Transient Material Accumulation in Rotary Drums

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Abstract
An empirical start-up transient model was developed for rotary drums using the data obtained from a pilot scale apparatus. The structure of the model is composed of dimensionless groups which reflect the effects of important operational variables on transient material filling-up inside the drum. The model is composed of two parts. One for predicting the displacing time of the material apex to any specified length of drum in an initially empty drum. The other one predicts the variations of the bed depth of material during filling-up period. It was shown that the model can be used, also, for steady state condition of material bed depth profile inside rotary drums.

Keywords
Bed Depth Data, Material Accumulation, Material Displacement

1. Introduction
Flow of solid materials inside rotary drums includes complicated flow patterns. This complexity is due to the axial and radial-angular motions caused by gravity and rotating wall of the drum. These motions have great importance and impose special effects on the patterns of materials flow and operation of the rotary drum. Solid materials transport in axial direction is important for its contribution to the residence time of the materials inside the drum. For instance, residence time of materials in rotary cement kilns affects the formation and quality of clinker. Axial transport of the materials, depend on parameters like feed flow rate, rotation speed, bed depth of solid materials, axis inclination and material physical properties including: particle size, density and dynamic angle of repose of the powder. Variations in axial velocity directly affect the bed depth in the bulk of materials. A lot of studies have been contributed to the axial and transversal motions of materials in rotary drums. A traditional practice in modeling and simulating rotary drums is that the bed depth of materials is supposed to be constant along the drum; Spang (1972) and Mastorakos et al. (1999). This approximation results in constant retention time and constant axial velocity of materials in the drum. However, a lot of mathematical models have been presented for describing the material flow and accumulation inside the rotary drums.

In this paper an empirical transient model for start-up conditions of rotary drums was developed. After testing various model structures to find the most adapted one, the parameters of the selected model were fitted to the transient data obtained from a pilot scale apparatus. Start-up transient models are less considered in the literature due to the fact that these conditions are operated almost by operators. However, a reliable start-up model for material transport and accumulation can help to implement automation of the start-up periods of the equipment. On the other hand, some needs and industry requirements, such as disorders or fluctuations in feeding systems, intensify the importance of using transient models in rotary drums. The modeling structure used by Chattergee et al., part I and part II (1983), is the starting point for modeling in this paper. They have studied the motion of solid materials in rotary kilns used for sponge iron manufacture. They investigated the effects of operating parameters such as rotation speed, drum slope and presence of dam on filling degree of solid materials inside the drum. They tried to find a logical relation between the ratio of length to diameter \( \frac{L}{D} \) and some meaningful parameters such as hold up value and retention time of solid materials within a drum in low temperature operation condition. They also presented an empirical correlation for prediction of retention time of solid materials through the drum according to the operating parameters. The developed correlation is the product of some dimensionless groups, including geometric and operating groups, as follows:

\[
T = \frac{0.1026L^3}{Q_v} \left( \frac{\gamma}{\beta} \right)^{1.054} \left( \frac{L}{D} \right)^{1.11} \left( \frac{Q_v}{L^2n} \right)^{0.981}
\]  

(1)

In Equation (1), \( \frac{\gamma}{\beta} \) and \( \frac{L}{D} \) are the dimensionless groups for slope and longitudinal distances inside the drum, while the \( \frac{Q_v}{L^2n} \) is the dimensionless group which includes both the feed flow rate and rotation speed variables together. The transient model developed in this paper is based on the similar dimensional analysis.

The developed model was used, also, for predicting the steady state material bed-depth profile inside the drum. So, it can give a thorough insight into the performance of the steady state models proposed in the literature.
Experimental and computational attempts in modeling and predicting steady state axial motion and bed depth of materials have been devoted by Saeman (1951), Kramers and Croockewit (1952), Peron and Bui (1990) and Spurling et al. (2001). The steady-state model of Kramers and Croockewit (1952) is shown in Equation (2). Since that in this paper the model of Kramers and Croockewit is frequently referred to, for the sake of simplicity it is abbreviated as K-C model all through the paper.

\[ Q_v = \frac{4\pi R^3}{3} \left( \frac{\tan(\beta)}{\sin(\gamma)} + \frac{d}{dx} \cot(\gamma) \right) \left( \frac{2}{R} - \frac{h}{R} \right)^{\frac{3}{2}} \]  

(2)

This equation represents the relation between volumetric flow rate of solid materials, \( Q_v \), and operating variables of rotary drums at steady-state condition. The variables include axis inclination of the drum, \( \beta \); material angle of repose, \( \gamma \); radius of the drum, \( R \); rotation speed, \( n \); bed depth of materials, \( h \), and the distance from discharge end, \( x \). Equation (2) can be rewritten as follow:

\[ \frac{dh}{dx} = C_A Q_v \left( 2R - h \right)^{\frac{3}{2}} - C_B \]  

(3)

\[ C_A = \frac{3\tan \gamma}{4Rn} \]  

\[ C_B = \frac{\tan \beta}{\cos \gamma} \]  

(4)

wherein \( C_A \) and \( C_B \) are two variables depending on the parameters; \( \beta \), \( \gamma \) and \( n \). As boundary condition for Equation (3), K-C model assumed that the bed depth of materials is equal to the size of a single particle (near to zero) at discharge end of the drum.

\[ h(x = 0) = h_0 \]  

(5)

Where, \( h_0 \) is taken as a very small value at the discharge end of the drum ( \( x = 0 \)). Specht et al. (2010) showed that the bed depth of material at the discharge section is much larger than a single particle size. They introduced the dimensionless, Bed Depth Number, in place of the single particle size. But, they did not develop a model with their own boundary condition. In this work the steady state model of K-C was used for checking the results of steady state predictions of the developed model.

