experiments were designed as per Taguchi method. The optimization of the experimentally obtained results was carried out by trying three mathematical models, namely; a multiple linear regression model without interaction effect, a multiple linear regression model with interactions effect, and a second-order polynomial regression model. The three models were evaluated using the experimental data, the coefficient of multiple determination R², and Standard error of the regression (S), were used as the evaluation criteria of the models. The polynomial model was chosen, and optimized using a Genetic Algorithm. The optimal value of the joint strength of 411 MPa was obtained at the highest value of the time (44.9 sec.) and the pressure of 112 MPa and the speed of 1349 r.p.m.

Keywords  Friction Welding, Taguchi Method, Regression Model, Coefficient of Multiple Determination R², Standard Error of Regression (S)

1. Introduction

High-speed steel HSS is one of the main tool materials. In order to save expensive steel, the cutting tool is produced as bimetallic: the working part is produced from the HSS steel, the tail part is produced from medium carbon steel. The joints between the working and tail parts are produced by friction welding which is the most productive and economically efficient process. However, this method of joining components is associated with a number of difficulties reducing the strength of the welded joint. In particular, this is associated with the presence of defects in the form of shiny slip bands on the side of the high-speed steel and a ferritic interlayer on the side of structural steel[1]. In conventional friction welding, the main parameters of the welding conditions are the speed of rotation, the extension of the components, the welding allowance, welding time, heating and forging force and time [2]. In his work titled "An Experimental Investigations On Friction Welding Of Dissimilar Metals" A.B Abdelsalam et.al [3] used a modified lathe machine as a direct drive friction welding machine to weld specimens of high speed steel, to medium carbon steel. The specimens were welded at different friction pressures, and different friction times, then heat treated. All the specimens were subjected to tensile tests. The study revealed that, a satisfactory joint efficiency was obtained by welding of high speed steel to carbon steel, the joint efficiency came to be about 47%. This work is aimed at developing a mathematical model for results obtained experimentally, then optimizing this model in order to find the optimum friction phase parameters that maximize the tensile strength.

ASSUMPTIONS: In continuous friction welding, the main parameters of the welding conditions are rotational speed, friction pressure and friction time in addition to forging force and time. In this study, the forging force, and time were assumed to have a minor effect on the joint strength, and were fixed to a forging force of 37.5 kN (resulting in a forging pressure of 187 Mpa) and a forging time of 15 seconds. The mentioned values of forging force and time are based on a previous study, carried out by A.B Abdelsalam et.al [3].

2. Experimental Work

Friction welding was carried out to joint M2 high speed steel and AISI 1040 carbon steel. Friction phase is affected by three factors (parameters): rotational speed, friction pressure and friction time. The three factors were chosen at three levels as shown in Table (1). The experiment was
designed to investigate the achievement of the optimal strength of the joint (Y). The experiment was designed, based on Taguchi method [4] with 3 replications. The design and the results are shown in Table (2). The experiments were carried out by modified lathe machine to work as continuous friction welding machine. The axial friction pressure was carried out by modified lathe machine to work as continuous and the results are shown in Table (2). The experiments were designed, and thereby to facilitate the optimization of the friction welding process. With these mathematical models, the objective function and process constraints can be formulated, and the optimization problem can then be solved by using Evolutionary Algorithms. The linear models can be expressed as follows: [5]

1. Model 1: Without interaction effect the multiple linear regression models is:
   \[ Y (X_1, X_2, X_3) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \epsilon \]

2. Model 2: With interaction effect the multiple linear regression models is:
   \[ Y (X_1, X_2, X_3) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_12 X_1 X_2 + \beta_13 X_1 X_3 + \beta_23 X_2 X_3 + \beta_123 X_1 X_2 X_3 + \epsilon \]

3. Model 3: The polynomial regression second-order mean function is given by
   \[ Y (X_1, X_2, X_3) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_12 X_1 X_2 + \beta_13 X_1 X_3 + \beta_23 X_2 X_3 + \beta_123 X_1 X_2 X_3 + \beta_111 X_1^2 + \beta_222 X_2^2 + \beta_333 X_3^2 + \epsilon \]

Where: \( \beta_i \) = coefficients, and obtained by means of LEAST SQUARE
\( \epsilon = \) Residual or Error, and is the difference between the fitted value and the predicted value

ANOVA: The general ANOVA of regression is shown in Table (3)

