Semiempirical Method for Temperature Determination in a High-Pressure Chamber at Nonstationary Heating Regimes and a New Facet of the Newtonian Law of Cooling

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Abstract  Experimental data and model calculations of the temperature behavior in high-pressure chambers upon stepped changes in the heating power are presented. A variant of formulation of the Newtonian law of cooling corresponding to specific cooling (or heating) conditions of samples in a reactionary cell of the high-pressure chamber is proposed. A semiempirical formula for temperature determination in a reaction cell of the chamber at nonstationary heating regimes set by a programmed power supply is obtained. This formula takes into account thermal inertia of the cell within the framework of the proposed \( \tau \) approximation. The formula obtained is compared with the Duhamel integral. Examples of application of the method are presented.

Keywords  High Pressures, High Temperatures, Nonstationary Heating, Newtonian Law of Cooling, Fourier Law, Duhamel Integral

1. Introduction

During high-temperature treatment of samples in high-pressure chambers, a reaction cell is most frequently heated without permanently operating internal temperature sensors. Only the electric current power feded to the ohmic resistance of the cell heater is controlled. Such a method is convenient, if a series of scientific experiments is performed [1] or under production conditions [2]. The power-temperature dependence obtained, as a rule, through a preliminary calibration with the aid of thermocouple, which is introduced into the central part of the working volume, is used for assignment of the temperature in the chamber. Experimental data for the calibration are obtained with a time hold-up at individual measurement points in order to determine steady temperature values corresponding to fixed values of power. Since the cell is thermally inertial during heating power variations, the question arises: how to use the available data on the calibration of quasi-steady heating regime for estimating the temperature in the chamber during different nonstationary heating regimes.

Below we will derive a general formula relating the functional time dependence of the temperature in the chamber, \( T(t) \), to the time dependence of the temperature set in accordance with the quasi-stationary calibration, \( \theta(t) \). This derivation is based on the hypothesis about the applicability of the relaxation time approximation to transitional thermal processes in high-pressure chambers. To confirm this hypothesis, we will present experimental data characterizing the time dependence of the temperature in the chamber upon stepped changes in the heating power (which can be used for determining the characteristic time constant \( \tau \) for a temperature relaxation in the cell of the high-pressure chamber), as well as the results of calculation of the model problem of cooling-down of a heated rod between chamber matrices. Evidently, this hypothesis can be considered as a variant of interpretation of the Newtonian law of cooling with its formulation corresponding to specific conditions of cooling (heating) of samples in the reaction cell of a high-pressure chamber.

2. Experimental Technique and Data

We will analyze the processes of generation of high temperatures in the Conac-type device of high pressure and high temperature [3]. The block-matrices of the high-pressure chamber are shown in Fig. 1. Such devices are intended for the use in hydraulic presses in which supporting plates are isolated from each other and water-cooled. For generating pressure and temperature in the chamber, a
solid-state container with a reaction cell is placed between its matrices. Fig. 2 shows design features of the container and typical reaction cell used for high-temperature treatment of samples.

The reaction cell design makes it possible to feed electric current from an external power supply to the graphite heater through matrices of the chamber and internal metallic current-carrying elements. The position of the cell with the thermocouple in the chamber during its calibration against temperature under pressure and schemes of measurements are shown in Fig. 3. The cell was heated with the use of the programmed digital power supply (Digital DC Power Supply XDC 10-600), whose intrinsic pulse rise time (front duration) during stepped power variation was less than 100 milliseconds. The thermal emf was recorded with the digital voltmeter or with analog XY recorder. In the experiences described, we used the chromel-alumel thermocouple with the wire diameters of 0.3 and 0.5 mm, whose junction was placed at the reaction cell center in the cylindrical sample turned from a hexagonal boron nitride rod. The heater was made from super purity graphite.

The presented experimental data of temperature measurements were obtained with the use of chambers having different working volumes, namely, the Conac 28 and Conac 48M chambers. These chambers had different dimensions of matrices and reaction cells (the two-digit number in the name of the chamber modification means the diameter expressed in millimeters of the central bowl-shaped recess on the working surface of the matrix). The temperature was measured at the same pressures in the chambers (about 3 GPa at room temperature). According to our measurements of the distances between the matrices in the Conac 28 and Conac 48M chambers, the reaction cell heights were equal to about 17.2 and 29.8 mm, respectively. The current-feeding coils were made from calibrated steel rods with the diameters 4 and 6 mm for small and large chambers, respectively.

The experimental data of temperature calibrations of the reaction cells of the Conac 28 and Conac 48M chambers are presented in Fig. 4. The reaction cells were calibrated with a time hold-up at each point of measurements. In this figure the ordinate axis shows stationary temperature values and is designated as $\theta$, the abscissa axis is the axis of variable values of the heating power $W$. The experimental data permit their linear approximation in the temperature range under consideration. (At higher temperatures, the temperature dependences of the power are inclined at steeper angles). It turns out that the sloping coefficients of the lines obtained are with a high degree of accuracy proportional to the ratio of the reaction cell height to the thermal contact area of the current-feeders in the chambers. This fact can be explained by solving a simple problem of stationary heating of the rod between the chamber matrices, which will be considered below.
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Figure 4. Data of temperature calibrations of the Conac 28 (■) and Conac 48M (○) high-pressure chambers through the linear approximation of the experimental points (solid lines). Data are obtained under a stationary heating regime at pressure about 3 GPa (at room temperature) with a chromel-alumel thermocouple (without regard for the pressure effect on thermocouple readings).

Figure 5. Time dependence of the difference between the actual temperature in the reaction cell center $T$ and room temperature $T_r \approx 20 ^\circ C$ after switching-off the heating power for the Conac 48M chamber. Open circles are experimental data on the temperature relaxation; solid line is the result of the regression analysis of these data. The formula describing the regression curve $F(t)$, as well as the determination coefficient $R^2$, are also presented.

The preliminarily constructed calibration curve for a chamber $\theta = f(W)$ is usually used in the subsequent work for setting a necessary temperature in the chamber by the heating power, however only in the case, when the chosen power value acts sufficiently long on the heater. If the heating power will be changed, for example with the aid of programmed power supply, setting a certain function $W(t)$, thereby the time dependence of stationary temperature values (this is an oxymoronic notion) in the chamber $\theta(t) = f(W(t))$ (as combined function) will be also set. However, such a dependence will not represent a functional time dependence of the temperature obtained in the chamber $T(t)$.