The K-C model does not consider the effect of counter flow of gas stream on material flow. However, since this effect is trivial on the hold up of materials, this model is well useable in simulating the steady-state bed depth profile of materials inside the drums. Lebas et al. (1995) published the experimental data of bed depth and residence time for a drum that was 0.6 m in diameter and compared their results with the K-C model. Experimental investigations on fitness of the K-C model performed by Mujumdar (2008), has revealed that the model could predict the steady state bed depth at different feed flow rate and rotational speed. Yousefi and Shirvani (2014) used a pilot scale apparatus to investigate the errors of the model of K-C for various sections of the drum including the feeding, discharge and intermediate sections. The main innovation of this work is that it considers the problem of material accumulation profile along the drum in a transient condition. This is almost an ignored problem in the literature, due to its complexity as well as less operational importance. However, in addition to its benefit for describing the start-up conditions of the drums it can be used for applying automation for the system when disorders in feeding system happens during normal operating conditions.

2. Apparatus

The rotary drum used in the experiments was made from transparent plastic material to provide the possibility of measuring and monitoring the variations in the half filling angle (or bed depth) by optical sensors installed around the desired cross section. Figure 1 shows a scheme of the drum with the optical sensors and projectors. In the following section the basic geometric relations required for calculations are presented. The experimental apparatus includes a transparent rotating drum of 14 cm inside diameter and 195 cm length, adjustable in axis inclination, with programmable gear-motors for feeding and rotating the drum, and the lateral equipments including: feed and discharge storage tanks, drum inclination regulating motor and the other facilities for materials transport between discharge and feed storage tanks. Other auxiliary equipments including dust removal cyclone and bag filter, as well as the draft fan were provided for material transportation from discharge tank to storage tank. For creating surface friction and prevention of solid materials from gliding on the inner wall, longitudinal narrow bands 3 mm width and 1 mm thickness made of transparent plastic material were stick on the inner wall surface of the drum in distances of about 30 mm from each other.

Two different types of powder materials were used in the experiments with the specifications listed in Table 1. The results of experiments for determining the effects of axis inclination and rotation speed and feed flow rate on bed depth of material were the same for both of materials used.

<table>
<thead>
<tr>
<th>Powder</th>
<th>Mean Particle size (mm)</th>
<th>Density (Kg/m3)</th>
<th>Angle of repose</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silica</td>
<td>0.244</td>
<td>2580</td>
<td>30° ± 0.5°</td>
</tr>
<tr>
<td>Grinded Zeolite Catalyst</td>
<td>0.152</td>
<td>2390</td>
<td>32° ± 0.5°</td>
</tr>
</tbody>
</table>

Table 1. Specifications of the material used in the experiments.
Optical sensors were used in different sections of the drum to measure the variations of bed depth of materials that can be used for determination of other required parameters.

All geometric parameters at any specified cross section could be calculated by measuring the two end points of the upper surface of the bed of materials designated by points A and B in Figure 2. For this purpose a series of photo resistors made from cadmium sulfide connected in series, with 0.10 mm resolution, were used. The obtained data from optical sensors were processed by the PLC. Three sets of 500 Watt projectors were employed as light sources that were adjustable axially to be put over the sensor locations in different cross sections along the drum. Moreover, the apparatus was covered by a curtain around it to provide dark room condition for preventing sensors malfunctioning affected from outdoor lights. This provides also, a reference light condition for calibrating the measuring system; that can be done by use of the measurements from straight readings of points A and B from outside of the drum.

3. Geometric Relations

Variables including the half filling angle of materials and the angle of repose of materials are of significant importance when considering material transport in rotary drums. The relations provided in this section illustrate the way of calculating these parameters at a cross section like the one which is shown in Figure 2. This figure shows a cross section around which the optical sensors are installed in a fixed and
stationary ring type frame such that the drum slides and rotates inside it. According to this figure the half filling angle \( \varphi \) can be calculated by measuring the length of arc \( BC + C'A' \).

\[
\varphi = \frac{1}{2R} (BC + C'A') \tag{6}
\]

Bed depth of materials, filling degree and the angle of repose of materials can be calculated from the half filling angle as indicated in Equations (7), (8) and (9), respectively.

\[
h = R(1 - \cos \varphi) \tag{7}
\]

\[
\gamma = \varphi - \varphi_1 \tag{9}
\]

The dimensionless Froude number \( " Fr " \), used for determining flow regime is calculated from rotation speed and drum radius as:

\[
Fr = \frac{(2\pi n)^3 R}{g} \tag{10}
\]

Variations in accumulation of materials at any cross section of the drum can be determined by continual measurement of points "A" and "B" in Figure 2. These points are measured with reference to the fixed point "C" by a series of optical sensors under the light of a projector mounted over them. From the measurements of "A" and "B" all the other required parameters can be calculated using the relations above. The mathematical details of the above equations are provided in appendix A.