### Table 1. Factors and their levels and values

<table>
<thead>
<tr>
<th>Factors</th>
<th>code</th>
<th>Levels</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotational Speed</td>
<td>X1</td>
<td>1000</td>
<td>Rpm</td>
</tr>
<tr>
<td>Friction Time</td>
<td>X2</td>
<td>25</td>
<td>Second</td>
</tr>
<tr>
<td>Friction Pressure</td>
<td>X3</td>
<td>62.5</td>
<td>M Pa</td>
</tr>
</tbody>
</table>

### Table 2. the design and the results of the experiments

<table>
<thead>
<tr>
<th>X1</th>
<th>X2</th>
<th>X3</th>
<th>Ultimate Tensile Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1000</td>
<td>25</td>
<td>338.232 298.44 298.44</td>
</tr>
<tr>
<td>2</td>
<td>1000</td>
<td>25</td>
<td>109.428 363.102 89.532*</td>
</tr>
<tr>
<td>3</td>
<td>1000</td>
<td>25</td>
<td>258.648 258.648 363.102</td>
</tr>
<tr>
<td>4</td>
<td>1000</td>
<td>35</td>
<td>288.492 288.492 258.648</td>
</tr>
<tr>
<td>5</td>
<td>1000</td>
<td>35</td>
<td>258.648 174.09 218.856</td>
</tr>
<tr>
<td>6</td>
<td>1000</td>
<td>35</td>
<td>348.18 353.154 323.31</td>
</tr>
<tr>
<td>7</td>
<td>1000</td>
<td>45</td>
<td>238.752 139.272* 248.7</td>
</tr>
<tr>
<td>8</td>
<td>1000</td>
<td>45</td>
<td>407.868 149.22* 437.712</td>
</tr>
<tr>
<td>9</td>
<td>1000</td>
<td>45</td>
<td>392.946 333.258 343.206</td>
</tr>
<tr>
<td>10</td>
<td>1400</td>
<td>25</td>
<td>248.7 298.44 253.674</td>
</tr>
<tr>
<td>11</td>
<td>1400</td>
<td>25</td>
<td>358.128 353.154 313.362</td>
</tr>
<tr>
<td>12</td>
<td>1400</td>
<td>25</td>
<td>389.012* - 338.232</td>
</tr>
<tr>
<td>13</td>
<td>1400</td>
<td>35</td>
<td>328.284 397.92 358.128</td>
</tr>
<tr>
<td>14</td>
<td>1400</td>
<td>35</td>
<td>64.662* 149.22 238.752</td>
</tr>
<tr>
<td>15</td>
<td>1400</td>
<td>35</td>
<td>333.258 397.92 104.454*</td>
</tr>
<tr>
<td>16</td>
<td>1400</td>
<td>45</td>
<td>363.102 353.154 358.128</td>
</tr>
<tr>
<td>17</td>
<td>1400</td>
<td>45</td>
<td>407.868 492.426 502.374</td>
</tr>
<tr>
<td>18</td>
<td>1400</td>
<td>45</td>
<td>437.712 397.92 432.738</td>
</tr>
<tr>
<td>19</td>
<td>2000</td>
<td>25</td>
<td>189.012 174.09 109.428</td>
</tr>
<tr>
<td>20</td>
<td>2000</td>
<td>25</td>
<td>358.128 353.154 308.388</td>
</tr>
<tr>
<td>21</td>
<td>2000</td>
<td>25</td>
<td>238.752 253.674 293.466</td>
</tr>
<tr>
<td>22</td>
<td>2000</td>
<td>35</td>
<td>248.7 338.232 392.946</td>
</tr>
<tr>
<td>23</td>
<td>2000</td>
<td>35</td>
<td>392.946 333.258 343.206</td>
</tr>
<tr>
<td>24</td>
<td>2000</td>
<td>35</td>
<td>382.998 338.232 353.154</td>
</tr>
<tr>
<td>25</td>
<td>2000</td>
<td>45</td>
<td>359.972 378.024 333.258</td>
</tr>
<tr>
<td>26</td>
<td>2000</td>
<td>45</td>
<td>363.102 373.05 437.712</td>
</tr>
<tr>
<td>27</td>
<td>2000</td>
<td>45</td>
<td>348.18 338.232 -</td>
</tr>
</tbody>
</table>

Note: Highlights indicate odd readings, attributed to experimental errors
Regression Model testing:

Test for Significance of Regression

The appropriate hypotheses are:

\[ H_0 : \beta_1 = \beta_2 = \ldots = \beta_k = 0 \]
\[ H_1 : \beta_k \neq 0 \text{ for at least one } j \] (12.17)

Using Minitab 16.1 software, to model the experimental data of table (2), The regression equations of the three models are:

**MODEL1:** The multiple linear regression models without interaction

The regression equation is:

\[ Y = 104 - 0.0034 x_1 + 3.83 x_2 + 1.08 x_3 \]

\[ S = 62.6655 \quad R^2 = 30.5\% \quad R^2(adj) = 21.4\% \]

**Model 2:** The multiple linear regression models with interaction effect

The regression equation is:

\[ Y = 1727 - 1.12 x_1 - 43.3 x_2 - 15.4 x_3 + 0.0324 x_1 x_2 + 0.0114 x_1 x_3 + 0.481 x_2 x_3 - 0.000331 x_1 x_2 x_3 \]

\[ S = 61.8514 \quad R^2 = 44.0\% \quad R^2(adj) = 23.4\% \]

**Model 3:** The polynomial regression second-order

The regression equation is:

\[ Y = 1790 - 0.767 x_1 - 69.5 x_2 - 12.5 x_3 + 0.0324 x_1 x_2 + 0.0114 x_1 x_3 + 0.481 x_2 x_3 - 0.000331 x_1 x_2 x_3 - 0.000117 x_1 x_1 + 0.374 x_2 x_2 - 0.0169 x_3 x_3 \]

\[ S = 60.7201 \quad R^2 = 54.6\% \quad R^2(adj) = 26.2\% \]

All models are validated within the following range of parameters:

- \(1000 \leq X_1 \leq 2000\)
- \(25 \leq X_2 \leq 50\)
- \(50 \leq X_3 \leq 115\)
- Sample diameter is 16mm

The three models were evaluated by computing the values of the coefficient of multiple determination \(R^2\), and Standard error of the regression (S) for each model. The results is shown in table (4). The polynomial model (model3) was chosen since it has the largest value of \(R^2\) and the least value of S, compared to model1 and model2.

### Table 4. \(S, R^2,\) and \(R^2(adj)\) for the three models

<table>
<thead>
<tr>
<th>Model number</th>
<th>(S)</th>
<th>(R^2)</th>
<th>(R^2(adj))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model1</td>
<td>62.6655</td>
<td>35.5%</td>
<td>21.4%</td>
</tr>
<tr>
<td>Model2</td>
<td>61.8514</td>
<td>44.0%</td>
<td>23.3%</td>
</tr>
<tr>
<td>Model3</td>
<td>60.7201</td>
<td>54.6%</td>
<td>26.2%</td>
</tr>
</tbody>
</table>

Using Minitab 16.1 software, to model the experimental data of table (2), the polynomial regression second-order equation is:

\[ Y = 1790 - 0.767 x_1 - 69.5 x_2 - 12.5 x_3 + 0.0324 x_1 x_2 + 0.0114 x_1 x_3 + 0.481 x_2 x_3 - 0.000331 x_1 x_2 x_3 - 0.000117 x_1 x_1 + 0.374 x_2 x_2 - 0.0169 x_3 x_3 \]

### Table 5. ANOVA results of friction welding process parameters of the model3

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Coef</th>
<th>SE Coef</th>
<th>T</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>1790.1</td>
<td>958.1</td>
<td>1.87</td>
<td>0.080</td>
</tr>
<tr>
<td>(x_1)</td>
<td>-0.7671</td>
<td>0.6347</td>
<td>-1.21</td>
<td>0.244</td>
</tr>
<tr>
<td>(x_2)</td>
<td>-69.49</td>
<td>29.09</td>
<td>-2.39</td>
<td>0.030</td>
</tr>
<tr>
<td>(x_3)</td>
<td>-12.49</td>
<td>11.64</td>
<td>-1.07</td>
<td>0.299</td>
</tr>
<tr>
<td>(x_1x_2)</td>
<td>0.03244</td>
<td>0.01533</td>
<td>2.12</td>
<td>0.050</td>
</tr>
<tr>
<td>(x_1x_3)</td>
<td>0.011383</td>
<td>0.006132</td>
<td>1.86</td>
<td>0.082</td>
</tr>
<tr>
<td>(x_2x_3)</td>
<td>0.4807</td>
<td>0.2599</td>
<td>1.85</td>
<td>0.083</td>
</tr>
<tr>
<td>(x_1x_2x_3)</td>
<td>-0.0003312</td>
<td>0.0001706</td>
<td>-1.94</td>
<td>0.070</td>
</tr>
<tr>
<td>(x_1x_1)</td>
<td>-0.0001166</td>
<td>0.0001040</td>
<td>-1.12</td>
<td>0.279</td>
</tr>
<tr>
<td>(x_2x_2)</td>
<td>0.3740</td>
<td>0.2479</td>
<td>1.51</td>
<td>0.151</td>
</tr>
<tr>
<td>(x_3x_3)</td>
<td>-0.01687</td>
<td>0.03966</td>
<td>-0.43</td>
<td>0.676</td>
</tr>
</tbody>
</table>