For determining temperature values in the reactionary cell of the chamber during the heating power change or immediately after it, one must know the parameter characterizing the temperature inertia of the cell. Such a parameter can be approximately determined from analysis of the time dependence of the temperature in the cell after a step-like power change. The data of temperature measurements in the cells of the Conac 48M and Conac 28 chambers after the heating power was switched-off are presented in Fig. 5 and Fig. 6, respectively. (The initial temperatures in these cells were about 900 and 1000 °C, respectively).

Figure 6. Time-dependent readings of the chromel-alumel thermocouple (thermal emf $E$ vs. time $t$) after switching-off the heating power for the Conac 28 chamber (filled circles) and the regression curve constructed by experimental points (solid line). The formula describing the regression curve $F(t)$, as well as the determination coefficient $R^2$, are also presented. (Cold ends of the thermocouple wires were kept at room temperature)

As it is seen from Fig. 5 and Fig. 6 the temperature relaxation in the high-pressure chambers is of the exponential character (the determination coefficients are surprisingly close to unit). The temperature in Celsius degrees and the thermal emf in millivolts were plotted on the ordinate axes of Fig. 5 and Fig. 6, respectively. The use of the latter is evidently more convenient for constructing the relaxation plots, because during the experiments, cold ends of the thermocouple wires were at room temperature, and the calibration characteristic (temperature-thermal emf) of the chromel-alumel thermocouple is virtually linear.

The experimental dependences obtained can be presented as:

$$ [T(t) - T_r] = (T_0^* - T_r) \exp \left(\frac{-t}{\tau}\right) $$

where $T(t)$ is the temperature of the measured point of the cell at the time moment $t$; $T_0^*$ is the temperature value at $t = 0$ for the regression curve; $T_r$ is the room temperature value (which is the same as the temperature value for the contacts between the heater current-feeders and the matrices); and $\tau$ is the time constant of temperature relaxation of the cell, which characterizes its temperature inertia.
The formulas of curves presented in Fig. 5 and Fig. 6 make it possible to calculate that at specified pressure, the values of \( \tau \) for the reaction cell of the Conac 48M and Conac 28 chambers are 1/0.033233 \( \approx 30.1 \) and 1/0.102689 \( \approx 9.74 \) sec, respectively. Note that these \( \tau \) values, taking into account measurement errors, can be assumed as proportional to the squares of heights of the reactionary cells in the chambers, because the numerical relation 30.1/9.74 \( \approx (29.8)^2/(17.2)^2 \) is fulfilled. In the left-hand side of this approximate equality is the ratio of the \( \tau \) values obtained for our cells, and its right-hand side, the ratio of the squares of heights of the respective cells.

It was established in our experiments that the exponential dependence (1) is the best approximation of temperature behavior during cooling the cell of the high-pressure chamber. This brings-up the question: What solution of this problem can be obtained by methods of the analytical theory of heat conduction? It is important to find out the error, with which exponential dependence (1) can replace the "exact" solution of the thermal problem of cooling of the cell presented by a simplified model.

The exponential dependence of temperature (temperature difference between the object and the environment) versus cooling time was for the first time established by Isaac Newton. In his work, I. Newton extended the range of temperature measurements, having applied the extrapolation of his data to the region of high temperatures in order to estimate the solidification points of some alloys and pure metals. However, his method has led to erroneous results of measurements in the high-temperature region and to underestimation of solidification points of these substances [6]. The author of paper [6] established two main reasons of the discrepancy between the present-day data and the Newtonian data: the nonlinearity of the oil thermometer scale and the radiation energy losses of a hot iron bar, which sharply increase with temperature and substantially decrease the time of its cooling in the high-temperature region. In our experiments performed in the high-pressure chambers, the cell was surrounded by a nontransparent solid-state medium, evidently ensuring multiple shielding of thermal radiation.

In the model calculations, we will take into account the heat transfer only through the heat conduction in accordance with the Fourier law. In addition, the problem is assumed to be one-dimensional.

### 3. Model Calculations

First, we will show that under stationary heating the temperature distribution along the graphite heater, which is in contact with the chamber matrices at the constant temperature \( T_r \), has the form of the quadratic parabola. Such a distribution will be used as the initial condition in the problem of cooling the cell in the chamber, when the heating power is switched off. Also we will determine the dependence of the temperature in the chamber on the electric heating power and elucidate a role of the current-feeding metallic elements (paradoxical, at first glance, role of heat insulators for the graphite heater).

Let us choose for consideration the simplest schemes of heaters shown in Fig. 7a and Fig. 7b. In the first case (Fig. 7a), the graphite heater in the shape of a cylindrical rod (or tube) with the cross section area \( S \) and length \( 2L \) is in direct contact with the chamber matrices. In the second case (Fig. 7b), the graphite heater contacts with the matrices through the metallic current-feeders, which have the relatively small height \( \ell \) and the same cross section (distance between the matrices remains unchanged). Suppose that the temperature at the ends of simple and composite rods with matrices is \( T_r \) (temperature of constants with cooled matrices), and their cylindrical surfaces are completely heat-insulated. Our task will consist in finding the temperature distributions along the heaters under the passage of electric current. Additionally, we can compare the presented heater schemes from standpoint of the electric power necessary for heating to the temperature \( T_r \) in the chamber center.

![Figure 7. Simplified schemes of the graphite heaters with the boundary conditions for the one-dimensional problem of heat conduction: (a) at direct contact of the heater with the matrices, and (b) with the use of metallic current-feeding elements in the heater assembly](image)

We will consider only the upper symmetric parts of the heaters, placing the origin of coordinates in the chamber center, as shown in Fig. 7. For the first scheme (Fig. 7a), the stationary equation of heat conduction

\[
\text{div}(\lambda \nabla T) = -f(M)
\]

where \( f(M) \) is the distribution function of the power density of heat sources (depending on the spatial coordinates of point \( M \)), can be written down in the following way:

\[
\frac{d}{dz} \left( \lambda \frac{dT}{dz} \right) = -\frac{I^2 \rho_g}{S} \tag{2}
\]

Here \( I \) is the heating current, \( S \) is the cross section area of the heater, \( \rho_g \) and \( \lambda_g \) are the specific electric resistance and

the thermal conductivity of graphite (which, in general case, depend on temperature and, consequently, on the coordinate z). In order to obtain a clearer analytical solution of this problem, \( \rho_g \) and \( \lambda_g \) will be assumed constant parameters.