In the next sections of the paper after presenting a thorough discussion on the errors of steady-state K-C model, the transient experimental data and start up transient modeling results for the bed depth of material at various cross sections in the drum are presented. Also, the steady-state material bed depths attained from the developed transient model are compared with the results of simulating the K-C model.

4. Model Development

The modeling procedures, as well as the data are comprised of two separate sections. One section relates to the displacement of the apex of materials, during feeding the drum in an initially empty condition, up to any specified cross section. The other set is related to the bed depth prediction during filling the drum up to the point of achievement of steady state condition.

The developed model can be used, also, for predicting the overall time for filling-up of the drum during start-up period. The overall time for filling-up of the drum can be divided into two parts. One is the time period required for the apex of materials to reach to the specified cross section in the empty drum. It is denoted here as "Material-Apex-Displacing-Time" (MADT). The other one is the Filling Time (FT) spent in filling the drum up to the achievement of steady state.

During modeling procedures it was tried to consider the whole of the effective parameters with the same functionality. The following dimensionless groups consisting of the main operating and design parameters of the drum are correlated for obtaining the transient model. They are used in a similar procedure both for obtaining the MADT and the bed depth, as well.

\[
\zeta_{Fl} = \frac{Q_s}{\rho n L^3} \tag{11}
\]

\[
\zeta_{Loc} = \frac{L}{R} \tag{12}
\]

\[
\zeta_{Sl} = 3 + \tan \beta \left( \frac{180}{\pi} \right) \tag{13}
\]

Where "Fl", "Loc" and "Sl" are denoted for Flow, Location and Slope, respectively. In order to find a proper relationship between MADT and the dimensionless groups, each of the parameters were examined while the other two being kept constant. Meanwhile, various functions such as polynomial, logarithmic, exponential and etc., with different arrangements were checked for determining the functionality between MADT and the dimensionless groups. As a result, a unique function which can be used for description of required functionality was found. Equations (14) to (16) are the simplest general functionality that was obtained between the MADT and the dimensionless groups relating to Flow, Location and Slope, abbreviated as: "Fl", "Loc" and "Sl", respectively.

\[
\phi_{\text{MADT}}^\text{FL} = \alpha_{\text{FL}} + \beta_{\text{FL}} \times (\zeta_{FL})^{\tau_{FL}} \tag{14}
\]

\[
\phi_{\text{MADT}}^\text{LOC} = \alpha_{\text{LOC}} + \beta_{\text{LOC}} \times (\zeta_{LOC})^{\tau_{LOC}} \tag{15}
\]

\[
\phi_{\text{MADT}}^\text{SL} = \alpha_{\text{SL}} + \beta_{\text{SL}} \times (\zeta_{SL})^{\tau_{SL}} \tag{16}
\]

Where, \( \alpha_i, \beta_i \) and \( \tau_i \) are constant coefficients related to the effects of the three dimensionless groups: flow, location and slope, respectively. Equation (17) is the resulting description of MADT with respect to the individual functionalities of dimensionless groups.

\[
\text{MADT} = \frac{\rho z^2}{Q_s} \left( \phi_{\text{MADT}}^\text{FL} \times \phi_{\text{MADT}}^\text{LOC} \times \exp^{\phi_{\text{MADT}}^\text{SL}} \right) \tag{17}
\]

Where, "z" is the distance from feeding side. Comparing to
the linear dependence for the two dimensionless groups \( \zeta_{Fl} \), \( \zeta_{Loc} \) the exponential function used for the slope in Equation (17) is an indication of the more intensive effect of slope on MADT. This was revealed during simulation procedures. Meanwhile, by applying an optimization procedure for determining the coefficients of Equation (17), it became apparent that this equation is not appropriate for inclusion of the effects of powder density. Further investigations showed that the slope functionality should be corrected for material density as:

\[
\phi_{Sl,Cor}^{MADT} = \phi_{Cor}^{MADT} \times \phi_{Sl}^{MADT}
\]  

(18)

Where,

\[
\phi_{Cor}^{MADT} = \alpha_{MADT,Cor} + \beta_{MADT,Cor} \times (\zeta_{Mat,Cor})^{\tau_{MADT,Cor}}
\]  

(19)

\[
\zeta_{Mat,Cor} = \frac{\rho}{\rho_{Water}}
\]  

(20)

The model structures selected in equations (17) and (19) are based on trials and the deduced behavior of parameter effects on MADT. \( \zeta_{Mat,Cor} \) is a dimensionless group for material density. The overall equation for estimation of MADT becomes:

\[
MADT = \frac{\rho z^3}{Q_s} \left( \phi_{Fl}^{MADT} \times \phi_{Loc}^{MADT} \times \exp \phi_{Sl,Cor}^{MADT} \right)
\]  

(21)

Fitting the model in Equation (21) to the experimental data gives the coefficients of the model \( \alpha_j \), \( \beta_j \) and \( \tau_j \), \( j = 1,2,3,4 \), while, the interactions between the affecting parameters are incorporated in the model.