### Table 6. Analysis of Variance for Testing Significance of Regression for Model3

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>10</td>
<td>70877</td>
<td>7088</td>
<td>1.92</td>
<td>0.118</td>
</tr>
<tr>
<td>Residual Error</td>
<td>16</td>
<td>58991</td>
<td>3687</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>26</td>
<td>129868</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4. Optimization

The Basic Optimization Problem

• A fitness (objective) function $F$ must be derived in terms of $n$ parameter, that influence the responses $x_1, x_2, \ldots, x_n$ as: [6]

$$F = f(x_1, x_2, \ldots, x_n)$$

• The most basic optimization problem is to adjust variables $x_1, x_2, \ldots, x_n$ in such a way as to minimize quantity $F$. This problem can be stated mathematically as

Minimize $F = f(x_1, x_2, \ldots, x_n)$

• For finding the maximum of the objective function.

$$\text{Max } [f(x)] = -\min [-f(x)]$$

Many algorithms are used for optimization, in this research Genetic Algorithm was adapted to find the optimum parameters that maximize the tensile strength of the polynomial model. For the genetic algorithms, the chromosomes represent set of genes, which code the independent variables. Every chromosome represents a solution of the given problem. A set of different chromosomes (individuals) forms a generation by means of evolutionary operators, like selection, recombination and mutation an offspring population [7].

Genetic Algorithm Toolbox: Genetic algorithm has been implemented as a Matlab Toolbox, i.e. a group of related functions, named GAOT. The basic function is the $ga$ function, which runs the simulated evolution. The command used in matlab command window is gatool.

Optimization Procedure: Optimization (maximization) of the tensile strength and determination of the process parameters were performed by Matlab genetic algorithm toolbox. The obtained mathematical model was used as the fitness function. The boundary values for process parameters are the following: speed ($x_1$) between 1000 and 2000, time ($x_2$) between 25 and 50 and friction pressure ($x_3$) between 50 and 115. Optimal forming conditions for a maximal tensile strength were achieved for the following evolutionary parameters:

- Population size 100
- Selection operator stochastic uniform
- Crossover probability 0.8
- Mutation probability 0.2
- Fitness parameter Tensile strength

Optimization Result: The optimal condition values were obtained as following:

- Objective function value = 411.3835469718085 MPa
- Speed ($x_1$) = 1349.6665351878491 rpm
- Time ($x_2$) = 44.94291192583553 second
- Friction Pressure = 111.95643558295001 MPa

5. Discussion

The p-value in the Analysis of Variance in table (6), which is (0.118) shows that the model estimated by the regression procedure is significant at $\alpha$ -level of 0.10. This indicates that at least one coefficient is different from zero. The p-values for the estimated coefficients of $X_2, X_1X_2, X_1X_3, X_2X_3$ and $X_1x_2X_3$ are both, indicating that they are significantly related to the Joint Strength. The p-value for $X_3$ and $X_1$, indicating that they are not related to Joint Strength at a-level of 0.10. The $R^2$ value indicates that the predictors explain 54% of the variance in tensile strength. The adjusted $R^2$ is 26%, which accounts for the number of predictors in the model. Both values indicate that the model does not fit the
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data well. Observations 14, is identified as unusual because the absolute value of the standardized residuals are greater than 100. This may indicate they are outliers or experimental error. The histogram and the probability plot of the plot of the residual confirm this as shown by the bar on the far left side of the histogram of Fig. (4).

6. Conclusions

Among the three regression models tried on the experimental results, the polynomial model was found to be the fitness function for the simulation. The model fitness of data could be improved if outliers were removed, e.g. if observations no. 14 was removed from the polynomial model, the value of $R^2$ would increase from 54% to 69% and $S$ from 60.7 to 48.2.

Time has significant effect on the strength of the joint, Pressure has next effect on the joint and Speed has less effect.

The ANOVA analysis showed that, the speed ($x_1$) has no significant effect on the model.

To maximize the joint strength, the trend is to increase the time and pressure and hold the speed at specific value.

The optimal value of the joint strength of 411 MPa was obtained at the highest value of the time (44.9 sec.) and the pressure of 112 MPs and the speed of 1349 r.p.m.

![Figure 3.](optimized results in Genetic Algorithm)

![Figure 4.](Graph of Residual Plots for Tensile Strength for the Model)
REFERENCES