After integrating Eq. (2) with respect to \( z \) from zero to \( z \) with allowance for the absence of temperature gradient at the origin of coordinates, we obtain:

\[
\lambda_g \frac{dT}{dz} = \frac{I^2 \rho_g}{S^2} z dz
\]

The secondary integration of the equation of heat conduction leads to the formula:

\[
T(z) = \frac{I^2 \rho_g}{2S^2 \lambda_g} z^2 + C
\]

where \( C \) is the constant of integration. Taking into account the boundary condition \( T(L) = T_r \), we obtain the formula describing the temperature distribution along the graphite rod:

\[
T(z) = T_r + \frac{I^2 \rho_g}{2S^2 \lambda_g} \left(L^2 - z^2\right)
\]

The obtained dependence \( T(z) \) (3) represents the quadratic parabola with the temperature maximum in the center of the rod \( (z = 0) \):

\[
T_c = T_r + \frac{I^2 \rho_g L^2}{2S^2 \lambda_g}
\]

If in the first case, we designate the heating electric power, which must be released in the entire graphite rod for obtaining the temperature \( T_c \) in its center, as \( W_1 \), then expression (4) can be rewritten as:

\[
T_c = T_r + \frac{L}{4S \lambda_g} W_1
\]

We used the formula for the power

\[
W_i = \frac{I^2 \rho_g L}{S}
\]

Note that in this case, the temperature in the chamber linearly dependence on the heating power, and the coefficient in linear dependence \( T_r(W_1) \) is proportional to the ratio \( L/S \).

We want to compare the heating powers necessary for obtaining the same temperatures in the chamber centers for the two presented schemes of heaters; therefore we will express \( W_1 \) from expression (5) separately:

\[
W_1 = 4\lambda_g \left(T_c - T_r\right) \frac{S}{L}
\]

For the second scheme of the heater (Fig. 7b), the problem of heat conduction is solved for the rod consisting of different materials. For the graphite part with the length \( L_1 \), the heating power (designated as \( W_2 \)) can be written, by analogy with the previous case, as follows:

\[
W_2 = 4\lambda_g \left(T_c - T_i\right) \frac{S}{L_1}
\]

where \( T_i \) is the intermediate temperature at graphite-metal boundary, and \( L_1 = L - L_2 \).

For the metallic part (current-feeder), the heat release during the current passage will be neglected, because the electric resistivity of graphite exceeds by far the electric resistivity of steel current-feeder. Therefore, applying the Fourier law, we can write for the site \( L_1 \leq z \leq L_2 \) the following equation:

\[
\frac{W_2}{2S} \frac{dT}{dz} = -\lambda_{met} \frac{dT}{dz}
\]

where \( \lambda_{met} \) is the thermal conductivity of the material of current-feeder. (The left-hand side of the equation is the density of the heat flow through the upper current-feeder). After integration of this equation, we find the intermediate temperature \( T_i \):

\[
T_i = T_r + \frac{W_2 L}{2S \lambda_{met}}
\]

Equations (7) and (8) give the following formula for the heating power consumed in the second case (Fig. 7b):

\[
W_2 = 4\lambda_g \left(T_c - T_i\right) \frac{S}{L_2} \left(1 + \frac{2\lambda_g L}{\lambda_{met} L_1}\right)^{-1}
\]

Using formulas (6) and (9), we can find the ratio of the heating powers \( W_1 \) and \( W_2 \):

\[
\frac{W_1}{W_2} = 1 + \left(\frac{2\lambda_g}{\lambda_{met}} - 1\right) \frac{L}{L_1}
\]

In practice, the heating power in designs of heaters with metallic current-feeding elements is considerably smaller than in heaters where graphite is in direct contact with matrices of the high-pressure chamber. According to the obtained formula (10), this effect is caused by the relation \( 2\lambda_g > \lambda_{met} \), at which \( W_1 \) will exceed \( W_2 \).

In the second case, under consideration, the temperature distribution along the heater will consist of two sites – parabolic and linear:

\[
T(z) = T_i + \frac{W_2}{4\lambda_g L_2} \left(L_2^2 - z^2\right), \quad 0 \leq z \leq L_1,
\]

\[
T(z) = T_i + \frac{W_2}{2S \lambda_{met}} (L - z), \quad L_1 \leq z \leq L.
\]

The calculations are illustrated by Fig. 8 comparing the temperature distributions in models with two heater types, namely, in the model where graphite is in direct contact with matrices and in the model with metallic current-feeders.
Figure 8. Plots of temperature distributions along the heaters. The dashed curve is the parabolic dependence of temperature in the model of heater, where graphite is in direct contact with the matrices. The solid line is temperature distribution in the model of heater with metallic current-feeders, consisting of the parabolic and linear sites.

For constructing the plots, we assumed the condition of the equality of temperatures at the central points of heaters and the following relations: \( \lambda_{\text{met}} = \lambda_{g}/2 \) and \( \ell = L/4 \). (The relation between the heat conduction coefficients for graphite and steel was assumed after an approximate estimation of these quantities from the available published data at atmospheric pressure [7, 8]).

Note that at the specified parameters of the construction of plots, the ratio of electric powers \( W_1/W_2 \), according with the formula (10), is 1.75. At the same time the presented temperature distributions do not seem to differ so significantly.

Let us return to the question about the temperature behavior in the chamber during cooling of the reactionary cell after switching-off the heating power. Now we know that, if a simplified model of heater in the form of a homogeneous rod is considered as the initial condition for solving the nonstationary equation of heat conduction, the parabolic temperature dependence along its height between the matrices should be chosen.

We will consider the problem, whose scheme is submitted in Fig. 9. The cooling rod is located along the Oz axis and has length 2L. The origin of coordinates is in the lowermost point of the rod. The lateral surface of the rod is heat-insulated, and the constant temperature \( T_r \) is maintained at its ends. In order to simplify the subsequent records, the temperature of rod-matrix contacts is assumed to be zero. The right-hand part of the figure shows the initial temperature distribution in the rod, which represents the quadratic parabola with the temperature maximum \( T(L,0) = kL^2 \), where \( k \) is the dimensional coefficient.