The model in Equation (21) predicts the time required for displacement of the apex of materials "MADT" to reach to any specified cross section at distance \( x \) from the feeder. After that the bed depth of materials increases up until it reaches to the ultimate steady state condition. This is also dependent on the operating conditions. In the same way as of the modeling MADT period, during modeling the variations of the bed depth \( h \) the effects of each of the operating variables were studied separately, while the other variables were kept constant. For all the times during MADT period the bed depth of materials in the considered cross section is zero and after that the spent time for bed depth increasing up can be expressed in a general form as Equation (22).

\[
h(t) = f \left( \zeta_{h,Fl}, \zeta_{h,Loc}, \zeta_{h,Sl}, \zeta_{h,Tm} \right) u \left( t - MADT \right)
\]  

(22)

In the same way as on modeling the MADT, the functionalities of the non-dimensional groups can be defined as:

\[
\phi_{h,Fl}^{h} = \alpha_{h,Fl} + \beta_{h,Fl} \times (\zeta_{h,Fl})^{\tau_{h,Fl}}
\]  

(23)

\[
\phi_{h,Loc}^{h} = \alpha_{h,Loc} + \beta_{h,Loc} \times (\zeta_{h,Loc})^{\tau_{h,Loc}}
\]  

(24)

\[
\phi_{h,Sl}^{h} = \alpha_{h,Sl} + \beta_{h,Sl} \times (\zeta_{h,Sl})^{\tau_{h,Sl}}
\]  

(25)

In Equation(22), \( u(t - MADT) \) is the unit step function defined as:

\[
u(t - MADT) = \begin{cases} 
0, & MADT < 0 \\
1, & MADT \geq 0 
\end{cases}
\]  

(26)

Where, \( t \) is the total time required for an empty drum to become filled-up to the steady state condition. The function \( u(t - MADT) \) is used to remove the MADT time period from the filling time period which is denoted by \( FT \). \( \zeta_{h,Tm} \) is a corrected dimensionless time group which is considered for the effect of time on the bed depth of materials. It is defined according to the dimensionless time group:

\[
z_{h,Tm} = \frac{(t - MADT)}{MADT}
\]  

(27)

The reason for using the corrected dimensionless time is that during modeling procedure it was realized that filling rates have different filling time behaviors in different locations. Thus, it was necessary to add a correction coefficient to the model for considering the effect of axial location on the filling time. Various correction structures were tested and the best results were obtained by using the following form of equation:

\[
\phi_{h,Tm,Cor}^{h} = \phi_{h,Tm,Cor}^{h} \times \left( \zeta_{h,Tm} \right)^{\tau_{h,Tm,Cor}}
\]  

(28)

\[
\phi_{h,Fl}^{h} = \alpha_{h,Fl} + \beta_{h,Fl} \times (\zeta_{h,Fl})^{\tau_{h,Fl}}
\]  

(29)

In the above equations the function \( \phi_{h,Tm,Cor}^{h} \) is used for inserting the location correction into the dimensionless time. The same procedure like that was used to find the most suitable mathematical structure for Equation (21) is applied for determining the most suitable correlation for bed depth filling-up period. In a try and error procedure, while considering the effect of operating variables, the following structure was found as the general form of Equation (22).

\[
h = k(z) \times \left\{ \phi_{h,Fl}^{h} \times \phi_{h,Loc}^{h} \times \phi_{h,Sl}^{h} \times \left( 1 - \exp^{-\zeta_{h,Tm}} \right) \right\} \times u(t - MADT)
\]  

(30)

Where \( k(z) \) is expressed as:
Where, "Z" is the distance from feeding end. In Equation (31) the constant 195 is the total length of the drum and the constant 19 is the length of feeding section which is not considered during data collection due to the excessive material accumulation in this region.

Calculation of the time required for achievement of steady state material bed depth can be done in an algorithmic method with consideration of a minimum limiting value for bed depth variations. i.e., after reaching to the specified limiting value it can be supposed that the term $\exp^{-\zeta_{h,Tm}}$ in Equation (30) equals to zero and in this way the spent time for achievement of steady state is calculated. However, determination of the spent time for achieving any amount of bed depth is possible during simulation of the model according to the Equation (30). In modeling the filling-up period, the equations corresponding to Equations (11) and (12), for "$\zeta_{Fl}$", "$\zeta_{Loc}$" and "$\zeta_{Loc}$" are defined as:

$$\zeta_{h,Fl} = \frac{Q_s}{\rho n [k(z)]}$$

(32)

$$\zeta_{h,Loc} = \frac{k(z)}{R}$$

(33)

Also, the dimensionless group for slope in this stage is defined as:

$$\zeta_{h,Sl} = 3 + \tan(\frac{180}{\pi})(_{Sl,Cor}^{h})$$

(34)

Where, the correction function $_{Sl,Cor}^{h} \phi$ is used for incorporating the effect of material density on the slope dimensionless group and is defined as:

$_{Sl,Cor}^{h} \phi = \alpha_{Sl,cor} + \beta_{Sl,cor} \times (\zeta_{Mat,Cor})$$

(35)

It is notable that the dimensionless group $\zeta_{Mat,Cor}$ used in Equation (35) is the same as the one used in Equation (19). But, the coefficients appearing in Equation (35) are not the same as those in Equation (19). After determination of governing model structure and considering the functionalities, the existing constant coefficients in Equations (21) and (30) were determined in an optimizing procedure. There are 12 coefficients for MADT model and 15 coefficients for material bed depth accumulation time, resulting in totally 27 constants to be determined from experimental data. The equation used as the objective function for parameter tuning was non-linear and complex. It was derived using Mean Square Error (MSE) between model estimations and experimental data defined as:

$$MSE = \frac{1}{N} \sum_{i=1}^{N} (y_{i}^{exp} - y_{i}^{est})^2$$

(36)

A powerful optimizer tool was required for finding the optimal values of the constant coefficients. Exploring among optimizer tools showed that genetic algorithm (GA) has the best compatibility with the existing non-linear problem. Genetic algorithm works by searching in a large feasible search space, possible solutions for the problem at hand. It has been built on the basis of natural biological evolution, Lawrence (1991). In this method in each step of iterations a large set of possible solutions is generated, and then all of the generated solutions are evaluated for investigation of their proximity to the optimal solution. Each set of the solutions which is named generation, is used to generate the next generation of solutions which is closer to the optimal point. GA operators, mainly crossover and mutation, are used to achieve the next solutions, Mitchell (1998). The crossover operator combines features of two solutions, named chromosomes, to form two new chromosomes in the next generation by swapping corresponding segments of parents, Winter et al. (1996). For preventing convergence of the algorithm around a local optimum point one or more genes, i.e. parameters within a solution, should be changed randomly; this is done by mutation operator, Michalewicz (1994). Tables 2 and 3 represent the estimated values of coefficients of the model.

<table>
<thead>
<tr>
<th>Table 2. Model parameters for Material Apex Displacement Time (MADT).</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\zeta$</td>
</tr>
<tr>
<td>Flow</td>
</tr>
<tr>
<td>Location</td>
</tr>
<tr>
<td>Slope</td>
</tr>
<tr>
<td>Slope Correction</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 3. Model parameters for Material Bed Depth $^{h} \zeta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\zeta$</td>
</tr>
<tr>
<td>Flow</td>
</tr>
<tr>
<td>Location</td>
</tr>
<tr>
<td>Slope</td>
</tr>
<tr>
<td>Slope Correction</td>
</tr>
</tbody>
</table>

5. Model Validation

In the following sections the results of comparisons
between the simulation of model and experimental data are presented to show the level of validity of the developed model for transient start-up operating conditions of the drum. Simulations are presented both for the displacing time of the apex of materials as well as the bed depth variations under the effect of operating parameters. Also, comparison between the predictions of steady state bed depth and the results obtained from the developed model and the K-C model are presented. Finally, the results of model predictions for overall time required of the bed depth of materials to reach the steady state values from an initially empty drum condition are compared with those of experimental data. In Figures 3 and 4 the predicted values of MADT and bed depth are plotted in comparison with the experimental values.

<table>
<thead>
<tr>
<th>CRITERION</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSE</td>
<td>192.38</td>
</tr>
<tr>
<td>R²</td>
<td>0.99</td>
</tr>
<tr>
<td>MSRE</td>
<td>0.01</td>
</tr>
<tr>
<td>RMSE</td>
<td>13.87</td>
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Table 4. Statistics of fitness for MADT model.

<table>
<thead>
<tr>
<th>CRITERION</th>
<th>VALUE</th>
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</thead>
<tbody>
<tr>
<td>MSE</td>
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<tr>
<td>R²</td>
<td>0.98</td>
</tr>
<tr>
<td>MSRE</td>
<td>2.16</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Table 5. Statistics of fitness for bed depth model.

The various error criteria were calculated according to the following equations:

\[
MSE = \frac{\sum_{i=1}^{N} (y_{est} - y_{exp})^2}{N} \tag{36}
\]

\[
RMSE = \sqrt{\frac{\sum_{i=1}^{N} (y_{est} - y_{exp})^2}{N}} \tag{37}
\]

\[
R^2 = \left[ \frac{\sum_{i=1}^{N} (y_{est} - y_{est,ave})(y_{exp} - y_{exp,ave})}{\sqrt{\sum_{i=1}^{N} (y_{est} - y_{est,ave})^2 \times \sum_{i=1}^{N} (y_{exp} - y_{exp,ave})^2}} \right]^2 \tag{38}
\]

\[
MSRE = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{y_{est} - y_{exp}}{y_{exp}} \right)^2 \tag{39}
\]

Where, "y_{est}" is the estimated value by model, "y_{est,ave}" is the average value of estimations, "y_{exp}" is the experimental data and "y_{exp,ave}" is the average value of data. The values of "R^2" named "correlation coefficient", shown in Figs. 3 and 4 reflect the level of fitness of data with the developed model in Equations (21) and (30).