The problem consists in finding the solution for one-dimensional homogeneous equation of heat conduction

\[
\frac{\partial T}{\partial t} = \chi \frac{\partial^2 T}{\partial z^2}
\]

for \( t \geq 0 \) at the boundary conditions \( T(0,t) = T(2L,t) = 0 \) and initial condition \( T(z,0) = k(z(2L - z)) \), where \( \chi \) is the thermal diffusivity coefficient of the rod material. Such a problem can be solved by the method of separation of variables (the Fourier method) with presentation of the final result as the infinite series of functions decreasing with time:

\[
T(z,t) = \frac{32}{\pi^3} kL^2 \sum_{n=0}^{\infty} \sin \left( \frac{\pi(2m+1)z}{2L} \right) \times
\]

\[
\times \frac{1}{(2m+1)^2} \exp \left( -\frac{\pi^2 (2m+1)^2 \chi t}{4L^2} \right).
\]

(A detailed solution of such a problem is given, for example, in textbook [9]). The time dependence of temperature for the central points of the rod (\( z = L \)) will have the form:

\[
T(L,t) = \frac{32}{\pi^3} kL^2 \sum_{n=0}^{\infty} \left( -\frac{1}{2m+1} \right)^n \times
\]

\[
\times \exp \left[ -\frac{\pi^2 (2m+1)^2 \chi t}{4L^2} \right].
\]

If we pass to the dimensionless quantities:

\[
x = \frac{\pi^2 \chi t}{4L^2},
\]

\[
Y(x) = \frac{T(L,x)}{T(L,0)},
\]

where \( T(L,0) = kL^2 \), then expression (11) can be rewritten in the form more convenient for its analysis:

\[
Y(x) = \frac{32}{\pi^3} \sum_{n=0}^{\infty} \left( -\frac{1}{2m+1} \right)^n e^{-\frac{(2m+1)^2 x}{4}}.
\]
Note that for the initial time moment $Y(0) = 1$, because

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2m+1)} = 1 - \frac{1}{3} + \frac{1}{5} - \cdots = \frac{\pi^2}{32}$$

The derivative $Y'(0)$ is expressed in terms of the Leibniz series (which is curious to obtain from solving a physical problem) and will be:

$$Y'(0) = -\frac{32}{\pi^2} \sum_{n=0}^{\infty} \frac{(-1)^n}{2m+1} = -\frac{32}{\pi^2} \left(1 - \frac{1}{3} + \frac{1}{5} - \cdots \right) = -\frac{32}{\pi^2} \cdot \frac{\pi}{4} = -\frac{8}{\pi^2}$$

Figure 10 shows the plots of the dependence $Y(x)$ with the use of several first terms (partial sums of the fourth and fifth orders) from formula (12), which are compared with the plot of temperature drop along the exponent $Y(x) = \exp(-x)$ (in accordance with the Newtonian law of cooling).

The value of the coefficient of determination for the function $y(x) = 1.02163 \exp(-0.992763x)$ obtained as the result of the analysis is very close to unity, $R^2 \approx 0.9999$. This means that the replacement of series (12) by the exponential dependence expressed by formula (1) (assuming that $T_r = 0$), i.e., by the dependence having the form:

$$T(L, t) = T_0^* \exp \left( -\frac{t}{\tau} \right)$$

where

$$T_0^* \approx 1.022T(L, 0),$$

is quite correct. The temperature error of such a replacement does not exceed 2.2%, which is the maximal temperature deviation (in percent) from the calculated value at the initial point.

To estimate the time constant of the temperature relaxation $\tau$ in the process of cooling of the rod (with the accuracy exceeding 1%), we can use the algebraical expression:

$$\tau = \frac{4L^2}{\pi^2 \chi}$$

Previously, we mentioned that $\tau$ is proportional to the square of height in the cells of high-pressure chambers, which followed from experimental values $\tau$, obtained for the Conac 28 and Conac 48M chambers. Now, using the formula (15), it becomes possible to obtain the numerical value of the averaged coefficient of thermal diffusivity for...
the materials of the reaction cells within the framework of the model of a heat-conducting homogeneous rod. Calculations show that \( \chi \approx 3 \cdot 10^{-6} \text{ m}^2/\text{sec} \) for both of the chambers. (According to reference books, the thermal diffusivity coefficients of some grades of stainless steel and some types of graphite are similar).

Note that the initial stage of cooling of the rod, for \( x \) values in the range about 0 – 0.15, is best approximated by a linear time dependence of temperature, which is illustrated in Fig. 12. It is seen from this figure that the initial stage is rather short and is equal only to \((0.14 - 0.15)\). The value of the derivative \( Y'(0) = -8/\pi^2 \) from (13) coincides with the coefficient of the regression line slope with the sixth decimal digit.

![Graph](image)

**Figure 12.** Result of the regressive analysis of data obtained at the initial stage of decreasing the temperature of the rod during its cooling. Filled circles are the data calculated on the formula (12) for \( x \) values from 0 to 0.15 at an interval of 0.01. The solid straight line is the best approximation of these data. The dashed line is the exponential dependence from Fig. 11 shown here for comparison. The formula describing the regression line at the initial stage \( y(t) \) and the coefficient of determination \( R^2 \) are also presented.

It should be noted that during the experimental determination of the curve of temperature decrease in the chamber after switching-off the heating power, the initial stage of the curve about 1-2 sec in duration is usually disregarded, because transitional processes in the equipment and instrumentation, as well as the phenomenon of self-induction in the electric circuit of the heater, which prevent sharp step-like changes in the heating power, can distort the validity of the experiment. (The final stage of cooling of the cell is also disregarded, because at low cooling rates, the accuracy of \( \tau \) determination decreases, and slow processes of cooling of the external container become to a larger degree affect the process of temperature relaxation in the reaction cell).

We considered the problem about the temperature behavior of a heat-conducting rod in its center after a step-like switch-off of the heating power in passing from one stationary thermal state to another. A similar model problem for the process of a step-like switch-on of the rod heating power was considered in [2], when the temperature rise was simulated in a high-pressure chamber with a cell for synthesis of diamond powders. If our coordinate system and designation of physical quantities were retained, this problem would consist in solving the inhomogeneous equation of heat conduction

\[
\frac{\partial T}{\partial t} = \chi \frac{\partial^2 T}{\partial z^2} + \frac{1}{c \rho} \frac{W}{2LS}
\]

for the rod at the boundary conditions \( T(0,t) = T(2L,t) = 0 \) and initial condition \( T(z,0) = 0 \). Here, \( W \) is the total power of the rod heating, \( S \) is the cross section area of the rod, \( c \) is the specific heat capacity of the rod material, and \( \rho \) is the density of the rod material. As a result, the authors of [2] obtained for the temperature in the chamber (rod) center the following expression:

\[
T(L,t) = \frac{LW}{4LS} \times \\
\left(1 + \sum_{m=1}^{\infty} \frac{32}{\pi^2(m+1)^2} \exp \left[ \frac{-\pi^2(2m+1)^2 \chi t}{4L^2} \right]\right)
\]

(16)

where

\[
\frac{LW}{4LS} = T(L,t \to \infty) = T_{max}
\]

is the maximum temperature value corresponding to the regime of stationary heating. The functional series in expressions (11) and (16) coincide. The authors of [2] indicated that time dependence of temperature (16) can be approximated by the formula:

\[
T(L,t) = T_{max} \left[1 - \exp \left(\frac{-L^2}{\tau} \right)\right]
\]

or

\[
T(L,t) = T_{max} \left[1 - \exp \left(\frac{-t}{\tau} \right)\right]
\]

(17)

This formula corresponded to the type of experimental curves for various high-pressure chambers described in work [2].