6. Model Validation for MADT

Figures 5 to 8 show the effect of operational variables and powder density on the time of MADT period. These figures clearly show that the MADT decreases with the increase of the feed flow rate, rotation speed and slope, but increases with increase of material density.
Figure 5. Comparison of data and model predictions for the MADT period versus drum slope.

\[ Q = 10.33 \, \text{g/s}, \quad \gamma = 30 \pm 0.50(\text{deg}), \quad \rho = 2580(\text{kg/m}^3) \]

Figure 6. Comparison of data and model predictions for the MADT period versus rotation speed.

\[ Q = 10.33 \, \text{g/s}, \quad \gamma = 30 \pm 0.50(\text{deg}), \quad \beta = 1.00(\text{deg}), \quad \rho = 2580(\text{kg/m}^3) \]
7. Model Validation for FT

In Figures 9 through 12 the transient data for filling the drum up to the ultimate bed depth (steady state condition) are compared with the model predictions for axis inclination variations in the range of zero up to 3.00 (deg.); at 90.00 (cm) distance from the feeder. Also, the calculated ultimate bed depth resulted from the developed model are compared with the estimations of steady state predictions obtained from the K-C model. During simulation of the developed model the achievement of steady state was defined in the computational procedures as the point in which the variation of material bed
depth with time becomes less than 0.625 (mm/min). It can be seen that at lower axis inclination the model of K-C better fits the data. However, the model very well fits the data over the entire range of axis inclination. Table 6 summarizes the errors between the data and the transient model as well as the errors between the data and K-C model for various axis inclinations. It can be seen that the trend of error variance is such that it increases with increase of slope.

![Figure 9](image9.png)

**Figure 9.** Transient bed depth variation and comparison of the ultimate bed depth predictions from model and data at zero (deg.) of drum slope.

![Figure 10](image10.png)

**Figure 10.** Transient bed depth variation and comparison of the ultimate bed depth predictions from model and data at 1.00 (deg.) of drum slope.
Figure 11. Transient bed depth variation and comparison of the ultimate bed depth predictions from model and data at 2.00 (deg.) of drum slope.

\[ Q_s = 10.33 \text{ (g/s)} , \gamma = 30 \pm 0.50 \text{(deg.)} , n = 6.00 \text{(rpm)} , \rho = 2580 \text{(kg/m}^3 \text{)} , \text{Sensor Location} = 0.90 \text{(m)} \]

Figure 12. Transient bed depth variation and comparison of the ultimate bed depth predictions from model and data at 3.00 (deg.) of drum slope.

\[ Q_s = 10.33 \text{ (g/s)} , \gamma = 30 \pm 0.50 \text{(deg.)} , n = 6.00 \text{(rpm)} , \rho = 2580 \text{(kg/m}^3 \text{)} , \text{Sensor Location} = 0.90 \text{(m)} \]

<table>
<thead>
<tr>
<th>Axis inclination (deg.)</th>
<th>Data and K-C model</th>
<th>Data and developed model</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>1.64</td>
<td>1.57</td>
</tr>
<tr>
<td>1.00</td>
<td>12.88</td>
<td>5.36</td>
</tr>
<tr>
<td>2.00</td>
<td>14.54</td>
<td>3.47</td>
</tr>
<tr>
<td>3.00</td>
<td>14.94</td>
<td>2.98</td>
</tr>
</tbody>
</table>
Figures 13 to 15 are prepared to show the effect of rotation speed on transient predictions of the model and data as well as the comparisons of ultimate bed depth predictions with that of the K-C model. These are prepared also, at 90.00 (cm) distance from the feeder and in the range of 3.33 to 7.00 rpm. The results shown in Figure 10 can also be used as one set of data in the range of interest in these series of figures. This figure presents the results for 6.00 rpm rotation speed and 10.33 g/s feed flow rate. It is seen that the mid values of rotation speed are the better fitted conditions with the predictions of K-C model. However, the transient model and data are well fitted at an entire rotation range of interest. Table 7 summarizes the errors for the effect of variations in rotation speed corresponding to the data of Figures 10, 13, 14 and 15. It can be seen that with increase of rotation speed the magnitudes of the errors are increasing, while, the trend of variation of errors with respect to increase of rotation speed are not monotone increasing like that of axis inclination.

Figure 13. Transient bed depth variation and comparison of the ultimate bed depth predictions from model and data at 3.33 (rpm) rotation speed.
\[ Q_s = 10.33 (g/s), \gamma = 30 \pm 0.50 (deg.), \beta = 1.00 (deg.), \rho = 2580 (kg/m^3), \text{Sensor Location} = 0.90 (m) \]

Figure 14. Transient bed depth variation and comparison of the ultimate bed depth predictions from model and data at 4.29 (rpm) rotation speed.
\[ Q_s = 10.33 (g/s), \gamma = 30 \pm 0.50 (deg.), \beta = 1.00 (deg.), \rho = 2580 (kg/m^3), \text{Sensor Location} = 0.90 (m) \]
Figure 15. Transient bed depth variation and comparison of the ultimate bed depth predictions from model and data at 7.00 (rpm) rotation speed. 

$$Q_s = 10.33 \text{ (g/s)} \quad \gamma = 30 \pm 0.50 \text{(deg.)} \quad \beta = 1.00 \text{(deg.)} \quad \rho = 2580 \text{ (kg/m}^3\text{)} \quad \text{Sensor Location = 0.90 (m)}$$

<table>
<thead>
<tr>
<th>Rotation speed (n)</th>
<th>Data and K-C model</th>
<th>Data and transient model</th>
</tr>
</thead>
<tbody>
<tr>
<td>n = 3.33(rpm)</td>
<td>9.33</td>
<td>1.48</td>
</tr>
<tr>
<td>n = 4.29(rpm)</td>
<td>0.84</td>
<td>1.22</td>
</tr>
<tr>
<td>n = 6.00(rpm)</td>
<td>12.88</td>
<td>5.36</td>
</tr>
<tr>
<td>n = 7.00(rpm)</td>
<td>14.02</td>
<td>1.58</td>
</tr>
</tbody>
</table>

Table 7. Steady state errors between data and model for variation in rotation speed.