Therefore, model calculations of the temperature behavior in a high-pressure chamber at step-like changes in the heating power (both during the power switch-off and switch-on) lead, at a fairly high accuracy, to the exponential temperature relaxation in passing from one stationary thermal state in the chamber to another stationary state. The transitional process can be characterized by the time constant of temperature relaxation \( \tau \), which is determined experimentally (for a certain range of working temperatures and a certain range of heating/cooling rate of samples in the chamber cell).
4. Discussions

Let us call the $\tau$ approximation the approximate description of the temperature behavior during step-like changes in the heating power in a high-pressure chamber with the use of simple exponential dependences (14) and (17). (Speaking about the temperature in the chamber, we will mean the temperature in its center). The plots of these ideal dependences are shown in Fig. 13. The time dependences of the specified temperature $\theta(t)$ during the switch-off and switch-on of the heating power in the chamber indicated in the plots. The point is that functional dependences (14) and (17) are solutions of the same differential equation.

\[
\frac{dT}{dt} = -\frac{T(t) - \theta(t)}{\tau} \quad (18)
\]

if the functions of the specified temperature $\theta(t)$ are substituted into this equation: $\theta(t) = 0 \, (t>0)$ is the substitution for the process of power switch-off, and $\theta(t) = T_m \, (t>0)$ is the substitution for the process of power switch-on, with the corresponding initial conditions.

Formulas (14) and (17) make it possible to take into account the temperature inertia of the cell of the high-pressure chamber for continuous processes of heating or cooling as well. The direct method for realizing such a possibility is the division of the process of temperature specification into separate steps (with small steps over short time intervals). The method is of "natural origin", because it is this method that is used in practice for a step-like control of the heating process with the aid of digital power supplies or computer programs.

Now, we will demonstrate the application of this method using the specification of the linear temperature trend in the chamber as an example. Let the temperature in the chamber is specified with the aid of a programmed power supply against the chamber calibration obtained in a quasi-stationary heating regime. Let us call the $\tau$ approximation the approximate description of the temperature behavior during step-like changes in the heating power in a high-pressure chamber with the use of simple exponential dependences (14) and (17). (Speaking about the temperature in the chamber, we will mean the temperature in its center). The plots of these ideal dependences are shown in Fig. 13. The time dependences of the specified temperature $\theta(t)$ during the switch-off and switch-on of the heating power in the chamber indicated in the plots. The point is that functional dependences (14) and (17) are solutions of the same differential equation.

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\frac{dT}{dt} = -\frac{T(t) - \theta(t)}{\tau} \quad (18)
\]

if the functions of the specified temperature $\theta(t)$ are substituted into this equation: $\theta(t) = 0 \, (t>0)$ is the substitution for the process of power switch-off, and $\theta(t) = T_m \, (t>0)$ is the substitution for the process of power switch-on, with the corresponding initial conditions.

The method is of "natural origin", because it is this method that is used in practice for a step-like control of the heating process with the aid of digital power supplies or computer programs.

Now, we will demonstrate the application of this method using the specification of the linear temperature trend in the chamber as an example. Let the temperature in the chamber is specified with the aid of a programmed power supply against the chamber calibration obtained in a quasi-stationary heating regime have the form $\theta(t) = V_T t$, where $V_T$ is the temperature rise rate. Suppose that the temperature is set in a step-like manner: at equal time intervals $\Delta t$, we increase the specified temperature by $\Delta\theta$ so that $\Delta\theta/\Delta t = V_T$, as it is shown in Fig. 14 for the initial site of the temperature rise.

![Figure 14](image)

Figure 14. Scheme of the replacement of the linear function of the temperature rise by its successive step-like increments in short time intervals at the mean rise rate $V_T = \Delta\theta/\Delta t$.

Then, using dependence (17) and assuming that $\tau = \text{const}$, the succession of temperature values obtained in the chamber in discretely increasing time intervals $\Delta t$, $2\Delta t$, ..., $n\Delta t$, and designated, respectively, as $T_1$, $T_2$, ..., $T_n$ can be written in the following way:

\[
T_1 = \Delta\theta \left(1 - e^{-\frac{\Delta t}{\tau}}\right) = \Delta\theta - \Delta\theta e^{-\frac{\Delta t}{\tau}}
\]

\[
T_2 = T_1 + \left(\Delta\theta + \Delta\theta e^{-\frac{\Delta t}{\tau}}\right) \left(1 - e^{-\frac{\Delta t}{\tau}}\right) = 2\Delta\theta - \Delta\theta e^{-\frac{\Delta t}{\tau}} + \Delta\theta e^{-\frac{2\Delta t}{\tau}}
\]

\[
T_n = n\Delta\theta - \Delta\theta \left(e^{-\frac{\Delta t}{\tau}} + e^{-\frac{2\Delta t}{\tau}} + \cdots + e^{-\frac{n\Delta t}{\tau}}\right)
\]

Since in the last equality, the sum of exponents in parentheses is the sum of terms of the geometric progression, the general expression for the temperature obtained after the $n$-th step by the time moment $t = n\Delta t$ can be rewritten as follows:

\[
T_n = n\Delta\theta - \Delta\theta \left(1 - e^{-\frac{n\Delta t}{\Delta t}}\right) = t \frac{\Delta\theta}{\Delta t} - \Delta\theta \frac{1 - e^{-\frac{t}{\Delta t}}}{e^{-\frac{t}{\Delta t}} - 1}
\]

Under the condition $\Delta t \ll \tau$, we find the following time dependence of the temperature obtained in the chamber:

\[
T(t) = tV_T - tV_T \left(1 - e^{-\frac{t}{\Delta t}}\right) \quad (19)
\]

Function (19) is plotted in Fig. 15. Note that for $t \gg \tau$, the difference between the values of the specified $\theta(t)$ and obtained $T(t)$ temperatures will be tended to $tV_T$, and the shift between the lines of temperature rise in time will be equal to $\tau$. 
It is obvious that graphic presentation of dependence (19) in the experiment will allow us to determine the numerical value of the time constant of temperature relaxation in the high-pressure chamber. But what value of this constant will be obtained in the experiment, the value of the constant (from experiment with step-like changes in the heating power in the chamber or somewhat larger? This question will be answered below, after solving with the aid of Duhamel integral the problem of temperature reaction in the cell in the case, when the linear trend of temperature rise in the chamber is specified.

It can be noted that dependence (19) is also the solution of differential equation (18)

$$\frac{dT}{dt} = \frac{T(t) - \vartheta(t)}{\tau}$$

if the function of the specified temperature for the heating process considered in this case $\theta(t) = Vt/\tau$ is substituted into this equation with the corresponding initial condition.

Based on the heating processes under consideration, we can generalize the notion of the $\tau$ approximation by putting $\tau = \omega^{-1}$. This hypothesis can be formulated in the following form:

If the power of heat sources, ensuring heating of the reactionary cell in high-pressure chamber, changes, the rate of temperature variations inside the cell will be proportional to the difference between the temperature value at the given time moment and that temperature value, which would take place inside the cell under a stationary heating regime at the power of heat sources specified for this moment.

Equation (18) is the linear differential equation of the first order. Its general solution has the form:

$$T(t) = e^{-\frac{\omega}{\tau}t} \int \frac{\vartheta(t)}{\tau} e^{\frac{\omega}{\tau}t} dt + C$$

where $C$ is the constant of integration determined from the initial condition. Recall that the general solution of the linear differential equation of the first order

$$y' + f(x)y = g(x)$$

can be presented as:

$$y = e^{-\int f(x) dx} \left[ \int g(x) e^{\int f(x) dx} dx + C \right]$$

$$F(x) = \int f(x) dx.$$

Therefore, within the framework of the $\tau$ approximation, it is possible to obtain general formula (20) for calculating the temperature $T(t)$ in the high-pressure chamber at an arbitrarily specified heating regime $\theta(t)$. This formula is semiempirical based on experimental data of the preliminary chamber calibration and data of measurement of the characteristic time of temperature relaxation $\tau$ in the cell. However, the application of this formula does not require the knowledge of such characteristics of assemblage elements of the cell as heat conductivity, mass density, and specific heat capacity, the knowledge of which is necessary, when the differential equation of heat conduction is used.

It should be noted that the presented general formula makes it possible to obtain the solution of the thermal problem in the analytic form, if the indefinite integral in this formula is expressed in terms of elementary functions, when the specified dependence $\theta(t)$ is substituted. The dependences $\theta(t)$, which satisfy this requirement, can be found from tables of indefinite integrals: the dependence $\theta(t)$ can be an arbitrary polynomial, goniometric functions sine and cosine (their integer powers), etc.

Let us determine, for example, the temperature response in the chamber cell to the specified temperature having a magnitude that varies as a sine curve. Suppose, that in a conventional temperature scale $\theta(t) = 1 + \sin(\omega t)$, where $\omega$ is the angular frequency. In this case, using general formula (20), we obtain the temperature in the chamber:

$$T(t) = e^{-\frac{\omega}{\tau}t} \left\{ \frac{1}{\tau} \left[ \int [1 + \sin(\omega t)] e^{\frac{\omega}{\tau}t} dt + C \right] \right\} =$$

$$= 1 + \frac{\sin(\omega t) - \tau \omega \cos(\omega t)}{\tau^2 \omega^2 + 1} + C e^{-\frac{\omega}{\tau}t}.$$
temperature in the chamber and the temperature obtained from formula (21) at different values of the time constant of temperature relaxation \( \tau \). Note that the extremes of the obtained temperature are located at the intersections with the plot of the specified temperature, because, according to the condition of \( \tau \)-approximation (18), the equation \( \theta(t) = T(t) \) follows from the equality \( dT/dt = 0 \) for the extreme points.

Figure 16. Plots of the specified temperature (thin line) and the obtained temperature \( T(t) \) at different values of the time constant of temperature relaxation: \( \tau = \pi/4 \omega \) (thick line) and \( \tau = \pi/\omega \) (dashed line)

The method of dividing the process of temperature specification into steps, which was used in describing the linear temperature rise in the chamber, can be used for the temperature specification in an arbitrary graphical form. Such a method is applied in derivation of the Duhamel integral formula for calculating the response of arbitrarily changing in time input signal.

In our case, when the temperature in the cell of the high-pressure chamber is specified by an arbitrary differentiable function \( \theta(t) \), the formula for the obtained temperature with the Duhamel integral can be presented as:

\[
T(t) = \theta(0) \cdot h(t) + \int_{0}^{t} \theta'(t*) \cdot h(t-t*) \, dt *
\]  

(22)

or, having written down the Duhamel integral in another form (using the convolution properties), as:

\[
T(t) = h(0) \cdot \theta(t) + \int_{0}^{t} \theta'(t*) \cdot h'(t-t*) \, dt *
\]  

(23)

where \( t* \) is the variable of integration, \( \theta'(t*) \) is the derivative of the function specified temperature with respect to time, \( h(t) \) is the transitional function, which represent the temperature response of the cell to the unit step: \( \theta(t) = 1 \) at \( t \geq 0 \) and \( \theta(t) = 0 \) at \( t < 0 \) (the Heaviside function), and

\[
h'(t-t*) = \frac{dh(t-t*)}{dt}
\]

Note that formulas (22) and (23) are valid only for passive systems, i.e., for \( T(0) = 0 \).

It is easy to demonstrate the derivation of formula (22). Let \( \theta(t) \) have the form of the function presented in Fig. 17. Having divided the current time period \([0, t]\) into \( n \) equal intervals \( \Delta t^* \), replace the initial curve by the step-like line. After that, summarize the temperature responses in the cell from all steps from the time moment \( t^* = m \Delta t^* \) (where \( m = 1, 2, \ldots, n-1 \)) to the time moment \( t \).