The effect of material feed flow rate on bed depth predictions are shown in Figures 16 and 17. Figure 10 can be used, again, as a set of data to investigate the effect of feed flow rate and the errors of the K-C model. Table 8 summarizes the errors corresponding to the data of Figures 10, 16 and 17. It is seen that the trend of variation of errors with respect to increase of feed flow rate are almost monotone decreasing.

Figure 16. Transient bed depth variation and comparison of the ultimate bed depth predictions from model and data at 6.80 (g/s) feed flow rate. 

$$\gamma = 30 \pm 0.50 \text{(deg.)} \quad \beta = 1.00 \text{(deg.)} \quad n = 6.00 \text{(rpm)} \quad \rho = 2580 \text{ (kg/m}^3\text{)} \quad \text{Sensor Location = 0.90 (m)}$$
Figure 17. Transient bed depth variation and comparison of the ultimate bed depth predictions from model and data at 13.60 (g/s) feed flow rate. 
\[ \gamma = 30 \pm 0.50(\text{deg.}) \, , \, \beta = 1.00(\text{deg.}) \, , \, n = 6.00(\text{rpm}) \, , \, \rho = 2580(\text{kg} / \text{m}^3) \, , \, \text{Sensor Location} = 0.90(\text{m}) \]

Table 8. Steady state errors between data and model for variation in feed flow rate.

<table>
<thead>
<tr>
<th>Feed flow rate ((Q_s))</th>
<th>Data and K-C model</th>
<th>Data and transient model</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Q_s = 6.80(\frac{g}{s}))</td>
<td>0.44</td>
<td>4.16</td>
</tr>
<tr>
<td>(Q_s = 10.33(\frac{g}{s}))</td>
<td>12.88</td>
<td>5.36</td>
</tr>
<tr>
<td>(Q_s = 13.60(\frac{g}{s}))</td>
<td>6.7</td>
<td>3.99</td>
</tr>
</tbody>
</table>

Also, Figures 18 in combination with Figure 10 shows the effect of density on bed depth. Table 9 summarizes the errors at steady state between data and the developed model as well as between data and K-C model.

Figure 18. Transient bed depth variation and comparison of the ultimate bed depth predictions from model and data at 2390 (kg/m³) density 
\[ Q_s = 10.33(\text{g} / \text{s}) \, , \, \gamma = 32 \pm 0.50(\text{deg.}) \, , \, \beta = 1.00(\text{deg.}) \, , \, n = 6.00(\text{rpm}) \, , \, \text{Sensor Location} = 0.90(\text{m}) \]
Table 9. Steady state errors between data and model for variation in density.

<table>
<thead>
<tr>
<th>Powder density ($\rho$)</th>
<th>Data and K-C model</th>
<th>Data and transient model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho = 2580\frac{\text{Kg}}{\text{m}^3}$</td>
<td>12.88</td>
<td>5.36</td>
</tr>
<tr>
<td>$\rho = 2390\frac{\text{Kg}}{\text{m}^3}$</td>
<td>1.9</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Figures 19 through 22 are drawn to show the predictions of model concerning the total time required for the bed depth of material to reach to the steady state condition from initially empty drum at various distances from the feeder. These are plotted for variations of the different operating parameters including axis inclination, rotation speed, feed flow rate as well as the density of the material. The total time includes of the time required for the movement of the apex of materials up to the specified cross section (MADT), plus the Filling Time (FT). According to these figures, the total time for filling up and achievement of steady state decreases with increase of axis inclination, rotation speed and feed flow rate. But, it is increasing with the increase of density.

$$t_{ss} = t_{MADT} + t_{FT}$$  \hspace{1cm} (40)

During experiments the establishment of the steady state condition was checked by a digital balance provided at the material exiting section of the drum.

Figure 19. Effect of drum slope on the time required for filling the drum up to ultimate value.

$Q_s = 10.33 \text{(g/s)}$, $\gamma = 30 \pm 0.50\text{(deg.)}$, $n = 6.00\text{(rpm)}$, $\rho = 2580\text{(kg/m}^3\text{)}$
Figure 20. Effect of rotation speed on the period of time required for filling the drum up to ultimate value.

\[ Q_s = 10.33 \text{ (g/s)} \], \[ \gamma = 30 \pm 0.50 \text{(deg.)} \], \[ \beta = 1.00 \text{(deg.)} \], \[ \rho = 2580 \text{ (kg/m}^3 \text{)} \]

Figure 21. Effect of feed flow rate on the period of time required for filling the drum up to ultimate value.

\[ \gamma = 30 \pm 0.50 \text{(deg.)} \], \[ \beta = 1.00 \text{(deg.)} \], \[ n = 6.00 \text{ (rpm)} \], \[ \rho = 2580 \text{ (kg/m}^3 \text{)} \]
8. Conclusions