Figure 17. Schematic presentation of the replacement of the initial function of the specified temperature by its step-like approximation, when the current time period \([0, t]\) is divided into \( n \) equal time intervals \( \Delta t^* \)

As a result, we obtain:

\[
T(t) = \theta(0) \cdot h(t) + \sum_{m=1}^{n-1} \Delta T(t^*, t)
\]

where \( \theta(0) \cdot h(t) \) is the temperature response to the jump of the initial value of the specified temperature in the time interval from 0 to \( t \). The temperature contribution from an individual step \( \Delta \theta(t^*) \) of the specified temperature, beginning from the time moment \( t^* \) to the time moment \( t \) is designated as \( \Delta T(t^*, t) \). The contribution from the step \( \Delta \theta(t^*) \) can be written as follows:

\[
\Delta T(t^*, t) = \Delta \theta(t^*) \cdot h(t-t*) \approx \theta'(t^*) \cdot \Delta t^* \cdot h(t-t*)
\]

Then

\[
T(t) \approx \theta(0) \cdot h(t) + \sum_{m=1}^{n-1} \theta'(t^*) \cdot \Delta t^* \cdot h(t-t*)
\]

Or, passing from the sum to integral at \( n \to \infty \), we obtain one of the forms of the Duhamel integral, i.e., formula (22):

\[
T(t) = \theta(0) \cdot h(t) + \int_{0}^{t} \theta'(t*) \cdot h(t-t*) \, dt *
\]

For using the Duhamel integral, it is necessary to specify or preliminarily calculate the transition function of the cell \( h(t) \). In principle, we have already done this for the cell of the high-pressure chamber. It follows from expression (16) (assuming that \( T_{\text{max}} = 1 \)) that the accurate model calculations give:
After the integration, we come to the formula:

$$θ_{\text{specified temperature}} = \int \text{response in the chamber to the linearly increasing with time}$$

The problem about the calculation of the temperature integral makes it possible to obtain a more exact solution of the transitional function of any form. For example, the Duhamel formula (20) in its generality, i.e., it is valid for the particular initial condition. Let the specified temperature is the function of asymptote as:

$$T(0) = V_T t$$

The plot of this time dependence of temperature (Fig. 18) differs only slightly from the plot presented in Fig. 15. However, the asymptote for function (27) in the plot is shifted with respect to the line of specified temperature rise to a larger degree than in Fig. 15.

Indeed, if $t >> \tau$, the formula (27) is approximated by the expression:

$$T(t) \approx V_T (t - 1.028 \tau).$$

Calculating approximately the sum of the rapidly convergent sing-alternating series, we can write the equation of asymptote as:

$$T(t) \approx V_T (t - 1.028 \tau).$$

Therefore, at $t >> \tau$, the time shift between the lines of rise of the specified and obtained temperatures will be about 1.028$t$. If the time constant of temperature relaxation for the chamber cell will be determined from the linear temperature rise, it can exceed the $\tau$ value obtained from the experiment with step-like changes in heating power by about 2.8%. Now, we answered the question raised earlier: What value of time constant we would obtain from the experiment with the specified linear temperature rise in the chamber. (But the experimental conditions would correspond to the one-dimensional model problem under consideration).

Let us now consider the example of application of our general formula (20) and the Duhamel integral for calculating the temperature in the chamber at a nonzero initial condition. Let the specified temperature is the function $θ(t) = V_T t$ and initial temperature in the chamber have other value than zero $T(0) = T_0 \neq 0$. Substituting the time
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dependence of the specified temperature into (20), we obtain:

\[ T(t) = e^{-\frac{t}{\tau}} \left( \int_0^\infty \frac{V_T}{\tau} e^{\frac{t}{\tau}} dt + C \right) = e^{-\frac{t}{\tau}} \left[ V_T (t - \tau) e^{\frac{t}{\tau}} + C \right]. \]

We can determine from the initial condition that \( C = T_0 + \tau V_T \). In this case, the time dependence of the obtained temperature has the form:

\[ T(t) = V_T t - V_T \tau \left( 1 - e^{-\frac{t}{\tau}} \right) + T_0 e^{-\frac{t}{\tau}} \]

(28)

The plot of this dependence is represented in Fig.19.

Figure 19. Time dependence of temperature (28) is shown as thick line. The thin solid line is the specified temperature, dashed line is the inclined asymptote to the plot of obtained temperature, which intersects the abscissa axis at the point \( t = \tau \). \( T_0 \) is the value of initial temperature in the cell of high-pressure chamber.

For applying Duhamel integral (22) in this case, it is necessary to pass to the coordinate system, which is shifted on the temperature scale by \( T_0 \), having introduced, for example, the variable \( T^*(t) = T(t) - T_0 \), whose initial condition satisfies the equality \( T^*(0) = 0 \). In the new coordinate system, the specified temperature takes the form \( \theta(t) = V_T t - T_0 \). Then, using formula (25) for the transitional function, we obtain from (22):

\[ T^*(t) = -T_0 + T_0 e^{-\frac{t}{\tau}} + V_T t - V_T \tau \left( 1 - e^{-\frac{t}{\tau}} \right). \]

That is, we come to the same formula (28) for variable \( T(t) = T^*(t) + T_0 \).

5. Examples of Solution of Practical Problems

Problems associated with the determination of maximal temperature values in the chamber under the action of various heating pulses can be met in practice. The delta-shape and saw-tooth pulses, as well as the calculated curves of the temperature responses corresponding to them, which were obtained with allowance for the temperature relaxation in the cell of high-pressure chamber during the heating process, are presented in Fig. 20.

Figure 20. Calculated curves of the obtained temperature in the chamber depending on time (thin lines) at the specified temperature pulses of the heating (thick straight lines): (a) symmetrical triangular pulse, and (b) saw-tooth pulse. Maxima of heating pulses have the time axis coordinate \( t_m = 3 \tau \) (as an example).

Calculations (in the \( \tau \) approximation) show that for the heating scheme shown in Fig. 20a, the difference between the maximal temperatures \( \Delta T_{m,\text{max}} \) (the peak values of the specified and obtained temperatures in the plot) is determined from the formula:

\[ \Delta T_{m,\text{max}} = \tau V_T \ln \left[ 2 - \exp \left( -\frac{t_m}{\tau} \right) \right] \approx \tau V_T \ln 2. \]

Here, \( V_T \) is the absolute value of the rate of temperature change for the symmetrical triangular pulse, and \( t_m \) is the time, at which the maximum of the specified temperature is attained.

For the heating scheme with saw-tooth peak of the specified temperature (Fig. 20b), the difference between the maxima of the specified and obtained temperatures will be determined from the formula:

\[ \Delta T_{m,\text{max}} = \tau V_T \left[ 1 - \exp \left( -\frac{t_m}{\tau} \right) \right] \approx \tau V_T. \]

Here, \( V_T \) is the rate of temperature rise for the saw-tooth pulse of the specified temperature.