Modeling of the transient start-up condition of material transport inside rotary drums was considered. An explicit model, using dimensionless groups, was developed based on experimental data. The developed model reflects the effects of geometric and operating parameters, and also the effect of material density on transient filling-up of materials inside the rotary drums. It also predicts the total time required for filling the drum up to the steady state condition from initially empty drum status. The model consists of two parts including: 1) a correlation for predicting the required time for the apex of materials to displace from feeding section up to any specified section in front of it, 2) a correlation for prediction of bed depth increasing up to the steady state condition. With consideration of the above distinguished times the total transient time of filling-up the drum was calculated and investigated for determining the effect of operating parameters. The results of experimental investigations showed that:

1) The developed correlation well described the transient start-up behavior of the drum up to the steady state conditions of the drum.
2) The developed model can describe the steady state material profile of rotary drums in a range of parameters variations which is compatible with industrial operational conditions.

As an appreciation to further attempts for extending the results and application of the method used in this work, one may consider the condition of material accumulation behind a dam installed inside the drum. In practicing industrial conditions material clogging or ring build-ups inside the rotary drums is one of the important operational problems. This causes severe operational problems as well as control problems. In such cases a material bed accumulation model describing the variations of the bed depth behind the clogged section will be of great application for operational purposes.

Appendix A

According to Figure 2 Points $A'$ and $B'$ are the first and the last sensors in the series of sensors and $C'$ is a reference fixed sensor located in a minimum point under the bed of material. Surface of the bed can be shown with two points in cross sectional area of the drum as $A$ and $B$. The angles $\varphi_1$ and $\varphi_2$ can be determined by measuring $AC'$ and $BC'$ lengths using optical sensors as follows:

\[ \varphi_1 = \frac{AC'}{R} \]  
\[ \varphi_2 = \frac{BC'}{R} \]

According to Figure 1 the following relations can be written:
\[ \varphi + \alpha = \pi/2 \quad (A.5) \]
\[ \alpha + \gamma + \varphi_1 = \pi/2 \quad (A.6) \]

According to Equations (A.5) and (A.6), dynamic angle of repose can be calculated as:
\[ \gamma = \varphi - \varphi_1 \quad (A.7) \]

Furthermore, bed depth of the drum "h" can be calculated as:
\[ y = R \cos \varphi \quad (A.8) \]
\[ h = R - y \quad (A.9) \]
\[ h = R(1 - \cos \varphi) \quad (A.10) \]

Filling degree of materials in a specified cross section can be defined as the ratio of the covered surface by the solid materials to cross sectional area of the drum.
\[ S_{\text{solid area}} = \pi R^2 \left( \frac{\varphi}{\pi} - \frac{y \times (AB)}{2} \right) \quad (A.11) \]
\[ S_{\text{drum}} = \pi R^2 \quad (A.12) \]
\[ AB = 2R \sin \varphi \quad (A.13) \]

According to Figure 1 calculations of filling degree "j" can be done as following:
\[ j = \frac{S_{\text{solid area}}}{S_{\text{drum}}} \times 100 = \frac{\pi R^2 \left( \frac{\varphi}{\pi} - \frac{y \times AB}{2} \right)}{\pi R^2} \times 100 \quad (A.14) \]
\[ j = \left( \frac{\varphi}{\pi} - \frac{R \cos \varphi R \sin \varphi}{\pi R^2} \right) \times 100 \]
\[ = \frac{1}{\pi} \left( \varphi - \sin 2\varphi \right) \times 100 \quad (A.15) \]

**Nomenclature**

- \(D\) = Inside diameter, \(m\)
- \(h\) = Material bed depth, \(m\)
- \(j\) = Filling degree, dimensionless
- \(L\) = Drum length, \(m\)
- \(n\) = Rotation speed, \(\text{rev}/s\)
- \(Q_v\) = Volumetric flow rate of transmitted solid material, \(m^3/s\)
- \(Q_s\) = Mass flow rate of transmitted solid material, \(g/s\)
- \(R\) = Drum radius, \(m\)
- \(t\) = Time, \(s\)
- \(T\) = Residence time, \(s\)
- \(\chi\) = Distance from discharge end, \(m\)
- \(z\) = Distance from feeding side, \(m\)

**Greek letters**

- \(\alpha, \beta, \tau\) = Model Coefficients in Equations (21) and (30), dimensionless
- \(\gamma\) = Angle of repose of material, deg.
- \(\varphi\) = Maximum half angle subtended by the bed at the drum axis, deg.
- \(\beta\) = Axis inclination of cylinder, deg.
- \(\phi\) = Dimensionless Functionality Groups
- \(\rho\) = Bulk density, \(kg/m^3\)

**Subscripts**

- \(\text{Cor}\) = Correction
- \(\text{Fl}\) = Feed flow rate and rotation speed
- \(\text{Loc}\) = Longitudinal location
- \(s\) = Mass
- \(\text{Sl}\) = Slope
- \(ss\) = Steady state
- \(\text{Tm}\) = Time

**Abbreviations**

- MADT = Material Apex Displacement Time
REFERENCES