The solution of the technical problem of short-duration high-temperature heating of diamond crystals in the cell of the high-pressure chamber with the use of the above formula for taking into account the temperature inertia of the cell can be found in the description of patent [10].

Figure 21 and Figure 22 show an examples of the experimental records of thermocouple readings during the delta-shape and saw-tooth heating pulse, respectively, of the
cell of the Conac 48M chamber. In the case of Fig. 22, the sharp temperature peak ensures the possibility of short-time heating of samples in the chamber cell in the high-temperature region.

Figure 21. Experimental record of temperature response (points in the plot) on delta-shape heating pulse (dashed lines) in the cell of high-pressure chamber (Conac 48M). \( t_0 \) is the time of beginning the linear rise of the temperature \( \theta(t) \). The temperature \( T(t) \) is recorded in 0.5 sec.

Figure 22. Sharp temperature peak in the cell of the high-pressure chamber (Conac 48M) obtained as a result of the saw-tooth heating scheme (with the initial step-like site of heating). Points in the plot mean temperature reading recorded in 0.5 sec. The horizontal dashed line indicates the initial temperature in the cell (room temperature), the vertical dashed line shows the time of beginning the linear rise of heating power. The scheme of setting the heating power (heating power \( W(t) \) or temperature \( \theta(t) \)) is shown in the inset.

We will give one more example, which illustrates the problem of obtaining a certain temperature in the chamber for a required time period with allowance for the temperature relaxation. The process scheme presented in Fig. 23 can be chosen for solving this problem. At the initial stage of the process, for the time interval \( 0 \leq t \leq t_1 \), we specify the linear temperature rise \( \theta(t) = V \theta \cdot t \) to the value exceeding the required temperature value in the chamber by \( \tau V \theta \cdot [1 - \exp(-t_1/\tau)] \). Further, with the downward jump by \( \tau V \theta \cdot [1 - \exp(-t_1/\tau)] \), we specify the constant temperature equal to the required temperature for the time interval from \( t = t_1 \) to \( t = t_2 \). At the end, we specify the temperature \( \theta(t) = 0 \), \( t > t_2 \). As a result, we will obtain in the chamber the trapezoidal temperature pulse with a constant temperature value in the cell:

Figure 23. Time dependence of the temperature in the chamber in form of trapezoidal pulse (thick line). The dashed lines indicate the temperatures specified at the consecutive sites of the time scale. \( \theta(t) \) is piecewise continuously differentiable function of the specified temperature. The lines of the specified and obtained temperatures coincide at the site \( t_1 < t < t_2 \).

\[
T(t) = \mathcal{G}(t) = V \theta \cdot t_1 - V \theta \cdot \tau \left[ 1 - \exp\left(-\frac{t}{\tau}\right) \right]
\]

in the specified time interval \( t_1 < t < t_2 \).

The experimental temperature record in the Conac 48M high-pressure chamber for the presented scheme of heating is shown in Fig. 24.

Note that the use of formula (20) for finding the temperature in the chamber in the case, when the specified temperature function \( \theta(t) \) or its derivative contain discontinuities, necessitates the division of the time scale into sites, for which their own constants of integration \( C \) must be determined through correlating the initial conditions at the sites with the temperatures obtained at their boundaries.
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Figure 24. Truncated temperature peak in the cell of the Conac 48M high-pressure chamber obtained as a result of the heating scheme (heating power $W$ or temperature $\theta$ vs. time) presented in the inset. Points in the plot mean thermocouple readings recorded at an interval of 0.5 sec.

6. Conclusions

This work advanced the hypothesis about the applicability of the approximation of relaxation time to thermal processes in high-pressure chambers and about the rate of temperature changes in a high-pressure chamber at a varying power of heating of its reaction cell. This hypothesis is related to the Newtonian law of cooling, according to which the rate of change in the temperature of a body during its cooling is proportional to the difference between the temperature of the body (body surface) $T(t)$ and the ambient temperature $T_a$ [5]:

$$\frac{dT(t)}{dt} \propto -[T(t) - T_a].$$

In accordance with the offered hypothesis, the rate of temperature changes in a cell of the high-pressure chamber during its heating or cooling is proportional to the difference between the temperature value existing at the given time moment $T(t)$ and the temperature value $\theta(t)$, which is specified for this time moment against the quasi-stationary calibration of the chamber:

$$\frac{dT(t)}{dt} \propto -[T(t) - \theta(t)].$$

This means that the temperature in the cell tends exponentially quickly to reach the own stationary value due to heat conduction at any time moment. The proposed hypothesis has made it possible to find general formula (20) relating the time dependences of the obtained and specified temperatures in the chamber with allowance for the temperature relaxation in the cell:

$$T(t) = e^{-\frac{t}{\tau}} \left[ \int \frac{\theta(t)}{\tau} e^\frac{t}{\tau} dt + C \right].$$

Calculations of the temperature behavior during cooling of the rod, which serves as a model of the cell in the chamber, and their analysis allowed us to estimate the error caused by the replacement of the “exact” solution of the heat conduction problem, representing an infinite series of exponential functions, by the functional temperature dependence on time containing a single exponent. This error does not exceed three percent. As it turned out, our general formula is directly related to the Duhamel integral, which is widely used for determining the response of passive linear systems to the specified input signal. Applying the Duhamel integral for the solution of the problem about the temperature response in the cell to the specified temperature linearly increasing with time, and using the exact transitional function in the form of an infinite series, we were able to refine the time constant of temperature relaxation in the cell for the one-dimensional model problem. The time constant in such a heating process can exceed the time constant of temperature relaxation in the process of step-like changes in the heating power, but again by no more than three percent ($\approx 2.8\%$).

Of course, Newtonian law of cooling can be violated in the high-temperature region, if the environment of hot object is transparent for thermal radiation. However, if the heat is transferred from a heated body to other bodies only through heat conduction, the exponential Newtonian law of cooling, according to which for a heated body during its cooling “temperature differences … were in a geometrical progression when the times are in an arithmetical progression” [5], is fulfilled within a high accuracy. Moreover, we believe that the exponential time dependence of temperature, in passing from one stationary thermal state of an object to another, can be considered as an extreme principle of heat conduction of solids. When cooling process are analyzed, the $\tau$ approximation yields a lower temperature estimate in the high-pressure chamber, and for heating process, the temperature estimate will be an upper one.

The accounting for the temperature relaxation in a high-pressure chamber in accordance with the general formula obtained permits to control more precisely the process of heating of the reaction cell of the high-pressure chamber with the use of programmed power supplies.

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